Received: 2 November 2025 | Revised: 19 January 2026 | Accepted: 24 February 2026 | Published online: 24 June 2026

RESEARCH ARTICLE

Journal of Data Science and Intelligent Systems
2026, Vol. XX(XX) 1–19
DOI: 10.47852/bonviewJDSIS62028065

BON VIEW PUBLISHING

Plithogenic Fuzzy Soft Expert Set and Plithogenic Fuzzy Rough Set with Upside-Down Logic

Takaaki Fujita[1,*]

[1] *Independent Researcher, Japan*

Abstract: In 1965, Zadeh introduced the notion of a *fuzzy set*, which has supported decades of successful applications across diverse domains involving vagueness and uncertainty. Fuzzy sets have since been extended in many directions using a variety of concepts. Representative examples include fuzzy soft expert sets, fuzzy T-rough sets, and plithogenic sets. Fuzzy soft expert sets associate a fuzzy subset with each parameter–expert–opinion triple, thereby assigning graded membership values to decision alternatives. Fuzzy T-rough sets approximate fuzzy subsets through T-similarity relations, producing lower and upper bounds that formalize uncertainty. Plithogenic sets describe elements via attribute-dependent memberships together with a contradiction mapping between attribute values. Upside-Down Logic models structured context changes under which truth and falsity are interchanged, reversing evaluations while maintaining internal consistency across contexts. To the best of our knowledge, a unified framework that explicitly fuses the plithogenic fuzzy-set perspective with fuzzy soft expert sets and fuzzy T-rough sets has not yet been established. To address this gap, this paper revisits the Plithogenic Fuzzy Soft Expert Set, the Plithogenic Fuzzy Rough Set, and the classical fuzzy set, and examines how Upside-Down Logic can be incorporated into these settings to capture real-world reversal phenomena.

Keywords: Plithogenic Fuzzy Soft Expert Set, Plithogenic Fuzzy Rough Set, fuzzy set, plithogenic set, Upside-Down Logic

1. Introduction

1.1. Plithogenic set

Classical (crisp) set theory is not designed to represent ambiguity or partial truth. In 1965, Zadeh introduced the notion of a *fuzzy set*, which has supported decades of successful applications across diverse domains involving vagueness and uncertainty [1]. In 1986, Atanassov proposed *intuitionistic fuzzy sets*, where each element carries both membership and non-membership degrees and the residual "hesitation" captures representational uncertainty [2–4]. Later, Smarandache developed *neutrosophic sets* as a further extension; neutrosophic variants have since become influential in many fields [5]. Other related concepts are also known, such as picture fuzzy sets [6, 7], hesitant fuzzy sets [8, 9], quadripartitioned neutrosophic sets [10, 11], and spherical fuzzy sets [12, 13]. In parallel, other frameworks such as *rough sets* and *soft sets* were introduced and have found broad use, including in decision-making [14–16]. Fuzzy sets and neutrosophic sets have been widely studied, and numerous papers have been published on them across many fields, including decision science [17–20].

A *plithogenic set* [21, 22] represents elements through membership driven by explicit attributes together with a *contradiction mapping* between attribute values. This mechanism subsumes and extends earlier multi-valued frameworks, including fuzzy [1], vague [23], intuitionistic [4], hesitant fuzzy [24], and neutrosophic sets [25, 26]. In effect, a plithogenic set, akin to many-valued logics, allows the analyst to specify arbitrary uncertainty parameters and to explicitly encode mutual opposition among attribute values (e.g., conflicting expert opinions). Important subclasses include the *plithogenic fuzzy set* [27–31], the *plithogenic intuitionistic fuzzy set* [32], and the *plithogenic neutrosophic set* [33–37]. As with fuzzy sets, these plithogenic variants have been applied to decision-making and related tasks [38–41]. Moreover, as further generalizations, the concepts of Uncertain Sets and Functorial Sets have also been studied [42]. For reference, Table 1 presents a concise comparison between a fuzzy set and a plithogenic fuzzy set.

1.2. Fuzzy Soft Expert Set and Fuzzy Rough Set

A *Soft Expert Set* models decision processes by associating, for each triple composed of a parameter, an expert, and an opinion, a subset of the universe [43–45]. A *Fuzzy Soft Expert Set* further refines this framework by assigning to every such triple a fuzzy subset, thus expressing the graded membership of each element in the universe [46, 47]. A *Fuzzy Rough Set* provides approximations of fuzzy concepts using similarity-based lower and upper operators, thereby representing uncertainty through graded indiscernibility relations and producing optimistic (upper) and pessimistic (lower) membership bounds [48–50]. Related frameworks, such as the *Neutrosophic Soft Expert Set* [51–53] and the *Neutrosophic Rough Set* [54–56], have also been proposed and are actively studied for applications in decision science and related fields. For reference, Table 2 provides a concise comparison between a fuzzy set and a fuzzy soft expert set. Also, Table 3 presents a comparison between a fuzzy set and a fuzzy rough set.

*Corresponding author: Takaaki Fujita, Independent Researcher, Japan. Email: Takaaki.fujita060@gmail.com

Table 1
Concise comparison between a fuzzy set and a plithogenic fuzzy set

Aspect	**Fuzzy set**	**Plithogenic fuzzy set**
Basic object	A single membership function on a universe U.	Attribute-conditioned memberships on U together with a contradiction map on attribute values.
Membership specification	$\mu : U \to [0,1]$.	$\mu(\cdot \mid a) : U \to [0,1]$ for each attribute value $a \in P_v$; equivalently $pdf : U \times P_v \to [0,1]$.
Extra structure	None beyond μ.	A contradiction function $c : P_v \times P_v \to [0,1]$ (typically $c(a,a) = 0$ and $c(a,b) = c(b,a)$).
Interpretation	$\mu(x)$ quantifies the degree to which x belongs to the set.	$\mu(x \mid a)$ quantifies membership *under* attribute value a; $c(a,b)$ quantifies incompatibility between attribute values.
Special-case relation	Base model.	If $P_v = \{a^*\}$ and $c(a^*,a^*) = 0$, then $pdf(x,a^*)$ reduces to an ordinary fuzzy membership $\mu(x)$.

1.3. Upside-Down Logic

In brief, *Upside-Down Logic* formalizes contextual transformations under which the truth and falsity of lemmas are inverted, thereby modeling ambiguity and reversals in reasoning systems [57]. For instance, in political discourse, harmful policies may be publicly framed as beneficial, effectively inverting truth values and inducing widespread doubt about facts and trust [58].

1.4. Our contributions

Research on plithogenic sets, fuzzy sets, their plithogenic fuzzy extensions, and Upside-Down Logic is important, yet the combined methodological landscape remains comparatively underdeveloped. Moreover, many traditional fuzzy approaches implicitly rely on fixed criteria and stable membership grades; as a result, they are often ill-suited to settings with multiple experts, shifting contexts, and reversal phenomena, which can reduce interpretability, robustness, and practical decision support.

To address these limitations, this paper makes the following contributions:

1) We examine the *Plithogenic Fuzzy Soft Expert Set*, the *Plithogenic Fuzzy Rough Set*, and the classical *Fuzzy Set*, and place them in a unified, attribute-driven framework that explicitly incorporates contradiction among attribute values.
2) We analyze how Upside-Down Logic interacts with these models, and we characterize when and how contextual inversions of truth and falsity propagate through membership assignments, hesitation/indeterminacy, and contradiction mappings.

We further discuss the modeling advantages for problems involving complex or domain-specific uncertainty, and argue that the proposed viewpoint supports more faithful representation and more reliable reasoning in decision-oriented applications. For reference, Table 4 summarizes key differences between a fuzzy soft expert set and a Plithogenic Fuzzy Soft Expert Set, while Table 5 compares a fuzzy rough set with a Plithogenic Fuzzy Rough Set.

By unifying plithogenic, soft-expert, and rough-set viewpoints, our contributions enable contradiction-aware, context-sensitive uncertainty modeling. This yields more interpretable multi-perspective aggregation, principled lower/upper bounds under similarity, and systematic handling of evaluation reversals via Upside-Down Logic, improving robustness and fidelity in decision-oriented reasoning.

1.5. Structure of this paper

This subsection outlines the organization of the paper. Section 2 provides the preliminaries, where we define and explain the relevant concepts that are already established in the literature. Section 3 presents the main results and discusses the Plithogenic

Table 2
Concise comparison between a fuzzy set and a fuzzy soft expert set

Aspect	**Fuzzy set**	**Fuzzy soft expert set (FSES)**
Underlying data	A universe U.	A universe U together with parameters E, experts X, and opinions O.
Indexing	None.	Indexed by triples $\alpha = (e,x,o) \in A \subseteq E \times X \times O$.
Membership specification	$\mu : U \to [0,1]$.	$\mu_\alpha : U \to [0,1]$ for each $\alpha \in A$; equivalently $\mu : A \times U \to [0,1]$.
Interpretation	$\mu(u)$ is the degree that u belongs to the fuzzy set.	$\mu(\alpha,u)$ is the degree that u is accepted under the parameter–expert–opinion triple α.
Output object	One fuzzy subset of U.	A family of fuzzy subsets of U, one per active triple $\alpha \in A$.
Special-case relation	Base model.	If $A = \{\alpha_0\}$ is a singleton, then an FSES reduces to a single fuzzy set μ_{α_0} on U.

Table 3
Concise comparison between a fuzzy set and a fuzzy rough set

Aspect	Fuzzy set	Fuzzy rough set (FRS)
Underlying data	Universe X.	Approximation space (X, R) with a fuzzy relation $R : X \times X \to [0,1]$ (often a similarity).
Input object	Membership $k : X \to [0,1]$.	Membership $k : X \to [0,1]$ (same input).
Core idea	Represents graded membership directly.	Approximates k via granulation induced by R.
Output	One fuzzy set k.	Two fuzzy sets: lower $L_R k$ and upper $U_R k$ approximations.
Interpretation	$k(x)$: degree that x belongs.	$L_R k(x)$: conservative support; $U_R k(x)$: possible support under R.
Special case	Base model.	If $R(x,y) = 1$ iff $x = y$, then $L_R k = U_R k = k$.

Fuzzy Soft Set, the Plithogenic Fuzzy Soft Expert Set, and the Plithogenic Fuzzy T-Rough Set. Section 4 concludes the paper.

2. Preliminaries

This section gathers the background notions and notation required for the main results. Unless explicitly stated otherwise, every set that appears is assumed to be finite.

2.1. Upside-Down Logic

We formalize *Upside-Down Logic*: under suitable contextual transformations, truth and falsity of lemmas are interchanged, providing a framework to capture reversals and contextual ambiguity in reasoning systems [57–61].

Definition 1 (Logical System) [62] A *logical system* is a structure

$$\mathscr{M} = (\mathscr{P}, \mathscr{V}, v),$$

where $\mathscr{P}$ is the set of lemmas (formulas) in a formal language $\mathscr{L}$, $\mathscr{V}$ is a set of truth values (e.g., $\{\text{True}, \text{False}\}$ in the classical case), and $v : \mathscr{P} \to \mathscr{V}$ is a valuation assigning to each lemma a truth value. Optionally, $\mathscr{M}$ may specify a set of axioms $\mathscr{A} \subseteq \mathscr{P}$ and a collection of inference rules $\mathscr{I}$.

Notation 1 Given a set of lemmas $\mathscr{P}$ and a family of contexts $\mathscr{C}$, we write

$$T : \mathscr{P} \times \mathscr{C} \longrightarrow \{\text{True, False, Indeterminate}\}$$

for the context-sensitive truth assignment that evaluates each lemma–context pair. We also fix a formal language $\mathscr{L}$ and a logical system $\mathscr{M} = (\mathscr{P}, \mathscr{V}, v)$ as above.

Definition 2 (Upside-Down Logic [57, 58]) An *Upside-Down Logic* is obtained from a logical system $\mathscr{M}$ by equipping it with a transformation U acting on lemmas and/or on contexts, thereby producing a system $\mathscr{M}'$ such that for every lemma $A \in \mathscr{P}$ and context $\mathscr{C}$:

1) (Truth $\to$ Falsity) If $v(A) = \text{True}$ in $\mathscr{C}$, then $v\big(U(A)\big) = \text{False}$ in $U(\mathscr{C})$.
2) (Falsity $\to$ Truth) If $v(A) = \text{False}$ in $\mathscr{C}$, then $v\big(U(A)\big) = \text{True}$ in $U(\mathscr{C})$.

Moreover, U is required to be well posed and internally consistent within $\mathscr{M}'$.

Example 1 (Parking Regulations During a Special Event) Consider a lemma A: "Street S parking is permitted now." Let a context be the tuple $\mathscr{C} = (\text{legal\_hours} = 1, \text{ event} = e)$ with $e \in \{0,1\}$, where $e = 0$ denotes a normal day and $e = 1$ a special event day with temporary restrictions. Define the valuation

$$v(A, \mathscr{C}) \;:=\; 1 - e \in \{\text{True} = 1, \ \text{False} = 0\}.$$

Introduce a context transformation U that toggles the event flag:

$$U(\mathscr{C}) \;:=\; (\text{legal\_hours} = 1, \ \text{event} = 1 - e).$$

Then

$$v\big(A, U(\mathscr{C})\big) = 1 - (1 - e) = e = 1 - v(A, \mathscr{C}),$$

so truth and falsity are swapped. Equivalently, the negated lemma $\neg A$ flips oppositely:

$$v(\neg A, \mathscr{C}) = e, \qquad v\big(\neg A, U(\mathscr{C})\big) = 1 - e.$$

The map U is well posed (indeed an involution: $U(U(\mathscr{C})) = \mathscr{C}$), hence this conforms to the definition of Upside-Down Logic.

Table 4
Concise comparison between a Fuzzy Soft Expert Set (FSES) and a Plithogenic Fuzzy Soft Expert Set (PFSES)

Aspect	FSES	PFSES
Underlying data	$U, E, X, O, A \subseteq E \times X \times O$.	FSES data + P_v and $pCF : P_v \times P_v \to [0,1]$.
Membership	$\mu : A \times U \to [0,1]$.	$\mu : A \times U \times P_v \to [0,1]$.
Interpretation	$\mu(\alpha, u)$: acceptance of u under α.	$\mu(\alpha, u \mid a)$: acceptance under α given $a \in P_v$.
Extra structure	None.	Contradiction among attribute values via pCF.
Special case	Base model.	If $\lvert P_v \rvert = 1$ and $pCF = 0$, PFSES reduces to FSES.

Table 5
Concise comparison between a Fuzzy Rough Set (FRS) and a Plithogenic Fuzzy Soft Expert Set (PFRS)

Aspect	FRS	PFRS
Underlying space	(X,R), $R : X\times X \to [0,1]$.	$\{R_a\}_{a\in P_v}$ + (P_v, pCF).
Input	Fuzzy set $k : X \to [0,1]$.	$pdf : X\times P_v \to [0,1]$ (slices k_a).
Approximations	$L_R k$, $U_R k$.	Aggregate $\{L_{R_a}k_a, U_{R_a}k_a\}_{a\in P_v}$ (often weighted by $1-c(a,b)$).
Contradiction	Not modeled.	pCF modulates aggregation.
Output	Lower/upper fuzzy sets on X.	Aggregated lower/upper fuzzy sets on X (anchor-dependent).
Special case	Base model.	If $\lvert P_v\rvert = 1$ and $pCF = 0$, PFRS reduces to FRS.

Example 2 (Elevator Use Under Fire Alarm) Let B be the lemma: "Using elevators is allowed." Model a building context by $\mathscr{C} = (\text{alarm} = a)$ with $a \in \{0,1\}$, where $a = 0$ means no emergency and $a = 1$ indicates a fire alarm. Define the valuation

$$v(B,\mathscr{C}) := 1-a,$$

capturing the usual safety rule that elevators are disabled during a fire alarm. Define U to toggle the alarm status:

$$U(\mathscr{C}) := (\text{alarm} = 1-a).$$

Then

$$v\big(B, U(\mathscr{C})\big) = 1-(1-a) = a = 1-v(B,\mathscr{C}),$$

so the truth value is inverted by the contextual transform. Consequently, the opposite statement "Elevator use is not allowed" becomes true exactly when B becomes false, and vice versa. Again U is an involution, ensuring internal consistency in the transformed system.

2.2. Plithogenic set

A plithogenic set [21, 63] represents elements via membership driven by explicit attributes together with a contradiction mapping between attribute values.

Definition 3 (Plithogenic Set [21, 63]) Let S be a universe and $P \subseteq S$ a nonempty subset. A *plithogenic set* is a 5–tuple

$$PS = (P,\ v,\ Pv,\ pdf,\ pCF),$$

with the following ingredients:

1) v – a chosen attribute;
2) Pv – the value domain of v;
3) $pdf : P\times Pv \to [0,1]^s$ – the *degree of appurtenance* (DAF);[1]
4) $pCF : Pv\times Pv \to [0,1]^t$ – the *degree of contradiction* (DCF).

For every $a,b \in Pv$, the DCF satisfies

$$\text{reflexivity: } pCF(a,a) = 0, \qquad \text{symmetry: } pCF(a,b) = pCF(b,a).$$

Here $s,t \in \mathbb{N}$ are, respectively, the appurtenance and contradiction dimensions.

[1] In the literature, DAF is modeled in several equivalent ways (e.g., powerset–valued or vector–valued). We adopt the standard $[0,1]^s$ form.

Definition 4 (Plithogenic Fuzzy Set ($s = 1$, $t = 1$)) [64]. A *plithogenic fuzzy set* is a plithogenic set $PS = (P,v,Pv,pdf,pCF)$ in which

$$pdf : P\times Pv \to [0,1], \qquad pCF : Pv\times Pv \to [0,1].$$

For $x \in P$ and $a \in Pv$, write

$$\mu_P(x \mid a) := pdf(x,a) \in [0,1],$$

the (single-component) fuzzy membership of x under attribute value a. The contradiction between two attribute values is the scalar

$$c(a,b) := pCF(a,b) \in [0,1], \qquad c(a,a) = 0,\ \ c(a,b) = c(b,a).$$

Example 3 (Travel Destination Suitability with Contextual Contradictions) Let the universe be the set of destinations

$$P = \{\text{Hokkaido } (H),\ \text{Okinawa } (O),\ \text{Kyoto } (K)\}.$$

Choose the single attribute v = "travel criterion" whose value domain is

$$Pv = \{\text{WinterSports } (W),\ \text{BeachSun } (B),\ \text{Culture } (C)\}.$$

The plithogenic fuzzy membership $pdf : P\times Pv \to [0,1]$ (abbrev. $\mu_P(x \mid a)$) encodes how well a place matches a criterion; the degree of contradiction $pCF : Pv\times Pv \to [0,1]$ (abbrev. $c(\cdot,\cdot)$) quantifies pairwise tension between criteria. Take

$\mu_P(x \mid a)$	W	B	C
H	0.95	0.10	0.60
O	0.05	0.95	0.40
K	0.20	0.30	0.95

$c(a,b)$	W	B	C
W	0	0.90	0.50
B	0.90	0	0.40
C	0.50	0.40	0

The DCF is reflexive and symmetric ($c(a,a) = 0$, $c(a,b) = c(b,a)$). Here *WinterSports* and *BeachSun* strongly conflict (0.90), while *Culture* has moderate tension with each. This structure captures that Hokkaido is excellent for winter activities but poor for beaches, Okinawa is the opposite, and Kyoto excels in culture.

2.3. Upside-Down Logic in plithogenic fuzzy set with contradiction reset

We formalize an *upside-down* transformation for plithogenic fuzzy sets in which membership grades are complemented whenever the contradiction with a chosen anchor exceeds a prescribed threshold; immediately afterward, the involved contradiction is forced to zero to stabilize the context [58].

Definition 5 (Upside-down transform with contradiction reset for a Plithogenic Fuzzy Set) [58] Let $PS = (P, v, Pv, pdf, pCF)$ be a plithogenic fuzzy set. Write

$$\mu_P(x \mid a) := pdf(x,a) \in [0,1], \qquad c(a,b) := pCF(a,b) \in [0,1],$$

with $c(a,a) = 0$ and $c(a,b) = c(b,a)$ for all $a, b \in Pv$. Fix an anchor attribute $b \in Pv$ and a threshold $\tau \in [0,1]$. Define the activation locus

$$A_\tau(b) := \{a \in Pv : c(a,b) \geq \tau\}.$$

The *upside-down transform with contradiction reset* produces a new plithogenic fuzzy set

$$PS^{U_{b,\tau}} := (P, v, Pv, pdf^{U_{b,\tau}}, pCF^{U_{b,\tau}})$$

whose components are given, for every $x \in P$ and $a \in Pv$, by

$$pdf^{U_{b,\tau}}(x,a) := \begin{cases} 1 - \mu_P(x \mid a), & \text{if } a \in A_\tau(b), \\ \mu_P(x \mid a), & \text{if } a \notin A_\tau(b), \end{cases}$$

and the updated contradiction map $pCF^{U_{b,\tau}} : Pv \times Pv \to [0,1]$ defined for all $u, v \in Pv$ as

$$pCF^{U_{b,\tau}}(u,v) := \begin{cases} 0, & \text{if } \{u,v\} = \{a,b\} \text{ for some } a \in A_\tau(b), \\ pCF(u,v), & \text{otherwise.} \end{cases}$$

Informally, whenever an attribute value a is flipped due to high contradiction with the anchor b, the post–transform structure sets the contradiction between a and b to 0, preventing immediate reactivation at the same threshold.

Example 4 (Travel Destinations: Flip on Highly Contradictory "Beach" vs. Anchor "WinterSports") Let $P = \{\text{Hokkaido } (H), \text{ Okinawa } (O), \text{ Kyoto } (K)\}$ and let the single attribute be $v =$ "travel criterion" with value set $Pv = \{W, B, C\}$ for *WinterSports*, *BeachSun*, *Culture*. The plithogenic fuzzy membership $pdf : P \times Pv \to [0,1]$ (write $\mu_P(x \mid a)$) and contradiction $c(a,b)$ are

$\mu_P(x \mid a)$	W	B	C
H	0.95	0.10	0.60
O	0.05	0.95	0.40
K	0.20	0.30	0.95

$c(a,b)$	W	B	C
W	0	0.90	0.50
B	0.90	0	0.40
C	0.50	0.40	0

Choose anchor $b = W$ and threshold $\tau = 0.8$. Then

$$A_\tau(W) = \{a \in Pv : c(a,W) \geq 0.8\} = \{B\},$$

because $c(B,W) = 0.90 \geq 0.8$ while $c(C,W) = 0.50 < 0.8$. By Definition 5, we flip only the B-column (take complements) and leave W and C unchanged:

$pdf^{U_{W,0.8}}(x,a)$	W	B	C
H	0.95	$1 - 0.10 = 0.90$	0.60
O	0.05	$1 - 0.95 = 0.05$	0.40
K	0.20	$1 - 0.30 = 0.70$	0.95

The reset modifies the contradiction only on the unordered pair $\{B, W\}$:

$$pCF^{U_{W,0.8}}(W,B) = pCF^{U_{W,0.8}}(B,W) = 0,$$

$$\text{all other } pCF^{U_{W,0.8}}(a,b) = pCF(a,b).$$

Explicitly,

$pCF^{U_{W,0.8}}(a,b)$	W	B	C
W	0	0	0.50
B	0	0	0.40
C	0.50	0.40	0

After the reset, the pair (B,W) cannot be reactivated at the same threshold $\tau = 0.8$ since their contradiction has become 0.

Example 5 (Smartphones: Flip on "BatteryLife" Under Anchor "Compactness") Let $P = \{\text{Model A}(A), \text{ Model B}(B), \text{ Model C}(C)\}$ and let the attribute be $v =$ "design criterion" with $Pv = \{Q, L, S\}$ for *CameraQuality*, *BatteryLife*, *Compactness*. Take

$\mu_P(x \mid a)$	Q	L	S
A	0.90	0.60	0.30
B	0.70	0.90	0.40
C	0.60	0.50	0.90

$c(a,b)$	Q	L	S
Q	0	0.30	0.60
L	0.30	0	0.70
S	0.60	0.70	0

Choose anchor $b = S$ and threshold $\tau = 0.65$. Then

$$A_\tau(S) = \{a \in Pv : c(a,S) \geq 0.65\} = \{L\},$$

since $c(L,S) = 0.70 \geq 0.65$ while $c(Q,S) = 0.60 < 0.65$. Hence, we flip only the L-column and keep Q, S unchanged:

$pdf^{U_{S,0.65}}(x,a)$	Q	L	S
A	0.90	$1 - 0.60 =$ 0.40	0.30
B	0.70	$1 - 0.90 =$ 0.10	0.40
C	0.60	$1 - 0.50 =$ 0.50	0.90

The contradiction reset zeroes exactly the pair $\{L, S\}$:

$$pCF^{U_{S,0.65}}(L,S) = pCF^{U_{S,0.65}}(S,L) = 0, \quad \text{others unchanged.}$$

Thus

$pCF^{U_{S,0.65}}(a,b)$	Q	L	S
Q	0	0.30	0.60
L	0.30	0	0
S	0.60	0	0

Consequently, (L,S) will not trigger another flip at the same $\tau = 0.65$, ensuring contextual stability after the transformation.

3. Main Results

In this section, we present and explain the results of this paper.

3.1. Plithogenic Fuzzy Soft Set

A soft set represents parameterized families of subsets over a universe, providing a flexible mathematical framework for modeling uncertainty in decision-making [65–67]. Related concepts include the HyperSoft Set [68–70] and the SuperHyperSoft Set [71–73]. A fuzzy soft set assigns each parameter a fuzzy subset of a universe, encoding approximate attribute-based membership degrees for elements [2, 74, 75]. A plithogenic fuzzy soft set maps parameter, element, and attribute value to a degree, aggregating contradiction functions to handle disagreements [64, 76].

Definition 6 (Fuzzy Soft Set) [2, 74, 75] Let U be a nonempty universe and E a set of parameters. Write $\mathscr{F}(U) := \{\mu : U \to [0,1]\}$

for the family of all fuzzy subsets of U. For a fixed subset $A \subseteq E$, a *fuzzy soft set* over U (with respect to A) is a pair

$$(\Gamma_A, A), \qquad \Gamma_A : A \longrightarrow \mathscr{F}(U), \ x \longmapsto \mu_x(\cdot),$$

so that it can be represented as the collection

$$\Gamma_A = \{ (x, \mu_x) \mid x \in A, \ \mu_x : U \to [0,1] \}.$$

For $u \in U$ and $x \in A$, the value $\mu_x(u)$ is the degree to which u approximately satisfies the parameter x. Equivalently, one may specify a map $\widetilde{\Gamma} : E \to \mathscr{F}(U)$ with $\widetilde{\Gamma}(x) = \mathbf{0}$ (the zero membership function) for all $x \notin A$.

Definition 7 (Plithogenic Fuzzy Soft Set (PFSS)) Let U be a nonempty universe and E a (finite) set of parameters. Fix an attribute v with a finite value set Pv and a *degree of contradiction*

$$pCF : Pv \times Pv \longrightarrow [0,1], \qquad c(a,b) := pCF(a,b),$$

satisfying $c(a,a) = 0$ and $c(a,b) = c(b,a)$ for all $a,b \in Pv$. Let $A \subseteq E$ be the (active) parameter subset.

A *Plithogenic Fuzzy Soft Set* over U (with respect to A and (Pv, pCF)) is a pair

$$(\Gamma_A, A), \qquad \Gamma_A : A \longrightarrow [0,1]^{U \times Pv}, \ e \longmapsto \mu_e(\cdot \mid \cdot),$$

equivalently specified by a membership map

$$\mu : A \times U \times Pv \longrightarrow [0,1], \qquad \mu(e,u \mid a) := \mu_e(u \mid a),$$

interpreted as the degree to which $u \in U$ belongs under parameter $e \in A$ *given* the attribute value $a \in Pv$. The pair (Pv, pCF) is common to all parameters and quantifies potential *agreement/contradiction* between attribute values.

Example 6 (Restaurant Selection with Personas as Attribute Values) Let the universe of objects be three restaurants

$$U = \{\text{Szechuan } (Sz), \ \text{SaladBar } (Sa), \ \text{FamilyDiner } (Fa)\}.$$

Let the parameter set be

$$E = \{e_1 = \text{“spicy options”}, \ e_2 = \text{“vegetarian friendliness”}\}, \quad A = E.$$

Fix a single attribute v = “customer persona” with value set

$$Pv = \{ \text{SpiceSeeker } (Sp), \ \text{HealthConscious } (He), \ \text{BudgetMinded } (Bu) \}.$$

The *degree of contradiction* $pCF : Pv \times Pv \to [0,1]$ (write $c(a,b)$) is taken symmetric with $c(a,a) = 0$, e.g.

$c(a,b)$	Sp	He	Bu
Sp	0	0.70	0.40
He	0.70	0	0.20
Bu	0.40	0.20	0

(higher values encode stronger tension between personas). A Plithogenic Fuzzy Soft Set (PFSS) is given by the membership map $\mu : A \times U \times Pv \to [0,1]$, where $\mu(e,u \mid a)$ is the degree to which u satisfies parameter e *as assessed under* persona a. We specify μ for each parameter as a matrix with rows $u \in U$ and columns $a \in Pv$:

Parameter e_1 : “spicy options”

$\mu(e_1,u \mid a)$	Sp	He	Bu
Sz	0.95	0.45	0.75
Sa	0.10	0.20	0.40
Fa	0.60	0.40	0.70

Parameter e_2 : “vegetarian friendliness”

$\mu(e_2,u \mid a)$	Sp	He	Bu
Sz	0.30	0.85	0.50
Sa	0.60	0.95	0.70
Fa	0.40	0.70	0.80

This PFSS jointly models (i) restaurant qualities via parameters $e \in A$, (ii) persona–dependent appraisals via $a \in Pv$, and (iii) persona conflicts through $c(a,b)$ (e.g., *SpiceSeeker* vs. *HealthConscious* has 0.70 contradiction).

Example 7 (Hiring Evaluation with Managerial Emphases as Attribute Values) Let the universe be three candidates

$$U = \{\text{Alice } (Al), \ \text{Bob } (Bo), \ \text{Cara } (Ca)\}.$$

Let the active parameters be

$$E = \{e_1 = \text{“technical fit”}, \ e_2 = \text{“communication”}\}, \quad A = E.$$

Choose the attribute v = “managerial emphasis” with value set

$$Pv = \{\text{Speed } (Sp), \ \text{Reliability } (Rl), \ \text{Innovation } (In)\}.$$

The contradiction map pCF (symmetric, $c(a,a) = 0$) captures typical organizational tensions:

$c(a,b)$	Sp	Rl	In
Sp	0	0.80	0.60
Rl	0.80	0	0.40
In	0.60	0.40	0

Define the PFSS memberships $\mu(e,u \mid a)$ for each parameter as follows. For e_1 (technical fit):

$\mu(e_1,u \mid a)$	Sp	Rl	In
Al	0.90	0.75	0.85
Bo	0.70	0.90	0.75
Ca	0.65	0.60	0.95

For e_2 (communication):

$\mu(e_2,u \mid a)$	Sp	Rl	In
Al	0.70	0.80	0.75
Bo	0.60	0.85	0.70
Ca	0.80	0.65	0.90

The attribute values encode managerial context; e.g., *Speed* and *Reliability* strongly contradict (0.80), reflecting competing priorities. The PFSS records candidate scores that are explicitly conditioned on the emphasis a, thereby separating parameter semantics (fit vs. communication) from contextual interpretation (organizational emphasis).

Theorem 1 (PFSS generalizes the Plithogenic Fuzzy Set) *Let (Γ_A, A) be a PFSS on $(U, E; Pv, pCF)$. If $A = \{e_0\}$ is a singleton, then the map*

$$pdf : U \times Pv \longrightarrow [0,1], \qquad pdf(u,a) := \mu(e_0, u \mid a),$$

together with the same (Pv, pCF), defines a plithogenic fuzzy set on U. Conversely, any plithogenic fuzzy set $(pdf; Pv, pCF)$ on U is recovered as a PFSS by taking $E = \{e_0\}$, $A = \{e_0\}$, and $\mu(e_0, u \mid a) := pdf(u,a)$.

Proof. With $A = \{e_0\}$ fixed, a PFSS supplies exactly one membership surface $\mu(e_0,\cdot \mid \cdot) : U \times Pv \to [0,1]$; rename it pdf. Since (Pv, pCF) is already part of the PFSS structure, $(pdf; Pv, pCF)$ is a plithogenic fuzzy set by definition. For the converse, given $pdf : U \times Pv \to [0,1]$ and (Pv, pCF), define $E = A = \{e_0\}$ and set $\mu(e_0, u \mid a) := pdf(u,a)$. This yields a PFSS, and the two constructions are mutually inverse up to the trivial relabeling of e_0.

Theorem 2 (PFSS generalizes the Fuzzy Soft Set) *Let* (Γ_A, A) *be a PFSS on* $(U, E; Pv, pCF)$. *If* $Pv = \{a^*\}$ *is a singleton and* $pCF(a^*, a^*) = 0$, *then*

$$\Gamma_A^{\mathrm{FSS}} : A \to [0,1]^U, \qquad \Gamma_A^{\mathrm{FSS}}(e)(u) := \mu(e,u \mid a^*),$$

defines a fuzzy soft set over U with respect to A. Conversely, any fuzzy soft set $(\Gamma_A^{\mathrm{FSS}}, A)$ *is obtained as a PFSS by taking* $Pv = \{a^*\}$, $pCF \equiv 0$, *and* $\mu(e,u \mid a^*) := \Gamma_A^{\mathrm{FSS}}(e)(u)$.

Proof. When $|Pv| = 1$, the attribute dimension carries no variability: for each $e \in A$ and $u \in U$ the single value $\mu(e,u \mid a^*)$ is a standard fuzzy membership degree. Thus the assignment $e \mapsto \big(u \mapsto \mu(e,u \mid a^*)\big)$ is a map $A \to [0,1]^U$, i.e., a fuzzy soft set. Conversely, given $\Gamma_A^{\mathrm{FSS}} : A \to [0,1]^U$, define $Pv = \{a^*\}$, set $pCF(a^*, a^*) := 0$, and put $\mu(e,u \mid a^*) := \Gamma_A^{\mathrm{FSS}}(e)(u)$. This is a PFSS whose projection along the (trivial) attribute dimension returns the original fuzzy soft set.

Definition 8 (Upside-Down Logic with contradiction reset on PFSS) Fix an *anchor* attribute $b \in Pv$ and a threshold $\tau \in [0,1]$. Define the activation set

$$A_\tau(b) := \{a \in Pv : pCF(a,b) \geq \tau\}.$$

Given a PFSS $(\mu; Pv, pCF)$, its *upside-down transform with contradiction reset* is the PFSS

$$(\mu^{U_{b,\tau}};\ Pv,\ pCF^{U_{b,\tau}})$$

defined, for every $e \in A$, $u \in U$, $a \in Pv$, by

$$\mu^{U_{b,\tau}}(e,u \mid a) := \begin{cases} 1 - \mu(e,u \mid a), & a \in A_\tau(b), \\ \mu(e,u \mid a), & a \notin A_\tau(b), \end{cases}$$

and the updated contradiction map

$$pCF^{U_{b,\tau}}(u,v) := \begin{cases} 0, & \{u,v\} = \{a,b\} \text{ for some } a \in A_\tau(b), \\ pCF(u,v), & \text{otherwise.} \end{cases}$$

Example 8 (Course Recommendation: Flip under Anchor "Work–Life Balance") Let the universe of items (courses) be

$$U = \{\text{Algorithms } (Al),\ \text{HCI } (Hc),\ \text{Data Science } (Ds)\}.$$

Let the parameter set be

$$E = \{e_1 = \text{"difficulty fit"},\ e_2 = \text{"schedule flexibility"}\}$$

and take $A = E$. Fix a single attribute $v =$ "learner type" with value set

$$Pv = \{\text{Analytic } (An),\ \text{HandsOn } (Ho),\ \text{WorkLife } (Wl)\}.$$

The contradiction map $pCF : Pv \times Pv \to [0,1]$ (write c) is symmetric with $c(a,a) = 0$; choose

$c(a,b)$	An	Ho	Wl
An	0	0.60	0.35
Ho	0.60	0	0.80
Wl	0.35	0.80	0

so that Ho and Wl are highly contradictory.

A Plithogenic Fuzzy Soft Set (PFSS) is specified by $\mu : A \times U \times Pv \to [0,1]$. For e_1 (difficulty fit) and e_2 (schedule flexibility) we set:

Parameter e_1 :

$\mu(e_1,u \mid a)$	An	Ho	Wl
Al	0.95	0.70	0.45
Hc	0.40	0.90	0.80
Ds	0.85	0.75	0.60

Parameter e_2 :

$\mu(e_2,u \mid a)$	An	Ho	Wl
Al	0.50	0.40	0.30
Hc	0.70	0.85	0.95
Ds	0.60	0.55	0.85

Upside-down with reset. Choose anchor $b = Wl$ and threshold $\tau = 0.75$. Then the activation set is

$$A_\tau(Wl) = \{a \in Pv : c(a, Wl) \geq 0.75\} = \{Ho\},$$

because $c(Ho, Wl) = 0.80$ while $c(An, Wl) = 0.35$. By Definition 8, we flip (complement) only the Ho-column for each parameter and reset pCF on the unordered pair $\{Ho, Wl\}$:

Parameter e_1 after flip :

$\mu^{U_{Wl,0.75}}(e_1,u \mid a)$	An	Ho	Wl
Al	0.95	$1-0.70=0.30$	0.45
Hc	0.40	$1-0.90=0.10$	0.80
Ds	0.85	$1-0.75=0.25$	0.60

Parameter e_2 after flip :

$\mu^{U_{Wl,0.75}}(e_2,u \mid a)$	An	Ho	Wl
Al	0.50	$1-0.40=0.60$	0.30
Hc	0.70	$1-0.85=0.15$	0.95
Ds	0.60	$1-0.55=0.45$	0.85

The updated contradiction becomes

$$pCF^{U_{Wl,0.75}}(Ho, Wl) = pCF^{U_{Wl,0.75}}(Wl, Ho) = 0,$$

with all other entries identical to pCF. Hence the highly conflicting pair (Ho, Wl) will not trigger another flip at the same threshold, ensuring contextual stability after transformation.

Example 9 (Laptop Choice: Flip under Anchor "Portability") Let the universe be three laptops

$$U = \{\text{L1, L2, L3}\}.$$

Take parameters $E = \{e_1 =$ "performance", $e_2 =$ "battery/mobility"$\}$ with $A = E$. Let the attribute be $v =$ "user profile" with

$$Pv = \{\text{Gaming } (Ga),\ \text{Portability } (Po),\ \text{Budget } (Bu)\}.$$

Use the symmetric contradiction map (zeros on the diagonal)

$c(a,b)$	Ga	Po	Bu
Ga	0	0.78	0.62
Po	0.78	0	0.35
Bu	0.62	0.35	0

so that Ga conflicts strongly with Po.

Define the PFSS memberships as follows. For e_1 (performance):

$\mu(e_1,u\mid a)$	Ga	Po	Bu
$L1$	0.95	0.55	0.70
$L2$	0.80	0.70	0.65
$L3$	0.60	0.85	0.60

For e_2 (battery/mobility):

$\mu(e_2,u\mid a)$	Ga	Po	Bu
$L1$	0.45	0.90	0.70
$L2$	0.65	0.95	0.80
$L3$	0.75	0.98	0.85

Upside-down with reset. Choose anchor $b = Po$ and threshold $\tau = 0.70$. Then $A_\tau(Po) = \{Ga\}$, since $c(Ga,Po) = 0.78 \geq 0.70$ but $c(Bu,Po) = 0.35 < 0.70$. We therefore complement only the Ga-column and reset the pair $\{Ga,Po\}$:

Parameter e_1 after flip :

$\mu^{U_{Po,0.70}}(e_1,u\mid a)$	Ga	Po	Bu
$L1$	$1-0.95=0.05$	0.55	0.70
$L2$	$1-0.80=0.20$	0.70	0.65
$L3$	$1-0.60=0.40$	0.85	0.60

Parameter e_2 after flip :

$\mu^{U_{Po,0.70}}(e_2,u\mid a)$	Ga	Po	Bu
$L1$	$1-0.45=0.55$	0.90	0.70
$L2$	$1-0.65=0.35$	0.95	0.80
$L3$	$1-0.75=0.25$	0.98	0.85

The contradiction update sets

$$pCF^{U_{Po,0.70}}(Ga,Po) = pCF^{U_{Po,0.70}}(Po,Ga) = 0,$$

while leaving all other entries unchanged. Thus, with the anchor fixed at *Portability*, the severe tension with *Gaming* is neutralized after one flip, and the same threshold will not reactivate the pair.

Lemma 1 (Well-definedness and basic properties) *Let* $(\mu;Pv,pCF)$ *be a PFSS and* $(\mu^{U_{b,\tau}};Pv,pCF^{U_{b,\tau}})$ *be given by Definition 8. Then:*

1) *For all* (e,u,a), $\mu^{U_{b,\tau}}(e,u\mid a) \in [0,1]$.
2) $pCF^{U_{b,\tau}} : Pv \times Pv \to [0,1]$ *satisfies* $pCF^{U_{b,\tau}}(a,a) = 0$ *and* $pCF^{U_{b,\tau}}(a,b) = pCF^{U_{b,\tau}}(b,a)$ *for all* $a,b \in Pv$.
3) *If* $\tau > 0$, *then for every* $a \in A_\tau(b)$ *one has* $pCF^{U_{b,\tau}}(a,b) = 0 < \tau$, *hence* $A_\tau(b)$ *becomes empty for the transformed contradiction map, i.e.* $A_\tau^U(b) = \varnothing$ *w.r.t.* $pCF^{U_{b,\tau}}$.

Proof. (1) Since $\mu(\cdot) \in [0,1]$, also $1-\mu(\cdot) \in [0,1]$; the case distinction preserves the range. (2) By construction $pCF^{U_{b,\tau}}$ is either 0 or pCF, hence remains in $[0,1]$. Reflexivity holds because if $u = v$ then the first case cannot happen (the unordered pair $\{u,u\}$ cannot equal $\{a,b\}$ with $a \neq b$), so $pCF^{U_{b,\tau}}(u,u) = pCF(u,u) = 0$. Symmetry is immediate from the definition by cases and the symmetry of pCF. (3) Directly from the definition, $pCF^{U_{b,\tau}}(a,b) = 0$ for all $a \in A_\tau(b)$, making them ineligible for re-activation under the same $\tau > 0$.

Theorem 3 (PFSS upside-down with reset generalizes the PFS version) *Let* $(\mu;Pv,pCF)$ *be a PFSS and fix any* $e_0 \in A$. *Define* $pdf : U \times Pv \to [0,1]$ *by* $pdf(u,a) := \mu(e_0,u\mid a)$. *Let* $b \in Pv$, $\tau \in [0,1]$, *and let* $(\mu^{U_{b,\tau}};pCF^{U_{b,\tau}})$ *and* $(pdf^{U_{b,\tau}};pCF^{U_{b,\tau}})$ *be given by Definitions 8 and 5, respectively. Then, for all* $u \in U$ *and* $a \in Pv$,

$$pdf^{U_{b,\tau}}(u,a) \;=\; \mu^{U_{b,\tau}}(e_0,u\mid a),$$

and both use the same updated contradiction map $pCF^{U_{b,\tau}}$. *In particular, when* $|A| = 1$, *the PFSS transform* reduces *to the PFS transform.*

Proof. Fix $u \in U$ and $a \in Pv$. There are two cases.

Case 1: $a \in A_\tau(b)$. By Definition 5,

$$pdf^{U_{b,\tau}}(u,a) = 1 - pdf(u,a) = 1 - \mu(e_0,u\mid a).$$

By Definition 8,

$$\mu^{U_{b,\tau}}(e_0,u\mid a) = 1 - \mu(e_0,u\mid a).$$

Hence $pdf^{U_{b,\tau}}(u,a) = \mu^{U_{b,\tau}}(e_0,u\mid a)$.

Case 2: $a \notin A_\tau(b)$. Then

$$pdf^{U_{b,\tau}}(u,a) = pdf(u,a) = \mu(e_0,u\mid a) = \mu^{U_{b,\tau}}(e_0,u\mid a),$$

again by the case distinction in both definitions. In both cases the equality holds. The update rule for $pCF^{U_{b,\tau}}$ is identical in Definitions 8 and 5, hence they coincide. If $|A| = 1$ with $A = \{e_0\}$, the identification $pdf(\cdot,\cdot) = \mu(e_0,\cdot\mid\cdot)$ shows the PFSS transform is exactly the PFS transform.

Remark (Effect of reset and (non)involutivity) If $\tau > 0$, Lemma 1 3) implies that once pairs (a,b) with $a \in A_\tau(b)$ are flipped, their contradiction is reset to 0; under the same (b,τ) a second application produces no further flips. Therefore the transform is generally *not* an involution when the reset is applied.

3.2. Plithogenic Fuzzy Soft Expert Set

A soft expert set models parameter–expert–opinion triples assigning subsets of universe, enabling structured decision-making across diverse perspectives [43, 45]. A fuzzy soft expert set assigns each parameter–expert–opinion triple a fuzzy subset, quantifying membership degrees for decision elements [46]. A plithogenic fuzzy soft expert set extends fuzzy soft expert sets with contradiction functions managing attribute conflicts in decision contexts.

Definition 9 (Fuzzy Soft Expert Set) [46, 77] Let U be a nonempty universe, E a set of parameters, X a set of experts, and O a set of opinions (e.g., $O = \{1,0\}$). Put $Z := E \times X \times O$ and let $A \subseteq Z$. A *fuzzy soft expert set* over U is a pair (F,A) where

$$F : A \longrightarrow \{\,\mu : U \to [0,1]\,\}, \qquad \alpha \longmapsto \mu_\alpha(\cdot),$$

so that each $\alpha = (e,x,o) \in A$ is associated with a fuzzy subset $\mu_\alpha : U \to [0,1]$. Equivalently, it is specified by a membership map

$$\mu : A \times U \longrightarrow [0,1], \qquad \mu(\alpha,u) := \mu_\alpha(u),$$

interpreted as the degree to which $u \in U$ is accepted under the parameter–expert–opinion triple $\alpha \in A$.

Definition 10 (Plithogenic Fuzzy Soft Expert Set (PFSES)) Let U be a nonempty universe, E a set of parameters, X a set of experts, and O a set of opinions. Put $Z := E \times X \times O$ and let $A \subseteq Z$

be the active index set of parameter–expert–opinion triples. Fix a plithogenic structure (Pv, pCF) as above.

A *Plithogenic Fuzzy Soft Expert Set* over U is a pair (F,A) where

$$F : A \longrightarrow [0,1]^{U\times Pv}, \qquad \alpha \longmapsto \mu_\alpha(\cdot \mid \cdot),$$

equivalently specified by a membership map

$$\mu : A \times U \times Pv \longrightarrow [0,1], \qquad \mu(\alpha,u \mid a) := \mu_\alpha(u \mid a).$$

Here $\mu(\alpha,u \mid a)$ is the degree to which $u \in U$ is accepted under the parameter–expert–opinion triple $\alpha = (e,x,o) \in A$, *given* the attribute value $a \in Pv$. The pair (Pv, pCF) is common to all α and quantifies potential agreement/contradiction among attribute values.

Example 10 (University Admissions with Expert–Opinion Triples and Attribute Contradictions) Let the universe be the set of applicants

$$U = \{\text{A1, A2, A3}\}.$$

Let the parameter set be

$$E = \{e_1 = \text{“academic strength”},\ e_2 = \text{“extracurricular impact”}\},$$
$$A \subseteq E \times X \times O.$$

Take experts $X = \{\text{Prof, Dean}\}$ and opinions $O = \{1,0\}$, where 1 denotes *approve* and 0 denotes *reject*. Fix the attribute $v =$ “evaluation criterion” with value set

$$Pv = \{G = \text{GPA},\ R = \text{Research},\ L = \text{Leadership}\}.$$

The degree of contradiction $pCF : Pv \times Pv \to [0,1]$ (abbrev. c) is symmetric with $c(a,a) = 0$:

$c(a,b)$	G	R	L
G	0	0.30	0.50
R	0.30	0	0.70
L	0.50	0.70	0

(higher values encode stronger tension between criteria; e.g., R vs. L is highly contradictory).

We specify a Plithogenic Fuzzy Soft Expert Set (PFSES) by giving the membership map

$$\mu : A \times U \times Pv \to [0,1], \qquad \mu(\alpha,u \mid a) =$$

degree that u meets parameter of α given a.

Choose the active index set

$$A = \{(e_1,\text{Prof},1),\ (e_2,\text{Prof},1),\ (e_1,\text{Dean},0),\ (e_2,\text{Dean},0)\}.$$

For each $\alpha = (e,x,o) \in A$, we list $\mu(\alpha,u \mid a)$ as a matrix whose rows are $u \in U$ and columns are $a \in Pv$.

Parameter e_1 (academic strength), expert Prof, opinion 1:

$\mu((e_1,\text{Prof},1),u \mid a)$	G	R	L
A1	0.92	0.80	0.40
A2	0.78	0.60	0.35
A3	0.70	0.55	0.30

Parameter e_2 (extracurricular impact), expert Prof, opinion 1:

$\mu((e_2,\text{Prof},1),u \mid a)$	G	R	L
A1	0.45	0.50	0.88
A2	0.40	0.55	0.80
A3	0.38	0.48	0.75

Parameter e_1 (academic strength), expert Dean, opinion 0:

$\mu((e_1,\text{Dean},0),u \mid a)$	G	R	L
A1	0.80	0.65	0.35
A2	0.68	0.50	0.30
A3	0.62	0.45	0.25

Parameter e_2 (extracurricular impact), expert Dean, opinion 0:

$\mu((e_2,\text{Dean},0),u \mid a)$	G	R	L
A1	0.42	0.48	0.82
A2	0.38	0.50	0.78
A3	0.35	0.46	0.72

This PFSES captures (i) parameterized judgments (academic vs. extracurricular), (ii) expert and opinion variability, and (iii) attribute-level contradictions $c(a,b)$ that formalize tradeoffs among GPA/Research/Leadership.

Example 11 (Medical Treatment Selection with Expert Endorse/Oppose and Evidence Attributes) Let the universe be candidate treatments

$$U = \{\text{T1, T2, T3}\}.$$

Parameters are

$$E = \{e_1 = \text{“effectiveness”},\ e_2 = \text{“tolerability”}\},$$

$$A \subseteq E \times X \times O.$$

Experts $X = \{\text{Oncologist, Pharmacologist}\}$ and opinions $O = \{1,0\}$ denote *endorse* and *oppose*. Take the attribute $v =$ “evidence source” with values

$$Pv = \{R = \text{RCT},\ W = \text{Real-World},\ C = \text{Consensus}\}.$$

Define a symmetric contradiction map pCF (zeros on the diagonal):

$c(a,b)$	R	W	C
R	0	0.40	0.25
W	0.40	0	0.55
C	0.25	0.55	0

reflecting moderate tension between Real-World and Consensus reports.

Choose the active index set

$$A = \{(e_1,\text{Oncologist},1),\ (e_2,\text{Oncologist},1),$$
$$(e_1,\text{Pharmacologist},0),\ (e_2,\text{Pharmacologist},0)\}.$$

List the PFSES memberships $\mu(\alpha,u \mid a)$.

We present in Table 6 the PFSES membership values $\mu(\alpha,u \mid a)$ for treatments $u \in \{\text{T1,T2,T3}\}$ under evidence attributes $a \in \{R,W,C\}$ (RCT, Real-World, Consensus).

This PFSES records treatment assessments conditioned by (i) parameter semantics (effectiveness vs. tolerability), (ii) expert identity and stance (endorse vs. oppose), and (iii) attribute-dependent evidence sources with quantified contradictions $c(a,b)$ that encode differences among RCT, real-world data, and consensus.

Theorem 4 (PFSES generalizes the Plithogenic Fuzzy Set) *Let (F,A) be a PFSES on $(U;Pv,pCF)$ and fix any $\alpha_0 \in A$. Then*

$$pdf : U \times Pv \longrightarrow [0,1], \qquad pdf(u,a) := \mu(\alpha_0,u \mid a),$$

together with (Pv,pCF), defines a plithogenic fuzzy set on U. Conversely, any plithogenic fuzzy set $(pdf;Pv,pCF)$ on U is obtained as a PFSES by taking $E = \{e_0\}$, $X = \{x_0\}$, $O = \{o_0\}$, $A = \{\alpha_0\}$ with $\alpha_0 = (e_0,x_0,o_0)$, and

$$\mu(\alpha_0,u \mid a) := pdf(u,a).$$

Table 6
PFSES membership values $\mu(\alpha, u \mid a)$ for treatments $u \in \{T1, T2, T3\}$ under evidence attributes $a \in \{R, W, C\}$ (RCT, Real-World, Consensus)

Setting	Item	R	W	C
e_1 **(effectiveness), Oncologist,** $o = 1$				
	T1	0.88	0.70	0.75
	T2	0.76	0.65	0.68
	T3	0.60	0.55	0.58
e_2 **(tolerability), Oncologist,** $o = 1$				
	T1	0.70	0.75	0.72
	T2	0.82	0.85	0.80
	T3	0.88	0.90	0.86
e_1 **(effectiveness), Pharmacologist,** $o = 0$				
	T1	0.80	0.68	0.70
	T2	0.70	0.62	0.65
	T3	0.58	0.52	0.55
e_2 **(tolerability), Pharmacologist,** $o = 0$				
	T1	0.68	0.72	0.70
	T2	0.80	0.84	0.78
	T3	0.86	0.88	0.84

Proof. With α_0 fixed, the PFSES provides exactly one membership surface $\mu(\alpha_0, \cdot \mid \cdot) : U \times Pv \to [0,1]$; rename it pdf. The plithogenic structure (Pv, pCF) is unchanged, hence $(pdf; Pv, pCF)$ is a PFS by definition. Conversely, given $pdf : U \times Pv \to [0,1]$ and (Pv, pCF), the singleton construction produces a PFSES whose unique component reproduces pdf verbatim. Both transformations are immediate by definition.

Theorem 5 (PFSES generalizes the Plithogenic Fuzzy Soft Set) *Let (F,A) be a PFSES and fix an expert $x^\star \in X$ and an opinion $o^\star \in O$. Let $A_s \subseteq E$ and set*

$$A' := \{ (e, x^\star, o^\star) : e \in A_s \} \subseteq A.$$

Define $\Gamma_{A_s} : A_s \to [0,1]^{U \times Pv}$ by

$$\Gamma_{A_s}(e)(u,a) := \mu\big((e, x^\star, o^\star), u \mid a\big).$$

Then (Γ_{A_s}, A_s) is a plithogenic fuzzy soft set over U with the same (Pv, pCF). Conversely, any plithogenic fuzzy soft set (Γ_{A_s}, A_s) is realized as a PFSES by taking $X = \{x^\star\}$, $O = \{o^\star\}$, $A = \{(e, x^\star, o^\star) : e \in A_s\}$ and

$$\mu\big((e, x^\star, o^\star), u \mid a\big) := \Gamma_{A_s}(e)(u,a).$$

Proof. By construction, for each $e \in A_s$ the map $(u,a) \mapsto \mu((e, x^\star, o^\star), u \mid a)$ is a membership surface on $U \times Pv$, hence an element of $[0,1]^{U \times Pv}$. Thus Γ_{A_s} has the required codomain, and (Γ_{A_s}, A_s) is a PFSS with the inherited (Pv, pCF). The converse is immediate by embedding the soft index e into the expert–opinion singleton layer, so the PFSS becomes a PFSES with trivial expert and opinion dimensions.

Theorem 6 (PFSES generalizes the Fuzzy Soft Expert Set) *Let (F,A) be a PFSES. If $Pv = \{a^*\}$ is a singleton and $pCF(a^*, a^*) = 0$, then*

$$G : A \to [0,1]^U, \qquad G(\alpha)(u) := \mu(\alpha, u \mid a^*),$$

defines a fuzzy soft expert set (G,A) over U. Conversely, any fuzzy soft expert set (G,A) is obtained as a PFSES by choosing $Pv = \{a^*\}$, $pCF \equiv 0$, and setting $\mu(\alpha, u \mid a^*) := G(\alpha)(u)$.

Proof. When $|Pv| = 1$, the attribute dimension carries no variability: $\mu(\alpha, \cdot \mid a^*)$ is a usual fuzzy set on U for each $\alpha \in A$. Thus $G(\alpha) := \mu(\alpha, \cdot \mid a^*)$ defines $G : A \to [0,1]^U$, i.e., an FSES. Conversely, embedding an FSES into PFSES with a singleton attribute set reproduces G verbatim.

Definition 11 (Upside-Down Logic with contradiction reset on PFSES) Fix an anchor attribute $b \in Pv$ and a threshold $\tau \in [0,1]$. Define the activation set

$$A_\tau(b) := \{ a \in Pv : c(a,b) = pCF(a,b) \geq \tau \}.$$

The *upside-down transform with contradiction reset* of the PFSES is the pair

$$\left(\mu^{U_{b,\tau}},\ pCF^{U_{b,\tau}}\right),$$

where, for every $\alpha \in A$, $u \in U$, $a \in Pv$,

$$\mu^{U_{b,\tau}}(\alpha, u \mid a) := \begin{cases} 1 - \mu(\alpha, u \mid a), & a \in A_\tau(b), \\ \mu(\alpha, u \mid a), & a \notin A_\tau(b), \end{cases}$$

and the updated contradiction map $pCF^{U_{b,\tau}} : Pv \times Pv \to [0,1]$ is

$$pCF^{U_{b,\tau}}(u,v) := \begin{cases} 0, & \{u,v\} = \{a,b\} \text{ for some } a \in A_\tau(b), \\ pCF(u,v), & \text{otherwise.} \end{cases}$$

Example 12 (University Admissions PFSES: Flip under Anchor "Leadership") **Setup.** Let $U = \{A1, A2, A3\}$, parameters $E = \{e_1 = \text{"academic strength"}, e_2 = \text{"extracurricular impact"}\}$, experts $X = \{\text{Prof}, \text{Dean}\}$, opinions $O = \{1, 0\}$ (approve/reject), and attribute values $Pv = \{G = \text{GPA},\ R = \text{Research},\ L = \text{Leadership}\}$. The contradiction map pCF (write c) is symmetric with $c(a,a) = 0$:

$c(a,b)$	G	R	L
G	0	0.30	0.50
R	0.30	0	0.70
L	0.50	0.70	0

Active indices

$$A = \{(e_1, \text{Prof}, 1), (e_2, \text{Prof}, 1), (e_1, \text{Dean}, 0), (e_2, \text{Dean}, 0)\}$$

. Memberships $\mu(\alpha, u \mid a)$ (rows $u \in U$, columns $a \in Pv$):

(i) $(e_1, \text{Prof}, 1)$

	G	R	L
A1	0.92	0.80	0.40
A2	0.78	0.60	0.35
A3	0.70	0.55	0.30

(ii) $(e_2, \text{Prof}, 1)$

	G	R	L
A1	0.45	0.50	0.88
A2	0.40	0.55	0.80
A3	0.38	0.48	0.75

(iii) $(e_1, \text{Dean}, 0)$

	G	R	L
A1	0.80	0.65	0.35
A2	0.68	0.50	0.30
A3	0.62	0.45	0.25

(iv) $(e_2, \text{Dean}, 0)$

	G	R	L
A1	0.42	0.48	0.82
A2	0.38	0.50	0.78
A3	0.35	0.46	0.72

Upside-down with reset. Choose anchor $b = L$ and threshold $\tau = 0.65$. Then $A_\tau(L) = \{R\}$ because $c(R,L) = 0.70 \geq 0.65$ while $c(G,L) = 0.50 < 0.65$. By Definition 11, flip only the R-column (complements $1 - \cdot$) for every $\alpha \in A$:
(i) $(e_1, \text{Prof}, 1)$ *after transform*

	G	R	L
A1	0.92	$1-0.80=0.20$	0.40
A2	0.78	$1-0.60=0.40$	0.35
A3	0.70	$1-0.55=0.45$	0.30

(ii) $(e_2, \text{Prof}, 1)$

	G	R	L
A1	0.45	$1-0.50=0.50$	0.88
A2	0.40	$1-0.55=0.45$	0.80
A3	0.38	$1-0.48=0.52$	0.75

(iii) $(e_1, \text{Dean}, 0)$

	G	R	L
A1	0.80	$1-0.65=0.35$	0.35
A2	0.68	$1-0.50=0.50$	0.30
A3	0.62	$1-0.45=0.55$	0.25

(iv) $(e_2, \text{Dean}, 0)$

	G	R	L
A1	0.42	$1-0.48=0.52$	0.82
A2	0.38	$1-0.50=0.50$	0.78
A3	0.35	$1-0.46=0.54$	0.72

Contradiction reset: set $pCF^{U_{L,0.65}}(R,L) = pCF^{U_{L,0.65}}(L,R) = 0$; all other entries remain as in pCF:

$pCF^{U_{L,0.65}}(a,b)$	G	R	L
G	0	0.30	0.50
R	0.30	0	0
L	0.50	0	0

Hence the (R,L) pair cannot reactivate at the same threshold.

Example 13 (Medical PFSES: Flip under Anchor "Consensus" Evidence) **Setup.** Let $U = \{\text{T1}, \text{T2}, \text{T3}\}$, parameters

$$E = \{e_1 = \text{"effectiveness"},\ e_2 = \text{"tolerability"}\},$$

experts

$$X = \{\text{Oncologist},\ \text{Pharmacologist}\},$$

opinions $O = \{1, 0\}$, and attribute values

$$Pv = \{R = \text{RCT},\ W = \text{Real-World},\ C = \text{Consensus}\}.$$

Contradiction map:

$c(a,b)$	R	W	C
R	0	0.40	0.25
W	0.40	0	0.55
C	0.25	0.55	0

Active indices

$$A = \{(e_1, \text{Oncologist}, 1), (e_2, \text{Oncologist}, 1),$$
$$(e_1, \text{Pharmacologist}, 0), (e_2, \text{Pharmacologist}, 0)\}.$$

Memberships (rows u, columns a):
(i) $(e_1, \text{Oncologist}, 1)$

	R	W	C
T1	0.88	0.70	0.75
T2	0.76	0.65	0.68
T3	0.60	0.55	0.58

(ii) $(e_2, \text{Oncologist}, 1)$

	R	W	C
T1	0.70	0.75	0.72
T2	0.82	0.85	0.80
T3	0.88	0.90	0.86

(iii) $(e_1, \text{Pharmacologist}, 0)$

	R	W	C
T1	0.80	0.68	0.70
T2	0.70	0.62	0.65
T3	0.58	0.52	0.55

(iv) $(e_2, \text{Pharmacologist}, 0)$

	R	W	C
T1	0.68	0.72	0.70
T2	0.80	0.84	0.78
T3	0.86	0.88	0.84

Upside-down with reset. Choose anchor $b = C$ and threshold $\tau = 0.50$. Then $A_\tau(C) = \{W\}$ because $c(W,C) = 0.55 \geq 0.50$ while $c(R,C) = 0.25 < 0.50$. Flip (complement) the W-column for each α:
(i) $(e_1, \text{Oncologist}, 1)$ *after transform*

	R	W	C
T1	0.88	$1-0.70=0.30$	0.75
T2	0.76	$1-0.65=0.35$	0.68
T3	0.60	$1-0.55=0.45$	0.58

(ii) $(e_2, \text{Oncologist}, 1)$

	R	W	C
T1	0.70	$1-0.75=0.25$	0.72
T2	0.82	$1-0.85=0.15$	0.80
T3	0.88	$1-0.90=0.10$	0.86

(iii) $(e_1, \text{Pharmacologist}, 0)$

	R	W	C
T1	0.80	$1-0.68=0.32$	0.70
T2	0.70	$1-0.62=0.38$	0.65
T3	0.58	$1-0.52=0.48$	0.55

(iv) $(e_2, \text{Pharmacologist}, 0)$

	R	W	C
T1	0.68	$1-0.72=0.28$	0.70
T2	0.80	$1-0.84=0.16$	0.78
T3	0.86	$1-0.88=0.12$	0.84

Contradiction reset: set $pCF^{U_{C,0.50}}(W,C) = pCF^{U_{C,0.50}}(C,W) = 0$; all other entries unchanged:

$pCF^{U_{C,0.50}}(a,b)$	R	W	C
R	0	0.40	0.25
W	0.40	0	0
C	0.25	0	0

Thus the (W,C) pair will not re-trigger flips at the same threshold, ensuring stability.

Lemma 2 (Well-definedness and structural properties) *For any PFSES and any (b,τ), the transform in Definition 11 satisfies:*

1. $\mu^{U_{b,\tau}}(\alpha,u \mid a) \in [0,1]$ *for all* α,u,a.
2. $pCF^{U_{b,\tau}}(a,a) = 0$ *and* $pCF^{U_{b,\tau}}(a,b) = pCF^{U_{b,\tau}}(b,a)$ *for all* $a,b \in Pv$.
3. *If $\tau > 0$, then for every $a \in A_\tau(b)$ we have $pCF^{U_{b,\tau}}(a,b) = 0 < \tau$, hence $A^U_\tau(b) = \varnothing$ with respect to $pCF^{U_{b,\tau}}$ (no immediate re-activation).*

Proof. (1) Since $\mu(\cdot) \in [0,1]$, also $1-\mu(\cdot) \in [0,1]$; the case distinction preserves the range. (2) If $u = v$, the first case of $pCF^{U_{b,\tau}}$ cannot occur (the unordered pair is not $\{a,b\}$ with $a \neq b$), so $pCF^{U_{b,\tau}}(u,u) = pCF(u,u) = 0$. Symmetry is immediate by the definition via unordered pairs and symmetry of pCF. (3) By construction, whenever $a \in A_\tau(b)$, the post-transform pair (a,b) has contradiction 0, hence cannot meet the threshold $\tau > 0$ again.

Theorem 7 (PFSES upside-down with reset generalizes the PFSS version) *Let (F,A) be a PFSES on $(U;Pv,pCF)$ with membership μ. Fix an expert $x^\star \in X$ and an opinion $o^\star \in O$. Let $A_s \subseteq E$ and embed it as*

$$A' := \{(e,x^\star,o^\star) : e \in A_s\} \subseteq A.$$

Define the associated PFSS (Γ_{A_s},A_s) by

$$\Gamma_{A_s}(e)(u,a) := \mu\big((e,x^\star,o^\star),u \mid a\big), \qquad e \in A_s.$$

Then for every $e \in A_s$, $u \in U$, $a \in Pv$,

$$\mu_e^{U_{b,\tau}}(u \mid a) = \mu^{U_{b,\tau}}\big((e,x^\star,o^\star),u \mid a\big)$$

and the updated contradiction maps coincide. In particular, the PFSES transform reduces to the PFSS transform under the expert-opinion singleton restriction.

Proof. Fix $e \in A_s$, $u \in U$, $a \in Pv$. There are two cases.

Case $a \in A_\tau(b)$: By the PFSS rule,

$$\mu_e^{U_{b,\tau}}(u \mid a) = 1-\mu_e(u \mid a) = 1-\mu\big((e,x^\star,o^\star),u \mid a\big).$$

By Definition 11,

$$\mu^{U_{b,\tau}}\big((e,x^\star,o^\star),u \mid a\big) = 1-\mu\big((e,x^\star,o^\star),u \mid a\big).$$

Thus the two values are equal.

Case $a \notin A_\tau(b)$: By PFSS,

$$\mu_e^{U_{b,\tau}}(u \mid a) = \mu_e(u \mid a) = \mu\big((e,x^\star,o^\star),u \mid a\big),$$

which matches the PFSES value by Definition 11. The contradiction map update is identical in both settings by construction (same unordered-pair reset), hence it also coincides. Therefore the transforms agree componentwise for all (e,u,a).

3.3. Plithogenic Fuzzy T-Rough Set

A rough set approximates vague concepts using lower and upper bounds derived from indiscernibility relations, managing uncertainty without external parameters [78–80]. A fuzzy T-rough set approximates fuzzy subsets using T-similarities, providing lower and upper bounds of uncertain knowledge representation [81]. A Plithogenic Fuzzy T-Rough Set extends fuzzy T-rough sets with attribute contradictions, aggregating multi-valued approximations under contextual conflicts.

Definition 12 (Triangular norm and residuum [81]) Let $T : [0,1]^2 \to [0,1]$ be a (lower semi-continuous) triangular norm. Its residuated implication (residuum) $\Rightarrow_T : [0,1]^2 \to [0,1]$ is

$$a \Rightarrow_T c := \sup\{h \in [0,1] : T(a,h) \le c\}.$$

Definition 13 (Fuzzy T-similarity [81]) For a nonempty set X, a fuzzy relation $R : X \times X \to [0,1]$ is a *fuzzy T-similarity* if, for all $x,y,z \in X$,

$$R(x,x) = 1, \qquad R(x,y) = R(y,x), \qquad T\big(R(x,z),R(z,y)\big) \le R(x,y).$$

Definition 14 (Fuzzy T-rough set; upper/lower approximations [81]) Let (X,R) be a fuzzy T-approximation space (i.e., R is a fuzzy T-similarity on X). For a fuzzy set $k \in [0,1]^X$ (i.e., $k : X \to [0,1]$), the *upper* and *lower* approximation operators $U_R, L_R : [0,1]^X \to [0,1]^X$ are defined pointwise by

$$(U_Rk)(x) := \sup_{u \in X} T\big(R(u,x),k(u)\big),$$

$$(L_Rk)(x) := \sup_{u \in X} \big(R(u,x) \Rightarrow_T k(u)\big) \qquad (x \in X).$$

These operators satisfy, for all k, the basic properties

$$L_Rk \le k \le U_Rk, \qquad U_R \circ U_R = U_R, \qquad L_R \circ L_R = L_R,$$

and the duality relations (via $\Rightarrow_T$)

$$L_Rk = \sup_{a \in [0,1]} \big(U_R \mathbf{1}_{\{k \ge a\}} \Rightarrow_T a\big),$$

$$U_Rk = \sup_{a \in [0,1]} \big(L_R \mathbf{1}_{\{k \ge a\}} \Rightarrow_T a\big),$$

where $\mathbf{1}_{\{k \ge a\}}(u) = 1$ if $k(u) \ge a$ and 0 otherwise.

Example 14 (Concrete computation of U_Rk and L_Rk on a small space) **Setting.** Let $X = \{x_1,x_2,x_3\}$. We use the Gödel (minimum) T-norm $T(a,b) = \min(a,b)$ and its residuum

$$a \Rightarrow_T c = \begin{cases} 1, & a \le c, \\ c, & a > c. \end{cases}$$

Consider the fuzzy T-similarity $R : X \times X \to [0,1]$ (symmetric, $R(x,x) = 1$), given by

$$R = \begin{array}{c|ccc} & x_1 & x_2 & x_3 \\ \hline x_1 & 1 & 0.8 & 0.7 \\ x_2 & 0.8 & 1 & 0.7 \\ x_3 & 0.7 & 0.7 & 1 \end{array}$$

which satisfies $T(R(x,z),R(z,y)) = \min(R(x,z),R(z,y)) \le R(x,y)$ for all x,y,z. Let the fuzzy set $k \in [0,1]^X$ be

$$k(x_1) = 0.20, \qquad k(x_2) = 0.70, \qquad k(x_3) = 0.50.$$

Upper approximation $U_R k$**.** For each $x \in X$,

$$(U_R k)(x) = \sup_{u\in X} \min\big(R(u,x), k(u)\big) = \max_{u\in X} \min\big(R(u,x), k(u)\big).$$

Compute pointwise:

$$\begin{aligned}(U_R k)(x_1) &= \max\{\min(1,0.20),\ \min(0.8,0.70),\ \min(0.7,0.50)\}\\ &= \max\{0.20,\ 0.70,\ 0.50\} = 0.70,\end{aligned}$$

$$\begin{aligned}(U_R k)(x_2) &= \max\{\min(0.8,0.20),\ \min(1,0.70),\ \min(0.7,0.50)\}\\ &= \max\{0.20,\ 0.70,\ 0.50\} = 0.70,\end{aligned}$$

$$\begin{aligned}(U_R k)(x_3) &= \max\{\min(0.7,0.20),\ \min(0.7,0.70),\ \min(1,0.50)\}\\ &= \max\{0.20,\ 0.70,\ 0.50\} = 0.70.\end{aligned}$$

Hence

$$U_R k = \big(0.70,\ 0.70,\ 0.70\big).$$

Lower approximation $L_R k$**.** For each $x \in X$,

$$(L_R k)(x) = \sup_{u\in X} \big(R(u,x) \Rightarrow_T k(u)\big).$$

Using the Gödel residuum $a \Rightarrow_T c \in \{1,c\}$:

$$\begin{aligned}(L_R k)(x_1) &= \max\{\,1 \Rightarrow_T 0.20,\ 0.8 \Rightarrow_T 0.70,\ 0.7 \Rightarrow_T 0.50\,\}\\ &= \max\{0.20,\ 0.70,\ 0.50\} = 0.70,\end{aligned}$$

$$\begin{aligned}(L_R k)(x_2) &= \max\{\,0.8 \Rightarrow_T 0.20,\ 1 \Rightarrow_T 0.70,\ 0.7 \Rightarrow_T 0.50\,\}\\ &= \max\{0.20,\ 0.70,\ 0.50\} = 0.70,\end{aligned}$$

$$\begin{aligned}(L_R k)(x_3) &= \max\{\,0.7 \Rightarrow_T 0.20,\ 0.7 \Rightarrow_T 0.70,\ 1 \Rightarrow_T 0.50\,\}\\ &= \max\{0.20,\ 0.70,\ 0.50\} = 0.70.\end{aligned}$$

Therefore

$$L_R k = \big(0.70,\ 0.70,\ 0.70\big).$$

With $T = \min$ and the above R and k, both the upper and lower fuzzy T-rough approximations at each point equal 0.70, i.e.,

$$U_R k = L_R k = (0.70, 0.70, 0.70).$$

This explicit calculation illustrates the definitions: the upper uses min-aggregation of similarity and membership followed by a pointwise supremum, while the lower uses the residuated implication followed by a pointwise supremum.

Plithogenic Fuzzy Rough Set is defined as follows.

Definition 15 (Plithogenic Fuzzy Rough Set (PFRS)) Let X be a nonempty (finite) universe. Fix a plithogenic structure (P_v, pCF), where P_v is a finite set of attribute values and

$$c(a,b) := pCF(a,b) \in [0,1] \qquad (a,b \in P_v)$$

satisfies $c(a,a) = 0$ and $c(a,b) = c(b,a)$. Let

$$pdf:\ X \times P_v \longrightarrow [0,1]$$

be a plithogenic fuzzy set on X, and write $k_a \in [0,1]^X$ for its a-slice:

$$k_a(x) := pdf(x,a) \qquad (x \in X,\ a \in P_v).$$

For each $a \in P_v$, let $R_a : X \times X \to [0,1]$ be a fuzzy relation (often taken as a fuzzy similarity). Define the classical fuzzy rough upper and lower approximations of k_a with respect to R_a by

$$(U_{R_a} k_a)(x) := \sup_{y\in X} \min\big(R_a(y,x),\, k_a(y)\big),$$

$$(L_{R_a} k_a)(x) := \inf_{y\in X} \max\big(1 - R_a(y,x),\, k_a(y)\big) \qquad (x \in X).$$

Fix an *anchor* (context) value $b \in P_v$. Using contradiction-aware weights

$$w_{a|b} := 1 - c(a,b) \in [0,1], \qquad W_b := \sum_{a\in P_v} w_{a|b},$$

assume $W_b > 0$ and define the *plithogenic* upper and lower approximations at anchor b by

$$U^{\mathrm{pl}}_{\mathbf{R}}(x \mid b) := \frac{1}{W_b} \sum_{a\in P_v} w_{a|b}\, (U_{R_a} k_a)(x),$$

$$L^{\mathrm{pl}}_{\mathbf{R}}(x \mid b) := \frac{1}{W_b} \sum_{a\in P_v} w_{a|b}\, (L_{R_a} k_a)(x) \qquad (x \in X),$$

where $\mathbf{R} := \{R_a\}_{a\in P_v}$.

The pair of fuzzy sets

$$\big(L^{\mathrm{pl}}_{\mathbf{R}}(\cdot \mid b),\ U^{\mathrm{pl}}_{\mathbf{R}}(\cdot \mid b)\big)$$

is called the PFRS of pdf at anchor b (with respect to $\mathbf{R}$ and pCF).

Plithogenic Fuzzy T-Rough Set is defined as follows.

Definition 16 (Attribute-indexed T-approximation space) Let $\mathbf{R} = \{R_a\}_{a\in Pv}$ be a family of fuzzy T-similarities on X. For each $a \in Pv$ and $k_a \in [0,1]^X$, define the *attribute-wise* upper/lower T-approximations:

$$(U_{R_a} k_a)(x) := \sup_{u\in X} T\big(R_a(u,x),\, k_a(u)\big),$$

$$(L_{R_a} k_a)(x) := \sup_{u\in X} \big(R_a(u,x) \Rightarrow_T k_a(u)\big).$$

Definition 17 (Plithogenic aggregation functional) For each *anchor* (context) $b \in Pv$, let

$$\mathscr{A}_b :\ [0,1]^{Pv} \longrightarrow [0,1]$$

be a monotone functional such that, for every vector $z = (z(a))_{a\in Pv} \in [0,1]^{Pv}$,

$$\min_a z(a) \le \mathscr{A}_b(z) \le \max_a z(a).$$

Examples. (i) Projection $\mathscr{A}_b(z) = z(b)$; (ii) Contradiction-weighted mean $\mathscr{A}_b(z) = \dfrac{\sum_a (1 - c(a,b))\, z(a)}{\sum_a (1 - c(a,b))}$; (iii) Plithogenic fold using a binary operator $\odot_{c(a,b)}$ (defined via a T/S interpolation), applied in any fixed order.

Definition 18 (Plithogenic Fuzzy T-Rough Set (PFT-rough set)) Let X be a universe, (Pv, pCF) a plithogenic structure, $pdf : X \times Pv \to [0,1]$ a plithogenic fuzzy set with components $k_a(\cdot) := pdf(\cdot, a)$, and $\mathbf{R} = \{R_a\}_{a\in Pv}$ a family of fuzzy T-similarities. Fix a family of aggregation functionals $\{\mathscr{A}_b\}_{b\in Pv}$.

For each anchor $b \in Pv$, the *plithogenic upper* and *plithogenic lower* T-approximations of pdf are the fuzzy sets on X defined by

$$U^{\mathrm{pl}}_{\mathbf{R}}(x \mid b) := \mathscr{A}_b\Big(\big\{\, (U_{R_a} k_a)(x)\, \big\}_{a\in Pv}\Big),$$

$$L^{\mathrm{pl}}_{\mathbf{R}}(x \mid b) := \mathscr{A}_b\Big(\big\{\, (L_{R_a} k_a)(x)\, \big\}_{a\in Pv}\Big).$$

A comparison between PFRS and PFT-RS is provided for reference in Table 7.

Several concrete examples are presented below.

Table 7
Concise comparison between Plithogenic Fuzzy Rough Sets (PFRS) and Plithogenic Fuzzy T–Rough Sets (PFT–RS)

Aspect	Plithogenic Fuzzy Rough Set (PFRS)	Plithogenic Fuzzy T-Rough Set (PFT-RS)
Base uncertainty object	Plithogenic fuzzy set $pdf : X \times P_v \to [0,1]$ with slices $k_a(\cdot) = pdf(\cdot, a)$.	Same: $pdf : X \times P_v \to [0,1]$ with slices k_a.
Relation layer	Family of fuzzy relations $\{R_a : X \times X \to [0,1]\}_{a \in P_v}$ (often similarities).	Family of fuzzy T-similarities $\{R_a\}_{a \in P_v}$ (satisfying T-transitivity).
Attribute-wise upper operator	$(U_{R_a} k_a)(x) = \sup_{y \in X} \min\big(R_a(y,x), k_a(y)\big)$.	$(U_{R_a} k_a)(x) = \sup_{y \in X} T\big(R_a(y,x), k_a(y)\big)$.
Attribute-wise lower operator	$(L_{R_a} k_a)(x) = \inf_{y \in X} \max\big(1 - R_a(y,x), k_a(y)\big)$.	$(L_{R_a} k_a)(x) = \sup_{y \in X} \big(R_a(y,x) \Rightarrow_T k_a(y)\big)$ (residuum $\Rightarrow_T$).
Plithogenic aggregation across $a \in P_v$	Typically a contradiction-weighted average at an anchor b: $\sum_a \frac{1 - c(a,b)}{\sum_{a'}(1 - c(a',b))}(\cdot)$.	General monotone functional $\mathscr{A}_b : [0,1]^{P_v} \to [0,1]$ (e.g., projection, contradiction–weighted mean, plithogenic fold).
Role of the T–norm	Implicitly uses the min/max pair (a specific choice of conjunction/disjunction).	Explicitly parameterized by a chosen T-norm (and its residuum), enabling different logical/aggregation behaviors.
Special-case reduction	If $\|P_v\| = 1$ (and $c = 0$), reduces to the classical fuzzy rough set w.r.t. R.	If $\|P_v\| = 1$, reduces to the classical fuzzy T–rough set w.r.t. R and T.

Example 15 (PFT-rough Set with Projection Aggregation (Reduction to the Anchor)) **Setting.** Let $X = \{x_1, x_2, x_3\}$ and $Pv = \{A, B, C\}$. Fix any triangular norm T and, for each $a \in Pv$, take the identity (Kronecker) fuzzy T–similarity

$$R_a(u,x) = \begin{cases} 1, & u = x, \\ 0, & u \neq x. \end{cases}$$

Let the plithogenic fuzzy set $pdf : X \times Pv \to [0,1]$ be specified by

$$k_A = \big(pdf(\cdot, A)\big) = (0.90,\, 0.60,\, 0.20),$$

$$k_B = (0.40,\, 0.70,\, 0.50),$$

$$k_C = (0.50,\, 0.50,\, 0.90).$$

By the identity similarity, for every $a \in Pv$ and $x \in X$,

$$(U_{R_a} k_a)(x) = (L_{R_a} k_a)(x) = k_a(x).$$

Choose the anchor $b = B$ and use the projector aggregation

$$\mathscr{A}_B(z) = z(B) \qquad \big(z \in [0,1]^{Pv}\big).$$

PFT-rough approximations. For each $x \in X$,

$$U^{\mathrm{pl}}_{\mathbf{R}}(x \mid B)$$

$$= \mathscr{A}_B\big(\{(U_{R_a} k_a)(x)\}_{a \in Pv}\big) = (U_{R_B} k_B)(x) = k_B(x),$$

and similarly $L^{\mathrm{pl}}_{\mathbf{R}}(x \mid B) = k_B(x)$. Therefore, numerically,

$$U^{\mathrm{pl}}_{\mathbf{R}}(\cdot \mid B) = L^{\mathrm{pl}}_{\mathbf{R}}(\cdot \mid B) = k_B = (0.40,\, 0.70,\, 0.50).$$

This example illustrates the reduction property: with identity similarities and projection aggregation, the PFT-rough approximations at anchor B coincide with the anchor slice k_B.

Example 16 (PFT-rough Set with Contradiction-Weighted Mean Aggregation) **Setting.** Use the same universe $X = \{x_1, x_2, x_3\}$, attributes $Pv = \{A, B, C\}$, and memberships

$$k_A = (0.90,\, 0.60,\, 0.20),$$

$$k_B = (0.40,\, 0.70,\, 0.50),$$

$$k_C = (0.50,\, 0.50,\, 0.90).$$

Let T be any triangular norm and, as before, take identity T-similarities R_a so that

$$(U_{R_a} k_a)(x) = (L_{R_a} k_a)(x) = k_a(x) \qquad (\forall a \in Pv,\ \forall x \in X).$$

Equip Pv with a plithogenic contradiction map pCF (symmetric, $pCF(a,a) = 0$) given here on pairs with B:

$$c(A,B) = 0.80, \qquad c(B,B) = 0, \qquad c(C,B) = 0.30.$$

Aggregation at anchor $b = B$. Use the contradiction-weighted mean

$$\mathscr{A}_B(z) = \frac{\sum_{a \in Pv} (1 - c(a,B))\, z(a)}{\sum_{a \in Pv} (1 - c(a,B))}.$$

The weights are $w_A = 1 - 0.80 = 0.20$, $w_B = 1 - 0 = 1.00$, $w_C = 1 - 0.30 = 0.70$ with total $W = 1.90$. Thus, for each $x \in X$,

$$U^{\mathrm{pl}}_{\mathbf{R}}(x \mid B) = \mathscr{A}_B\big(\{k_a(x)\}_{a \in Pv}\big) = \frac{0.20\, k_A(x) + 1.00\, k_B(x) + 0.70\, k_C(x)}{1.90},$$

and $L^{\mathrm{pl}}_{\mathbf{R}}(x \mid B)$ equals the same expression (since $U_{R_a} k_a = L_{R_a} k_a = k_a$).

Numerical values.

$$U^{\mathrm{pl}}_{\mathbf{R}}(x_1 \mid B) = \frac{0.20 \cdot 0.90 + 1.00 \cdot 0.40 + 0.70 \cdot 0.50}{1.90} = \frac{0.18 + 0.40 + 0.35}{1.90} = \frac{0.93}{1.90} \approx 0.4895,$$

$$U^{\mathrm{pl}}_{\mathbf{R}}(x_2 \mid B) = \frac{0.20 \cdot 0.60 + 1.00 \cdot 0.70 + 0.70 \cdot 0.50}{1.90} = \frac{0.12 + 0.70 + 0.35}{1.90} = \frac{1.17}{1.90} \approx 0.6158,$$

$$U^{\mathrm{pl}}_{\mathbf{R}}(x_3 \mid B) = \frac{0.20 \cdot 0.20 + 1.00 \cdot 0.50 + 0.70 \cdot 0.90}{1.90} = \frac{0.04 + 0.50 + 0.63}{1.90}$$

$$= \frac{1.17}{1.90} \approx 0.6158.$$

Hence

$$U^{\mathrm{pl}}_{\mathbf{R}}(\cdot \mid B) = L^{\mathrm{pl}}_{\mathbf{R}}(\cdot \mid B) \approx (0.4895,\ 0.6158,\ 0.6158).$$

This example shows how the plithogenic contradiction with the anchor ($c(\cdot, B)$) reweights attribute-wise T-rough evaluations to yield an aggregated approximation sensitive to inter-attribute disagreement.

Lemma 3 (Basic properties) *For every anchor $b \in Pv$ and every $x \in X$,*

$$L^{\mathrm{pl}}_{\mathbf{R}}(x \mid b) \;\le\; U^{\mathrm{pl}}_{\mathbf{R}}(x \mid b).$$

Moreover, if $k_a \le k'_a$ for all $a \in Pv$, then

$$U^{\mathrm{pl}}_{\mathbf{R}}(\cdot \mid b;k) \;\le\; U^{\mathrm{pl}}_{\mathbf{R}}(\cdot \mid b;k'), \qquad L^{\mathrm{pl}}_{\mathbf{R}}(\cdot \mid b;k) \;\le\; L^{\mathrm{pl}}_{\mathbf{R}}(\cdot \mid b;k').$$

Proof. For each a, $L_{R_a}k_a \le U_{R_a}k_a$ holds for fuzzy T-rough sets. Monotonicity of $\mathscr{A}_b$ yields the first inequality. The second statement follows componentwise from the monotonicity of U_{R_a}, L_{R_a} and of $\mathscr{A}_b$.

Theorem 8 (Reduction to the classical fuzzy T-rough set) *Assume $|Pv| = 1$, say $Pv = \{a^*\}$, and let $R_{a^*} = R$ be a fuzzy T-similarity on X. Then, for any choice of $\mathscr{A}_{a^*}$,*

$$U^{\mathrm{pl}}_{\mathbf{R}}(x \mid a^*) = (U_R k_{a^*})(x), \qquad L^{\mathrm{pl}}_{\mathbf{R}}(x \mid a^*) = (L_R k_{a^*})(x),$$

i.e. the PFT-rough set reduces to the usual fuzzy T-rough set of k_{a^*} w.r.t. R.

Proof. With $Pv = \{a^*\}$, the vectors given to $\mathscr{A}_{a^*}$ are singletons: $\{(U_R k_{a^*})(x)\}$ and $\{(L_R k_{a^*})(x)\}$. By the bounds of $\mathscr{A}_{a^*}$, $\mathscr{A}_{a^*}$ returns the unique entry.

Theorem 9 (Reduction to the plithogenic fuzzy set) *Assume that, for every $a \in Pv$, R_a is the Kronecker (identity) similarity*

$$R_a(u,x) = \begin{cases} 1, & u = x, \\ 0, & u \ne x. \end{cases}$$

Then $U_{R_a}k_a = L_{R_a}k_a = k_a$ for all a. If, in addition, the aggregation functional is the projector $\mathscr{A}_b(z) = z(b)$, then

$$U^{\mathrm{pl}}_{\mathbf{R}}(x \mid b) = L^{\mathrm{pl}}_{\mathbf{R}}(x \mid b) = k_b(x) = pdf(x,b),$$

so the PFT-rough set collapses to the plithogenic fuzzy membership at anchor b.

Proof. With the identity similarity, for each a,

$$(U_{R_a}k_a)(x) = \sup_u T(R_a(u,x), k_a(u)) = T(1, k_a(x)) = k_a(x),$$

and

$$(L_{R_a}k_a)(x) = \sup_u \big(R_a(u,x) \Rightarrow_T k_a(u)\big) = (1 \Rightarrow_T k_a(x)) = k_a(x),$$

where we used $T(1,z) = z$ and $1 \Rightarrow_T z = z$. Applying $\mathscr{A}_b(z) = z(b)$ yields $U^{\mathrm{pl}}_{\mathbf{R}}(x \mid b) = k_b(x)$ and similarly for L^{pl}.

Definition 19 (Upside-down with contradiction reset on PFT-rough sets) Let $(X, \mathbf{R}, T)$ and (Pv, pCF) be as above, and let $\{\mathscr{A}_b\}_{b \in Pv}$ be fixed. For $b \in Pv$ and $\tau \in [0,1]$, with $A_\tau(b)$, define the *flipped* attribute-wise memberships

$$k_a^{U_{b,\tau}}(x) \;:=\; \begin{cases} 1 - k_a(x), & a \in A_\tau(b), \\ k_a(x), & a \notin A_\tau(b), \end{cases} \qquad (x \in X,\ a \in Pv),$$

and the *reset* contradiction map $pCF^{U_{b,\tau}}$. The *upside-down plithogenic upper/lower T-approximations* are then the fuzzy sets on X

$$U^{\mathrm{pl},U}_{\mathbf{R}}(x \mid b) := \mathscr{A}_b\Big(\big\{ (U_{R_a}k_a^{U_{b,\tau}})(x) \big\}_{a \in Pv} \Big),$$
$$L^{\mathrm{pl},U}_{\mathbf{R}}(x \mid b) := \mathscr{A}_b\Big(\big\{ (L_{R_a}k_a^{U_{b,\tau}})(x) \big\}_{a \in Pv} \Big),$$

where, for each $a \in Pv$ and $h \in [0,1]^X$,

$$(U_{R_a}h)(x) = \sup_{u \in X} T\big(R_a(u,x), h(u)\big),$$

$$(L_{R_a}h)(x) = \sup_{u \in X} \big(R_a(u,x) \Rightarrow_T h(u)\big).$$

Lemma 4 (Well-definedness and basic order) *For every $b \in Pv$ and $\tau \in [0,1]$:*

1. *$k_a^{U_{b,\tau}} : X \to [0,1]$ for all $a \in Pv$, and $pCF^{U_{b,\tau}}$ is symmetric with $pCF^{U_{b,\tau}}(a,a) = 0$.*
2. *For all $x \in X$, $L^{\mathrm{pl},U}_{\mathbf{R}}(x \mid b) \le U^{\mathrm{pl},U}_{\mathbf{R}}(x \mid b)$.*
3. *If $\tau > 0$ and $a \in A_\tau(b)$, then $pCF^{U_{b,\tau}}(a,b) = 0 < \tau$, hence $A^U_\tau(b) = \varnothing$ with respect to $pCF^{U_{b,\tau}}$.*

Proof. (1) Since $k_a \in [0,1]^X$, also $1 - k_a \in [0,1]^X$; the casewise definition preserves $[0,1]$. The reset keeps values in $[0,1]$, preserves symmetry by cases on unordered pairs, and $pCF^{U_{b,\tau}}(a,a) = pCF(a,a) = 0$. (2) For each a, $L_{R_a}h \le U_{R_a}h$ holds for fuzzy T-rough sets. Applying $\mathscr{A}_b$, which is monotone coordinatewise, yields the inequality. (3) Immediate from the reset definition.

Lemma 5 (Monotonicity w.r.t. memberships) *If for all $a \in Pv$ we have $k_a \le k'_a$ pointwise on X, then for all $x \in X$,*

$$U^{\mathrm{pl},U}_{\mathbf{R}}(\cdot \mid b;k) \;\le\; U^{\mathrm{pl},U}_{\mathbf{R}}(\cdot \mid b;k'), \qquad L^{\mathrm{pl},U}_{\mathbf{R}}(\cdot \mid b;k) \;\le\; L^{\mathrm{pl},U}_{\mathbf{R}}(\cdot \mid b;k').$$

Proof. For each a, the map $h \mapsto U_{R_a}h$ and $h \mapsto L_{R_a}h$ are monotone; the flip $h \mapsto h^{U_{b,\tau}}$ is either identity (if $a \notin A_\tau(b)$) or $h \mapsto 1 - h$, which reverses order. However, on the branch $a \in A_\tau(b)$, both U_{R_a} and L_{R_a} are applied to $1 - h$, and we compare the resulting vectors coordinatewise before applying the same monotone $\mathscr{A}_b$. A direct check shows: if $h \le h'$, then $1 - h' \le 1 - h$, so $U_{R_a}(1 - h') \le U_{R_a}(1 - h)$ and $L_{R_a}(1 - h') \le L_{R_a}(1 - h)$. Therefore the whole vector given to $\mathscr{A}_b$ is monotone, hence so is the aggregate.

Theorem 10 (Generalization to the PFS upside-down transform) *Assume that, for every $a \in Pv$, R_a is the Kronecker (identity) similarity*

$$R_a(u,x) = \begin{cases} 1, & u = x, \\ 0, & u \ne x, \end{cases}$$

and that the aggregation is the projector $\mathscr{A}_b(z) = z(b)$. Let $pdf^{U_{b,\tau}}$ and $pCF^{U_{b,\tau}}$. Then, for all $x \in X$ and anchors $b \in Pv$,

$$U^{\mathrm{pl},U}_{\mathbf{R}}(x \mid b) \;=\; L^{\mathrm{pl},U}_{\mathbf{R}}(x \mid b) \;=\; pdf^{U_{b,\tau}}(x,b)$$

and the contradiction maps coincide (the same reset on $\{a,b\}$ pairs).

Proof. With identity similarity, for any $h \in [0,1]^X$ and any a,

$$(U_{R_a}h)(x) = \sup_u T(R_a(u,x), h(u)) = T(1, h(x)) = h(x),$$

$$(L_{R_a}h)(x) = \sup_u (R_a(u,x) \Rightarrow_T h(u)) = (1 \Rightarrow_T h(x)) = h(x),$$

using $T(1,z) = z$ and $1 \Rightarrow_T z = z$. Hence $(U_{R_a} k_a^{U_{b,\tau}})(x) = (L_{R_a} k_a^{U_{b,\tau}})(x) = k_a^{U_{b,\tau}}(x)$ for all a. Applying $\mathscr{A}_b(z) = z(b)$,

$$U_{\mathbf{R}}^{\mathrm{pl},U}(x \mid b) = k_b^{U_{b,\tau}}(x) = pdf^{U_{b,\tau}}(x,b),$$

$$L_{\mathbf{R}}^{\mathrm{pl},U}(x \mid b) = k_b^{U_{b,\tau}}(x) = pdf^{U_{b,\tau}}(x,b).$$

The contradiction reset is defined identically in both settings, so the maps coincide.

Example 17 (Concrete numeric verification under the identity case) Let $X = \{x_1,x_2\}$, $Pv = \{\texttt{pro},\texttt{con}\}$ with $c(\texttt{pro},\texttt{con}) = c(\texttt{con},\texttt{pro}) = 0.8$ and $c(\texttt{pro},\texttt{pro}) = c(\texttt{con},\texttt{con}) = 0$. Fix $b = \texttt{pro}$ and $\tau = 0.7$, so $A_\tau(b) = \{\texttt{con}\}$. Let $R_{\texttt{pro}}$ and $R_{\texttt{con}}$ be identity similarities, and choose

	x_1	x_2
$pdf(\cdot,\texttt{pro})$	0.9	0.3
$pdf(\cdot,\texttt{con})$	0.2	0.6

Then

$$k_{\texttt{con}}^{U_{b,\tau}} = 1 - k_{\texttt{con}} = (0.8,0.4),$$

$$k_{\texttt{pro}}^{U_{b,\tau}} = k_{\texttt{pro}} = (0.9,0.3).$$

Since $U_{R_a} h = L_{R_a} h = h$ for identity similarities and $\mathscr{A}_b$ is the projector,

$$U_{\mathbf{R}}^{\mathrm{pl},U}(x_i \mid \texttt{pro}) = L_{\mathbf{R}}^{\mathrm{pl},U}(x_i \mid \texttt{pro})$$

$$= k_{\texttt{pro}}^{U_{b,\tau}}(x_i) = pdf^{U_{b,\tau}}(x_i,\texttt{pro}) \in \{0.9,0.3\}.$$

Moreover $pCF^{U_{b,\tau}}(\texttt{pro},\texttt{con}) = 0$ by the reset rule, preventing re-activation at the same threshold $\tau = 0.7$. This matches Theorem 10 exactly.

4. Conclusion

In this paper, we revisited the Plithogenic Fuzzy Soft Expert Set, the Plithogenic Fuzzy Rough Set, and the classical Fuzzy Set, and we examined how Upside-Down Logic can be incorporated into these frameworks. Our unified viewpoint supports decision science by explicitly modeling multi-expert disagreement, attribute-level contradictions, and context-driven reversals, thereby enabling more interpretable aggregation and more robust uncertainty bounds in complex, dynamic uncertain systems.

In future work, we expect further studies that extend these concepts by employing Graphs [82], HyperGraphs [83, 84], HyperFuzzy Sets [85], and SuperHyperGraphs [86–89]. Since this paper emphasizes theoretical foundations, we also anticipate that subsequent research will develop computational experiments and quantitative analyses to validate and extend the proposed ideas. We further expect progress on algorithm design and on analyzing the computational complexity of the proposed frameworks. In addition, it is promising to combine the proposed methods with machine-learning mechanisms to support enhanced decision-making under higher levels of uncertainty and complexity. We further expect that applications to domains such as healthcare, environmental science, and artificial intelligence will be actively explored.

Acknowledgments

We would like to express our sincere gratitude to everyone who provided valuable insights, support, and encouragement throughout this research. We also extend our thanks to the readers for their interest and to the authors of the referenced works, whose scholarly contributions have greatly influenced this study. Lastly, we are deeply grateful to the publishers and reviewers who facilitated the dissemination of this work.

Ethical Statement

This study does not contain any studies with human or animal subjects performed by the author.

Conflicts of Interest

The author declares that he has no conflicts of interest to this work.

Data Availability

Data sharing is not applicable to this article as no new data were created or analyzed in this study.

Author Contribution Statement

Takaaki Fujita: Conceptualization, Methodology, Validation, Formal analysis, Investigation, Writing – original draft, Writing – review & editing, Project administration.

Disclaimer

This work presents theoretical ideas and frameworks that have not yet been empirically validated. Readers are encouraged to explore practical applications and further refine these concepts. Although care has been taken to ensure accuracy and appropriate citations, any errors or oversights are unintentional. The perspectives and interpretations expressed herein are solely those of the authors and do not necessarily reflect the viewpoints of their affiliated institutions.

References

[1] Zadeh, L. A. (1965). Fuzzy sets. *Information and Control*, *8*(3), 338–353. https://doi.org/10.1016/S0019-9958(65)90241-X

[2] Das, S., & Kar, S. (2013). Intuitionistic multi fuzzy soft set and its application in decision making. In *Pattern Recognition and Machine Intelligence: 5th International Conference*, 587–592. https://doi.org/10.1007/978-3-642-45062-4_82

[3] Wu, M., Fu, J., & Fan, J. (2026). A three-state three-way decision model integrating intuitionistic fuzzy sets and improved D-S evidence theory for complex decision-making. *Applied Soft Computing*, *186*, 114196. https://doi.org/10.1016/j.asoc.2025.114196

[4] Atanassov, K. T. (2020). Circular intuitionistic fuzzy sets. *Journal of Intelligent & Fuzzy Systems*, *39*(5), 5981–5986. https://doi.org/10.3233/JIFS-189072

[5] Al Tahan, M., Al-Kaseasbeh, S., & Davvaz, B. (2024). Neutrosophic quadruple H_v-modules and their fundamental module. *Neutrosophic Sets and Systems*, *72*, 304–325.

[6] Gul, R., & Sarfraz, M. (2025). Enhancing artificial intelligence models with interval-valued picture fuzzy sets and

Sugeno-Weber triangular norms. *Spectrum of Engineering and Management Sciences*, *3*(1), 126–146. https://doi.org/10.31181/sems31202540g

[7] Nwokoro, C. O., & Ejegwa, P. A. (2025). A decade of picture fuzzy sets in multi-criteria decision-making: A comprehensive review of trends, gaps, and future directions. *Knowledge and Decision Systems with Applications*, *1*, 145–164. https://doi.org/10.59543/kadsa.v1i.14051

[8] Bihari, R., Jeevaraj, S., & Pamucar, D. (2025). A complete ranking principle for trapezoidal hesitant fuzzy sets and its application in solving trapezoidal hesitant fuzzy linear programming problems. *Complex & Intelligent Systems*, *11*(12), 486. https://doi.org/10.1007/s40747-025-02113-2

[9] Gao, F. (2025). An integrated multi criteria decision making method using dual hesitant fuzzy sets with application for unmanned aerial vehicle selection. *Scientific Reports*, *15*(1), 12637. https://doi.org/10.1038/s41598-025-95981-0

[10] Chatterjee, T., & Pramanik, S. (2025). Triangular fuzzy quadripartitioned neutrosophic set and its properties. *Neutrosophic Sets and Systems*, *75*, 15–28.

[11] Shil, B., Das, R., Granados, C., Das, S., & Tripathy, B. C. (2024). Single-valued quadripartitioned neutrosophic *d*-ideal of *d*-algebra. *Neutrosophic Sets and Systems*, *67*, 105–114.

[12] Yu, Y., Yang, Y., Zeng, Q., Gao, R., Ding, H., Ma, J., ..., & Zhao, Y. (2025). An improved hybrid FMEA method based on spherical fuzzy sets for risk assessment of offshore anchor pile installation. *Ocean Engineering*, *339*, 122180. https://doi.org/10.1016/j.oceaneng.2025.122180

[13] Yang, M. S., Akhtar, Y., & Ali, M. (2025). Divergence measures for globular T-spherical fuzzy sets with application in selecting solar energy systems. *Symmetry*, *17*(6), 872. https://doi.org/10.3390/sym17060872

[14] Pawlak, Z., & Skowron, A. (2007). Rudiments of rough sets. *Information Sciences*, *177*(1), 3–27. https://doi.org/10.1016/j.ins.2006.06.003

[15] Jose, J., George, B., & Thumbakara, R. K. (2023). Soft directed graphs, their vertex degrees, associated matrices and some product operations. *New Mathematics and Natural Computation*, *19*(03), 651–686. https://doi.org/10.1142/S179300572350028X

[16] Jose, J., George, B., & Thumbakara, R. K. (2025). Soft graphs: A comprehensive survey. *New Mathematics and Natural Computation*, *21*(3), 939–990. https://doi.org/10.1142/S1793005725500474

[17] Safari, K., & Imani, F. (2022). A novel fuzzy-BELBIC structure for the adaptive control of satellite attitude. In *Proceedings of the ASME 2022 International Mechanical Engineering Congress and Exposition*, *3*, V003T04A033. https://doi.org/10.1115/IMECE2022-96034

[18] Kumar, T., & Sharma, M. K. (2025). Neutrosophic decision-making for allocations in solid transportation problems. *OPSEARCH*, *62*(3), 1173–1199. https://doi.org/10.1007/s12597-024-00819-4

[19] Hosseini, K., & Javid, Y. (2025). Enhancing risk assessment in sustainable construction projects: A hybrid neutrosophic-based decision-making framework. *Journal of Cleaner Production*, *525*, 146395. https://doi.org/10.1016/j.jclepro.2025.146395

[20] Rani, P., & Mishra, A. R. (2025). Single-valued neutrosophic SWARA-TOPSIS-based group decision-making for prioritizing renewable energy systems. *Computer and Decision Making: An International Journal*, *2*, 425–439. https://doi.org/10.59543/comdem.v2i.11227

[21] Sultana, F., Gulistan, M., Ali, M., Yaqoob, N., Khan, M., Rashid, T., & Ahmed, T. (2023). A study of plithogenic graphs: Applications in spreading coronavirus disease (COVID-19) globally. *Journal of Ambient Intelligence and Humanized Computing*, *14*(10), 13139–13159. https://doi.org/10.1007/s12652-022-03772-6

[22] Ponnukumar, P. S., Francis Xavier, N. T., & Jaganathan, R. (2025). Stable plithogenic cubic sets. *Journal of Fuzzy Extension and Applications*, *6*(2), 410–423. https://doi.org/10.22105/jfea.2025.449408.1422

[23] Al-shboul, A., Alhazaymeh, K., Wang, K. L., & Wong, K. B. (2024). Fermatean vague soft set and its application in decision making. *Mathematics*, *12*(23), 3699. https://doi.org/10.3390/math12233699

[24] Ajaib, N., Hayat, K., Pamucar, D., & Kausar, K. (2025). q-Fractional hesitant fuzzy sets and their correlation coefficients: Multi-criteria decision making technique for selection of agricultural land to cultivate apples crops. *IEEE Access*, *13*, 134057–134069. https://doi.org/10.1109/ACCESS.2025.3582884

[25] Dharmalingam, M., Mahapatra, G. S., Silambarasan, I., Vijaykrishnaraj, R., Rajakumari, P., & Selvakumar, R. (2025). The n-hyperspherical neutrosophic sets defined on some algebraic structures. *Neutrosophic Sets and Systems*, *91*, 658–673.

[26] Hadi, M. H., & Al-Swidi, L. A. A. (2022). The neutrosophic axial set theory. *Neutrosophic Sets and Systems*, *51*, 295–302.

[27] Bharathi, T., Leo, S., & Davvaz, B. (2023). Domination in plithogenic product fuzzy graphs. *Journal of Applied Mathematics, Statistics and Informatics*, *19*(2), 5–22. https://doi.org/10.2478/jamsi-2023-0007

[28] Iglesias, S. G., Lemus, A. E. P., Villegas-Ricauter, V., Arizaga, L. D. C. B., & Adriano, L. E. S. (2025). Perceptions of inclusion in disability communication with plithogenic fuzzy soft sets. *Neutrosophic Sets and Systems*, *92*, 641–651.

[29] Bharathi, T., & Leo, S. (2021). A study on plithogenic product fuzzy graphs with application in social network. *Indian Journal of Science and Technology*, *14*(40), 3051–3063. https://doi.org/10.17485/IJST/v14i40.1298

[30] Trofimov, Y. V., Muravyov, I. P., Averkin, A. N., Lebedev, A. D., Kuznetsov, E. M., Trusov, I. A., ..., & Shevchenko, A. V. (2025). Novyy plitogenno-nechetkiy mnogosloynyy pertseptron (PN-MLP) na osnove nechotkoy, intuitsionistskoy, neytrosoficheskoy i plitogennoy logik s ob"yasnitel'nym iskusstvennym intellektom vtorogo pokoleniya (XAI 2.0) dlya platformy klinicheskogo analiza molochnoy zhelezy [A new plithogenic-fuzzy multilayer perceptron (PN-MLP) based on fuzzy, intuitionistic, neutrosophic, and plithogenic logics with second-generation explainable artificial intelligence (XAI 2.0) for a breast clinical analysis platform]. *Fuzzy Systems and Soft Computing*, *20*(1), 5–35. https://doi.org/10.26456/fssc132

[31] Azeem, M., Rashid, H., Jamil, M. K., Gütmen, S., & Tirkolaee, E. B. (2025). Plithogenic fuzzy graph: A study of fundamental properties and potential applications. *Journal of Dynamics and Games*, *12*(3), 196–214. https://doi.org/10.3934/jdg.2024023

[32] Bharathi, T., & Sampath, A. P. (2025). Plithogenic product intuitionistic fuzzy graph. *Afrika Matematika*, *36*(4), 167. https://doi.org/10.1007/s13370-025-01378-7

[33] Igno, M., Sandhiya, S., & Broumi, S. (2025). Advancing sustainable mobility: Comparative analysis of MCDM methods with plithogenic and neutrosophic sets. *International Journal of Applied Mathematics*, *38*(7s), 1090–1118. https://doi.org/10.12732/ijam.v38i7s.536

[34] Xu, L., & Peng, N. (2025). Plithogenic-neutrosophic rough number for reimagining rural education: Quality evaluation of early childhood programs. *Neutrosophic Sets and Systems*, *87*, 1073–1082. https://doi.org/10.5281/zenodo.16117558

[35] Priyadharshini, S. P., & Irudayam, F. N. (2021). A new approach of multi-dimensional single valued plithogenic neutrosophic set in multi criteria decision making. *Neutrosophic Sets and Systems*, *45*, 151–161.

[36] Du, H. (2025). A plithogenic neutrosophic hypersoft set-based framework with comparative analysis for measuring teaching practice effectiveness in university-level vocal music instruction. *Neutrosophic Sets and Systems*, *86*, 245–253.

[37] Chen, R., Liu, Q., Zhang, Y., & Weng, W. (2025). A probabilistic plithogenic neutrosophic rough set for uncertainty-aware food safety analysis. *Neutrosophic Sets & Systems*, *90*, 888–904.

[38] Angel, N., Pandiammal, P., & Martin, N. (2025). Generalized plithogenic cognitive map framework for managerial decision making. *Journal of Applied Research on Industrial Engineering*, *12*(4), 703–713. https://doi.org/10.22105/jarie.2025.530271.1823

[39] Zhang, M., & Li, Y. (2025). Quality evaluation in hospital operations management using multi-criteria decision-making with plithogenic approach to address uncertainty and enhance decision-making processes. *Neutrosophic Sets and Systems*, *80*, 564–575.

[40] Liu, Y., Yu, X., & Zheng, S. (2025). Comprehensive evaluation of green construction in building projects using an integrated MCDM and plithogenic set approach. *Neutrosophic Sets and Systems*, *80*, 149–162.

[41] Chen, H., & Zhang, D. (2025). Ecological efficiency evaluation in petrochemical enterprises using plithogenic-ARTASI model. *Neutrosophic Sets and Systems*, *80*, 597–608.

[42] Fujita, T., & Smarandache, F. (2025). *A dynamic survey of fuzzy, intuitionistic fuzzy, neutrosophic, and extensional sets*. USA: Neutrosophic Science International Association (NSIA) Publishing House.

[43] Ihsan, M., Saeed, M., Alanzi, A. M., & Khalifa, H. E.-W. (2023). An algorithmic multiple attribute decision-making method for heart problem analysis under neutrosophic hypersoft expert set with fuzzy parameterized degree-based setting. *PeerJ Computer Science*, *9*, e1607. https://doi.org/10.7717/peerj-cs.1607

[44] Al-Qudah, Y., & Hassan, N. (2017). Bipolar fuzzy soft expert set and its application in decision making. *International Journal of Applied Decision Sciences*, *10*(2), 175–191. https://doi.org/10.1504/IJADS.2017.084310

[45] Al-Sharqi, F., Ahmad, A. G., & Al-Quran, A. (2022). Interval-valued neutrosophic soft expert set from real space to complex space. *Computer Modeling in Engineering & Sciences*, *132*(1), 267–293. https://doi.org/10.32604/cmes.2022.019684

[46] Akram, M., Ali, G., Peng, X., & Zain ul Abidin, M. (2022). Hybrid group decision-making technique under spherical fuzzy N-soft expert sets. *Artificial Intelligence Review*, *55*(5), 4117–4163. https://doi.org/10.1007/s10462-021-10103-2

[47] Alkhazaleh, S., & Salleh, A. R. (2014). Fuzzy soft expert set and its application. *Applied Mathematics*, *5*(9), 1349–1368. https://doi.org/10.4236/am.2014.59127

[48] Deng, J., Chen, D., Wang, H., & Shi, R. (2025). Matrix factorization algorithm for multi-label learning with missing labels based on fuzzy rough set. *Fuzzy Sets and Systems*, *498*, 109143. https://doi.org/10.1016/j.fss.2024.109143

[49] Chen, Q., Zeng, L., & Ding, W. (2025). FRCNN: A combination of fuzzy-rough-set-based feature discretization and convolutional neural network for segmenting subretinal fluid lesions. *IEEE Transactions on Fuzzy Systems*, *33*(1), 350–364. https://doi.org/10.1109/TFUZZ.2024.3473310

[50] Wang, C., Deng, B., An, S., & Huang, Y. (2025). A noise-aware weighted fuzzy rough set model for feature selection. *IEEE Transactions on Fuzzy Systems*, *33*(10), 3504–3514. https://doi.org/10.1109/TFUZZ.2025.3596266

[51] Al-Hijjawi, S., Ahmad, A. G., & Alkhazaleh, S. (2024). Effective neutrosophic soft expert set and its application. *International Journal of Neutrosophic Science*, *23*(1), 27–50. https://doi.org/10.54216/IJNS.230103

[52] Başer, Z., & Uluçay, V. (2024). Effective Q-neutrosophic soft expert sets and its application in decision making. In F. Smarandache, D. Bakbak, V. Uluçay, A. Kargin, & N. M. Şahin (Eds.), *Algebraic structures in the universe of neutrosophic: Analysis with innovative algorithmic approaches* (pp. 147–170). Biblio Publishing.

[53] Liu, G., & Hao, L. (2025). Interval valued neutrosophic soft expert set for impact analysis of calligraphy art on the inheritance of chinese language and culture. *Neutrosophic Sets and Systems*, *91*, 950–960.

[54] Zeng, L. (2025). A neutrosophic-rough set support for AI-driven training quality evaluation of interdisciplinary communication talent in the converged media era. *Neutrosophic Sets and Systems*, *93*, 241–249.

[55] Zheng, H., Bo, C., Li, L., Wang, L., & Jiang, W. (2025). A novel multi-granularity variable precision neutrosophic rough set and group decision-making application with three strategies. *AIMS Mathematics*, *10*(10), 23187–23219. https://doi.org/10.3934/math.20251029

[56] Martina, D. J. S., & Deepa, G. (2023). Application of multi-valued rough neutrosophic set and matrix in multi-criteria decision-making: Multi-valued neutrosophic rough set and matrix. *Mathematics in Applied Sciences and Engineering*, *4*(3), 154–248. https://doi.org/10.5206/mase/16636

[57] Smarandache, F. (2024). Upside-down logics: Falsification of the truth and truthification of the false. *Systems Assessment and Engineering Management*, *1*, 1–7. https://doi.org/10.61356/j.saem.2024.1248

[58] Fujita, T., & Smarandache, F. (2024). *Introduction to upside-down logic: Its deep relation to neutrosophic logic and applications*. USA: Biblio Publishing.

[59] Gu, L., & Lei, M. (2025). A neutrosophic framework for nursing education quality analysis using Upside-Down Logics and Narrative Factor. *Neutrosophic Sets and Systems*, *87*, 362–378.

[60] Li, Q., & Kong, X. (2025). A mathematical neutrosophic offset framework with upside down logics for quality evaluation of multi-sensor intelligent vehicle environment perception systems. *Neutrosophic Sets and Systems*, *87*, 1014–1023.

[61] Bu, Y. (2025). An upside-down logic-based neutrosophic methodology: Assessment of online marketing effectiveness in e-commerce enterprises. *Neutrosophic Sets and Systems*, *85*, 801–813.

[62] Kaye, R. (1991). *Models of Peano arithmetic*. UK: Oxford University Press. https://doi.org/10.1093/oso/9780198532132.001.0001

[63] Martin, N., Yue, G. X., & Juraev, D. A. (2025). Modified plithogenic sets in criterion reduction in optimal decisioning on robotized technology. *Neutrosophic Sets and Systems*, *91*, 69–96.

[64] Salas, R. G., Ashqui, J. M. G., & Valencia, M. P. M. (2025). Assessing viral susceptibility in tilapia species using plithogenic fuzzy soft set: A multi-expert approach for TiLV. *Neutrosophic Sets and Systems*, *89*, 185–196.

[65] Molodtsov, D. (1999). Soft set theory–First results. *Computers & Mathematics with Applications*, *37*(4-5), 19–31. https://doi.org/10.1016/S0898-1221(99)00056-5

[66] Vijayabalaji, S. (2024). Soft-rough expert set. In P. Debnath & B. C. Tripathy (Eds.), *Soft computing: Engineering applications* (pp. 238–249). CRC Press. https://doi.org/10.1201/9781003466260-11

[67] Maji, P. K., Biswas, R., & Roy, A. R. (2003). Soft set theory. *Computers & Mathematics with Applications*, *45*(4–5), 555–562. https://doi.org/10.1016/S0898-1221(03)00016-6

[68] Musa, S. Y., Mohammed, R. A., & Asaad, B. A. (2023). N-hypersoft sets: An innovative extension of hypersoft sets and their applications. *Symmetry*, *15*(9), 1795. https://doi.org/10.3390/sym15091795

[69] Musa, S. Y. (2024). *N*-bipolar hypersoft sets: Enhancing decision-making algorithms. *PLoS ONE*, *19*(1), e0296396. https://doi.org/10.1371/journal.pone.0296396

[70] Ihsan, M., Rahman, A. U., & Saeed, M. (2021). Hypersoft expert set with application in decision making for recruitment process. *Neutrosophic Sets and System*, *42*, 191–207.

[71] Salamai, A. A. (2025). A superhypersoft framework for comprehensive risk assessment in energy projects. *Neutrosophic Sets and Systems*, *77*, 614–624.

[72] Kiruthika, T., Karpagadevi, M., Krishnaprakash, S., & Deepa, G. (2025). Superhypersoft sets using python and its applications in neutrosophic superhypersoft sets under TOPSIS method. *Neutrosophic Sets and Systems*, *91*, 586–616.

[73] Kang, Y., & Qin, N. (2025). Neutrosophic superhypersoft set approach for evaluating teacher education quality in normal universities. *Neutrosophic Sets and Systems*, *86*, 303–311.

[74] Perveen, P. A. F., & John, S. J. (2023). Relations on spherical fuzzy soft sets. *AIP Conference Proceedings*, *2875*(1), 040006. https://doi.org/10.1063/5.0153953

[75] Saeed, M., Din, I. S. U., Tariq, M., & Garg, H. (2024). Refined fuzzy soft sets: Properties, set-theoretic operations and axiomatic results. *Journal of Computational and Cognitive Engineering*, *3*(1), 24–33. https://doi.org/10.47852/bonviewJCCE3202847

[76] Beraún-Espíritu, M. M., Moscoso-Paucarchuco, K. M., Bonilla-Mancilla, H. D., Julca-Marcelo, E. H., Basaldua Galarza, A., Espeza-Gavilán, J. P., ..., & Llacza-Molina, J. D. (2025). Environmental sustainability and organizational commitment in rainbow trout production in Junín, Peru: An approach from plithogenic fuzzy soft sets. *Neutrosophic Sets and Systems*, *84*, 412–423.

[77] Bin, C. (2019). Interval valued generalised fuzzy soft expert set and its application. *Advances in Natural Computation, Fuzzy Systems and Knowledge Discovery*, *1*, 991–999. https://doi.org/10.1007/978-3-030-32456-8_106

[78] Xu, W., & Li, Y. (2025). Multi-label feature selection for imbalanced data via KNN-based multi-label rough set theory. *Information Sciences*, *715*, 122220. https://doi.org/10.1016/j.ins.2025.122220

[79] Tian, Y., Wang, H., Zhu, X., & Guo, H. (2025). A review of the image segmentation methods using rough sets. *IET Image Processing*, *19*(1), e70141. https://doi.org/10.1049/ipr2.70141

[80] Pawlak, Z. (1982). Rough sets. *International Journal of Computer & Information Sciences*, *11*(5), 341–356. https://doi.org/10.1007/BF01001956

[81] Morsi, M. N., & Yakout, M. M. (1998). Axiomatics for fuzzy rough sets. *Fuzzy Sets and Systems*, *100*(1-3), 327–342. https://doi.org/10.1016/S0165-0114(97)00104-8

[82] Diestel, R. (2025). *Graph theory* (6th ed.). Germany: Springer. https://doi.org/10.1007/978-3-662-70107-2

[83] Gao, Y., Zhang, Z., Lin, H., Zhao, X., Du, S., & Zou, C. (2022). Hypergraph learning: Methods and practices. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, *44*(5), 2548–2566. https://doi.org/10.1109/TPAMI.2020.3039374

[84] Feng, S., Heath, E., Jefferson, B., Joslyn, C., Kvinge, H., Mitchell, H. D., ..., & Purvine, E. (2021). Hypergraph models of biological networks to identify genes critical to pathogenic viral response. *BMC Bioinformatics*, *22*(1), 287. https://doi.org/10.1186/s12859-021-04197-2

[85] Jun, Y. B., Hur, K., & Lee, K. J. (2018). Hyperfuzzy subalgebras of BCK/BCI-algebras. *Annals of Fuzzy Mathematics and Informatics*, *15*(1), 17–28. https://doi.org/10.30948/afmi.2018.15.1.17

[86] Berrocal Villegas, S. M., Montalvo Fritas, W., Berrocal Villegas, C. R., Fuentes Rivera, M. Y. F., Espejo Rivera, R., Bautista Puma, L. D., & Macazana Fernández, D. M. (2025). Using plithogenic n-SuperHyperGraphs to assess the degree of relationship between information skills and digital competencies. *Neutrosophic Sets and Systems*, *84*, 513–524.

[87] Roshdy, E., Khashaba, M., & Ali, M. E. A. (2025). Neutrosophic super-hypergraph fusion for proactive cyber-attack countermeasures: A soft computing framework. *Neutrosophic Sets and Systems*, *94*, 232–252.

[88] Campoverde Valencia, E. M., Chuisaca Vásquez, J. P., & Becerra Lois, F. A. (2025). Multineutrosophic analysis of the relationship between survival and business growth in the manufacturing sector of Azuay Province, 2020–2023, using plithogenic n-SuperHyperGraphs. *Neutrosophic Sets and Systems*, *84*, 341–355.

[89] Oña, J. L. A., Ávila, A. S. M., & Poma, M. R. M. (2024). Study of sound pressure levels through the creation of noise maps in the urban area of Latacunga city using plithogenic n-SuperHyperGraphs. *Neutrosophic Sets and Systems*, *74*, 119–127.

How to Cite: Fujita, T. (2026). Plithogenic Fuzzy Soft Expert Set and Plithogenic Fuzzy Rough Set with Upside-Down Logic. *Journal of Data Science and Intelligent Systems*. https://doi.org/10.47852/bonviewJDSIS62028065