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Three-Mode Upside-Down Logic Within Plithogenic Neutrosophic Sets: Handling Contradiction and Truth Inversion for Applications in Decision Science

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Abstract: Reversal effects are common in real-world reasoning: a claim that is initially regarded as false may later be treated as true. Upside-Down Logic offers a formal way to model such reversals via context-dependent transformations that exchange truth and falsity, thereby capturing ambiguity and shifts in judgment over time. A (finite) Plithogenic Set describes elements through membership degrees conditioned on attribute values and is equipped with contradiction functions defined on those values, so it both subsumes and generalizes the (finite) classical fuzzy, intuitionistic, and neutrosophic frameworks. Established notions such as fuzzy sets and neutrosophic sets also admit plithogenic counterparts, including (finite) Plithogenic Fuzzy Sets and (finite) Plithogenic Neutrosophic Sets. De-Plithogenication denotes a systematic procedure for neutralizing contradictions in a plithogenic structure—for example, by resetting or transforming conflicting attribute relations into consistent, contradiction-free configurations. A Plithogenic Neutrosophic Set additionally assigns degrees of truth, indeterminacy, and falsity in the presence of such contradictions, thereby enriching neutrosophic semantics with explicit context sensitivity. We develop Three-Mode Upside-Down Logic, an enriched Upside-Down framework, alongside De-Plithogenication, and we investigate their interplay within Plithogenic Neutrosophic Sets. Because the present work is mainly theoretical, we expect future research to assess the practical effectiveness of these notions through computational experiments and related quantitative investigations.

Keywords: Upside-Down Logic, Plithogenic Neutrosophic Set, uncertain set, De-Plithogenication, Three-Mode Upside-Down Logic

1. Introduction

1.1. Crisp, fuzzy, neutrosophic, and related uncertain sets

Classical (crisp) set theory is not designed to represent ambiguity or partial truth. In 1965, Zadeh proposed the concept of a fuzzy set, which has since served as a foundational tool for modeling vagueness and uncertainty in a wide range of applications [1]. Later, Smarandache developed *neutrosophic sets* as a further generalization; they have become influential in a wide range of fields [2–5]. A neutrosophic set assigns to each element three mutually independent degrees, namely, truth, indeterminacy, and falsity, thereby providing a formal framework for representing information that may be incomplete, inconsistent, or inherently indeterminate. Related families include *vague sets* [6, 7], *bipolar neutrosophic sets* [8–10], *m-polar neutrosophic sets* [11, 12], *q-rung orthopair neutrosophic sets* [13, 14], *complex neutrosophic sets* [15, 16], *interval-valued neutrosophic sets* [17–19], *neutrosophic cubic sets* [20, 21], *Pythagorean neutrosophic sets* [22, 23], *Fermatean Neutrosophic Sets* [24–26], and *hesitant neutrosophic sets* [27, 28], among others. Like fuzzy sets, these frameworks have also seen extensive study and application in social science, control, and decision-making.

A *Plithogenic Set* is a unifying framework that, akin to multivalued logic, allows one to specify arbitrary uncertainty parameters while also handling explicit contradictions [29–32]. Here, a *contradiction value* encodes, for instance, opposing opinions among attributes. Plithogenic Sets encompass, as subclasses, the *Plithogenic Fuzzy Set* [33–36], the *plithogenic intuitionistic fuzzy set*, and the *Plithogenic Neutrosophic Set* [37, 38]. If the contradiction parameter is omitted from these subclasses, one obtains the standard *fuzzy*, *intuitionistic fuzzy*, and *neutrosophic* sets. For these reasons, research on Plithogenic Sets is of significant importance, and moreover, Plithogenic Set theory has been actively investigated in a wide range of recent publications [39–42]. For convenience, Table 1 summarizes the main features of classical, fuzzy, intuitionistic fuzzy, neutrosophic, and Plithogenic Sets.

1.2. Upside-Down Logic

In brief, *Upside-Down Logic* formalizes context-driven inversions of truth and falsity, thereby modeling ambiguity and reversal phenomena in reasoning systems [43–45]. For example, in political discourse, harmful policies may be framed as "helpful," effectively inverting truth/falsity assessments and eroding public trust. As another recurring scenario, a harmful policy is marketed as "safety reform," flipping truth and falsity; citizens may then accept surveillance as protection under crisis rhetoric. Upside-Down Logic, a concept recently proposed by Smarandache [45], has begun to be studied both theoretically and within existing uncertainty frameworks, including neutrosophic and plithogenic settings [44, 46]. Moreover, the operation that we call *De-Plithogenication* uses Upside-Down mechanisms to *reduce* contradiction parameters [46]. This helps in diagnosing truth–falsity inversions induced by perceived contradictions and, after applying the operation, enables a more fine-grained treatment of uncertainty.

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Table 1
Overview of classical, fuzzy, intuitionistic fuzzy, neutrosophic, and Plithogenic Sets

Concept	Membership	Short description
Classical set	$\mu_A : X \to \{0,1\}$	Crisp membership: each element either belongs to A or does not.
Fuzzy set	$\mu_A : X \to [0,1]$	Single degree in $[0,1]$; models gradual membership and partial truth.
Intuitionistic fuzzy	$(\mu_A, \nu_A) : X \to [0,1]^2$, $\mu_A(x) + \nu_A(x) \le 1$	Membership $\mu_A(x)$ and non-membership $\nu_A(x)$ are separated; the remainder is hesitation.
Neutrosophic set	$(T_A, I_A, F_A) : X \to [0,1]^3$	Independent degrees of truth $T_A(x)$, indeterminacy $I_A(x)$, and falsity $F_A(x)$, capturing incomplete and inconsistent information.
Plithogenic Set	$\mu_A(x,\alpha)$ with $pConF(\alpha,\beta)$	Element x is evaluated w.r.t. attribute values α; a contradiction function $pConF$ adjusts aggregation of conflicting assessments and generalizes the above models.

1.3. Motivation and our contribution

This paper investigates how Upside-Down Logic can be applied within the *Plithogenic Neutrosophic Set* framework, which blends the principles of neutrosophic and Plithogenic Sets to capture both uncertainty and explicit contradiction more faithfully. Concretely, we introduce and study *Three-Mode Upside-Down Logic*, an enhanced, context-triggered operator acting on plithogenic neutrosophic memberships, together with a complementary procedure called *De-Plithogenication*. For attributes activated by the context, Three-Mode Upside-Down Logic operates as summarized in Table 2.

These operators help in assessing whether an apparent contradiction is *genuine*, *resolvable (i.e., not a true contradiction)*, or *undetermined* within the available context. We analyze their behavior in plithogenic neutrosophic models and outline how the proposed mechanisms can be used to reason about conflict-laden information in complex, real-world decision processes. For reference, a comparison between Upside-Down Logic (UDL) and Three-Mode Upside-Down Logic (3M-UDL) is provided in Table 3.

2. Preliminaries

This section lays down the core definitions and symbols used throughout the paper. Unless explicitly indicated otherwise, all sets and structures considered below are finite.

2.1. Formal setup for Upside-Down Logic

We next set up a precise formal environment for defining and analyzing *Upside-Down Logic*. The aim is to model context-driven flips between truth and falsity; see [43–45, 47–49] for complementary perspectives.

Definition 1 (Logical system) [50]. A *logical system* is a tuple

$$\mathscr{M} \;=\; (\mathrm{LOG},\ \mathrm{P},\ \mathrm{VAL},\ v;\ \mathrm{AX},\ \mathrm{Rule}), \tag{1}$$

where LOG is a formal language, P is the set of well-formed propositions over LOG, VAL is a set of truth values (e.g. $\{\mathrm{T},\mathrm{F}\}$ or $\{\mathrm{T},\mathrm{F},\mathrm{I}\}$), and $v : \mathrm{P} \to \mathrm{VAL}$ is a valuation. In addition, $\mathrm{AX} \subseteq \mathrm{P}$ denotes a set of axioms, and Rule denotes a collection of inference rules.

Notation 1 (A set of contexts). Let CT denote a nonempty set of *contexts*. A *contextual valuation* is a function

$$T:\ \mathrm{P}\times\mathrm{CT} \longrightarrow \mathrm{VAL}, \qquad (A,c) \longmapsto T(A,c), \tag{2}$$

which evaluates each proposition $A \in \mathrm{P}$ under a specified context $c \in \mathrm{CT}$ by returning a truth value in VAL. A standard (context-independent) valuation v is obtained by fixing a reference context $c_0 \in \mathrm{CT}$ and defining

$$v(A) := T(A, c_0). \tag{3}$$

Definition 2 (Upside-Down Logic). Let $\mathscr{M}$ be a logical system together with a (finite) contextual valuation T as in Notation 1. A (finite) *Upside - Down Logic* derived from $\mathscr{M}$ is any structure

$$\mathscr{M}' \;=\; (\mathrm{LOG},\ \mathrm{P},\ \mathrm{VAL},\ T^U;\ \mathrm{AX}',\ \mathrm{Rule}') \tag{4}$$

obtained by choosing 1) a transformation U acting on propositions and/or contexts and 2) a designated "flip" permutation $\sigma : \mathrm{VAL} \to \mathrm{VAL}$ such that

$$\sigma(\mathrm{T}) = \mathrm{F}, \qquad \sigma(\mathrm{F}) = \mathrm{T}, \qquad \sigma(\mathrm{I}) = \mathrm{I}, \tag{5}$$

that is, σ exchanges T and F while fixing I (whenever $\mathrm{I} \in \mathrm{VAL}$).

The transformed contextual valuation is defined by

$$T^U(A,c) \;:=\; \sigma\big(T\big(U(A), U(c)\big)\big), \qquad (A \in \mathrm{P},\ c \in \mathrm{CT}). \tag{6}$$

We require that T^U be the total on $\mathrm{P}\times\mathrm{CT}$, and we choose $(\mathrm{AX}', \mathrm{Rule}')$ so that the induced proof calculus is consistent.

Example 1 (Traffic speed policy under a snow emergency). Let contexts be parameterized by a snow-severity index $s \in [0,1]$, where larger s indicates more severe conditions. Define the (effective) legal speed limit by

$$L(s) \;:=\; \begin{cases} 50 & \text{if } s < 0.80 \quad \text{(normal conditions)},\\ 30 & \text{if } s \ge 0.80 \quad \text{(snow emergency)}. \end{cases} \tag{7}$$

Table 2
Three-Mode Upside-Down Logic actions on activated attributes

Mode	**Action on the neutrosophic triplet** (T,I,F)
Keep	Preserves the triplet (T,I,F) (truth, indeterminacy, falsity).
Swap	Exchanges the truth and falsity components, mapping $(T,I,F) \mapsto (F,I,T)$, while leaving indeterminacy unchanged.
Absorb	Aggregates uncertainty into the indeterminacy component (e.g., consolidating support from T and F into I).

Table 3
Concise contrast between UDL and 3M-UDL

Aspect	Upside-Down Logic (UDL)	Three-Mode Upside-Down Logic (3M-UDL)
Basic action	Inverts truth and falsity (typically preserves indeterminacy).	On activated attributes: *Keep*, *Swap* ($T \leftrightarrow F$), or *Absorb* into I.
Trigger	Context/proposition transform U with a flip σ on truth values.	Contradiction-threshold activation $\mathsf{Actvt}(a \mid b, \lambda)$ plus mode selector $\mathrm{MO}(a)$.
Indeterminacy	Usually unchanged.	Unchanged (Keep/Swap) or increased (Absorb via a t-conorm S).
Contradiction map	Not intrinsic; can be coupled with optional reset in plithogenic models.	Built-in option: *no-reset* or *reset* on processed pairs (useful for De-Plithogenication).

Consider the propositions

A : "Driving at 50 km/h is permitted on segment S." and

B : "Driving at ≤ 30 km/h is *mandatory* on S."

Contextual valuation. For the context $c_s \in \mathrm{CT}$ interpreted as "severity s," define

$$T(A,c_s) = \begin{cases} \mathrm{T}, & L(s)=50, \\ \mathrm{F}, & L(s)=30, \end{cases} \quad T(B,c_s) = \begin{cases} \mathrm{T}, & L(s)=30, \\ \mathrm{F}, & L(s)=50. \end{cases} \tag{8}$$

Upside-Down transform. Use Definition 2 with the flip permutation $\sigma(\mathrm{T}) = \mathrm{F}$, $\sigma(\mathrm{F}) = \mathrm{T}$, $\sigma(\mathrm{I}) = \mathrm{I}$, and take U to be the identity on both propositions and contexts: $U(A) = A$, $U(B) = B$, and $U(c_s) = c_s$. Then, for any $X \in \{A,B\}$,

$$T^U(X,c_s) = \sigma\big(T(U(X),U(c_s))\big) = \sigma\big(T(X,c_s)\big). \tag{9}$$

Concrete computation. Pick $s_0 = 0.20$ (clear roads), so $L(s_0) = 50$ and hence

$$T(A,c_{s_0}) = \mathrm{T}, \qquad T(B,c_{s_0}) = \mathrm{F}. \tag{10}$$

Applying the transform yields

$$T^U(A,c_{s_0}) = \sigma(\mathrm{T}) = \mathrm{F} \tag{11}$$

(a true statement is flipped to false),

$$T^U(B,c_{s_0}) = \sigma(\mathrm{F}) = \mathrm{T} \tag{12}$$

(a false statement is flipped to true).

If indeterminate values are included in VAL, they remain unchanged because $\sigma(\mathrm{I}) = \mathrm{I}$.

Example 2 (Hospital visitation policy under outbreak thresholds). **Interpretation.** Let contexts be parameterized as $c_{(i,o;\lambda)} \in \mathrm{CT}$, where $i \in [0,1]$ is an infection index, $o \in [0,1]$ is ICU occupancy, and $\lambda \in (0,1]$ is the visitation risk threshold. Define the policy predicate

$$\mathrm{Allow}(i,o;\lambda) := \begin{cases} \mathrm{T}, & i \leq \lambda \text{ and } o \leq 0.85, \\ \mathrm{I}, & i \leq \lambda \text{ and } 0.85 < o < 0.95 \\ & \quad \text{(borderline capacity)}, \\ \mathrm{F}, & \text{otherwise}. \end{cases} \tag{13}$$

Consider the propositions

A : "An in-person ICU visit at 18:00 is permitted."

and

B : "A tele-visit at 18:00 is required."

with contextual valuation

$$T(A,c_{(i,o;\lambda)}) = \mathrm{Allow}(i,o;\lambda), \tag{14}$$

$$T(B,c_{(i,o;\lambda)}) = \begin{cases} \mathrm{T}, & \mathrm{Allow}(i,o;\lambda) = \mathrm{F}, \\ \mathrm{F}, & \mathrm{Allow}(i,o;\lambda) = \mathrm{T}, \\ \mathrm{I}, & \mathrm{Allow}(i,o;\lambda) = \mathrm{I}. \end{cases} \tag{15}$$

Upside-Down transform. As in Definition 2, take the same fixed flip σ and again let U be the identity (to isolate the flip effect): $U(A) = A$, $U(B) = B$, and $U(c_{(i,o;\lambda)}) = c_{(i,o;\lambda)}$. Then, for each $X \in \{A,B\}$,

$$T^U(X,c) = \sigma\big(T(X,c)\big) \qquad (c \in \mathrm{CT}). \tag{16}$$

Concrete computation. Choose the baseline context

$$c_0 = c_{(i_0,o_0;\lambda_0)} = c_{(0.15,\ 0.90;\ 0.20)}. \tag{17}$$

Here, $i_0 \leq \lambda_0$ and $0.85 < o_0 < 0.95$, so

$$T(A,c_0) = \mathrm{I} \quad \text{(borderline capacity)}, \qquad T(B,c_0) = \mathrm{I}. \tag{18}$$

Applying the transform gives

$$T^U(A,c_0) = \sigma(\mathrm{I}) = \mathrm{I} \quad \text{(indeterminacy preserved)}, \tag{19}$$

$$T^U(B,c_0) = \sigma(\mathrm{I}) = \mathrm{I}. \tag{20}$$

Now modify only the *context parameters* while keeping U the identity: set $o_1 = 0.78$ (capacity sufficient) with (i_0, λ_0) unchanged, i.e., $c_1 = c_{(0.15,\ 0.78;\ 0.20)}$. Then,

$$T(A,c_1) = \mathrm{T}, \qquad T(B,c_1) = \mathrm{F}, \tag{21}$$

and hence

$$T^U(A,c_1) = \sigma(\mathrm{T}) = \mathrm{F} \quad \text{(a true statement is flipped to false)}, \tag{22}$$

$$T^U(B,c_1) = \sigma(\mathrm{F}) = \mathrm{T} \quad \text{(a false statement is flipped to true)}. \tag{23}$$

Thus, with a fixed flip σ and identity U, the transform inverts permissions and requirements while leaving indeterminate judgments intact.

2.2. Plithogenic Set

A *Plithogenic Set* extends the standard notion of membership by explicitly attaching *attribute values* to assessments and by quantifying the *degree of contradiction* between such values. In this manner, it yields a versatile framework that subsumes and generalizes fuzzy, intuitionistic fuzzy, and neutrosophic settings [36, 51].

Definition 3 (Plithogenic Set) [51, 52]. Let S be a universe and let $P \subseteq S$ be a nonempty set. A *Plithogenic Set* is a quintuple

$$PS = \big(P,\ v,\ \mathrm{Dom}(v),\ pdf,\ pConF\big), \tag{24}$$

where v is a selected attribute and $\mathrm{Dom}(v)$ denotes the set of admissible values of v. The mapping

$$pdf:\ P \times \mathrm{Dom}(v) \longrightarrow [0,1]^s \tag{25}$$

is called the *degree of appurtenance function* (DAF), while

$$pConF:\ \mathrm{Dom}(v) \times \mathrm{Dom}(v) \longrightarrow [0,1]^t \tag{26}$$

is the *degree of contradiction function* (DegCF). For all $a, b \in \mathrm{Dom}(v)$, we assume

$$\text{(Reflexivity)}\ pConF(a,a) = 0, \tag{27}$$

$$\text{(Symmetry)}\ pConF(a,b) = pConF(b,a). \tag{28}$$

Here, $s,t \in \mathbb{N}$ denotes fixed dimensions. When $s > 1$, the range $[0,1]^s$ is interpreted coordinatewise.[1]

Example 3 (E-commerce shipping choice as a Plithogenic Set). Let $P = \{x\}$ with $x =$ "The chosen shipping method for order #12345 is appropriate." Take the attribute $v = \text{ShippingMethod}$ with

$$\mathrm{Dom}(v) = \{\text{Standard}(:= S),\ \text{Express}(:= E),\ \text{LockerPickup}(:= L)\}. \tag{29}$$

Use a two-dimensional degree of appurtenance ($s = 2$):

$$pdf(x,a) = \big(\text{BudgetFit},\ \text{Convenience}\big) \in [0,1]^2. \tag{30}$$

Specify

a	BudgetFit	Convenience
S	0.90	0.60
E	0.55	0.95
L	0.75	0.70

(31)

(each component independently in $[0,1]$).

Model the (scalar) degree of contradiction $pConF : \mathrm{Dom}(v) \times \mathrm{Dom}(v) \to [0,1]$ ($t = 1$), symmetric with $pConF(a,a) = 0$:

$pConF$	S	E	L
S	0	0.72	0.30
E	0.72	0	0.55
L	0.30	0.55	0

(32)

capturing, e.g., a strong tension between low-cost *Standard* and speed-focused *Express* (0.72), and a mild tension between *Standard* and *LockerPickup* (0.30). Thus, $PS = (P, v, \mathrm{Dom}(v), pdf, pConF)$ is a concrete Plithogenic Set for a typical checkout decision with multi-criteria membership and explicit contradictions among attribute values.

[1] Some variants in the literature allow powerset-valued (or hyper-) appurtenance. For concreteness, we work with the cube $[0,1]^s$.

Example 4 (Healthcare appointment modality as a Plithogenic Set). Let $P = \{y\}$ with $y =$ "Today's appointment modality for patient C is appropriate." Take the attribute $v = \text{VisitType}$ with

$$\mathrm{Dom}(v) = \{\text{InPerson}(:= I),\ \text{Telehealth}(:= T),\ \text{HomeVisit}(:= H)\}. \tag{33}$$

Use a three-dimensional degree of appurtenance ($s = 3$):

$$pdf(y,a) = \big(\text{ClinicalAdequacy},\ \text{Convenience},\ \text{Safety}\big) \in [0,1]^3. \tag{34}$$

Assign

a	Adequacy	Convenience	Safety
I	0.90	0.50	0.75
T	0.70	0.95	0.95
H	0.80	0.70	0.85

(35)

and define the scalar contradiction map ($t = 1$), symmetric with $pConF(a,a) = 0$:

$pConF$	I	T	H
I	0	0.68	0.35
T	0.68	0	0.55
H	0.35	0.55	0

(36)

reflecting, for instance, a substantial modality conflict between *InPerson* and *Telehealth* (0.68) and a moderate conflict between *Telehealth* and *HomeVisit* (0.55). Hence, $PS = (P, v, \mathrm{Dom}(v), pdf, pConF)$ concretely encodes multi-criteria suitability with explicit inter-modality contradictions for real clinical scheduling.

2.3. De-Plithogenication: contradiction reset via Upside-Down transforms

We make a precise, simple, finitely terminating procedure that eliminates pairwise contradictions among attribute values by iterated Upside - Down transforms with thresholded resets [46].

Definition 4 (Single-step transform with reset) [46]. Fix $b \in \mathrm{Dom}(v)$ (anchor) and $\lambda \in [0,1]$ (threshold). Define the *activation set*

$$\mathrm{Act}_{b,\lambda} := \{\, a \in \mathrm{Dom}(v) \mid pConF(a,b) \ \geq\ \lambda \,\}. \tag{37}$$

The transform $U_{b,\lambda}$ maps $PS = (P, v, \mathrm{Dom}(v), pdf, pConF)$ to

$$PS^{U_{b,\lambda}} = (P, v, \mathrm{Dom}(v), pdf^{U_{b,\lambda}}, pConF^{U_{b,\lambda}}) \tag{38}$$

by the following rules, for all $x \in P$ and $a, u, w \in \mathrm{Dom}(v)$:

DAF flip:

$$pdf^{U_{b,\lambda}}(x,a) = \begin{cases} \mathbf{1} - pdf(x,a), & a \in \mathrm{Act}_{b,\lambda}, \\ pdf(x,a), & a \notin \mathrm{Act}_{b,\lambda}, \end{cases}$$

DegCF reset:

$$pConF^{U_{b,\lambda}}(u,w) = \begin{cases} 0, & \text{if } \{u,w\} \cap \{b\} \neq \varnothing, \\ & \quad \text{and } (u \in \mathrm{Act}_{b,\lambda} \text{ or } w \in \mathrm{Act}_{b,\lambda}), \\ pConF(u,w), & \text{otherwise,} \end{cases}$$

where $\mathbf{1} \in [0,1]^s$ is the all-ones vector and the subtraction is componentwise when $s > 1$.

Remark (Scope of the DAF flip versus PNS operations). Definition 4 (the *DAF flip*) is introduced as an auxiliary, componentwise complement map acting on *plithogenic membership degrees* $\mathrm{pdf}(x,a) \in$

$[0,1]^3$ understood as a generic triplet of real degrees. In particular, within the subsequent development of *Plithogenic Neutrosophic Sets (PNS)*, we do *not* interpret the DAF flip as an operation on neutrosophic triplets (T,I,F) (truth, indeterminacy, falsity). Instead, the Upside-Down behavior in the PNS framework is governed exclusively by the three-mode actions (Keep/Swap/Absorb), which are tailored to the semantic roles of T, I, and F. Consequently, whenever a triplet is intended to represent a neutrosophic valuation, we employ the PNS-specific transformations and avoid applying the componentwise complement $1-\text{pdf}$ to (T,I,F).

Definition 5 (De-Plithogenication). A *De-Plithogenication* of *PS* is a finite composition

$$PS^{\text{dep}} := U_{b_k,\lambda_k} \circ \cdots \circ U_{b_2,\lambda_2} \circ U_{b_1,\lambda_1}(PS) \tag{39}$$

such that the resulting contradiction function is identically zero:

$$pConF^{\text{dep}}(u,w) \equiv 0 \qquad \text{for all } u,w \in \text{Dom}(v). \tag{40}$$

We call PS^{dep} the *contradiction-free normal form* of *PS*.

Example 5 (Supplier selection: De-Plithogenication to neutralize policy contradictions). **Setup.** Let $P=\{x\}$, where x denotes the statement "Supplier S is appropriate at the present time." Take $v = \text{PolicyFocus}$ and

$$\text{Dom}(v) = \{\text{CostEffective}(:=C), \tag{41}$$

$$\text{HighQuality}(:=Q),\ \text{FastDelivery}(:=F)\}. \tag{42}$$

Assign *three-component* plithogenic degrees $pdf(x,a) = (d_1,d_2,d_3) \in [0,1]^3$ (not interpreted here as a neutrosophic valuation):

a	d_1	d_2	d_3
C	0.70	0.10	0.20
Q	0.55	0.25	0.35
F	0.40	0.20	0.60

(43)

and consider a symmetric contradiction map $pConF$ with zero diagonal:

$pConF$	C	Q	F
C	0	0.62	0.80
Q	0.62	0	0.58
F	0.80	0.58	0

(44)

Step 1 (anchor $b_1 = C$, threshold $\lambda_1 = 0.60$). The activation set is

$$\mathsf{Actvt}_{C,0.60} = \{\, a \in \text{Dom}(v) \mid pConF(a,C) \geq 0.60 \,\} = \{Q,F\}. \tag{45}$$

Apply the DAF flip $pdf^{(1)} = U_{C,0.60}(pdf)$ to the activated attributes by the componentwise complement $\mathbf{1} - pdf$:

$$\begin{aligned} pdf^{(1)}(x,Q) &= \mathbf{1} - (0.55,0.25,0.35) = (0.45,0.75,0.65), \\ pdf^{(1)}(x,F) &= \mathbf{1} - (0.40,0.20,0.60) = (0.60,0.80,0.40), \\ pdf^{(1)}(x,C) &= (0.70,0.10,0.20) \quad \text{(unchanged)}. \end{aligned} \tag{46}$$

Reset the processed contradictions (Definition 4):

$$pConF^{(1)}(C,Q) = pConF^{(1)}(Q,C) = 0, \tag{47}$$

$$pConF^{(1)}(C,F) = pConF^{(1)}(F,C) = 0, \tag{48}$$

while $pConF^{(1)}(Q,F) = pConF(Q,F) = 0.58$ remains.

Step 2 (anchor $b_2 = F$, threshold $\lambda_2 = 0.58$). Under $pConF^{(1)}$, the activation set becomes

$$\mathsf{Actvt}_{F,0.58} = \{\, a \in \text{Dom}(v) \mid pConF^{(1)}(a,F) \geq 0.58 \,\} = \{Q\}. \tag{49}$$

Flip Q once more:

$$pdf^{(2)}(x,Q) = \mathbf{1} - (0.45,0.75,0.65) = (0.55,0.25,0.35), \tag{50}$$

and reset $pConF^{(2)}(F,Q) = pConF^{(2)}(Q,F) = 0$. Hence, all remaining contradictions vanish:

$$pConF^{(2)}(u,w) = 0 \quad \forall u,w \in \{C,Q,F\}. \tag{51}$$

Contradiction-free normal form.

$$\begin{aligned} & pdf^{\text{dep}}(x,C) = (0.70,0.10,0.20), \\ & pdf^{\text{dep}}(x,Q) = (0.55,0.25,0.35), \\ & pdf^{\text{dep}}(x,F) = (0.60,0.80,0.40), \qquad pConF^{\text{dep}} \equiv 0. \end{aligned} \tag{52}$$

In this two-step De-Plithogenication, the strong tensions C–Q and C–F are neutralized in Step 1, and the remaining Q–F contradiction is removed in Step 2, yielding a stable, contradiction-free representation of the supplier policy focus.

Example 6 (IT change management: De-Plithogenication across action choices). **Setup.** Let $P=\{y\}$, where y denotes the statement "Today's action regarding the production change is appropriate." Take $v = \text{Action}$ and

$$\text{Dom}(v) = \{\text{ProceedChange}(:=P), \tag{53}$$

$$\text{ScheduleMaintenance}(:=S),\ \text{Rollback}(:=R)\}. \tag{54}$$

Assign three-component degrees $pdf(y,a) = (d_1,d_2,d_3) \in [0,1]^3$:

a	d_1	d_2	d_3
P	0.50	0.20	0.45
S	0.60	0.15	0.35
R	0.30	0.25	0.70

(55)

and consider the contradiction map $pConF$ (symmetric, zero diagonal):

$pConF$	P	S	R
P	0	0.55	0.90
S	0.55	0	0.72
R	0.90	0.72	0

(56)

Step 1 (anchor $b_1 = P$, threshold $\lambda_1 = 0.70$). The activation set is

$$\mathsf{Actvt}_{P,0.70} = \{\, a \in \text{Dom}(v) \mid pConF(a,P) \geq 0.70 \,\} = \{R\}. \tag{57}$$

Flip R:

$$pdf^{(1)}(y,R) = \mathbf{1} - (0.30,0.25,0.70) = (0.70,0.75,0.30), \tag{58}$$

and reset $pConF^{(1)}(P,R) = pConF^{(1)}(R,P) = 0$ (all other pairs unchanged).

Step 2 (anchor $b_2 = S$, threshold $\lambda_2 = 0.70$). Under $pConF^{(1)}$, the activation set becomes

$$\mathsf{Actvt}_{S,0.70} = \{\, a \in \text{Dom}(v) \mid pConF^{(1)}(a,S) \geq 0.70 \,\} = \{R\}. \tag{59}$$

Flip R again:

$$pdf^{(2)}(y,R) = \mathbf{1} - (0.70,0.75,0.30) = (0.30,0.25,0.70), \tag{60}$$

and reset $pConF^{(2)}(S,R) = pConF^{(2)}(R,S) = 0$. The only remaining contradiction is $pConF^{(2)}(P,S) = 0.55$.

Step 3 (anchor $b_3 = P$, threshold $\lambda_3 = 0.55$). The activation set is

$$\mathsf{Actvt}_{P,0.55} = \{\, a \in \mathrm{Dom}(v) \mid pConF^{(2)}(a,P) \geq 0.55 \,\} = \{S\}. \quad (61)$$

Flip S:

$$pdf^{(3)}(y,S) = \mathbf{1} - (0.60, 0.15, 0.35) = (0.40, 0.85, 0.65), \quad (62)$$

and reset $pConF^{(3)}(P,S) = pConF^{(3)}(S,P) = 0$. Therefore, $pConF^{(3)} \equiv 0$.

Contradiction-free normal form.

$$\begin{aligned} pdf^{\mathrm{dep}}(y,P) &= (0.50, 0.20, 0.45), \\ pdf^{\mathrm{dep}}(y,S) &= (0.40, 0.85, 0.65), \\ pdf^{\mathrm{dep}}(y,R) &= (0.30, 0.25, 0.70), \qquad pConF^{\mathrm{dep}} \equiv 0. \end{aligned} \quad (63)$$

This three-step De-Plithogenication first resolves the strong antagonisms P–R and S–R, and then neutralizes the moderate P–S tension, producing a stable, contradiction-free assessment for IT change-management actions.

Proposition 1 (Finite elimination of contradictions). *Let* $\mathrm{Dom}(v) = \{\mathrm{F}_1, \ldots, \mathrm{F}_m\}$. *Choose anchors* $b_1, \ldots, b_m$ *as a permutation of* $\mathrm{Dom}(v)$ *and take thresholds* $\lambda_i := 0$ *for* $1 \leq i \leq m$. *Then, the composition* $U_{b_m,0} \circ \cdots \circ U_{b_1,0}$ *is a De-Plithogenication: for every unordered pair* $\{u,w\} \subseteq \mathrm{Dom}(v)$, *one has* $pConF^{\mathrm{dep}}(u,w) = 0$.

Proof. Write $pConF^{(0)} := pConF$ and $pConF^{(i)}$ for the DegCF after i transforms. By Definition 4 with $\lambda_i = 0$, at step i we reset to 0 every pair involving the anchor b_i:

$$pConF^{(i)}(b_i, a) = 0 \;\text{ for all } a \in \mathrm{Dom}(v), \quad (64)$$

$$pConF^{(i)}(u,w) = pConF^{(i-1)}(u,w) \text{ otherwise.} \quad (65)$$

Fix distinct $u, w \in \mathrm{Dom}(v)$ and let i be the index with $b_i = u$ (exists by construction). Then, $pConF^{(i)}(u,w) = 0$, and subsequent steps do not change zeros; hence, $pConF^{(m)}(u,w) = 0$. As $\{u,w\}$ was arbitrary, $pConF^{(m)} \equiv 0$.

Remark (Idempotence of the normal form). If $pConF^{\mathrm{dep}} \equiv 0$, then for any b, λ, one has $pConF^{U_{b,\lambda}} = pConF^{\mathrm{dep}}$ by Definition 4; thus, further applications of $U_{b,\lambda}$ leave the contradiction-free normal form unchanged.

2.4. Plithogenic Neutrosophic Set

A *neutrosophic set* specifies, for every element, three mutually independent membership degrees, namely, truth, indeterminacy, and falsity, each taking values in $[0,1]$, and thus generalizes the classical crisp and fuzzy set frameworks [53–55]. Besides the notions introduced in the Introduction, several closely related extensions have been proposed in the literature, including the Quadripartitioned Neutrosophic Set [56] and the Heptapartitioned Neutrosophic Set [57, 58]. A *Plithogenic Neutrosophic Set* further incorporates attribute values together with a contradiction degree among such values so that truth/indeterminacy/falsity assessments are expressed in a contradiction-sensitive, context-dependent manner [37, 38, 59, 60]. We now discuss how Upside-Down Logic can be applied in this setting.

Definition 6 (Neutrosophic Set) [53–55]. Let U be a nonempty (finite) set. A *Neutrosophic Set* (NS) NTS on U is specified by three membership functions

$$T_{\mathrm{NTS}} : U \to [0,1], \qquad I_{\mathrm{NTS}} : U \to [0,1], \qquad F_{\mathrm{NTS}} : U \to [0,1], \quad (66)$$

where, for each $x \in U$, the numbers $T_{\mathrm{NTS}}(x)$, $I_{\mathrm{NTS}}(x)$, and $F_{\mathrm{NTS}}(x)$ represent, respectively, the truth degree, the indeterminacy degree, and the falsity degree of x relative to NTS. These degrees satisfy the bound

$$0 \leq T_{\mathrm{NTS}}(x) + I_{\mathrm{NTS}}(x) + F_{\mathrm{NTS}}(x) \leq 3 \qquad \text{for all } x \in U. \quad (67)$$

Definition 7 (Plithogenic Neutrosophic Sets (the case $s = 3$, $t = 1$)) [61]. A (finite) *Plithogenic Neutrosophic Set* is a (finite) Plithogenic Set $PS = (P, v, \mathrm{Dom}(v), pdf, pConF)$ for which

$$pdf : \; P \times \mathrm{Dom}(v) \longrightarrow [0,1]^3, \quad (68)$$

$$pConF : \; \mathrm{Dom}(v) \times \mathrm{Dom}(v) \longrightarrow [0,1], \quad (69)$$

and for each $(u, \alpha) \in P \times \mathrm{Dom}(v)$,

$$pdf(u,\alpha) = \big(T_P(u \mid \alpha), I_P(u \mid \alpha), F_P(u \mid \alpha)\big), \quad (70)$$

$$T_P(u \mid \alpha), \; I_P(u \mid \alpha), \; F_P(u \mid \alpha) \in [0,1]. \quad (71)$$

A global normalization condition is not assumed; nevertheless, one may, if desired, require the single-valued bound

$$0 \leq T_P(u \mid \alpha) + I_P(u \mid \alpha) + F_P(u \mid \alpha) \leq 3 \qquad (\forall u \in P, \forall \alpha \in \mathrm{Dom}(v)). \quad (72)$$

The symbols T_P, I_P, and F_P represent, respectively, the truth, indeterminacy, and falsity degrees, each assessed with respect to the attribute value α. The contradiction function is a scalar-valued symmetric map with zero diagonal entries.

For reference, a brief overview of (finite) Neutrosophic Sets and (finite) Plithogenic Neutrosophic Sets is provided in Table 4.

In addition, a comparison between a Plithogenic Set (PS) and a Plithogenic Neutrosophic Set (PNS) is provided in Table 5.

Example 7 (E-commerce return resolution for a single order). Let $P = \{u\}$ with $u =$ "The chosen resolution for order #12345 is appropriate." Take the attribute $v = $ *Resolution* with value set

$$\mathrm{Dom}(v) = \{\text{Refund, Replace, Repair}\}. \quad (73)$$

Define the plithogenic neutrosophic degrees $pdf(u,\alpha) = (T, I, F) \in [0,1]^3$ as:

α	T	I	F
Refund	0.68	0.12	0.30
Replace	0.55	0.20	0.35
Repair	0.35	0.25	0.55

(74)

(independent components; no normalization required).

Model the contradiction degrees $pConF : \mathrm{Dom}(v) \times \mathrm{Dom}(v) \to [0,1]$ (symmetric, zero diagonal) by

$pConF$	Refund	Replace	Repair
Refund	0	0.35	0.78
Replace	0.35	0	0.60
Repair	0.78	0.60	0

(75)

so that Refund and Repair are highly contradictory (0.78) while Refund and Replace are only mildly contradictory (0.35).

Table 4
Neutrosophic Set vs. Plithogenic Neutrosophic Set

Aspect	Neutrosophic Set (NS)	Plithogenic Neutrosophic Set (PNS)
Domain	Element $u \in U$.	Pair $(u, \alpha) \in P \times \mathrm{Dom}(v)$.
Membership	$(T_{\mathrm{NTS}}, I_{\mathrm{NTS}}, F_{\mathrm{NTS}}) : U \to [0,1]^3$.	$pdf : P \times \mathrm{Dom}(v) \to [0,1]^3$.
Attributes	No explicit attribute values.	Degrees depend on attribute value α.
Contradiction	No contradiction function.	Uses $pConF : \mathrm{Dom}(v) \times \mathrm{Dom}(v) \to [0,1]$.

Concrete computation (bounded-sum t-conorm preview). With $S_{\mathrm{bs}}(u,v) = \min\{1, u+v\}$, the uncertainty mass that could be absorbed from truth/falsity for Repair is

$$S_{\mathrm{bs}}(T,F) \;=\; \min\{1,\, 0.35 + 0.55\} \;=\; 0.90, \tag{76}$$

which, if needed in a downstream "Absorb" step, would yield $I' = \min\{1, 0.25 + 0.90\} = 1.00$ while setting $T' = F' = 0$ (Definition 9). This example provides a fully specified PNS $(P, v, \mathrm{Dom}(v), pdf, pConF)$ grounded in a routine returns workflow.

Example 8 (Hiring decision for a candidate). Let $P = \{u\}$ with $u =$ "Today's action for candidate C is appropriate." Take the attribute $v = \textit{Action}$ with value set

$$\mathrm{Dom}(v) = \{\text{HireNow, InterviewAgain, Reject}\}. \tag{77}$$

Assign neutrosophic triplets (independent components in $[0,1]$):

α	T	I	F
HireNow	0.52	0.18	0.44
InterviewAgain	0.60	0.25	0.30
Reject	0.40	0.15	0.58

(78)

and a contradiction map (symmetric, $pConF(\alpha, \alpha) = 0$):

$pConF$	HireNow	InterviewAgain	Reject
HireNow	0	0.50	0.90
InterviewAgain	0.50	0	0.65
Reject	0.90	0.65	0

(79)

indicating that HireNow and Reject are strongly contradictory (0.90), whereas HireNow vs. InterviewAgain shows moderate tension (0.50).

Concrete computation (sanity checks). For $\alpha =$ InterviewAgain, one may verify potential uncertainty consolidation via

$$S_{\mathrm{bs}}(T,F) = \min\{1,\, 0.60 + 0.30\} = 0.90, \tag{80}$$

$$\text{and} \quad T + I + F = 0.60 + 0.25 + 0.30 = 1.15, \tag{81}$$

which is permitted in neutrosophic modeling (no global normalization constraint). This yields a practical PNS $(P, v, \mathrm{Dom}(v), pdf, pConF)$ for day-to-day hiring triage, explicitly encoding both support levels and pairwise contradictions among actions.

Definition 8 (Upside-Down Logic in a Plithogenic Neutrosophic Set (truth–falsity swap)). Let $PS = (P, v, \mathrm{Dom}(v), pdf, pConF)$ be a Plithogenic Neutrosophic Set such that

$$pdf(u,\alpha) = \big(T_P(u \mid \alpha),\, I_P(u \mid \alpha),\, F_P(u \mid \alpha)\big) \in [0,1]^3, \tag{82}$$

and define the (scalar) contradiction degree by

$$c(\alpha,\beta) := pConF(\alpha,\beta) \in [0,1], \tag{83}$$

so that $c(\alpha,\alpha) = 0$ and $c(\alpha,\beta) = c(\beta,\alpha)$ for all $\alpha, \beta \in \mathrm{Dom}(v)$. Fix an *anchor* $b \in \mathrm{Dom}(v)$ and a *threshold* $\lambda \in [0,1]$. We say that the flip is *activated* at $\alpha \in \mathrm{Dom}(v)$ precisely when

$$\mathsf{Actvt}(\alpha; b, \lambda) \iff c(\alpha, b) \geq \lambda. \tag{84}$$

The Upside-Down transform $U_{b,\lambda}$ acting on the membership map pdf is defined by

$$pdf^{U_{b,\lambda}}(u,\alpha) := \begin{cases} \big(F_P(u \mid \alpha),\, I_P(u \mid \alpha),\, T_P(u \mid \alpha)\big), & \text{if } \mathsf{Actvt}(\alpha; b, \lambda), \\ \big(T_P(u \mid \alpha),\, I_P(u \mid \alpha),\, F_P(u \mid \alpha)\big), & \text{otherwise}, \end{cases} \quad (u \in P,\ \alpha \in \mathrm{Dom}(v)). \tag{85}$$

Equivalently, whenever the contradiction between α and the anchor b is at least λ, the transform swaps the *truth* and *falsity* components while keeping the *indeterminacy* component unchanged.

Example 9 (School event under weather conditions: truth–falsity swap). Let $P = \{u\}$ with $u =$ "Holding the outdoor ceremony now is appropriate." Let the attribute be $v =$ Weather with

$$\mathrm{Dom}(v) = \{\text{Clear, LightRain, Thunderstorm}\}. \tag{86}$$

Table 5
Brief comparison between a Plithogenic Set (PS) and a Plithogenic Neutrosophic Set (PNS)

Aspect	**Plithogenic Set (PS)**	**Plithogenic Neutrosophic Set (PNS)**
DAF / membership	$pdf : P \times \mathrm{Dom}(v) \to [0,1]^s$ (general s-dimensional appurtenance).	$pdf : P \times \mathrm{Dom}(v) \to [0,1]^3$ with $pdf(u,\alpha) = \big(T(u \mid \alpha), I(u \mid \alpha), F(u \mid \alpha)\big)$.
Semantics	Generic attribute-conditioned membership degrees.	Explicit truth, indeterminacy, and falsity degrees under each attribute value.
DegCF / contradiction	$pConF : \mathrm{Dom}(v) \times \mathrm{Dom}(v) \to [0,1]^t$ (general t-dimensional contradiction).	Typically scalar $pConF : \mathrm{Dom}(v) \times \mathrm{Dom}(v) \to [0,1]$ (often $t = 1$), symmetric, zero diagonal.
Special case	Most general template (includes multiple subclasses by fixing s,t).	A specific instance of PS with $(s,t) = (3,1)$ (often used in uncertainty modeling).

Give neutrosophic degrees $pdf(u,\alpha) = (T,I,F) \in [0,1]^3$ by

α	T	I	F
Clear	0.75	0.10	0.20
LightRain	0.45	0.25	0.50
Thunderstorm	0.15	0.20	0.85

(87)

and a symmetric contradiction map $c(\cdot,\cdot) = pConF(\cdot,\cdot)$ (zero on the diagonal) by

$$c(\text{LightRain}, \text{Clear}) = 0.58, \tag{88}$$

$$c(\text{Thunderstorm}, \text{Clear}) = 0.93. \tag{89}$$

Fix anchor $b = \text{Clear}$ and threshold $\lambda = 0.80$. Activation:

$$\mathsf{Actvt}(\text{LightRain}; b, \lambda) = \mathbf{1}[0.58 \geq 0.80] = 0, \tag{90}$$

$$\mathsf{Actvt}(\text{Thunderstorm}; b, \lambda) = \mathbf{1}[0.93 \geq 0.80] = 1. \tag{91}$$

Under the Upside-Down transform $U_{b,\lambda}$, which exchanges T and F while leaving I unchanged for activated α,

$$\begin{aligned} pdf^U(u, \text{Clear}) &= (0.75, 0.10, 0.20) \quad \text{(not activated)}, \\ pdf^U(u, \text{LightRain}) &= (0.45, 0.25, 0.50) \quad \text{(not activated)}, \\ pdf^U(u, \text{Thunderstorm}) &= (0.85, 0.20, 0.15) \quad \text{(activated; } (T,F) \text{ swapped)}. \end{aligned} \tag{92}$$

Thus, under high contradiction with the anchor condition "Clear," the assessment for *Thunderstorm* flips from mostly false to mostly true for the proposition u.

Example 10 (Same-day delivery policy under inventory states: truth–falsity swap). Let $P = \{u\}$ with $u =$ "Offering same-day delivery is appropriate." Let the attribute be $v = \text{InventoryState}$ with

$$\text{Dom}(v) = \{\text{InStock, LowStock, Outage}\}. \tag{93}$$

Specify $pdf(u,\alpha) = (T,I,F)$ as

α	T	I	F
InStock	0.82	0.06	0.12
LowStock	0.58	0.22	0.33
Outage	0.28	0.30	0.72

(94)

and contradiction degrees

$$c(\text{LowStock}, \text{InStock}) = 0.65, \tag{95}$$

$$c(\text{Outage}, \text{InStock}) = 0.88. \tag{96}$$

Fix anchor $b = \text{InStock}$ and threshold $\lambda = 0.70$. Then,

$$\mathsf{Actvt}(\text{LowStock}; b, \lambda) = \mathbf{1}[0.65 \geq 0.70] = 0, \tag{97}$$

$$\mathsf{Actvt}(\text{Outage}; b, \lambda) = \mathbf{1}[0.88 \geq 0.70] = 1. \tag{98}$$

Applying $U_{b,\lambda}$ (swap T and F only on activated values),

$$\begin{aligned} pdf^U(u, \text{InStock}) &= (0.82, 0.06, 0.12) \quad \text{(not activated)}, \\ pdf^U(u, \text{LowStock}) &= (0.58, 0.22, 0.33) \quad \text{(not activated)}, \\ pdf^U(u, \text{Outage}) &= (0.72, 0.30, 0.28) \\ &\quad \text{(activated; } (T,F) \text{ swapped)}. \end{aligned} \tag{99}$$

Hence, when *Outage* is highly contradictory to the anchor *InStock*, the evaluation of same-day delivery reverses its truth/falsity while preserving indeterminacy.

3. A Three-Mode Upside-Down Operator for Plithogenic Neutrosophic Sets

In this section, we formalize a three-way Upside-Down mechanism for plithogenic neutrosophic memberships. The rule is driven by context through contradiction: once an attribute value is *activated* by exceeding a prescribed contradiction threshold, one may apply exactly one of the following operations. *Keep* leaves the triplet (T,I,F) intact, *Swap* exchanges the truth and falsity entries, and *Absorb* suppresses truth and falsity while moving their mass into indeterminacy.

Remark (A context-to-parameter policy map). To make the phrase *context-driven* mathematically explicit, one may fix a set of contexts CT and introduce a *policy map*

$$\Phi : \text{CT} \longrightarrow \text{Dom}(v) \times [0,1] \times \{\text{Keep, Swap, Absorb}\}^{\text{Dom}(v)}. \tag{100}$$

For each context $c \in \text{CT}$, write

$$\Phi(c) = \big(b(c), \lambda(c), MO_c\big), \tag{101}$$

where $b(c) \in \text{Dom}(v)$ is the context-selected anchor attribute, $\lambda(c) \in [0,1]$ is the context-selected contradiction threshold, and $MO_c : \text{Dom}(v) \to \{\text{Keep, Swap, Absorb}\}$ assigns, for each attribute $a \in \text{Dom}(v)$, the action to be applied under context c. In the present paper, the triplet (b, λ, MO) can be viewed as either 1) specified directly by the decision maker or 2) generated via such a map Φ; the three-mode definitions and results remain valid in both cases.

Definition 9 (Three-Mode Upside-Down Logic on a Plithogenic Neutrosophic Set). Fix a (finite) *anchor* $b \in \text{Dom}(v)$, a *threshold* $\lambda \in [0,1]$, and a (finite) *mode selector*

$$\text{MO} : \text{Dom}(v) \longrightarrow \{\text{Keep, Swap, Absorb}\}. \tag{102}$$

Activation is dictated by the contradiction with the anchor:

$$\mathsf{Actvt}(\alpha \mid b, \lambda) := \mathbf{1}[c(\alpha, b) \geq \lambda] \in \{0,1\}. \tag{103}$$

Let $S : [0,1]^2 \to [0,1]$ be an arbitrary *t-conorm* (s-norm) used to aggregate degrees into indeterminacy; in the single-valued case, the *bounded sum*

$$S_{\text{bs}}(x,y) := \min\{1, x+y\} \tag{104}$$

is a standard choice.

The induced *Three-Mode Upside-Down transform*

$$U^{(3)}_{b,\lambda,\text{MO},S} : PS \longrightarrow PS^U \tag{105}$$

acts only on those attribute values that are activated. The contradiction map may be left unchanged (*no-reset*) or, alternatively, set to zero on the pairs that are processed (*reset*; see below). For each $(u,\alpha) \in P \times \text{Dom}(v)$,

$$pdf^U(u,\alpha) = \big(T'_P(u \mid \alpha), I'_P(u \mid \alpha), F'_P(u \mid \alpha)\big) \tag{106}$$

is defined by

$$(T'_P, I'_P, F'_P) = \begin{cases} (T_P, I_P, F_P), & \mathsf{Actvt}(\alpha \mid b, \lambda) = 0 \\ & \text{(not activated)}, \\ (T_P, I_P, F_P), & \mathsf{Actvt}(\alpha \mid b, \lambda) = 1 \\ & \text{and } \text{MO}(\alpha) = \text{Keep}, \\ (F_P, I_P, T_P), & \mathsf{Actvt}(\alpha \mid b, \lambda) = 1 \\ & \text{and } \text{MO}(\alpha) = \text{Swap}, \\ \big(0, S(I_P, S(T_P, F_P)), 0\big), & \mathsf{Actvt}(\alpha \mid b, \lambda) = 1 \\ & \text{and } \text{MO}(\alpha) = \text{Absorb}, \end{cases} \tag{107}$$

where $T_P = T_P(u \mid \alpha)$, $I_P = I_P(u \mid \alpha)$, and $F_P = F_P(u \mid \alpha)$. Equivalently:

1) *Keep*: do nothing to (T,I,F) on activated α;

2) *Swap*: apply $(T,I,F) \mapsto (F,I,T)$ on activated α;

3) *Absorb*: enforce $T' = F' = 0$ and set $I' = S\big(I, S(T,F)\big)$, i.e., treat the case as *neither true nor false*.

Update of the contradiction map (two common options).

1) *No-reset (Keep/Swap remain involutive):* take $pConF^U = pConF$.

2) *Reset (processed pairs become fixed at zero):* for each activated α,

$$pConF^U(\alpha,b) = pConF^U(b,\alpha) = 0, \tag{108}$$

$$pConF^U(u,w) = pConF(u,w) \ \text{ otherwise.} \tag{109}$$

Both update rules are compatible with the definition of pdf^U; the selection depends on whether one wishes to retain contradictions as contextual information or to neutralize them after transformation.

Example 11 (Airport security alert levels: Keep & Swap modes). **Interpretation.** Let $P = \{u\}$ contain a single proposition $u =$ "Secondary screening at gate G is required." Let the attribute alphabet (policy alert level) be

$$\mathrm{Dom}(v) = \{\text{Green, Orange, Red}\}. \tag{110}$$

We model the plithogenic neutrosophic degrees for u under each alert level by

$$pdf(u,\alpha) = \big(T_P(u \mid \alpha), I_P(u \mid \alpha), F_P(u \mid \alpha)\big) \in [0,1]^3. \tag{111}$$

Contradiction degrees $c(\alpha,\beta) = pConF(\alpha,\beta)$ are symmetric with $c(\alpha,\alpha) = 0$.

Numerical setup (before transform). Choose anchor $b =$ Green and set the threshold $\lambda = 0.60$. Let the contradiction map satisfy

$$c(\text{Orange},\text{Green}) = 0.62, \qquad c(\text{Red},\text{Green}) = 0.85, \tag{112}$$

so both Orange and Red are *activated* because $c(\cdot,\text{Green}) \geq \lambda$. Take initial neutrosophic triplets:

$$\begin{aligned} pdf(u,\text{Green}) &= (0.20,\ 0.10,\ 0.70),\\ pdf(u,\text{Orange}) &= (0.55,\ 0.15,\ 0.35),\\ pdf(u,\text{Red}) &= (0.30,\ 0.20,\ 0.60). \end{aligned} \tag{113}$$

Mode selector. We choose

$$\mathrm{MO}(\text{Orange}) = \text{Keep}, \qquad \mathrm{MO}(\text{Red}) = \text{Swap}. \tag{114}$$

(Here, "Keep" means that (T,I,F) is left unchanged, whereas "Swap" means that the truth and falsity components are exchanged.)

Three-Mode UD transform (no-reset option). Applying Definition 9 with the no-reset choice for $pConF$:

$$pdf^U(u,\alpha) = \begin{cases} (T,I,F), & \alpha \text{ not activated},\\ (T,I,F), & \alpha \text{ activated and } \mathrm{MO}(\alpha) = \text{Keep},\\ (F,I,T), & \alpha \text{ activated and } \mathrm{MO}(\alpha) = \text{Swap}. \end{cases} \tag{115}$$

Hence,

$$\begin{aligned} pdf^U(u,\text{Green}) &= (0.20,\ 0.10,\ 0.70) \quad \text{(not activated)},\\ pdf^U(u,\text{Orange}) &= (0.55,\ 0.15,\ 0.35) \quad \text{(Keep)},\\ pdf^U(u,\text{Red}) &= (0.60,\ 0.20,\ 0.30)\\ &\quad \text{(Swap of } (0.30, 0.20, 0.60)). \end{aligned} \tag{116}$$

In words, under a *Red* alert, the previously mostly false statement becomes mostly true by swapping T and F, while the *Orange* setting is kept unchanged despite activation (e.g., policy decides not to overturn it). Because we selected the no-reset option, $pConF$ remains the same after the transform.

Example 12 (Loan approval triage: absorb mode with bounded-sum and reset). **Interpretation.** Consider a single application u and the decision attribute alphabet

$$\mathrm{Dom}(v) = \{\text{Approve, Deny, ManualReview}\}. \tag{117}$$

Let $pdf(u,\alpha) = (T,I,F)$ encode the neutrosophic support that the decision α is appropriate. We regard Approve as anchor b and measure contradiction $c(\alpha,b)$ between α and b.

Numerical setup (before transform). Take

$$\begin{aligned} pdf(u,\text{Approve}) &= (0.65,\ 0.10,\ 0.25),\\ pdf(u,\text{Deny}) &= (0.40,\ 0.20,\ 0.45),\\ pdf(u,\text{ManualReview}) &= (0.35,\ 0.30,\ 0.40), \end{aligned} \tag{118}$$

and contradiction degrees

$$c(\text{Deny},\text{Approve}) = 0.88, \qquad c(\text{ManualReview},\text{Approve}) = 0.50. \tag{119}$$

Set the threshold $\lambda = 0.80$ so that only Deny is *activated*. Choose the bounded-sum t-conorm $S_{\text{bs}}(x,y) = \min\{1, x+y\}$ and select

$$\mathrm{MO}(\text{Deny}) = \text{Absorb}. \tag{120}$$

Absorb mode (reset option). By Definition 9, for Absorb, we set $T' = F' = 0$ and move all support into I' via the t-conorm:

$$I' = S_{\text{bs}}\Big(I,\ S_{\text{bs}}(T,F)\Big). \tag{121}$$

For $\alpha =$ Deny, we compute

$$S_{\text{bs}}(T,F) = \min\{1,\ 0.40 + 0.45\} = 0.85, \tag{122}$$

$$I' = \min\{1,\ 0.20 + 0.85\} = 1.00, \tag{123}$$

hence

$$pdf^U(u,\text{Deny}) = (0,\ 1.00,\ 0). \tag{124}$$

Non-activated values remain unchanged:

$$pdf^U(u,\text{Approve}) = (0.65, 0.10, 0.25), \tag{125}$$

$$pdf^U(u,\text{ManualReview}) = (0.35, 0.30, 0.40). \tag{126}$$

With the *reset* choice for the contradiction map, the processed pair is neutralized:

$$pConF^U(\text{Deny},\text{Approve}) = pConF^U(\text{Approve},\text{Deny}) = 0, \tag{127}$$

while all other $pConF$-entries are left unchanged.

Interpretation of the update. Because crucial documents are missing (high contradiction with "Approve"), the system refuses to assert "Approve" or "Deny" as true/false for the Deny decision and *absorbs* the support into indeterminacy, signaling a state that is neither true nor false but requires resolution. The reset erases the immediate antagonism between Deny and Approve for this case, avoiding downstream oscillations in subsequent evaluations.

Proposition 2 (Well-posedness and range preservation). *Let $S : [0,1]^2 \to [0,1]$ be any t-conorm on $[0,1]$; that is, S is commutative, associative, and monotone in each argument, and satisfies the boundary conditions $S(x,0) = x$ and $S(x,1) = 1$. Then, for every $(u,\alpha) \in P \times \mathrm{Dom}(v)$, the transformed triplet $pdf^U(u,\alpha)$ belongs to $[0,1]^3$. In particular, for the bounded-sum t-conorm $S_{\mathrm{bs}}(x,y) = \min\{1,x+y\}$, the* Absorb *update yields*

$$0 \le I'_P(u \mid \alpha) \tag{128}$$

$$= S_{\mathrm{bs}}\Big(I_P(u \mid \alpha),\, S_{\mathrm{bs}}\big(T_P(u \mid \alpha), F_P(u \mid \alpha)\big)\Big) \tag{129}$$

$$= \min\big\{1,\, I_P(u \mid \alpha) + T_P(u \mid \alpha) + F_P(u \mid \alpha)\big\} \le 1, \tag{130}$$

and $T'_P(u \mid \alpha), F'_P(u \mid \alpha) \in [0,1]$ by construction. Consequently, $U^{(3)}_{b,\lambda,\mathrm{MO},S}$ is a well-defined endomorphism on the class of Plithogenic Neutrosophic Sets.

Proof. Fix $(u,\alpha) \in P \times \mathrm{Dom}(v)$ and write

$$(T,I,F) := \big(T_P(u \mid \alpha), I_P(u \mid \alpha), F_P(u \mid \alpha)\big) \in [0,1]^3. \tag{131}$$

We verify that the definition of $pdf^U(u,\alpha) = (T',I',F')$ produces a triplet in $[0,1]^3$ in each case.

Case 1: not activated. If $\mathsf{Actvt}(\alpha \mid b,\lambda) = 0$, then $(T',I',F') = (T,I,F) \in [0,1]^3$.

Case 2: activated and $\mathrm{MO}(\alpha) = \mathrm{Keep}$. The update is again the identity; hence, $(T',I',F') = (T,I,F) \in [0,1]^3$.

Case 3: activated and $\mathrm{MO}(\alpha) = \mathrm{Swap}$. The update permutes coordinates: $(T',I',F') = (F,I,T)$, which still lies in $[0,1]^3$ because each component of (T,I,F) lies in $[0,1]$.

Case 4: activated and $\mathrm{MO}(\alpha) = \mathrm{Absorb}$. By definition, $T' = F' = 0 \in [0,1]$ and

$$I' = S\big(I, S(T,F)\big). \tag{132}$$

Because $T,F \in [0,1]$ and S maps $[0,1]^2$ into $[0,1]$, we have $S(T,F) \in [0,1]$; applying S once more gives $I' \in [0,1]$. Thus, $(T',I',F') \in [0,1]^3$.

Therefore, in all branches, the membership update sends $[0,1]^3$ to $[0,1]^3$. Finally, the optional *reset* rule for the contradiction map replaces certain values $pConF(\cdot,\cdot) \in [0,1]$ by $0 \in [0,1]$ and leaves all other entries unchanged; hence, it preserves the codomain $[0,1]$. This shows that $U^{(3)}_{b,\lambda,\mathrm{MO},S}$ is well defined on Plithogenic Neutrosophic Sets.

4. Three-Mode De-Plithogenication for Plithogenic Neutrosophic Sets

A Three-Mode De-Plithogenication in a Plithogenic Neutrosophic Set is a procedure neutralizing contradictions by sequential Keep, Swap, and Absorb modes on activated attributes, aggregating uncertainty and optionally resetting contradiction degrees.

Definition 10 (Three-Mode De-Plithogenication in a Plithogenic Neutrosophic Set). Let $PS = (P,v,\mathrm{Dom}(v),pdf,pConF)$ be a Plithogenic Neutrosophic Set with

$$pdf(u,\alpha) = \big(T_P(u \mid \alpha), I_P(u \mid \alpha), F_P(u \mid \alpha)\big) \in [0,1]^3, \tag{133}$$

$$pConF : \mathrm{Dom}(v) \times \mathrm{Dom}(v) \to [0,1] \tag{134}$$

(symmetric, $pConF(\alpha,\alpha) = 0$). Fix:

1) an *anchor* $b \in \mathrm{Dom}(v)$ together with a *threshold* $\lambda \in [0,1]$;
2) a *mode-selection map* $\mathrm{MO} : \mathrm{Dom}(v) \to \{\mathrm{Keep},\mathrm{Swap},\mathrm{Absorb}\}$;
3) a t-conorm $S : [0,1]^2 \to [0,1]$ (a standard option is the bounded sum $S_{\mathrm{bs}}(x,y) = \min 1, x+y$).

Define activation by

$$\mathsf{Actvt}(\alpha \mid b,\lambda) := \mathbf{1}[\, pConF(\alpha,b) \ge \lambda\,]. \tag{135}$$

The *single-step three-mode De-Plithogenication operator*

$$\mathscr{D}^{(3)}_{b,\lambda,\mathrm{MO},S} :\ (pdf, pConF) \longmapsto (pdf^\star, pConF^\star) \tag{136}$$

is given pointwise for each $(u,\alpha) \in P \times \mathrm{Dom}(v)$ by

$$pdf^\star(u,\alpha) = \begin{cases} (T_P, I_P, F_P), \\ \mathsf{Actvt}(\alpha \mid b,\lambda) = 0, \\ (T_P, I_P, F_P), \\ \mathsf{Actvt}(\alpha \mid b,\lambda) = 1 \ \wedge\ \mathrm{MO}(\alpha) = \mathrm{Keep}, \\ (F_P, I_P, T_P), \\ \mathsf{Actvt}(\alpha \mid b,\lambda) = 1 \ \wedge\ \mathrm{MO}(\alpha) = \mathrm{Swap}, \\ \big(0, S\big(I_P, S(T_P,F_P)\big), 0\big), \\ \mathsf{Actvt}(\alpha \mid b,\lambda) = 1 \ \wedge\ \mathrm{MO}(\alpha) = \mathrm{Absorb}, \end{cases} \tag{137}$$

where $T_P = T_P(u \mid \alpha)$, $I_P = I_P(u \mid \alpha)$, $F_P = F_P(u \mid \alpha)$, and S is applied as a t-conorm. The *contradiction reset* is

$$pConF^\star(\beta,\gamma) = \begin{cases} 0, & \text{if } \{\beta,\gamma\} = \{\alpha,b\} \\ & \text{ and } \mathsf{Actvt}(\alpha \mid b,\lambda) = 1, \\ pConF(\beta,\gamma), & \text{otherwise.} \end{cases} \tag{138}$$

A *Three-Mode De-Plithogenication (sequence)* is a finite composition

$$PS^{\mathrm{dep3}} := \mathscr{D}^{(3)}_{b_k,\lambda_k,\mathrm{MO}_k,S} \circ \cdots \circ \mathscr{D}^{(3)}_{b_2,\lambda_2,\mathrm{MO}_2,S} \circ \mathscr{D}^{(3)}_{b_1,\lambda_1,\mathrm{MO}_1,S}(PS) \tag{139}$$

such that the final contradiction map is identically zero, i.e., $pConF^{\mathrm{dep3}}(\beta,\gamma) = 0$ for all $\beta,\gamma \in \mathrm{Dom}(v)$. We call PS^{dep3} the *three-mode de-plithogenicated normal form*.

Example 13 (Three-step De-Plithogenication on three attribute values). **Setup.** Let $P = \{u\}$ and $\mathrm{Dom}(v) = \{A,B,C\}$. The initial neutrosophic triplets (for decision/proposition u) are

$$pdf(u,A) = (0.62,\, 0.18,\, 0.25), \tag{140}$$

$$pdf(u,B) = (0.28,\, 0.22,\, 0.66), \tag{141}$$

$$pdf(u,C) = (0.45,\, 0.27,\, 0.41). \tag{142}$$

Initial contradictions:

$$pConF(A,B) = 0.81, \quad pConF(A,C) = 0.57, \quad pConF(B,C) = 0.69 \tag{143}$$

(symmetry and zero diagonal understood). Use the bounded-sum t-conorm S_{bs}.

Step 1 (anchor A, threshold $\lambda_1 = 0.65$). Activation against A:

$$\mathsf{Actvt}(B \mid A, 0.65) = \mathbf{1}[0.81 \ge 0.65] = 1, \tag{144}$$

$$\mathsf{Actvt}(C \mid A, 0.65) = \mathbf{1}[0.57 \ge 0.65] = 0. \tag{145}$$

Choose $\mathrm{MO}_1(B) = \text{Swap}$ (treat B as a genuine reversal), any choice for C is irrelevant (not activated). Then,

$$pdf^{(1)}(u,B) = (F,I,T) = (0.66,\ 0.22,\ 0.28), \tag{146}$$

$$pdf^{(1)}(u,A) = (0.62,\ 0.18,\ 0.25), \tag{147}$$

$$pdf^{(1)}(u,C) = (0.45,\ 0.27,\ 0.41). \tag{148}$$

Reset processed pair: $pConF^{(1)}(A,B) = 0$, with other entries unchanged:

$$pConF^{(1)}(A,C) = 0.57, \qquad pConF^{(1)}(B,C) = 0.69. \tag{149}$$

Step 2 (anchor C, threshold $\lambda_2 = 0.60$). Activation against C under $pConF^{(1)}$:

$$\mathsf{Actvt}(B \mid C, 0.60) = \mathbf{1}[0.69 \geq 0.60] = 1, \tag{150}$$

$$\mathsf{Actvt}(A \mid C, 0.60) = \mathbf{1}[0.57 \geq 0.60] = 0. \tag{151}$$

Choose $\mathrm{MO}_2(B) = \text{Absorb}$. Compute for B with S_{bs}:

$$S_{\mathrm{bs}}\big(T_B^{(1)}, F_B^{(1)}\big) = \min\{1,\ 0.28 + 0.66\} = 0.94, \tag{152}$$

$$I'_B = \min\{1,\ 0.22 + 0.94\} = 1.00, \tag{153}$$

so

$$pdf^{(2)}(u,B) = (0,\ 1.00,\ 0), \tag{154}$$

while A and C remain as in step 1. Reset the processed pair: $pConF^{(2)}(B,C) = 0$. At this point,

$$pConF^{(2)}(A,B) = 0, \tag{155}$$

$$pConF^{(2)}(B,C) = 0, \tag{156}$$

$$pConF^{(2)}(A,C) = 0.57. \tag{157}$$

Step 3 (anchor A, threshold $\lambda_3 = 0.57$). Activation against A: $\mathsf{Actvt}(C \mid A, 0.57) = \mathbf{1}[0.57 \geq 0.57] = 1$. Choose $\mathrm{MO}_3(C) = \text{Keep}$ (we only wish to neutralize the contradiction). Thus, $pdf^{(3)}(u,C) = pdf^{(2)}(u,C) = (0.45,\ 0.27,\ 0.41)$, and we reset

$$pConF^{(3)}(A,C) = 0. \tag{158}$$

Resulting normal form. All pairwise contradictions are now zero:

	A	B	C
A	0.00	0.00	0.00
B	0.00	0.00	0.00
C	0.00	0.00	0.00

(159)

with the final neutrosophic triplets

$$\begin{aligned} pdf^{\mathrm{dep3}}(u,A) &= (0.62,\ 0.18,\ 0.25),\\ pdf^{\mathrm{dep3}}(u,B) &= (0,\ 1.00,\ 0),\\ pdf^{\mathrm{dep3}}(u,C) &= (0.45,\ 0.27,\ 0.41). \end{aligned} \tag{160}$$

The sequence demonstrates all three modes: *Swap* (Step 1) for a genuine reversal, *Absorb* (Step 2) to encode "neither true nor false" by moving support into indeterminacy, and *Keep* (Step 3) to neutralize a residual contradiction without altering memberships.

Example 14 (Severe-weather school operations: Keep + Absorb to neutralize conflicts). **Interpretation.** Let $P = \{u\}$ with the proposition u = "Today's chosen operating mode is appropriate." Let the set of admissible attribute values be

$$\mathrm{Dom}(v) = \{\text{OpenCampus},\ \text{OnlineOnly},\ \text{CloseCampus}\}. \tag{161}$$

For each $\alpha \in \mathrm{Dom}(v)$, $pdf(u,\alpha) = (T,I,F) \in [0,1]^3$ is the neutrosophic support that option α is appropriate. The contradiction degrees $pConF$ satisfy $pConF(\alpha,\beta) = pConF(\beta,\alpha)$ and $pConF(\alpha,\alpha) = 0$. Use the bounded-sum t-conorm $S_{\mathrm{bs}}(x,y) = \min\{1, x+y\}$.

Initial data.

$$\begin{aligned} pdf(u,\text{OpenCampus}) &= (0.58,\ 0.12,\ 0.36),\\ pdf(u,\text{OnlineOnly}) &= (0.47,\ 0.28,\ 0.31),\\ pdf(u,\text{CloseCampus}) &= (0.26,\ 0.33,\ 0.63), \end{aligned} \tag{162}$$

$$\begin{aligned} pConF(\text{CloseCampus},\text{OpenCampus}) &= 0.86,\\ pConF(\text{OnlineOnly},\text{OpenCampus}) &= 0.61,\\ pConF(\text{OnlineOnly},\text{CloseCampus}) &= 0.46. \end{aligned} \tag{163}$$

Step 1 (anchor b = OpenCampus, threshold $\lambda_1 = 0.80$). Activation:

$$\mathsf{Actvt}(\text{CloseCampus} \mid b, \lambda_1) = \mathbf{1}[0.86 \geq 0.80] = 1, \tag{164}$$

$$\mathsf{Actvt}(\text{OnlineOnly} \mid b, \lambda_1) = \mathbf{1}[0.61 \geq 0.80] = 0. \tag{165}$$

Choose the mode *Absorb* for CloseCampus. Then,

$$S_{\mathrm{bs}}(T,F) = \min\{1,\ 0.26 + 0.63\} = 0.89, \tag{166}$$

$$I' = \min\{1,\ 0.33 + 0.89\} = 1.00, \tag{167}$$

so

$$pdf^{(1)}(u,\text{CloseCampus}) = (0,\ 1.00,\ 0), \tag{168}$$

while

$$pdf^{(1)}(u,\text{OpenCampus}) = (0.58, 0.12, 0.36), \tag{169}$$

$$pdf^{(1)}(u,\text{OnlineOnly}) = (0.47, 0.28, 0.31). \tag{170}$$

Reset the processed pair:

$$pConF^{(1)}(\text{CloseCampus},\text{OpenCampus}) = 0, \tag{171}$$

$$pConF^{(1)}(\text{OnlineOnly},\text{OpenCampus}) = 0.61, \tag{172}$$

$$pConF^{(1)}(\text{OnlineOnly},\text{CloseCampus}) = 0.46. \tag{173}$$

Step 2 (same anchor b = OpenCampus, threshold $\lambda_2 = 0.60$). Now $\mathsf{Actvt}(\text{OnlineOnly} \mid b, \lambda_2) = \mathbf{1}[0.61 \geq 0.60] = 1$. Choose *Keep* for OnlineOnly; hence, pdf is unchanged, and we reset

$$pConF^{(2)}(\text{OnlineOnly},\text{OpenCampus}) = 0. \tag{174}$$

The remaining contradiction is $pConF^{(2)}(\text{OnlineOnly},\text{CloseCampus}) = 0.46$.

Step 3 (anchor b = OnlineOnly, threshold $\lambda_3 = 0.46$). $\mathsf{Actvt}(\text{CloseCampus} \mid b, \lambda_3) = \mathbf{1}[0.46 \geq 0.46] = 1$. Choose *Keep* (we only wish to neutralize). Thus, pdf remains, and we reset

$$pConF^{(3)}(\text{OnlineOnly},\text{CloseCampus}) = 0. \tag{175}$$

Outcome (three-mode de-plithogenicated normal form). All pairwise contradictions are zero; the final triplets are

$$\begin{aligned} pdf^{\text{dep3}}(u,\text{OpenCampus}) &= (0.58,\ 0.12,\ 0.36),\\ pdf^{\text{dep3}}(u,\text{OnlineOnly}) &= (0.47,\ 0.28,\ 0.31),\\ pdf^{\text{dep3}}(u,\text{CloseCampus}) &= (0,\ 1.00,\ 0). \end{aligned} \tag{176}$$

This sequence uses *Absorb* to encode "neither true nor false yet" for closure, and *Keep* steps to neutralize the remaining contradictions without altering memberships.

Example 15 (Clinical triage plan: Swap + Absorb + Keep to stabilize recommendations). **Interpretation.** Let $P = \{u\}$ with $u =$ "The plan is appropriate now." Take

$$\text{Dom}(v) = \{\text{StartAntibiotics},\ \text{WatchfulWaiting},\ \text{SendToER}\}. \tag{177}$$

As before, $pdf(u,\alpha) = (T,I,F)$ and $pConF$ is symmetric with zero diagonal. Use S_{bs} as the t-conorm.

Initial data.

$$\begin{aligned} pdf(u,\text{StartAntibiotics}) &= (0.42,\ 0.26,\ 0.54),\\ pdf(u,\text{WatchfulWaiting}) &= (0.57,\ 0.22,\ 0.33),\\ pdf(u,\text{SendToER}) &= (0.32,\ 0.18,\ 0.70), \end{aligned} \tag{178}$$

$$\begin{aligned} pConF(\text{SendToER},\text{StartAntibiotics}) &= 0.91,\\ pConF(\text{SendToER},\text{WatchfulWaiting}) &= 0.74,\\ pConF(\text{StartAntibiotics},\text{WatchfulWaiting}) &= 0.58. \end{aligned} \tag{179}$$

Step 1 (anchor $b =$ StartAntibiotics, threshold $\lambda_1 = 0.85$). Activation:

$$\mathsf{Actvt}(\text{SendToER} \mid b,\lambda_1) = \mathbf{1}[0.91 \geq 0.85] = 1. \tag{180}$$

Choose *Swap* for SendToER to reflect a genuine reversal under red flags:

$$pdf^{(1)}(u,\text{SendToER}) = (F,I,T) = (0.70,\ 0.18,\ 0.32). \tag{181}$$

Reset the processed pair:

$$pConF^{(1)}(\text{SendToER},\text{StartAntibiotics}) = 0, \tag{182}$$

with the other entries unchanged.

Step 2 (anchor $b =$ WatchfulWaiting, threshold $\lambda_2 = 0.70$).

$$\mathsf{Actvt}(\text{SendToER} \mid b,\lambda_2) = \mathbf{1}[0.74 \geq 0.70] = 1. \tag{183}$$

Choose *Absorb* for SendToER to encode "neither true nor false until diagnostics":

$$S_{\text{bs}}\big(T^{(1)},F^{(1)}\big) = \min\{1,\ 0.70+0.32\} = 1.00, \tag{184}$$

$$I' = \min\{1,\ 0.18+1.00\} = 1.00, \tag{185}$$

hence

$$pdf^{(2)}(u,\text{SendToER}) = (0,\ 1.00,\ 0). \tag{186}$$

Reset the processed pair:

$$pConF^{(2)}(\text{SendToER},\text{WatchfulWaiting}) = 0. \tag{187}$$

Step 3 (anchor $b =$ StartAntibiotics, threshold $\lambda_3 = 0.58$).

$$\mathsf{Actvt}(\text{WatchfulWaiting} \mid b,\lambda_3) = \mathbf{1}[0.58 \geq 0.58] = 1. \tag{188}$$

Choose *Keep* for WatchfulWaiting (we only neutralize):

$$pdf^{(3)}(u,\text{WatchfulWaiting}) = (0.57, 0.22, 0.33), \tag{189}$$

$$pConF^{(3)}(\text{StartAntibiotics},\text{WatchfulWaiting}) = 0. \tag{190}$$

Outcome (three-mode de-plithogenicated normal form). All pairwise contradictions are zero, with final triples

$$\begin{aligned} pdf^{\text{dep3}}(u,\text{StartAntibiotics}) &= (0.42,\ 0.26,\ 0.54),\\ pdf^{\text{dep3}}(u,\text{WatchfulWaiting}) &= (0.57,\ 0.22,\ 0.33),\\ pdf^{\text{dep3}}(u,\text{SendToER}) &= (0,\ 1.00,\ 0). \end{aligned} \tag{191}$$

This sequence demonstrates *Swap* (genuine reversal), *Absorb* (uncertainty consolidation), and *Keep* (neutralization without reweighting), culminating in a stable, contradiction-free representation.

5. Conclusion

We present *Three-Mode Upside-Down Logic*, an enhanced form of Upside-Down Logic, alongside *De-Plithogenication*, and we analyze their behavior within the setting of Plithogenic Neutrosophic Sets. We believe that the proposed approach enables inversion phenomena arising in highly uncertain settings to be expressed more clearly and interpreted more transparently.

As future work, we anticipate further investigations of Plithogenic Neutrosophic Sets and Three-Mode Upside-Down Logic, including their integration with Graphs [62], HyperGraphs [63, 64], and SuperHyperGraphs [65–69]. Moreover, because the present study is primarily theoretical, we expect subsequent research to pursue quantitative assessments of the proposed notions through computational experiments and to further explore the design of related algorithms. Finally, we are also interested in whether additional extensions, such as Four-Mode Upside-Down Logic and more general Multi-Mode Upside-Down Logic, can be formulated and studied.

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Ethical Statement

This study does not contain any studies with human or animal subjects performed by the author.

Conflicts of Interest

The author declares that he has no conflicts of interest to this work.

Data Availability Statement

Data sharing is not applicable to this article as no new data were created or analyzed in this study.

Author Contribution Statement

Takaaki Fujita: Conceptualization, Methodology, Software, Validation, Formal analysis, Investigation, Resources, Data curation, Writing – original draft, Writing – review & editing, Visualization, Supervision, Project administration.

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