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Correction, Reconstruction, and Modeling of Experimental Data Using LI Transforms

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Abstract: This paper examines mathematical methods of using the linear invariant (LI) transforms for the correction, reconstruction, and modeling of experimental data—one-dimensional and multidimensional signals including images distorted during processing by LI systems, which in the 1D case are the linear stationary or time-invariant systems. Methods enable data processing with a minimum of initial information and computational resources; they are simple, require minimal resources for numerical calculations, and can be effectively used to process data of large volumes. LI methods have virtually no restrictions on the class of processing functions, which must be locally integrable in the domain under consideration. LI methods are effective and designed to solve some typical signal processing problems often encountered in practice. This paper provides examples of the practical application of LI methods for signal processing of electronic devices, time-of-flight neutron spectrometers, image processing, etc. Mathematical LI methods can improve the quality of data processing and enhance the effective parameters of processing systems without solving complex scientific, technical, and technological problems or creating expensive equipment.

Keywords: mathematical processing of signals and images, linear stationary and invariant systems, multidimensional convolution-type equation

1. Introduction

Imperfection of processing systems leads to distortions in the data they process. Hardware methods for reducing distortions deal with complex scientific, technical, and technological problems, as well as creating expensive equipment. Alternative mathematical methods for restoring corrupted data [1–5] are not universal and are not effective in all practical situations, so the development of new methods is still in demand.

Direct inversion methods are ineffective when solving ill-posed and ill-conditioned problems. Approximate methods, such as regularization, iterative methods, wavelet analysis, and so on, are usually used to solve these problems [6–12]. However, they are slow, computationally intensive, and ineffective when processing functions of very wide classes, such as discontinuous or generalized functions, which typically describe the transfer functions of processing systems.

The mathematical linear invariant (LI) methods proposed in this paper are intended for the correction, reconstruction, and modeling of signals and images distorted during their processing by LI systems, which are stationary or time-invariant in the one-dimensional case [12, 13]. LI methods belong to the methods of direct inversion. When solving a number of specific, but important and frequently encountered in practice problems of signal processing, they require a minimum of initial information, use minimal computational resources, and are faster than spectral methods or iterative techniques based on the solution of Toeplitz systems. For example, LI methods require less than $O(n \log n)$ operations than spectral methods typically require to reconstruct distorted digital signals of size n, and this processing does not require auxiliary matrices or additional data transformations. Being direct inversion methods, LI methods practically do not limit the class of reconstructed functions, which must be locally integrable in the domain under consideration, and are suitable for processing both continuous signals and discrete data.

LI transforms include linear operations and operations of direct shifts, so they are closely related to the deconvolution by the shift-and-subscribed schemes and step-by-step unfolding techniques, which are the particular cases of the presented approach in processing discrete signals.

LI transforms and main problems, which they can solve, are considered in Section 2. The solution of the basic LI problem of two shifted pulses superposition is considered in Section 3. The typical tasks of signal processing, reduced to the basic LI problem and frequently encountered in practice, are analyzed in Section 4. Parameters of LI methods when processing digital signals are compared with other numerical methods of direct inversion in Section 5. Practical examples of using LI methods are presented in Section 6. The results are discussed in Section 7. Conclusions and recommendations are also presented in the paper.

2. Linear Invariant Transforms

A functional system for processing one- and multidimensional signals, including images, is LI (linear stationary or time-invariant in the one-dimensional case) if its operation in the domain $x \in D$ is characterized by a transfer function $u(x)$ and its

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response $w(x)$ (output signal) to an external action $v(x)$ (input signal) is described by a convolution-type equation:

$$w(x) - \{\eta(x)\} = (u \times v)(x) = \int_D u(x-\xi)\, v(\xi)\, d\xi, \qquad (1)$$

which we will call the LI system equation, or, for short, the LI equation. Here, $\eta(x)$ is the uncertainty of the LI equation. LI Equation (1) follows from the convolution equation:

$$w_0(x) = (u_0 \times v_0)(x)$$

for deterministic functions $w_0(x) = (w - \Delta w)(x)$, $u_0(x) = (u - \Delta u)(x)$and $v_0(x) = (v - \Delta v)(x)$ with noise and uncertainties $\Delta w(x)$, $\Delta v(x)$, and $\Delta u(x)$. Thus, the total uncertainty of LI Equation (1) is:

$$\begin{aligned}\eta(x) &= (\Delta w - u \times \Delta v - \Delta u \times v + \Delta u \times \Delta v)(x) \\ &= (\Delta w - u_0 \times \Delta v - v \times \Delta u)(x).\end{aligned}$$

In various fields of science and technology, the transfer function $u(x)$ is called the hardware function, the system function, the impulse function, the distortion or superposition function, the transformation or distortion kernel, etc.

Introducing the LI convolution operator $\mathscr{S}$ defined by the generating function $s(x)$: $\mathscr{S} f(x) \equiv (s \times f)(x)$, LI Equation (1) can be represented in operator form:

$$\mathscr{U} v(x) = w(x) - \{\eta(x)\},$$

or, using Borel's convolution theorem, in spectral form:

$$W(\omega) - \{\mathscr{N}(\omega)\} = U(\omega)\, V(\omega),$$

where $U(\omega) = FT[\mathscr{U}] = FT[u(x)]$ is the spectrum (Fourier transform) of a function $u(x)$ or operator $\mathscr{U}$.

The convolution is linear: $(\alpha f + \beta g) \times h = \alpha(f \times h) + \beta(g \times h)$, commutative: $(f \times g) = (g \times f)$, and associative $f \times (g \times h) = (f \times g) \times h$ [12, 13], so the LI operators $\mathscr{S}$ of sequential applications $\mathscr{S} = \mathscr{S}_n \ldots \mathscr{S}_2 \mathscr{S}_1$ or linear combinations $\mathscr{S} = \sum_i \alpha_i \mathscr{S}_i$ of LI operators $\mathscr{S}_i$ are also the LI operators.

Applying LI operator $\mathscr{S}$ on both sides of the LI equation and using the mentioned convolution properties, we obtain the LI transform (LIT) equation:

$$\begin{aligned}\mathscr{S} w(x) - \{\mathscr{S}\eta(x)\} &= (s \times u \times v)(x) \\ &= (\mathscr{S} u \times v)(x) = (u \times \mathscr{S} v)(x),\end{aligned} \qquad (2)$$

which in the spectral domain is:

$$\begin{aligned}S(\omega)\, W(\omega) - \{S(\omega)\, \mathscr{N}(\omega)\} &= U(\omega)\, S(\omega)\, V(\omega) \\ &= [S(\omega)\, U(\omega)]\, V(\omega) = U(\omega)\, [S(\omega)\, V(\omega)].\end{aligned}$$

The error of signal processing by using the LI transforms directly depends on the uncertainty of the LIT equation and the applied LI transforms $\mathscr{S}$:

$$\mathscr{S}\eta(x) = (s \times \eta)(x),$$

which in the spectral domain is $S(\omega)\mathscr{N}(\omega)$.

According to the LIT Equation (2), the following problems can be solved using LI transformations:

1) Reconstruct the LI system response $\mathscr{S} w(x)$ to the input signal $\mathscr{S} v(x)$ using the LI system output $w(x)$ and input $v(x)$ signals if the transfer function is unknown.
2) Simulate the response $\mathscr{S} w(x)$ of LI systems with the transfer function $\mathscr{S} u(x)$ to the same input signal using the response $w(x)$ of the LI system with transfer function $u(x)$ for any unknown input signals.
3) Simulate the response $\mathscr{S}_2 \mathscr{S}_1 w(x)$ of the LI system the with transfer function $\mathscr{S}_1 u(x)$ to the input signal $\mathscr{S}_2 v(x)$ using the response $w(x)$ of LI systems with the transfer function $u(x)$ to the input signal $v(x)$ for any transfer functions $u(x)$ and input signals $v(x)$.

Representing multidimensional functions $f(x)$ in discrete form: $f = \sum_{i_1 i_2 \ldots} f_{i_1 i_2 \ldots} \delta_{i_1 i_2 \ldots}$, where $\delta_{i_1 i_2 \ldots}$ is the Kronecker delta function, which differs from 0 and is equal to 1 only on the $i_1 i_2 \ldots$-th partition of the domain D, for LI transforms $g(x) = \mathscr{S} f(x) = (s \times f)(x)$ in discrete form, we have:

$$\begin{aligned}g &= \sum_{m_1 m_2 \ldots} g_{m_1 m_2 \ldots} \delta_{m_1 m_2 \ldots} \\ &= \sum_{i_1 i_2 \ldots} \sum_{j_1 j_2 \ldots} s_{i_1 i_2 \ldots} f_{j_1 j_2 \ldots} \delta_{i_1 + j_1, i_2 + j_2, \ldots}.\end{aligned} \qquad (3)$$

Substituting $i_k = m_k - j_k$ or $j_k = m_k - i_k$ into (3) for the element $g_{m_1 m_2 \ldots}$, we obtain:

$$\begin{aligned}g_{m_1 m_2 \ldots} &= \sum_{j_1 j_2 \ldots} s_{m_1 - j_1, m_2 - j_2, \ldots} f_{j_1 j_2 \ldots} \\ &= \sum_{i_1 i_2 \ldots} s_{i_1 i_2 \ldots} f_{m_1 - i_1, m_2 - i_2, \ldots}.\end{aligned}$$

In the one-dimensional case: $f = \sum_i f_i \delta_i$, $g = \sum_i \sum_j s_i v_j \delta_{i+j}$, and $g_m = \sum_j s_{m-j} f_j = \sum_i s_i f_{m-i}$.

The LI transforms in the spectral domain satisfy the equation: $G(\omega) = S(\omega) F(\omega)$, so the discrete spectra of the signals are related as:

$$G_{i_1 i_2 \ldots} = S_{i_1 i_2 \ldots} F_{i_1 i_2 \ldots},$$

which in the one-dimensional case is $G_i = S_i F_i$.

The convolution of functions corresponds to the direct product of their spectra, so numerical calculations in the spectral domain are usually much faster than in the spatial domain. However, in practice, signals are usually considered in limited regions, outside of which they are unknown, and if these signals are nonzero and nonperiodic in external regions, then their spectra, depending on the Fourier integral over the entire space, cannot be determined. This circumstance limits the scope of application of spectral methods in practice and makes calculations in the spatial domain, which we will use further, relevant and in demand.

3. Basic LI Problem

LI methods are effective and designed to solve typical signal processing problems that can be reduced to the superposition of two shifted pulses, which we call the basic LI problem. Let us first consider the solution of the basic LI problem in the one-dimensional case.

Basic LI problem (superposition of two shifted one-dimensional signals). Reconstruct on the interval $x \in (-\infty, L)$ the LI system response $w(x)$ from the impulse $v(x)$, using the response $w_0(x)$ from two input pulses $v(x)$ shifted relative to each other by a distance (or time) $T > 0$: $v_0(x) = q_0 v(x) - q_1 v(x - T)$, where $q_0, q_1 \neq 0$ are some coefficients and $w_0(x) = v_0(x) = 0$ at $x < 0$.

Representing the input signal $v_0(x)$ as a convolution: $v_0(x) = (q \times v)(x)$, where $q(x) = q_0\delta(x) - q_1\delta(x - T)$, and using the commutativity and associativity of the convolution, we obtain $w_0(x) = (u \times v_0)(x) = (u \times (q \times v))(x) = (q \times w)(x)$. Applying consequently LIT $\mathscr{S}_k$ with the generating function $s_k(x) = \delta(x) + a^{2^{k-1}}\delta(x - 2^{k-1}T)$, $a = q_1/q_0$, $k = 1..K$, to the function $q(x)$, we get $\mathscr{S}q(x) = \mathscr{S}_K \cdots \mathscr{S}_1 q(x) = q_0\delta(x) - q_0 a^{2^K}\delta(x - 2^K T)$. Then, according to (2), $\mathscr{S}w_0(x) = (\mathscr{S}q \times w)(x) = q_0 w(x) - q_0 a^{2^K} w(x - 2^K T)$. Thus, when $2^K T > L$, in $K = [\log_2(L/T)] + 1$ steps on the interval $x \in (-\infty, L)$, we have $w(x) = \mathscr{S}w_0(x)/q_0$ (see Figure 1).

In the presence of uncertainty $\eta(x)$ in the initial equation, the reconstruction error is different in different intervals. According to (2), the reconstruction error $\eta_k(x)$ on the interval $\left[2^{k-1}T, 2^k T\right]$ is equal to $\eta_k(x) = \frac{1}{q_0}\mathscr{S}_k \cdots \mathscr{S}_1\eta(x)$, so its norm can be estimated as:

$$\|\eta_k\| \le \frac{\|\eta\|}{q_0}\left(1 + \sum_{i=1}^{k} a^{2^i}\right). \tag{4}$$

When $|q_0| > |q_1|$ and, consequently, $|a| < 1$, the error increase $\|\eta_k - \eta_{k-1}\| = a^{2^k}$ on the interval $[2^{k-1}T, 2^k T]$ rapidly decreases, and when $|q_0| < |q_1|$ and $|a| > 1$, on the contrary, it rapidly increases (see Table 1).

Thus, we can reconstruct the response of an unknown LI system to a single pulse on the interval $x \in (-\infty, L)$ from its response to two pulses separated by a distance T in $K = [\log_2(L/T)] + 1$in steps (or even less if at some step $k < K$ the

Figure 1
Reconstruction in overlapping of two one-dimensional signals

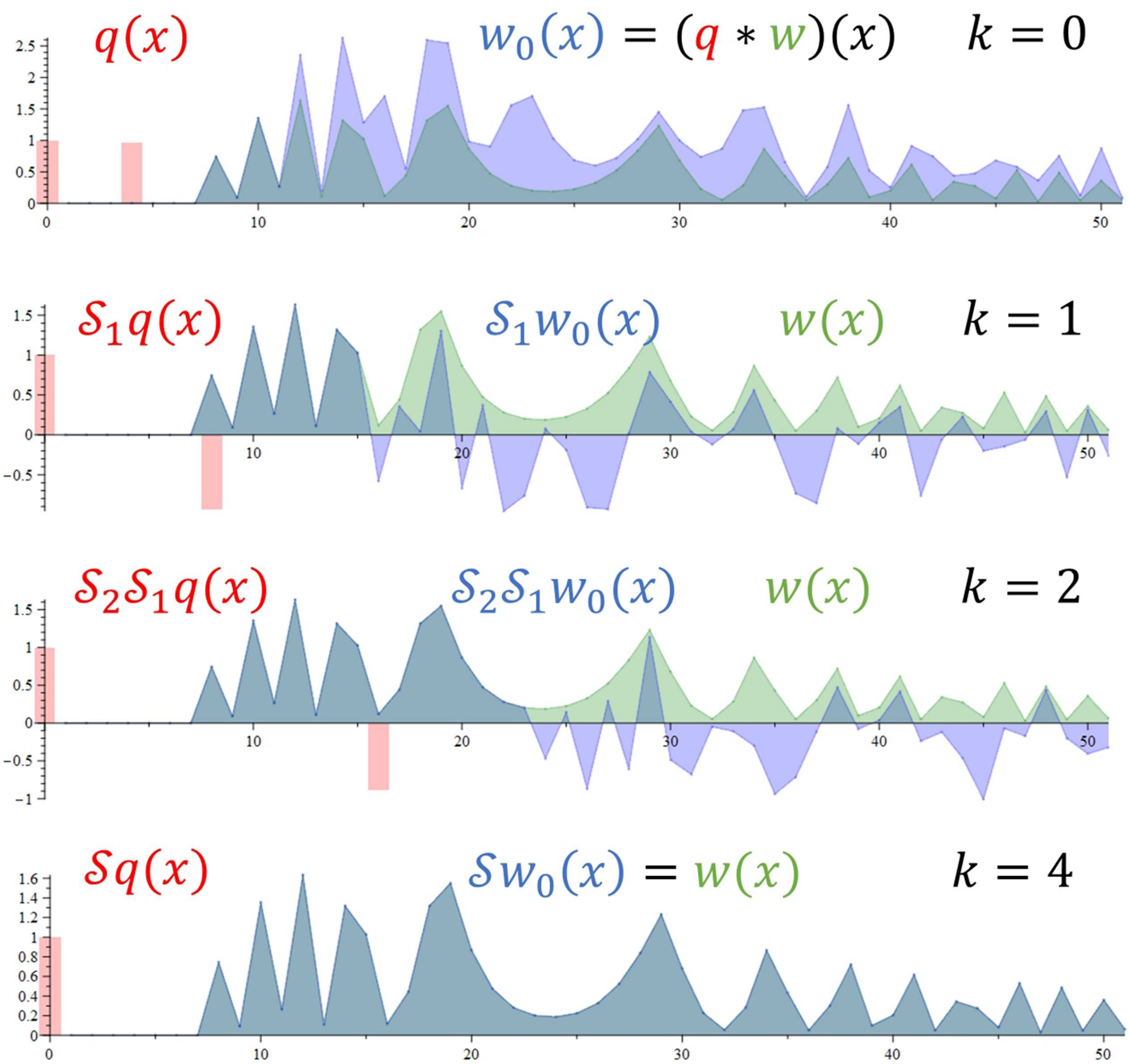


Table 1
The error increase $||\eta_k - \eta_{k-1}|| = a^{2^k}$ on the interval $[2^{k-1}T, 2^kT]$ at the k-th iteration step

k\a	*0.5*	*0.9*	*1.1*	*1.5*
1	0.25	0.81	1.21	2.25
3	3.9e-3	4.3e-1	2.14	25.6
5	2.3e-10	3.4e-2	21.1	4.3e5
7	2.9e-39	1.4e-6	1.9e5	3.5e22
9	7.5e-155	3.7e-24	1.6e21	1.4e90

error increase $||\eta_k - \eta_{k-1}|| = a^{2k}$ shown in Table 1 is small and can be neglected).

For digital signals of length n, one needs

$$N = \sum_{k=1}^{K} \left(n - 2^{k-1}m\right) < nK = n\left(\left[\log_2\left(\frac{n}{m}\right)\right] + 1\right) \quad (5)$$

operations for numerical signal recovery, which is even less than the spectral methods usually require. Here, $n = [L/\Delta] + 1$ and $m = [T/\Delta] + 1$, where Δ is a partition step.

LI methods are efficient and designed to solve specific, typical signal processing problems. When solving many of these problems, the number of required operations can be even smaller than when solving the basic problem of LI methods considered here (see Sections 4 and 5 for details).

Multidimensional signals. The solution to the basic problem in the case of multidimensional signals is similar to that of one-dimensional, but the functions are considered in the domain $x = \{x_i\} \in D \subset \mathbb{R}^n$, and the plane-parallel shift without rotations is given by a vector $T = \{T_i\}$ in space $\mathbb{R}^n$.

Figure 2 illustrates a process of reconstruction of two superimposed two-dimensional images. The algorithm and processing programs are described in Section 5.

In the multidimensional case, not all displacements are LI transforms. For example, the plane-parallel shifts in Cartesian coordinates are the LI transforms in the plane of displacement, while in polar coordinates, the rotations are the LI transforms. Combinations of plane-parallel shifts and rotations are not the LI transforms in both coordinates, and in general, for such distortions, the problem cannot be solved by LI methods.

4. Typical LI Problems

We will call signal processing problems typical if they can be reduced to the basic LI problem using some auxiliary LI transformations. In this section, we consider some typical LI problems frequently encountered in practice. For simplicity, we analyze one-dimensional cases, which, as shown in the previous section, can be easily extended to multidimensional cases.

Problem 1 (rectangular pulse). Reconstruct on the interval $x \in (-\infty, L)$ the response $w(x)$ of the LI system to a rectangular pulse $v(x)$ of duration T ($v(x) = 1$ for $x \in [0, T]$ and $v(x) = 0$ otherwise), if there is known the response of this system $w_0(x)$ ($w_0(x) = 0$ for $x < 0$) to the input rectangular pulse $v_0(x)$ of duration $T_0 > T$ ($v_0(x) = 1$ for $x \in [0, T_0]$ and $v_0(x) = 0$ otherwise).

Applying the auxiliary LI transform $\mathscr{S}_0 : \delta(x) - \delta(x - T)$ to the LI equation $w_0(x) = (u \times v_0)(x)$ and taking into account that $\mathscr{S}_0 v_0(x) = v(x) - v(x - T_0)$, we obtain $\mathscr{S}_0 w_0(x) = (u \times \mathscr{S}_0 v_0)(x) = w(x) - w(x - T_0)$ (see Figure 3(a)), which is the basic problem of superposition of two shifted pulses. The uncertainty of the problem will be $\eta(x)$ for $0 \le x < T_0$ and $\eta(x) - \eta(x - T)$ for $x \ge T$. If $T > L$, function $v_0(x)$ on the interval $x \in (-\infty, L)$ is stepwise and only one auxiliary LI transformation $\mathscr{S}_0$ is required for reconstruction.

The reconstruction error is minimal when the zeros of the spectra $V(\omega) = 2\sin(\omega T/2)/(\omega T/2)$ and $V_0(\omega) = 2\sin(\omega T_0/2)/(\omega T_0/2)$ of signals $v(x)$ and $v_0(x)$ coincide, that is, when $T_0 = mT$, $m = 1, 2, \ldots$ In the case of discrete signals where T is the sampling step, the result is the discrete transfer function of the LI system (see Figure 3 (a)).

Problem 2 (trapezoidal impulse). From the known on the interval $x \in (-\infty, L)$ the response of the LI system $w_0(x)$

Figure 2
Reconstruction in overlapping of two images

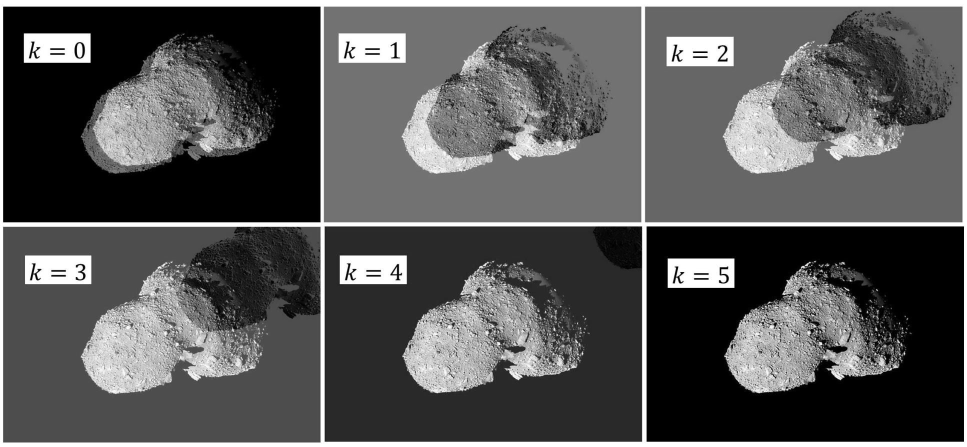
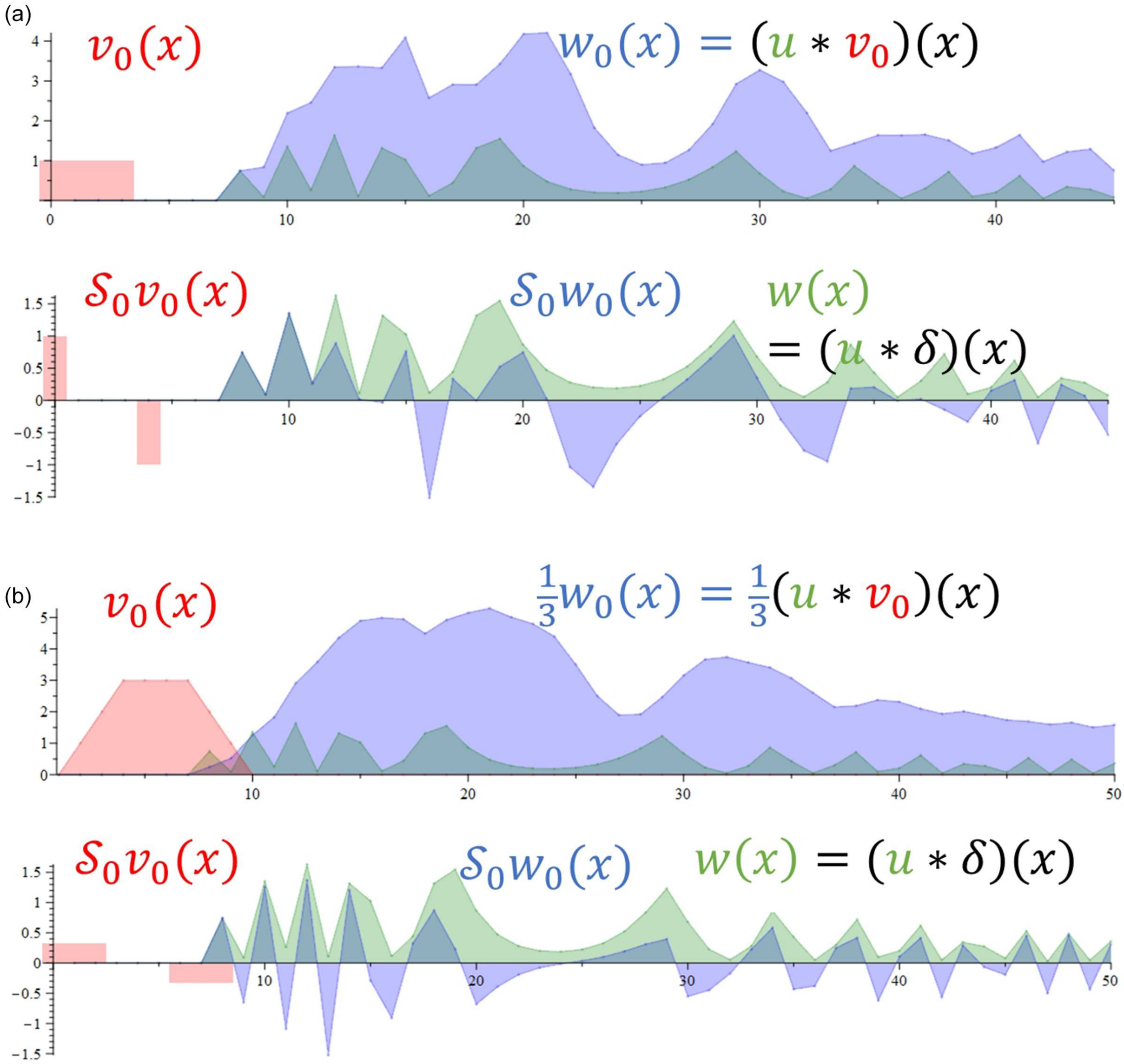


Figure 3
Auxiliary transformations in processing (a) rectangular and (b) trapezoidal signals



($w_0(x) = 0$ at $x < 0$) to a trapezoidal input pulse $v_0(x)$ with equal edges t ($v_0(x) = x/t$ at $x \in [0, t]$, 1 at $x \in [t, T_0]$ and $1-(x-T_0)/t$ at $x \in [T_0, T_0 + t]$ (see Figure 3(b)) reconstruct on this interval the response $w(x)$ of this LI system to:

a) the trapezoidal input pulse $v(x)$ of shorter duration $T < T_0$ ($T > t$);
b) the rectangular pulse $v(x)$ of duration t ($v(x) = 1$ at $x \in [0, t]$ and 0 at $x \notin [0, t)$.

a) Applying the auxiliary LI transformation $\mathscr{S}_0 : \delta(x) - \delta(x - T)$ to the LI equation $w_0(x) = (u \times v_0)(x)$ and taking into account that $\mathscr{S}_0 v_0(x) = v(x) - v(x - T_0)$, we obtain the equation: $rcl\mathscr{S}_0 w_0(x) = (u \times \mathscr{S}_0 v_0)(x) = w(x) - w(x - T_0)$, which is the basic problem of superposition of two shifted pulses. The uncertainty of the problem is $\eta(x)$ at $0 \leq x < T$ and $\eta(x) - \eta(x - T)$ at $x \geq T$. The reconstruction error is minimal when the spectral zeros of signals $v_0(x)$ and $v(x)$ coincide, that is, when $T_0 = m\,T$, $m = 1, 2. \ldots$

b) Applying the auxiliary LI transform of differentiation $\mathscr{S}_0 : d/dx$ to the LI equation $w_0(x) = (u \times v_0)(x)$, we obtain the equation: $\mathscr{S}_0 w_0(x) = (u \times \mathscr{S}_0 v_0)(x) = w(x) - w(x - T_0)$ (see Figure 3(b)), which is the basic LI problem with uncertainty $d\eta(x)/dx$. According to the Fourier transform, the spectrum of this uncertainty increases with frequency $FT[d\eta(x)/dx] = -I\omega FT[\eta(x)]$. The signal processing error decreases with increasing pulse T_0 and edge t durations and is minimal when $t \geq L$, which corresponds to a function $v(x)$ linearly increasing on the interval $x \in [0, L]$.

Problem 3 (exponential pulses). From the LI system response $w_0(x)$, $x \in (-\infty, L)$, to the input exponential pulse $v_0(x)$ ($w_0(x) = v_0(x) = 0$ at $x < 0$):

a) $v_0(x) = \exp(-x/B)$, $B > 0$, at $x \geq 0$, (see Figure 4(a))
b) $v_0(x) = \exp(-x/B) - \exp(-x/b)$, $B > b > 0$, when $x \geq 0$ (see Figure 4(b))

reconstruct the response $w(x)$ to the impulse $v(x) = v_0(x)$ at $x \in [0, T]$ and 0 at $x \notin [0, T]$.

1) Applying the auxiliary LI transformation of shift and subtraction $\mathscr{S}_0 : \delta(x) - \exp(-T/B)\delta(x-T)$ to the function $v_0(x)$ we obtain $\mathscr{S}_0 v_0(x) = v(x)$, so according to (2), $\mathscr{S}_0 w_0(x) = (u \times \mathscr{S}_0 v_0)(x) = (u \times v)(x) = w(x)$. The solution error is equal $\eta(x)$ at $x < T$ and $\eta(x) - \exp(-T/B)\eta(x-T)$ at $x \geq T$. In the case of discrete signals with a sampling step T, the reconstructed response $w(x)$ is equal to the discrete impulse response $u(x)$ of the LI system (see Figure 4(a)).
2) Applying auxiliary LI transformations $\mathscr{S}_0$: $\delta(x) - \exp(-T/B)\delta(x-T)$ and $\mathscr{S}_1$: $A[\delta(x) - \exp(-T/b)\,\delta(x-T)]$ $A = [\exp(-T/B) - \exp(-T/b)]^{-1}$, to the function $v_0(x)$, we obtain $\mathscr{S}_1\mathscr{S}_0 v_0(x) = v(x)$, then according to (2), we have $\mathscr{S}_1\mathscr{S}_0 w_0(x) = (u \times \mathscr{S}_1\mathscr{S}_0 v_0)(x) = (u \times v)(x) = w(x)$. The solution error is equal $\eta(x)$ at $x < T$ and $\mathscr{S}_1\mathscr{S}_0\eta(x)$ at $x \geq T$. In the case of discrete signals with a sampling step T, the reconstructed response $w(x)$ is equal to the discrete impulse response $u(x)$ of the LI system (see Figure 4(b)).

Problem 4 (arbitrary discrete distortions). From the LI system response of the $w_0(x)$, $x \in (-\infty, L)$, to the input signal $v_0(xt) = (q \times v)(x)$, where $q(x) = \sum_{k=0}^{n} q_k \delta(x - kt)$, $q_0 \neq 0$, $t > 0$, ($w_0(x) = v_0(x) = 0$ at $x < 0$) (see Figure 5) reconstruct the response of this system to the pulse $v(x)$ on the interval $x \in (-\infty, L)$.

Applying to the function $q(x)$ the LI transforms of shift and subtraction $\mathscr{S} = \mathscr{S}_n \cdots \mathscr{S}_1$, where $\mathscr{S}_k : \delta(x) - a_k\delta(x - kt)$, $a_k = q_{k+1}^k / q_0$, $k = 1..n$, we successively set the coefficient q_{k+1}^k in the function $q^k(x) = \mathscr{S}_k \cdots \mathscr{S}_1 q(x)$ to zero. At n-th iteration step, when $nt > L$ on the interval $x \in (-\infty, L)$ we get $q^n(x) = \mathscr{S} q(x) = q_0\delta(x)$. From here, according to (2), $\mathscr{S} w_0(x) = q_0 w(x)$ and $w(x) = \mathscr{S} w_0(x)/q_0$. The solution error on the interval $x \in [kt, kt+t]$ is $\mathscr{S}_k \cdots \mathscr{S}_1 \eta(x)/q_0$. In the case of discrete signals with sampling step t, the reconstructed response is the discrete impulse function of the LI system (see Figure 5).

An algorithm for numerical calculations using LI transformations can be implemented based on solving Problem 4 of arbitrary discrete pulse superposition. The advantage of this algorithm is that if the coefficient q_{k+1}^k is zero at the k-th iteration step, then you can skip this step and move on to the next one, which significantly reduces the calculation time. In the presence of noise and uncertainties, the condition $|q_{k+1}^k| < \varepsilon$ can be checked, where the coefficient ε depends on the noise level in the data, which not only reduces the calculation time but also reduces the

Figure 4
Auxiliary transformations in processing exponential pulses

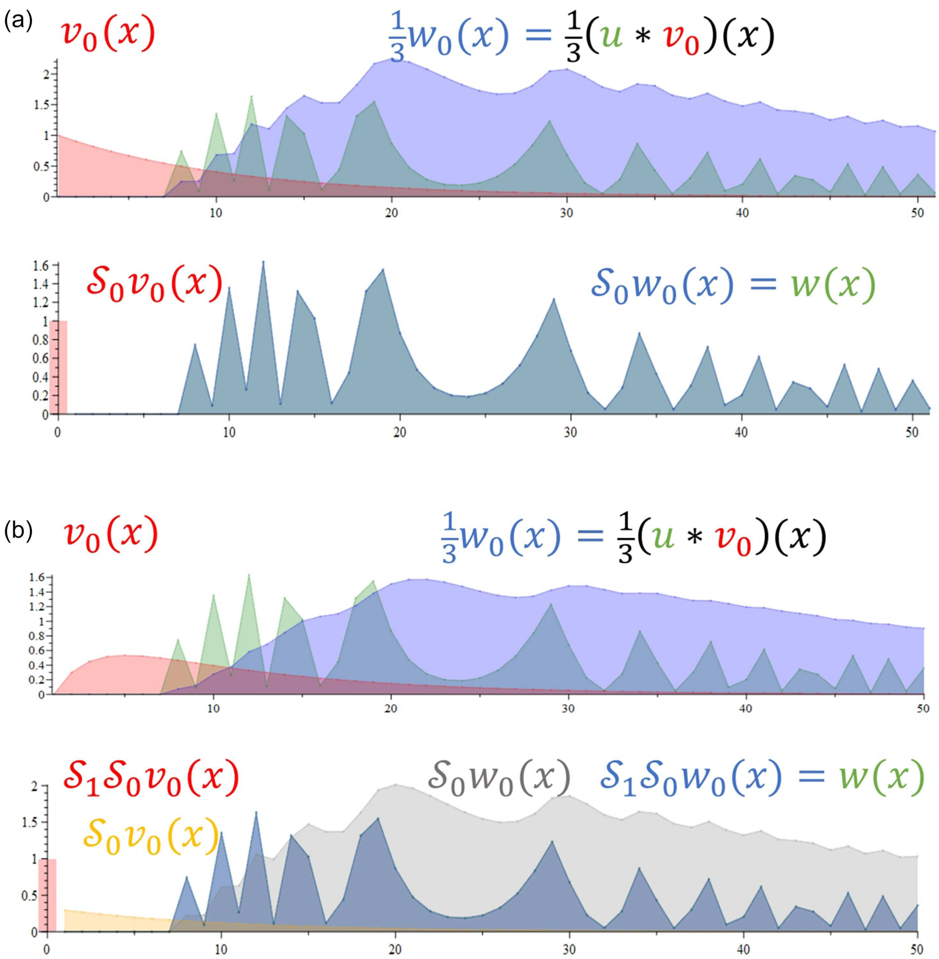
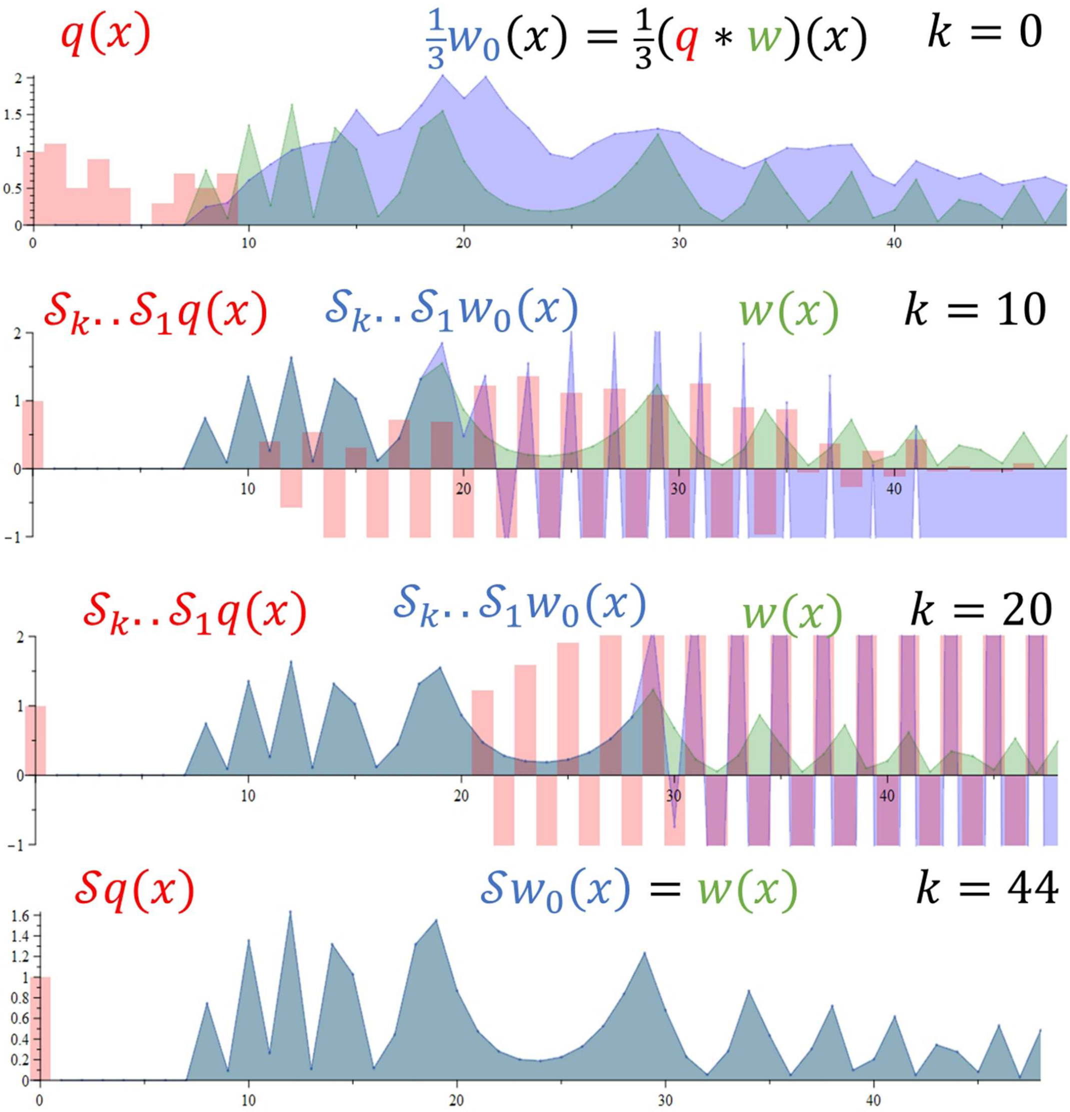


Figure 5
The reconstruction of signals with arbitrary discrete distortions



error, because the uncertainties in the skipped iteration steps do not add up.

For the signals in Figure 5, Problem 4 is ill-conditioned ($|q_0| < |q_1|$). Therefore, during numerical calculations, the signal amplitude increases rapidly at intermediate iteration steps, but within the interval under consideration, it does not overflow, which allows the reconstruction. However, with worse conditioning or over a longer reconstruction interval, the processing error can become very large. This problem can often be solved using auxiliary LI transformations.

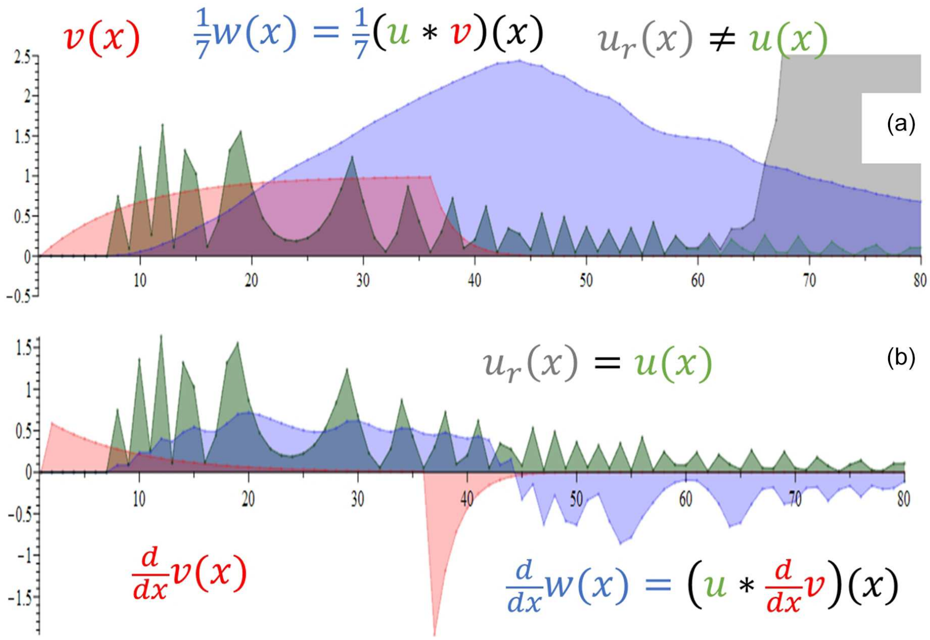
Problem 5 (ill-conditioning). Let the LI system response $w(x)$ to a trapezoidal input pulse $v(x)$ with exponential edges be known on some interval (see Figure 6(a)). Reconstruct the discrete transfer function $u(x)$.

If we represent the signals $w(x)$ and $v(x)$ in discrete form and solve Problem 5 using the approach of Problem 4, the solution to the Problem 5 diverges at the end of the interval (see Figure 6(a) and Table 1), and the relative error of the reconstruction $\|u_r - u\| / \|u\|$ reaches $1.68 \cdot 10^5\%$. However, if we apply the auxiliary LI transformation of differentiation $\mathscr{S}_0 : d/dx$ to the equation $w(x) = (u \times v)(x)$ and solve the equation $\mathscr{S}_0 w(x) = (u \times \mathscr{S}_0 v)(x)$ using the same method, we can obtain the solution with the relative error $1.97 \cdot 10^{-11}$ % (see Figure 6(b)).

Problems similar to Problem 5 are frequently encountered in practice, because real signals have nonzero edges and finite rise times, so the element v_0 of their discrete representation is usually smaller than the subsequent elements, leading to ill-conditioning of the reconstruction problem (see Table 1 and comments to Problem 4). Auxiliary differentiation operations (including higher-order differentiation operations) used in solving Problem 5 are not universal, and optimal auxiliary transformations may differ from these operations. For example, auxiliary differentiation operations could also be used to solve typical problems 3a and 3b, but the transformations given in the solutions to these problems are more efficient, as they allow problem 3a to be solved in just one iteration, and problem 3b in two steps.

The typical tasks discussed here are frequently encountered in practice, but they are not the only ones. There are many other tasks that can be reduced to the basic LI problem using suitable auxiliary LI transforms. For example, the test trials of processing

Figure 6
Auxiliary transformations in solving ill-posed and ill-conditioned problems



systems proposed in References [14] can also be considered as auxiliary LI transformations that enhance the capabilities of signal and image processing.

5. Comparison of Numerical Methods

The effectiveness in solving typical tasks of signal processing by LI methods is due to the possibility of reducing these tasks to the basic LI problem. From the estimates of the number of operations required to numerically solve the basic LI problem using LI methods given in Equation (5) of Section 3, one can see that LI methods are faster not only than many direct methods but also than methods using signal processing in the spectral domain.

This section compares the reconstruction time of solving the basic LI problem for discrete 1D signals of various lengths n by LI methods with some other methods of direct inversion. Numerical calculations were made in the Maple 2017.3 Waterloo Maple Inc. environment on an HP 255 G7 laptop with an AMD Ryzen 3, 2.5 GHz processor, and we used for comparison the optimized codes of direct inversion methods from the standard Maple 2017.3 packages *LinearSolve* and *LinearAlgebra*. These are Maple codes suitable for solving the linear system of equations $w = \boldsymbol{S}v$ with the Toeplitz matrix $\boldsymbol{S}$, corresponding to the basic LI problem: the backward Gauss–Jordan (GJ) method $v_{i+1} = \left(w_i - \sum_{j=1}^{i} s_{i-j}w_j\right)/s_0$, $i = 0..n$; the method of direct matrix inversion (DMI) $v = \boldsymbol{S}^{-1}w$; and the lower-upper (LU) and singular value decomposition (SVD) methods [15–20]. The calculation time in seconds for discrete signals of various lengths n are shown in Table 2.

The calculation time using the direct shift method is significantly less than the calculation time using the presented methods. For example, for a vector of length $n = 6400$, the recovery time using the SVD method is 2707 times longer than using the LI method.

LI methods are efficient and designed to solve specific, typical signal processing problems. When solving many of these problems, the number of required operations can be even smaller than when solving the basic LI method problem considered here. For example, solving typical problem 3a requires only one iteration step and, therefore, n operations when processing signals of length n, while solving problem 3b requires two iteration steps and $2n$ operations.

It should also be noted that another advantage of LI methods is that they require no additional arrays, matrices, or integral transforms to perform calculations, which significantly saves computing resources and allows processing the data of very large volumes.

Table 2
The time (sec) to solve the basic LI problem at different signal lengths n

n	LI	GJ	DMI	LU	SVD
200	0.015	0.032	0.140	0.297	0.250
400	0.047	0.094	0.578	0.891	1.125
800	0.062	0.266	2.375	2.750	4.375
1600	0.094	1.250	11.38	11.70	22.47
3200	0.266	6.109	53.656	55.234	133.4
6400	0.719	23.97	294.06	329.44	1946.5

The program codes based on the numerical solution to Problem 4 of Section 4 and used in this study are publicly available in the author's domain of the GitHub library at https://github.com/novikov-borodin/lst-data-proc.

6. Examples of Using LI Methods

Solutions to typical tasks can be used for a wide variety of LI systems. Figure 7 shows an example of simulating the output signals of electronic devices—of a capacitive voltage divider.

Here, according to the solution to typical problem 2a, the device's response to a trapezoidal signal with a duration of 30 ns is simulated from a response to a trapezoidal signal with a duration of 120 ns. The original signals shown in Figure 7 were calculated using the Micro-CAP 12.2.0.4 simulator of Spectrum Software; the circuit diagram of the device was taken from the standard library of this program. Simulating the signal of 30 ns duration from the signal of 120 ns duration in the range from 0 to 500 ns requires only three iterations by LI methods. The simulation error $1.8 \cdot 10^{-3}\%$ is primarily caused by numerical calculations, rather than the error of the LI method itself. If the initial signals are obtained experimentally, then analysis of the LI system, its circuit diagram, and parasitic parameters of its elements are not required. In addition, continuous signals can be processed in a similar way.

Figure 8 shows an example of using the LI methods in data processing of complex systems—correction of the spectra of the time-of-flight (TOF) neutron spectrometer at the Institute for Nuclear Research of the Russian Academy of Sciences [22]. The input signal is a pulse of neutrons with different energies, initiated in a neutron target by a short proton beam pulse from a linear accelerator (see Figure 8(a)). Neutrons with different energies are separated in time as they fly to the target, where they interact with the nuclei of the substance being studied, which emit γ quanta corresponding to the nuclear energy spectrum. As a result, the γ detector records the energy spectrum of the nuclei, which serves as the output signal. The relationship between the input and output signals satisfies the convolution Equation (1), making the spectrometer an LI system. However, since the data are obtained experimentally, consideration of the described complex physical processes of signal conversion within the system is not required, and only the experimental data (Figure 8(b)) are used in processing (Figure 8(c)).

The spectrometer's resolution directly depends on the proton pulse duration and the length of the TOF baseline. However, generating extremely short proton pulses is challenging due to beam stabilization issues, which led to various pulse distortions, such as spectral line broadening in the first series of measurements (Set 1) or the appearance of after-pulses in the second series of measurements (Set 2), as shown in Figure 8(a). Using the solution to the basic LI problem, the after-pulse distortions in the second series of measurements were successfully eliminated in just three iterations, yielding the actual spectra being studied.

It is possible to improve the effective parameters of such systems using LI methods without labor-intensive and costly equipment upgrades and design modifications. For example, the resolution of the considered neutron spectrometer directly depends on the proton pulse duration, and to improve it, it is necessary to minimize the pulse duration. But generating pulses with short duration t is one of the main problems in spectrometers

Figure 7
Modeling the response of a voltage divider

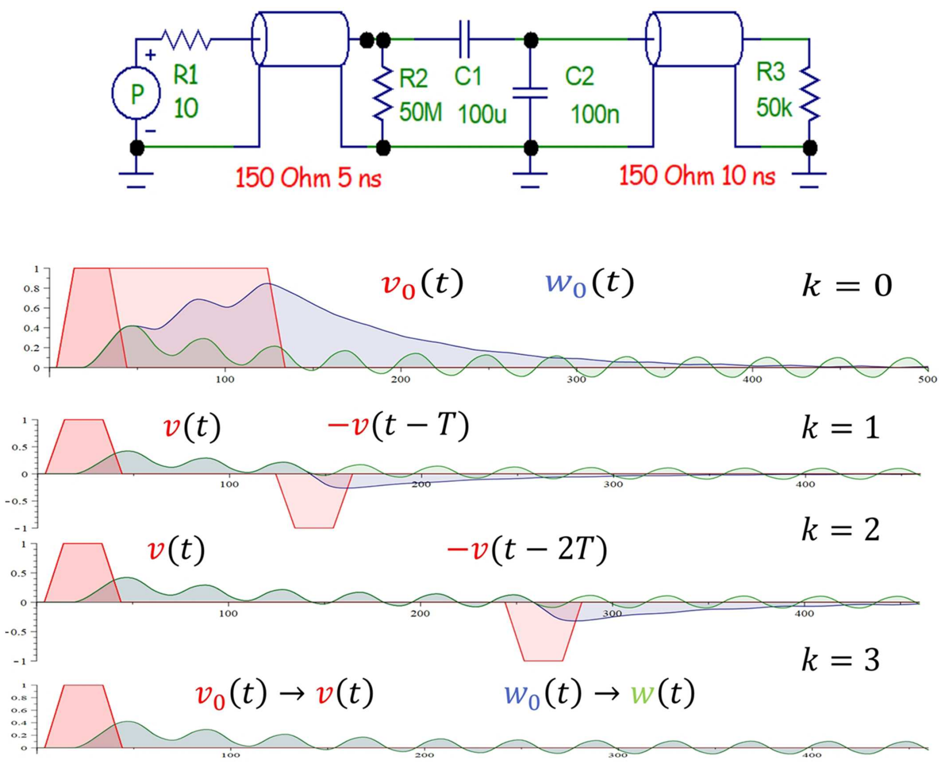
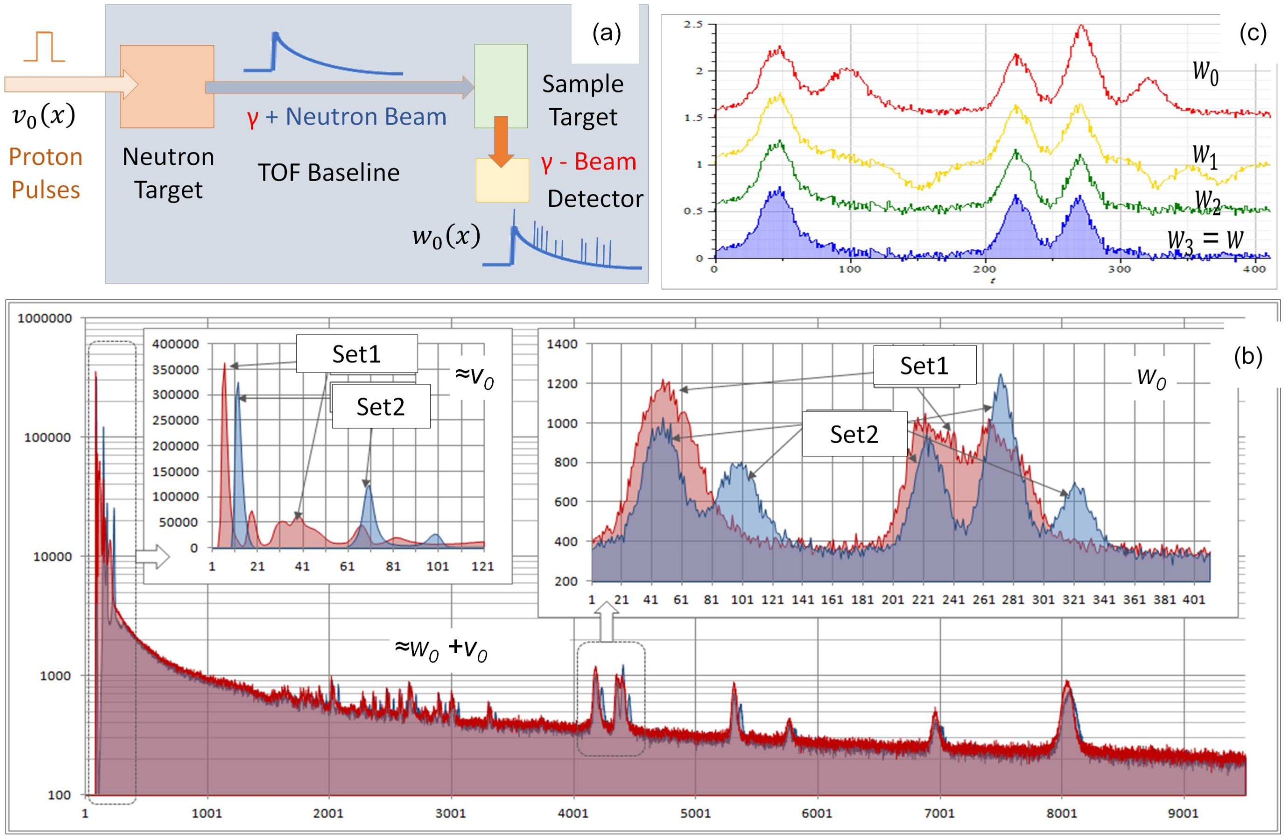


Figure 8
Correction of experimental data from the TOF neutron spectrometer: (a) initial data, (b) structural diagram of the spectrometer, and (c) the data correction process



of this type, and in practice, due to various physical limitations, the beam stabilization is only ensured with a pulse duration $T > t$. However, it is possible to make measurements with a pulse duration T and then, using the solution of typical tasks 1a or 2a, simulate the spectrometer response to a pulse of shorter duration t. Thus, both stable operation and improved resolution can be ensured.

Figure 9 illustrates the processing of multidimensional data using LI methods on the example of restoring a two-dimensional color image $W[j, i]$ of 90 × 100 pixels blurred uniformly horizontally by $T = 20$ pixels (typical problem 1a). The elimination of this distortion using the LI methods requires only four iterations: one auxiliary shift-subtraction transformation $\mathscr{S}_0 : W_0[j, i] = W[j, i] - W[j, i-1]$, which reduces the task to the basic LI problem, and three transformations $\mathscr{S}_k : W_k[j, i] = W_{k-1}[j, i] - W_{k-1}[j, i - 2^{k-1}T]$, and $k = 1..3$. No additional arrays are required for reconstruction.

For comparison, Figure 9(b) shows the process of reconstruction of the same image, but with arbitrary discrete blurring $q(x) = \sum_{k=0}^{n} q_k \delta(x - kt)$ (typical problem 4). In this case, reconstruction needs 100 iterations with transformations $\mathscr{S}_k$: $W_k[j, i] = W_{k-1}[j, i] - (q_{k+1}^{k}/q_0)W_{k-1}[j, i-k]$, $k = 1..100$. No additional arrays are also required. For simplicity of analysis, Figure 9 considers cases of blurring with plane-parallel image displacements along the horizontal axis, but, as was shown earlier in Figure 2 of Section 3, reconstruction is possible with blurring along arbitrary directions or other displacements that can be described by LI transforms.

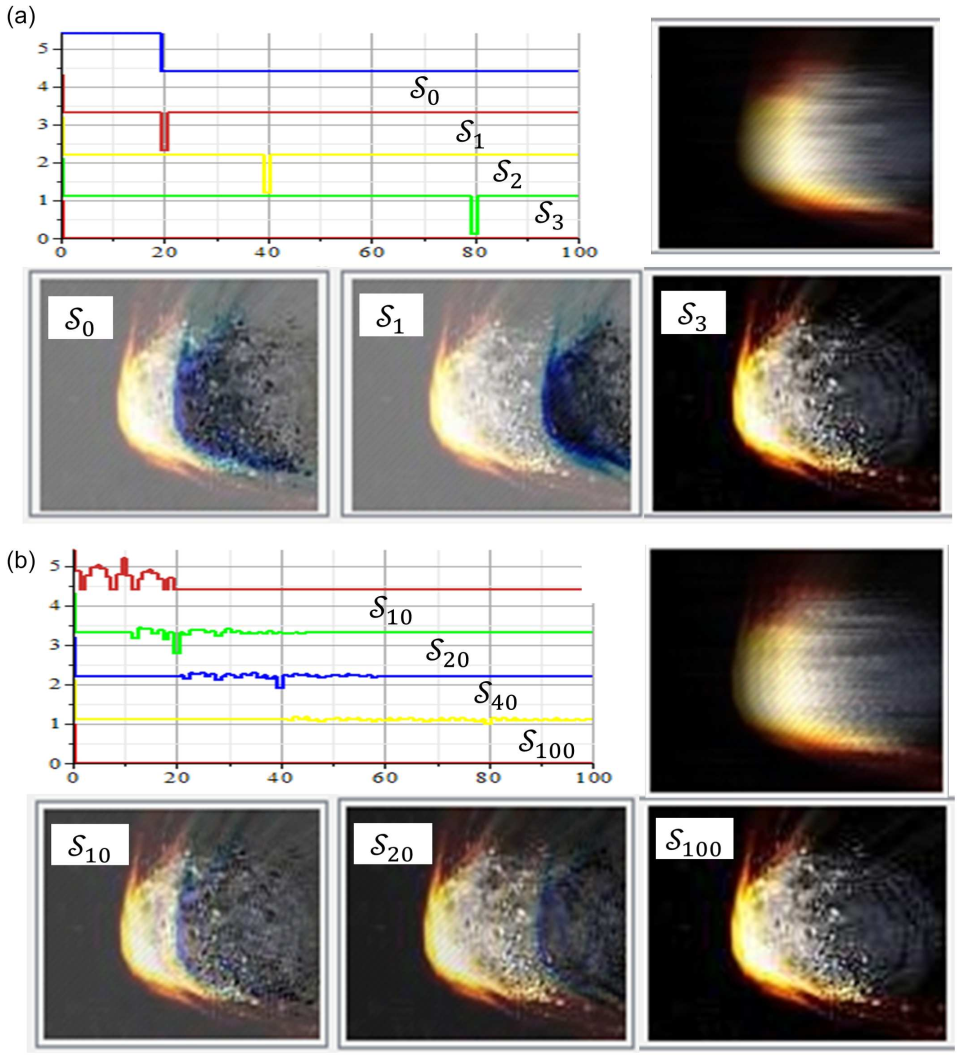
LI transforms can solve many other signal processing problems. It is, for example, a problem of super-resolution [23–26]. Combining consequently n signals from a device with a resolution T, shifted relative to each other by a step $t = T/n$ with coefficients q_k, we can obtain the signal, which is a convolution of the signal with resolution t with a distortion function $q(x) = \sum_{k=0}^{n} q_k \delta(x - kt)$, $q_0 \neq 0$. It is most efficient to use for super-resolution the distortion function of rectangular or exponential type from typical problems 1a or 3a, for which processing can be performed in a minimum number of iteration steps. Indeed, it needs only one iteration if the distortion corresponds to an exponential type.

LI methods have many other applications in quite different fields. For example, in Reference [27], LI transforms were used for modeling the irradiation zones of neoplasms during proton therapy, and in References by Jordanov and Jordanova [28] and Liu et al. [29], the unfolding technique correlated with the solution of the typical problem 3 was used for the registration and separation of digital pulse processing in high-intensity particle detectors.

7. Discussions

The main advantages of mathematical LI methods are that they allow for the rapid and efficient solution of problems involving reconstruction, correction, and modeling of experimental data, using minimal initial information and computational resources. Indeed, LI methods are quite simple to implement in numerical calculations and allow for significant savings in

Figure 9
Reconstruction of color images with (a) uniform and (b) arbitrary blur



computing resources when processing large volumes of data, since virtually no additional arrays other than the arrays of the signals and images themselves are required for their processing.

These advantages are due to the selection of optimal LI transformations and the reduction of their number, which has been demonstrated in examples of typical signal and image processing problems frequently encountered in practice. However, the typical problems considered in this paper are not the only ones. Many other signal processing problems exist that can be reduced to the basic LI problem using suitable auxiliary transformations. However, finding optimal auxiliary transformations is a complex task, unique to each specific case. Such non-universality of LI methods can be considered as their disadvantage.

8. Conclusions

The mathematical methods using LI transforms proposed in this paper are designed for the correction, reconstruction, and modeling of signals and images distorted during their processing by linear stationary and invariant systems. LI methods enable signal reconstruction and modeling with a minimum of initial information about the signals and processing systems and are applicable to the processing of both discrete and continuous signals, including multidimensional ones. LI methods belong to methods of direct inversion and can process a very wide class of functions, which must be locally integrable in the considered domain. Thus, they enable to process generalized and discontinuous functions, spectrometer signals with narrow spectral lines, high-contrast images with sharp boundaries, etc.

Unlike other methods of direct inversion, LI methods allow to minimize processing errors by selecting optimal LI transformations and reducing their number. Examples of using LI methods for solving so-called typical tasks frequently encountered in practice as well as similar or reducible to them are considered.

Among the disadvantages is that LI methods are not universal, because they are efficient and designed to solve specific, typical signal processing problems. Some typical tasks considered in this paper are frequently encountered in practice, but they are not the only ones. There are many other tasks that can be reduced to the basic LI problem using suitable auxiliary transformations. Finding optimal auxiliary transformations can be challenging and unique to each specific case, which is a direction for future work.

The proposed mathematical LI methods offer an affordable alternative to hardware methods associated with solving complex scientific and technical problems and creating expensive equipment. LI methods have broad practical application potential for improving the effective parameters of various equipment for signal processing.

Recommendations

The main advantage of mathematical LI methods is that they enable a minimum of initial information to solve the problems of reconstruction, correction, and modeling of experimental data. Methods also enable to process signals and images related to functions of a very wide class, including discontinuous functions.

Among the disadvantages is that LI methods are not universal, because they are efficient and designed to solve specific, typical signal processing problems. Some typical tasks considered in this paper are frequently encountered in practice, but they are not the only ones. There are many other tasks that can be reduced to the basic LI problem using suitable auxiliary transformations. However, finding optimal auxiliary transformations can be challenging and unique to each specific case.

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Ethical Statement

This study does not contain any studies with human or animal subjects performed by the author.

Conflicts of Interest

The author declares that he has no conflicts of interest to this work.

Data Availability Statement

The program codes used in this study are publicly available in the author's domain of the GitHub library at https://github.com/novikov-borodin/lst-data-proc. These codes can be used for free, but references to the author's work and the author's code domain are required.

Author Contribution Statement

Andrey Novikov-Borodin: Conceptualization, Methodology, Software, Validation, Formal analysis, Investigation, Resources, Writing – original draft, Writing – review & editing, Visualization, Supervision, Project administration, Funding acquisition.

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