

RESEARCH ARTICLE



Disturbance Observer-Based Distributed Formation-Tracking Control for Multi-Cluster Quadrotor Cooperative Suspended Payload Systems

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Abstract: Multi-Cluster Quadrotor Cooperative Suspended Payload Systems (MQCSPS) offer enhanced payload capacity and transportation flexibility, making them promising for autonomous aerial transportation in complex and uncertain environments. However, unknown external disturbances and the coupled dynamics among multiple quadrotors and the payload pose significant challenges to accurate formation tracking and stable cooperative transportation. To address the distributed formation-tracking control problem of an MQCSPS subject to unknown disturbances, a disturbance-observer-based distributed formation-tracking control method is proposed. Specifically, distributed sliding-mode formation controllers and sliding-mode tracking controllers are designed for the payload and quadrotors, respectively, enabling accurate trajectory tracking of the quadrotors while maintaining the desired distributed formation of the payload. A disturbance observer is incorporated to estimate and compensate for the effects of unknown disturbances, thereby improving the disturbance-rejection capability of the closed-loop system. Through Lyapunov-based stability analysis, the uniform ultimate boundedness of the closed-loop tracking errors is established. Simulation results demonstrate that the proposed method achieves accurate payload formation tracking and stable quadrotor motion in the presence of disturbances, validating its effectiveness for cooperative aerial transportation in complex environments.

Keywords: disturbance observer, distributed formation control, quadrotor cooperative suspended payload system, sliding-mode control

1. Introduction

In recent years, unmanned aerial vehicle (UAV) suspended transportation technology has attracted significant attention owing to its high maneuverability, rapid deployment capability, and operational flexibility, demonstrating broad application prospects in low-altitude economy, intelligent logistics, and emergency rescue scenarios [1, 2, 3]. However, the payload capacity of a single UAV is fundamentally constrained by its thrust and lift, making it insufficient for large-scale cargo transport. Cooperative transportation using multiple UAVs effectively enhances payload capacity, operational reliability, and energy efficiency, thereby emerging as an active research topic [4]. With the rapid advancement of cooperative control and autonomous decision-making technologies, multi-UAV transportation systems are evolving from small-scale cooperation to cluster-based collaborative transportation, enabling efficient aerial logistics over large areas and exhibiting great potential for complex applications such

as mountainous logistics and disaster relief. Nevertheless, with an increasing number of UAVs, the overall system exhibits strong nonlinearity, high dimensionality, and intense dynamic couplings, creating significant challenges for conventional control methods to simultaneously maintain transportation accuracy, formation stability, and robustness. Therefore, developing high-performance control strategies for cluster-based cooperative UAV transportation systems with suspended payloads subject to unknown disturbances is of significant theoretical and practical value.

In recent years, extensive efforts have been devoted to the modeling and control of cooperative multi-UAV suspended transportation systems, leading to substantial progress. Regarding dynamic modeling [5, 6, 7], comprehensive dynamic models have been established for systems involving flexible cables, rigid-flexible coupling, and cooperative transportation by multiple UAVs, providing a solid theoretical foundation for controller design. In terms of control strategies, robust control [8], backstepping control [9], and nonlinear control [10] have been extensively investigated to improve trajectory-tracking accuracy and disturbance-rejection performance. Among these approaches, sliding-mode control (SMC) has garnered considerable attention owing to its inherent robustness against parameter uncertainties and external

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disturbances, rendering it well suited for UAV suspended load systems [11]. Furthermore, to improve disturbance-rejection performance under complex operating conditions, disturbance-compensation approaches including the extended state observer [12] and the disturbance observer [13] have been integrated into UAV control systems for real-time estimation and compensation of unknown disturbances. Despite these advances, existing studies have primarily focused on single suspended load systems or centralized cooperative control frameworks, while distributed cooperative control for multi-cluster quadrotor cooperative suspended payload systems (MQCSPSs) remains relatively underexplored. On the one hand, the complex dynamic couplings and communication topology constraints among multiple UAVs significantly limit the applicability of centralized control schemes. As the system scale increases, both computational complexity and communication overhead grow rapidly, resulting in poor scalability. On the other hand, most existing distributed control approaches assume that external disturbances are either known or accurately modelable. In contrast, the combined effects of unknown external disturbances and modeling uncertainties are rarely considered. Consequently, transportation accuracy and formation stability cannot be effectively guaranteed in complex environments. Therefore, there is an urgent need to develop a novel distributed control strategy that simultaneously possesses distributed cooperative capability and effective compensation for unknown disturbances, thereby improving the cooperative transportation performance and robustness of MQCSPSs under complex operating conditions.

Motivated by the above observations, a distributed formation-tracking control method based on a disturbance observer is proposed. To simultaneously tackle cooperative payload formation, payload transportation, and quadrotor trajectory tracking, a distributed sliding-mode formation controller is developed for payloads. Meanwhile, a sliding-mode trajectory-tracking controller is devised for quadrotors, which facilitates the construction of a unified cooperative control framework. The key contribution of the proposed approach is to treat the payloads as distributed cooperative nodes and establish a distributed formation mechanism based solely on local communication among neighboring payloads. Through information exchange between payloads, multiple quadrotor-suspended transportation systems can achieve coordinated formation flight without requiring a global communication network among all quadrotors, thereby significantly reducing communication overhead and improving

the scalability and robustness of the overall control architecture. Furthermore, a disturbance observer is incorporated to online estimate and compensate for unknown disturbances, thereby improving both the accuracy of formation transportation and system stability under complex operating environments.

Notation Description: For matrices A and B , $A \times B$ and $A \otimes B$ denote the matrix cross product and the Kronecker product, respectively. $\lambda_{\min}(A)$ denotes the minimum eigenvalue of matrix

$$A. \text{ For the vector } X = [X_1, X_2, X_3]^T, X^\times = \begin{bmatrix} 0 & -X_3 & X_2 \\ X_3 & 0 & -X_1 \\ -X_2 & X_1 & 0 \end{bmatrix}$$

denotes the skew-symmetric matrix of X . I_3 and I_{3n} are identity matrices with dimensions 3 and $3n$, respectively.

2. Problem Description

2.1. Dynamics model

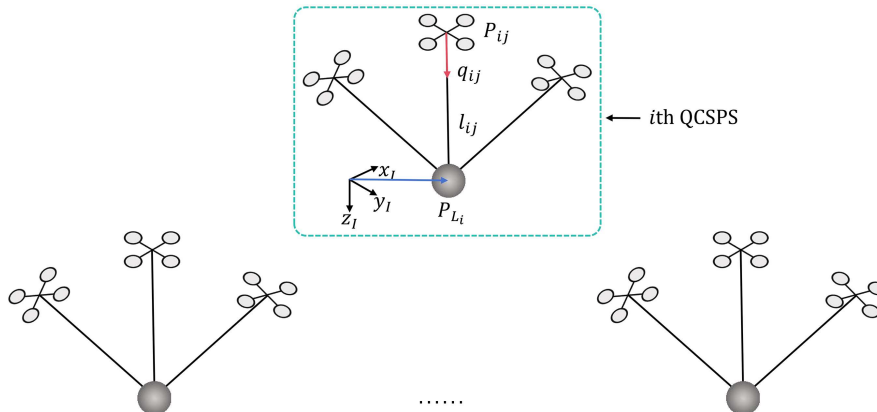
Figure 1 illustrates a schematic diagram of the MQCSPS, which consists of n quadrotor cooperative suspension-payload systems (QCSPS). Each QCSPS comprises m quadrotors connected to a payload via linkages. As shown in Figure 1, we define the inertial frame as $I = \{x_I, y_I, z_I\}$. The positions of the j th quadrotor and the payload in the i th QCSPS are denoted as $P_{ij} = [x_{ij}, y_{ij}, z_{ij}]^T (i = 1, \dots, n; j = 1, \dots, m)$ and $P_{Li} = [x_{Li}, y_{Li}, z_{Li}]^T$, with their respective masses denoted as m_{ij} and m_{Li} . The length and direction vector of the j th suspension rope in the i th QCSPS are denoted as l_{ij} and q_{ij} , respectively. According to Figure 1, under the assumption that both the payload and the suspension rope are rigid bodies [6], the quadrotor position in the i th QCSPS can be expressed as $P_{ij} = P_{Li} - l_{ij}q_{ij}$.

The control input u_{ij} for the j th quadrotor in the i th QCSPS is defined as $u_{ij} = u_{ij}^p + u_{ij}^v$, where u_{ij}^p and u_{ij}^v represent the orthogonal projections of u_{ij} along the direction of q_{ij} and onto the plane perpendicular to q_{ij} , respectively. Let μ_{ij} be the virtual control signal parallel to q_{ij} . Then, the parallel component of u_{ij} can be expressed as [14]

$$u_{ij}^p = m_{ij}l_{ij}||v_{ij}||^2 q_{ij} + m_{ij}q_{ij}q_{ij}^T \left(\frac{1}{m_{Li}} \sum_{j=1}^m \mu_{ij} \right) + \mu_{ij} \quad (1)$$

where v_{ij} represents the angular velocity of the j th suspension rope in the i th QCSPS.

Figure 1
Schematic diagram of MQCSPS



In the presence of unknown disturbances, the model equations of the i th QCSPPS are described as [14]

$$\ddot{P}_{Li} = ge_3 + \frac{1}{m_{Li}} \sum_{j=1}^m \mu_{ij} + D_{Li} \quad (2)$$

$$\dot{q}_{ij} = v_{ij} \times q_{ij} \quad (3)$$

$$\dot{v}_{ij} = \frac{1}{m_{Li}l_{ij}} q_{ij}^\times \sum_{j=1}^m \mu_{ij} - \frac{1}{m_{ij}l_{ij}} q_{ij}^\times u_{ij}^v + D_{ij} \quad (4)$$

where g represents the gravitational acceleration, e_3 denotes the unit vector, and D_{Li} and D_{ij} represent unknown disturbances.

Remark 1: The generalized control input u_{ij} of each quadrotor is decomposed into the components parallel and perpendicular to its suspension cable. The parallel component u_{ij}^p is responsible for regulating the translational motion and formation relationship among different Suspended Payload subsystems and is determined by the distributed payload formation controller. In contrast, the perpendicular component u_{ij}^v regulates the cable direction and is determined by the centralized quadrotor tracking controller within each subsystem. This input decomposition effectively decouples the inter-subsystem formation coordination from the intra-subsystem quadrotor coordination.

2.2. Graph theory

The network topology for payload formation is an undirected graph [15, 16, 17]. The communication architecture is represented by the graph $\mathcal{G} = (V_{\mathcal{G}}, E_{\mathcal{G}})$, comprising n vertices and L edges, where $V_{\mathcal{G}}$ and $E_{\mathcal{G}}$ are defined as $V_{\mathcal{G}} = \{1, \dots, n\}$ and $E_{\mathcal{G}} = \{(i, k) | i, k \in V_{\mathcal{G}}, i \neq k\}$, respectively. The neighbor set $N_i(E_{\mathcal{G}})$ of vertex i is defined as $N_i(E_{\mathcal{G}}) = \{k \in V_{\mathcal{G}} | (i, k) \in E_{\mathcal{G}}\}$. The adjacency matrix $\mathcal{A} = [a_{ik}]$ determines whether vertex i can receive information from vertex k , where $a_{ik} = 1$ indicates a connection, and $a_{ik} = 0$ otherwise. The degree matrix $D = \text{diag}(D_1, \dots, D_n)$ is obtained by summing the row elements of $\mathcal{A}$, and the Laplacian matrix $\mathcal{L}$ is defined as $\mathcal{L} = D - \mathcal{A}$.

Additionally, for a payload formation consisting of one leader and n followers, another graph $\tilde{\mathcal{G}}$ is introduced. The adjacency matrix of the leader is defined as $B = \text{diag}(b_1, \dots, b_n)$, where $b_i = 1$ indicates that the i th follower can receive data from the leader, and $b_i = 0$ otherwise.

2.3. Control objective

The control objective of this paper is to design a distributed formation-tracking control method based on a disturbance observer for the MQCSPS under unknown disturbances, achieving formation control of the payload and trajectory-tracking control of the quadrotors. To facilitate the controller design, the following assumptions are provided:

Assumption 1: All cables remain taut during the transportation process under consideration. Each cable is modeled as a massless and inextensible link with a constant length $l_{ij} > 0$, and all system states are measurable [18]. Moreover, the attachment point of each suspension cable is assumed to coincide with the center of mass of the corresponding quadrotor [4].

Assumption 2: The unknown disturbances D_{Li} and D_{ij} are bounded and continuously differentiable, with bounded first-order time derivatives, i.e., $\|\dot{D}_{Li}\| < \epsilon_{Li}$ and $\|\dot{D}_{ij}\| < \epsilon_{ij}$, where ϵ_{Li} and ϵ_{ij} are unknown bounded positive constants [19].

Assumption 3: The attitude/thrust inner loop of each quadrotor has a sufficiently fast response compared with the translational outer-loop dynamics [11]. Consequently, the desired attitude and thrust generated by the outer-loop controller can be tracked with sufficiently small bounded errors within the considered operating region.

Remark 2: Under Assumption 3, the attitude/thrust inner loop can sufficiently rapidly realize the force command generated by the outer-loop controller. Therefore, the inner-loop tracking error can be regarded as a bounded perturbation to the outer-loop dynamics. As a result, the uniformly ultimately bounded property established for the outer-loop system is preserved under bounded inner-loop tracking errors, although the corresponding ultimate bound may increase with the magnitude of the inner-loop error.

3. Controller Design

A distributed formation-tracking control method based on a disturbance observer is developed for the MQCSPS subject to unknown disturbances. First, distributed formation-tracking errors are formulated according to the distributed communication topology among the payloads. By integrating a disturbance observer with an SMC strategy, distributed formation-tracking controllers are designed to generate the desired position trajectories for the quadrotors. Subsequently, for each quadrotor in each QCSPPS, a disturbance-observer-based sliding-mode tracking controller is designed to accurately track the desired trajectories, thereby enabling cooperative multi-UAV payload transportation. The overall control framework of the MQCSPS is illustrated in Figure 2.

3.1. Distributed formation control of payloads

The formation-tracking error of the i th payload is defined as $e_{Li} = b_i(P_{Li} - P_t - P_{Li}^d) + \sum_{k \in N_i} a_{ik}((P_{Li} - P_{Lk} - P_{Li}^d))$, where P_t represents the given trajectory of the virtual leader and P_{Li}^d and P_{Lk}^d represent the desired relative position of the i th payload with respect to the virtual leader and the k th payload, respectively. By defining $e_L = [e_{L1}^T, \dots, e_{Ln}^T]^T$, the compact form of the payload tracking error is expressed as

$$e_L = [(\mathcal{L} + B) \otimes I_3](P_L - P_L^d) \quad (5)$$

where $P_L = [P_{L1}^T, \dots, P_{Ln}^T]^T$, $P_L^d = [P_{L1}^{dT}, \dots, P_{Ln}^{dT}]^T$, $P_{Li}^d = P_t + P_{Li}^d$.

In view of (2), the compacted form of the payload dynamics is expressed as

$$\ddot{P}_L = A_L + B_L F_L + D_L \quad (6)$$

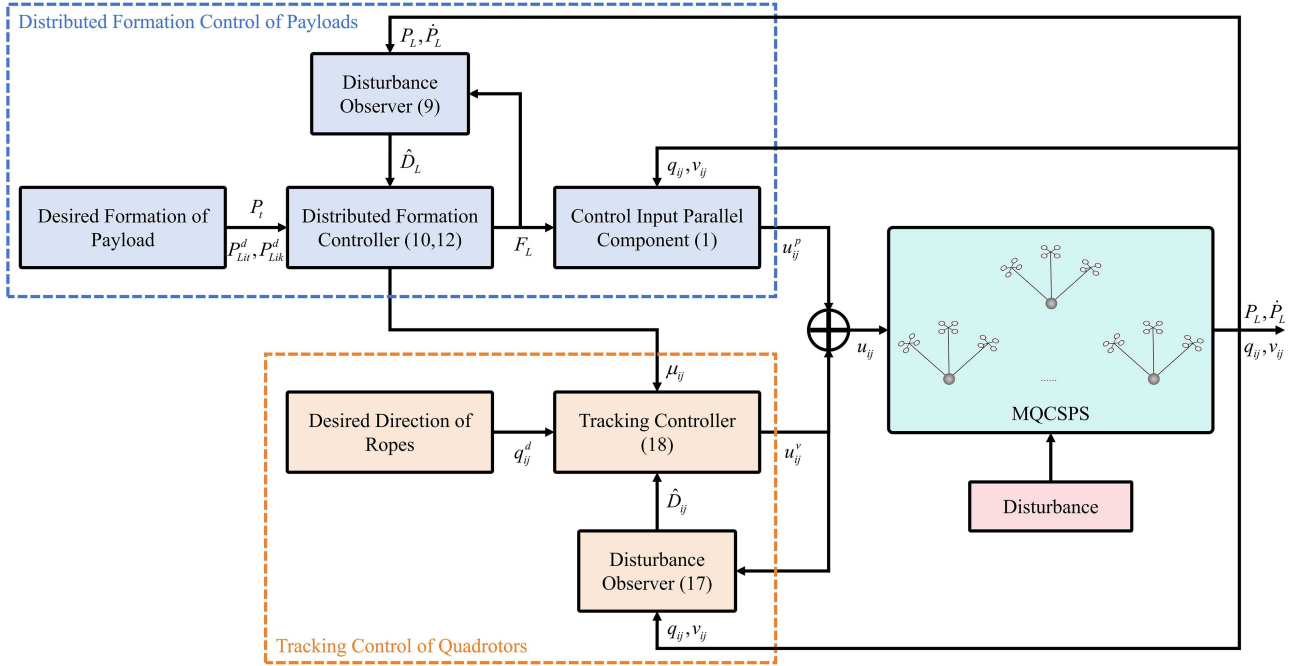
where $A_L = g[e_3^T, \dots, e_3^T]^T$, $B_L = \text{diag}(B_{L1}, \dots, B_{Ln}) \otimes I_3$, $F_L = [F_{L1}^T, \dots, F_{Ln}^T]^T$, $D_L = [D_{L1}^T, \dots, D_{Ln}^T]^T$, $B_{Li} = \frac{1}{m_{Li}}$, $F_{Li} = \sum_{j=1}^m \mu_{ij}$. The sliding-mode surface S_L is defined for the design of the payload formation controller:

$$S_L = K_L e_L + \dot{e}_L \quad (7)$$

where K_L represents the parameter matrix. Taking the derivative of (7) and combining it with (6) yields

$$\dot{S}_L = A_S + B_S F_L + [(\mathcal{L} + B) \otimes I_3] D_L \quad (8)$$

Figure 2
The overall control framework of the MQCSPS



where $A_S = K_L \dot{e}_L + [(\mathcal{L} + \mathcal{B}) \otimes I_3](A_L - \dot{P}_L^d)$ and $B_S = [(\mathcal{L} + \mathcal{B}) \otimes I_3]B_L$.

To address the impact of the disturbance D_L , the following disturbance observer is designed as

$$\begin{cases} \hat{D}_L = Z_L + \eta_L \dot{P}_L \\ \dot{Z}_L = [(\mathcal{L} + \mathcal{B}) \otimes I_3]^T S_L - \eta_L (A_L + B_L F_L + \hat{D}_L) \end{cases} \quad (9)$$

where η_L represents the gain matrix.

The virtual control signal F_L for the payload is designed as

$$F_L = -B_S^{-1}(K_{S_L} S_L + A_S + [(\mathcal{L} + \mathcal{B}) \otimes I_3] \hat{D}_L) \quad (10)$$

where K_{S_L} represents the parameter matrix.

Assumption 4: The cable configuration is assumed to satisfy $\text{rank}(W_i) = 3$, where $W_i = [q_{i1}, \dots, q_{im}] \in \mathbb{R}^{3 \times m}$. This condition requires at least three linearly independent cable direction vectors and guarantees that $W_i W_i^T$ is nonsingular.

Remark 3: Assumption 4 is physically reasonable for the considered multi-cable cooperative transportation system. Since $F_{Li} \in \mathbb{R}^3$ represents the desired three-dimensional force acting on the payload, at least three linearly independent cable directions are required to provide full three-dimensional force transmission. The condition $\text{rank}(W_i) = 3$ therefore excludes degenerate cable configurations, such as parallel or coplanar cable directions, in which the payload force cannot be arbitrarily distributed in $\mathbb{R}^3$. Moreover, this condition guarantees that $W_i W_i^T$ is positive definite and hence that the Moore–Penrose pseudoinverse can be expressed as $W_i^\dagger = W_i^T (W_i W_i^T)^{-1}$. Therefore, the desired payload force can be exactly reconstructed from the cable forces. This assumption is imposed only to ensure the well-posedness of the force-allocation problem and does not restrict the subsequent distributed control design to a specific number of cables.

Based on Assumption 4 and the force/moment allocation matrix properties established by Lee [6] and Wang et al. [20], the

desired payload force can be uniquely reconstructed when $m = 3$, whereas the force allocation problem becomes underdetermined when $m > 3$. The virtual control force assigned to the j th cable is given by

$$\mu_{ij} = [W_i^T (W_i W_i^T)^{-1} F_{Li}]_{\bar{j}}, \bar{j} \in 1, \dots, m \quad (11)$$

where $[H]_{\bar{j}}$ denotes the $\bar{j}$ th element of the vector H . This corresponds to the minimum parameter solution, which can be readily proven as [20]

$$\sum_{j=1}^m \mu_{ij} = W_i (W_i^T (W_i W_i^T)^{-1} F_{Li}) = F_{Li} \quad (12)$$

Remark 4: When $n \geq 3$, $\sum_{j=1}^m \mu_{ij}$ is always equal to F_{Li} , which permits arbitrary configurations of rope directions [20]. Accordingly, the desired trajectory P_{ij}^d for the j th quadrotor within the i th QCSPS is defined as $P_{ij}^d = P_{Li} - l_{ij} q_{ij}^d$, where q_{ij}^d denotes the desired direction vector of the suspension ropes. Consider the following Lyapunov function:

$$V_L = \frac{1}{2} S_L^T S_L + \frac{1}{2} \tilde{D}_L^T \tilde{D}_L \quad (13)$$

where $\tilde{D}_L = D_L - \hat{D}_L$ represents the disturbance estimation error.

By differentiating (13) and combining it with (6), (8), (9), and (10), the following is obtained:

$$\dot{V}_L = -S_L^T K_{S_L} S_L - \tilde{D}_L^T \eta_L \tilde{D}_L + \tilde{D}_L^T \dot{D}_L \quad (14)$$

3.2. Tracking control of quadrotors

The tracking error of the j th quadrotor within the i th QCSPS is defined as $e_{ij} = P_{ij} - P_{ij}^d = l_{ij}(q_{ij}^d - q_{ij})$. The

sliding-mode surface S_{ij} below is defined for the design of the quadrotor tracking controller:

$$S_{ij} = \mathcal{P}_{ij}(K_{ij}e_{ij} + \dot{e}_{ij}) \quad (15)$$

where K_{ij} represents the parameter matrix, and $\mathcal{P}_{ij} = I_3 - q_{ij}q_{ij}^T$ is an orthogonal projection matrix.

Taking the derivative of (13) and combining it with (3) and (4) yields

$$\begin{aligned} \dot{S}_{ij} &= \dot{\mathcal{P}}_{ij}(K_{ij}e_{ij} + \dot{e}_{ij}) + \mathcal{P}_{ij}\left(K_{ij}\dot{e}_{ij} + l_{ij}\ddot{q}_{ij}^d - l_{ij}\dot{v}_{ij}^\times \dot{q}_{ij} \right. \\ &\quad \left. + l_{ij}q_{ij}^\times \left(\frac{1}{m_{Li}l_{ij}} q_{ij}^\times \sum_{j=1}^m \mu_{ij} - \frac{1}{m_{ij}l_{ij}} q_{ij}^\times u_{ij}^v + D_{ij} \right) \right) \\ &= A_{ij} + B_{ij}u_{ij}^v + l_{ij}q_{ij}^\times D_{ij} \end{aligned} \quad (16)$$

where $A_{ij} = \dot{\mathcal{P}}_{ij}(K_{ij}e_{ij} + \dot{e}_{ij}) + \mathcal{P}_{ij}(K_{ij}\dot{e}_{ij} + l_{ij}\ddot{q}_{ij}^d - l_{ij}\dot{v}_{ij}^\times \dot{q}_{ij} + \frac{1}{m_{Li}} q_{ij}^\times q_{ij}^\times \sum_{j=1}^m \mu_{ij})$ and $B_{ij} = -\frac{1}{m_{ij}} q_{ij}^\times q_{ij}^\times$.

To address the impact of the disturbance D_{ij} , the following disturbance observer is designed as

$$\begin{cases} \hat{D}_{ij} = Z_{ij} + \eta_{ij}v_{ij} \\ \dot{Z}_{ij} = -\eta_{ij} \left(\frac{q_{ij}^\times}{m_{Li}l_{ij}} \sum_{j=1}^m \mu_{ij} - \frac{q_{ij}^\times}{m_{ij}l_{ij}} u_{ij}^v + \hat{D}_{ij} \right) + l_{ij}q_{ij}^\times S_{ij} \end{cases} \quad (17)$$

where η_{ij} represents the gain matrix.

The control signal u_{ij}^v for the quadrotor is designed in the following form:

$$u_{ij}^v = -m_{ij}F_{ij}^v \quad (18)$$

where $F_{ij}^v = K_{S_{ij}}S_{ij} + \mathcal{P}_{ij}A_{ij} + l_{ij}q_{ij}^\times \hat{D}_{ij}$ is a projected control term.

Remark 5: It should be noted that u_{ij}^v is constrained to be perpendicular to the cable direction q_{ij} [6]. For a unit vector q_{ij} , define $\mathcal{P}_{ij} = I_3 - q_{ij}q_{ij}^T = -q_{ij}^\times q_{ij}^\times$, which represents the orthogonal projection onto the two-dimensional subspace $q_{ij}^\perp$. Accordingly, $B_{ij} = \frac{1}{m_{ij}} \mathcal{P}_{ij}$. Given that $q_{ij}^T S_{ij} = 0$, $q_{ij}^T \mathcal{P}_{ij} A_{ij} = 0$, and $q_{ij}^T q_{ij}^\times \hat{D}_{ij} = 0$, it follows that $q_{ij}^T F_{ij}^v = 0$. This indicates that F_{ij}^v lies in the two-dimensional tangent space $q_{ij}^\perp$. Therefore, the control design is formulated directly within the admissible subspace $q_{ij}^\perp$, yielding $q_{ij}^T u_{ij}^v = -m_{ij}q_{ij}^T F_{ij}^v = 0$.

Substituting $B_{ij} = \frac{1}{m_{ij}} \mathcal{P}_{ij}$ into (18) yields $B_{ij}u_{ij}^v = -\mathcal{P}_{ij}F_{ij}^v$. Since $F_{ij}^v \in q_{ij}^\perp$, it follows that $\mathcal{P}_{ij}F_{ij}^v = F_{ij}^v$, and consequently, $B_{ij}u_{ij}^v = -F_{ij}^v$. Then, Eq. (16) is rewritten as

$$\mathcal{P}_{ij}\dot{S}_{ij} = \mathcal{P}_{ij}A_{ij} - F_{ij}^v + l_{ij}q_{ij}^\times D_{ij} \quad (19)$$

Substituting F_{ij}^v into (19) yields

$$\mathcal{P}_{ij}\dot{S}_{ij} = -K_{S_{ij}}S_{ij} + l_{ij}q_{ij}^\times \tilde{D}_{ij} \quad (20)$$

where $\tilde{D}_{ij} = D_{ij} - \hat{D}_{ij}$ denotes the disturbance estimation error. Since S_{ij} is orthogonal to q_{ij} , it follows that $S_{ij}^T \mathcal{P}_{ij} = S_{ij}^T$. Consequently, Eq. (20) can be rewritten as

$$S_{ij}^T \mathcal{P}_{ij} \dot{S}_{ij} = S_{ij}^T \dot{S}_{ij} = -S_{ij}^T K_{S_{ij}} S_{ij} + l_{ij} S_{ij}^T q_{ij}^\times \tilde{D}_{ij} \quad (21)$$

Consider the following Lyapunov function:

$$V_{ij} = \frac{1}{2} S_{ij}^T S_{ij} + \frac{1}{2} \tilde{D}_{ij}^T \tilde{D}_{ij} \quad (22)$$

By differentiating (22) and combining it with (17) and (21), the following is obtained:

$$\dot{V}_{ij} = -S_{ij}^T K_{S_{ij}} S_{ij} - \tilde{D}_{ij}^T \eta_{ij} \tilde{D}_{ij} + \tilde{D}_{ij}^T \dot{\tilde{D}}_{ij} \quad (23)$$

3.3. Stability analysis

Given the design of the aforementioned controller, Theorem 1 provides the stability analysis of the closed-loop system.

Theorem 1: Under the conditions of Assumptions 1–3, based on the system described by (2)–(4), the disturbance observers (9) and (17), the payload formation controller (10), and the quadrotor tracking controller (18) are designed, with the design parameters satisfying:

$$\begin{cases} \lambda_{\min}(2K_{S_L}) > 0, \lambda_{\min}(2\eta_L - I_{3n}) > 0, \\ \lambda_{\min}(2K_{S_{ij}}) > 0, \lambda_{\min}(2\eta_{ij} - I_3) > 0 \end{cases} \quad (24)$$

Consequently, the closed-loop system is uniformly ultimately bounded.

Proof: Consider the following Lyapunov function:

$$V = V_L + \sum_{i=1}^n \sum_{j=1}^m V_{ij} \quad (25)$$

By differentiating (25) and combining with (14) and (23), the following is obtained:

$$\begin{aligned} \dot{V} &= -S_L^T K_{S_L} S_L - \tilde{D}_L^T \eta_L \tilde{D}_L + \tilde{D}_L^T \dot{\tilde{D}}_L \\ &\quad + \sum_{i=1}^n \sum_{j=1}^m (-S_{ij}^T K_{S_{ij}} S_{ij} - \tilde{D}_{ij}^T \eta_{ij} \tilde{D}_{ij} + \tilde{D}_{ij}^T \dot{\tilde{D}}_{ij}) \end{aligned} \quad (26)$$

By virtue of Young's inequality, the following inequality holds:

$$\tilde{D}_L^T \dot{\tilde{D}}_L \leq \frac{1}{2} \tilde{D}_L^T \tilde{D}_L + \frac{1}{2} \|\dot{\tilde{D}}_L\|^2 \quad (27)$$

$$\tilde{D}_{ij}^T \dot{\tilde{D}}_{ij} \leq \frac{1}{2} \tilde{D}_{ij}^T \tilde{D}_{ij} + \frac{1}{2} \|\dot{\tilde{D}}_{ij}\|^2 \quad (28)$$

By substituting (27) and (28) into (26), the following is obtained:

$$\begin{aligned} \dot{V} &\leq -S_L^T K_{S_L} S_L - \tilde{D}_L^T (\eta_L - \frac{1}{2} I_{3n}) \tilde{D}_L + \frac{1}{2} \|\dot{\tilde{D}}_L\|^2 \\ &\quad + \sum_{i=1}^n \sum_{j=1}^m (-S_{ij}^T K_{S_{ij}} S_{ij} - \tilde{D}_{ij}^T (\eta_{ij} - \frac{1}{2} I_3) \tilde{D}_{ij} \\ &\quad + \frac{1}{2} \|\dot{\tilde{D}}_{ij}\|^2) \leq -KV + \Delta \end{aligned} \quad (29)$$

where $k = \min\{\lambda_{\min}(2k_{s_L}), \lambda_{\min}(2\eta_L - I_{3n}), \lambda_{\min}(2k_{s_{ij}}), \lambda_{\min}(2\eta_{ij} - I_3)\}$ and $\lambda_{\min}(2\eta_{ij} - I_3)$ and $\Delta = \frac{1}{2}(\|\dot{\tilde{D}}_L\|^2 + \|\dot{\tilde{D}}_{ij}\|^2)$. By Assumption 2, Δ is bounded, and there exists a positive constant ϵ_V satisfying $\Delta \leq \epsilon_V$. Therefore, Eq. (29) can be scaled to $\dot{V} \leq -KV + \epsilon_V$. Integrating $\dot{V}$ yields the following:

$$0 \leq V(t) \leq \left(V(0) - \frac{\epsilon_V}{K} \right) e^{-Kt} + \frac{\epsilon_V}{K} \quad (30)$$

which means that the signals S_L , $\tilde{D}_L$, S_{ij} , and $\tilde{D}_{ij}$ are bounded. Therefore, the tracking error of the closed-loop system satisfies uniform ultimate boundedness. $\square$

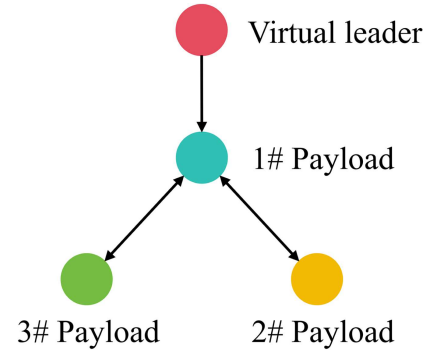
Remark 6: The disturbance observers in (9) and (17) do not employ any additional filtering or signal-conditioning schemes. The observers directly use the available measured and computed state information. Under the conditions of Theorem 1 and Assumption 2, the observer errors $\tilde{D}_L$ and $\tilde{D}_{ij}$ satisfy $\limsup_{t \rightarrow \infty} \|\tilde{D}_L(t)\| \leq \frac{\bar{d}_L}{\lambda_{\min}(\eta_L)}$ and $\limsup_{t \rightarrow \infty} \|\tilde{D}_{ij}(t)\| \leq \frac{\bar{d}_{ij}}{\lambda_{\min}(\eta_{ij})}$, respectively, where $\bar{d}_L$ and $\bar{d}_{ij}$ are bounded positive constants. In practical implementation, increasing η_L and η_{ij} can accelerate disturbance estimation but may also increase the sensitivity of the observers to high-frequency measurement noise.

4. Numerical Simulation

This subsection validates the effectiveness and advantages of the proposed control method via numerical comparative simulations. The MQCSPS consists of three QCSPS, where each QCSPS comprises three quadrotors connected to a payload via linkages. The communication topology structure among the payloads is shown in Figure 3. The trajectory of the virtual leader is denoted as $P_t = [8 \sin(\frac{\pi t}{20}), 8 \cos(\frac{\pi t}{20}), -0.25t - 0.5]^T$. The relative positions, as well as among the payloads, are specified as $P_{L1}^d = [0, 0, 0]^T$, $P_{L12}^d = 2[\sin(\frac{\pi t}{20}), \cos(\frac{\pi t}{20}), 0]^T$, and $P_{L13}^d = 2[-\sin(\frac{\pi t}{20}), -\cos(\frac{\pi t}{20}), 0]^T$. The desired direction vectors of the ropes are specified as $q_{i1}^d = [0.5, 0, 0.9]^T$, $q_{i2}^d = [-0.3, 0.4, 0.9]^T$, and $q_{i3}^d = [-0.3, -0.4, 0.9]^T$. The system model parameters are selected as $m_{ij} = 0.76(\text{kg})$, $m_{L_i} = 0.5(\text{kg})$, $l_{ij} = 1(\text{m})$ and $g = 9.8(\text{m/s}^2)$. The initial states of the system are set as $P_{L1}(0) = [0.25, 6.25, -0.25]^T$, $P_{L2}(0) = [0.25, 8.25, -0.25]^T$, and $P_{L3}(0) = [0.25, 4.25, -0.25]^T$. The control parameters are selected as $K_L = K_{S_L} = 5I_3$, $\eta_L = 10I_3$, $K_{ij} = I_3$,

$K_{S_{ij}} = 5I_3$, and $\eta_{ij} = 10I_3$. To validate the system's robustness, the disturbances D_{Li} and D_{ij} are set as $D_{L1} = 0.5 \sin(0.25t) + e^{-0.25t}$, $D_{L2} = -0.5 \sin(0.35t) - e^{-0.25t}$, $D_{L3} = 0.75 \sin(0.5t) + e^{-0.5t}$, $D_{1j} = 0.25 \cos(0.5t) + e^{-0.5t}$, $D_{2j} = 0.2 \cos(0.25t) - e^{-0.25t}$, and $D_{3j} = 0.25 \cos(0.75t) + e^{-0.35t}$. For ease of comparison, the proposed control method and the corresponding control method without the disturbance observer are denoted as sliding mode control based on disturbance observer (SMC-DO) and SMC, respectively. To ensure a fair comparison, the control parameters are kept identical to those of the SMC-DO. The only difference is that the disturbance estimation and compensation mechanism is removed from the SMC.

Figure 3
The communication topology structure



The simulation results of the SMC-DO and SMC are shown in Figures 4–7, which depict the control response trajectories, tracking errors, disturbance estimations, and control inputs, respectively. The root mean square (RMS) of the tracking

Figure 4
Control response trajectory of the MQCSPS

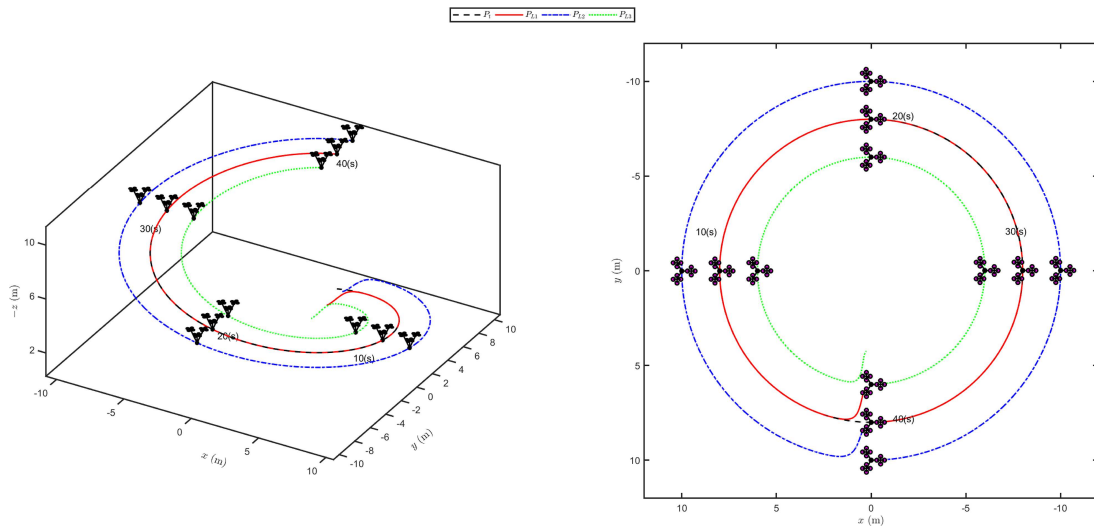


Figure 5
The 2-norm of the payload formation-tracking error e_L and the quadrotor tracking errors $e_i (i = 1, 2, 3)$

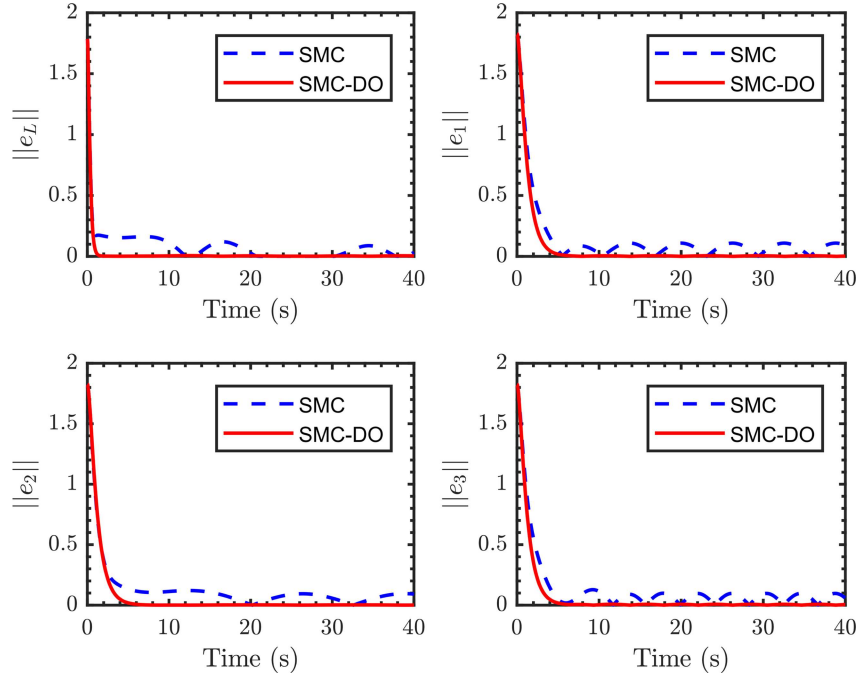
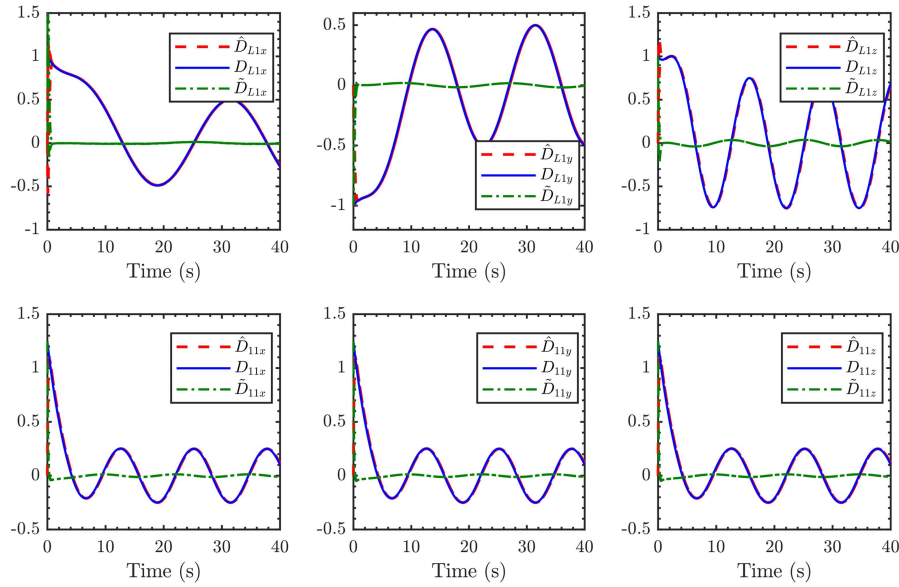
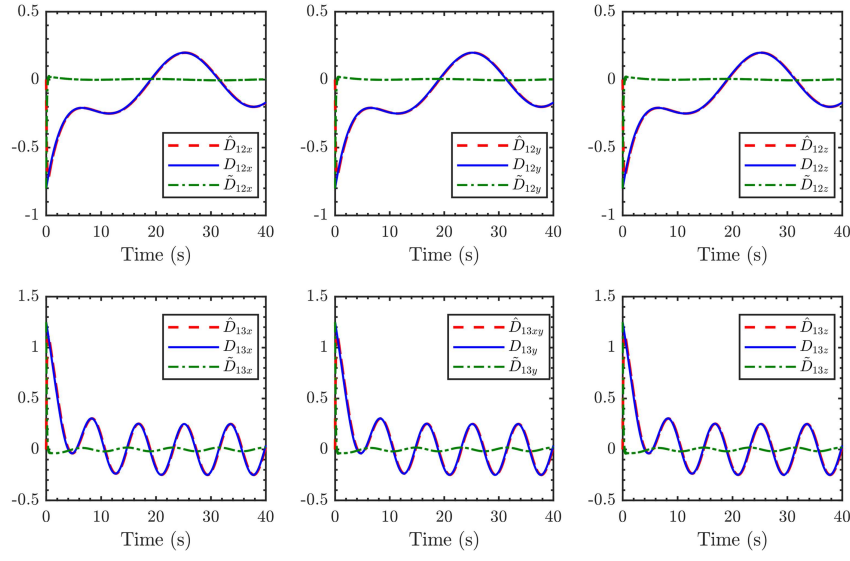
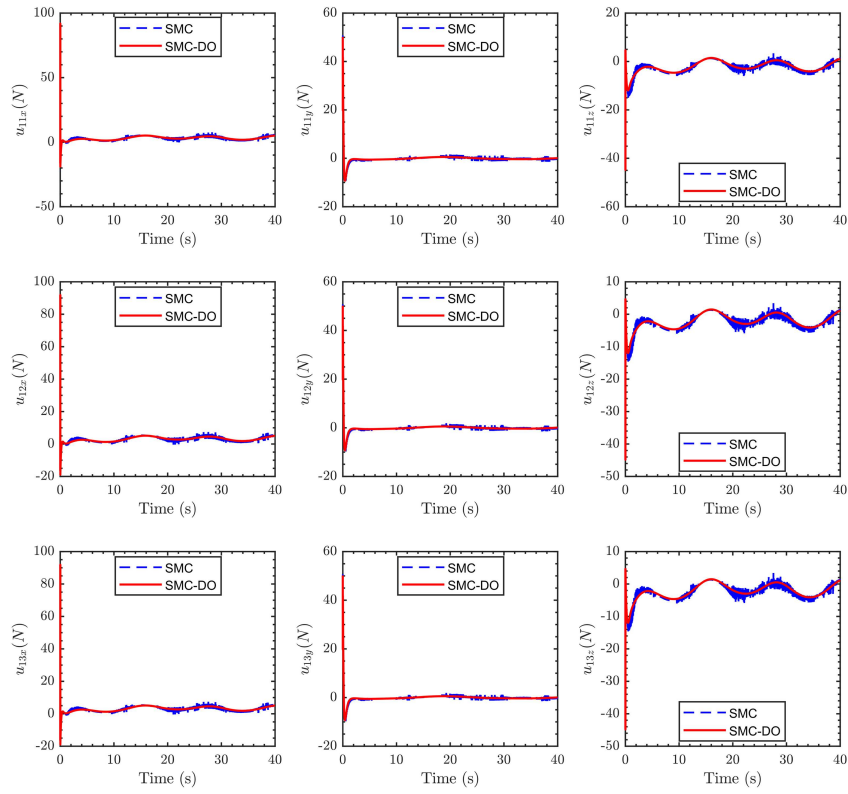


Figure 6
Estimation and estimation error of the disturbances D_{L1} and $D_{1j} (j = 1, 2, 3)$



error under the two control methods is shown in Table 1. As shown in Figures 4–7, compared with SMC, SMC-DO achieves smaller fluctuations in both the distributed formation-tracking errors of the payloads and the tracking errors of the quadrotors. Meanwhile, the disturbance observer accurately estimates the unknown disturbances, demonstrating its capability for effective disturbance estimation and compensation. These results indicate that incorporating the disturbance observer significantly

enhances the robustness of the MQCSPS. In addition, SMC-DO generates smoother control inputs, effectively mitigating the inherent chattering of SMC. The quantitative results demonstrate that, compared with the SMC, the SMC-DO reduces the RMS tracking errors by 16.93%, 10.78%, 4.92%, and 10.82% in the corresponding directions, respectively. Overall, the simulation results validate the effectiveness and superior performance of the proposed control method.

Figure 6
(Continued)

 Figure 7
Control input u_{1j} ($j = 1, 2, 3$)

 Table 1
Comparison of RMSs between SMC and SMC-DO

Method	$\text{RMS}(\ e_L\)$	$\text{RMS}(\ e_1\)$	$\text{RMS}(\ e_2\)$	$\text{RMS}(\ e_3\)$
SMC	0.1654	0.2856	0.2662	0.2857
SMC-DO	0.1374	0.2548	0.2531	0.2548

5. Conclusion

This paper proposes a disturbance observer-based distributed formation-tracking control method to solve the tracking control problem of the MQCSPS subject to unknown disturbances. Guided by sliding-mode control theory, distributed sliding-mode formation controllers and sliding-mode tracking controllers are designed to realize trajectory tracking of quadrotors as well as distributed formation transportation of the payload. A disturbance observer is adopted to estimate and compensate for unknown disturbances, so as to improve the robustness of the system. Numerical simulation results verify the effectiveness of the proposed control scheme.

The thrust and torque saturation constraints are not explicitly considered, and the generalized control input is assumed to remain within the feasible operating range. Future work will incorporate actuator constraints into the proposed distributed-centralized framework. Moreover, finite sampling, measurement noise, and sampled-data effects are not explicitly considered in the current continuous-time formulation. Developing a discrete-time control framework with enhanced noise robustness will therefore be pursued. Finally, the cables are assumed to remain taut, massless, and inextensible. Extending the framework to slack-taut transitions, cable tension constraints, and flexible-cable dynamics will be considered in future work.

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Ethical Statement

This study does not contain any studies with human or animal subjects performed by any of the authors.

Conflicts of Interest

The authors declare that they have no conflicts of interest to this work.

Data Availability Statement

Data sharing is not applicable to this article as no new data were created or analyzed in this study.

Author Contribution Statement

Wei Liu: Conceptualization, Methodology, Formal analysis, Resources, Data curation, Writing – original draft, Writing – review & editing, Visualization. **Xuejun Liu:** Software, Supervision, Funding acquisition. **Bo Liu:** Validation, Investigation, Project administration.

References

- [1] Estevez, J., Garate, G., Lopez-Guede, J. M., & Larrea, M. (2024). Review of aerial transportation of suspended-cable

- payloads with quadrotors. *Drones*, 8(2), 35. <https://doi.org/10.3390/drones8020035>
- [2] Li, S., Duong, T. T. H., & Zanotto, D. (2024). In-flight cable length control for improved quadrotor-based suspended load transportation. *IEEE Robotics and Automation Letters*, 9(1), 667–674. <https://doi.org/10.1109/LRA.2023.3335778>
- [3] Liang, X., Zhang, Z., Yu, H., Wang, Y., Fang, Y., & Han, J. (2022). Antiswing control for aerial transportation of the suspended cargo by dual quadrotor UAVs. *IEEE/ASME Transactions on Mechatronics*, 27(6), 5159–5172. <https://doi.org/10.1109/TMECH.2022.3174144>
- [4] Cai, J., & Xian, B. (2024). Robust hierarchical geometry control for the multiple UAVs aerial transportation system with a suspended payload. *Nonlinear Dynamics*, 112(6), 4551–4571. <https://doi.org/10.1007/s11071-024-09277-5>
- [5] Liang, X., Yu, H., Zhang, Z., Liu, H., Fang, Y., & Han, J. (2023). Unmanned aerial transportation system with flexible connection between the quadrotor and the payload: Modeling, controller design, and experimental validation. *IEEE Transactions on Industrial Electronics*, 70(2), 1870–1882. <https://doi.org/10.1109/TIE.2022.3163526>
- [6] Lee, T. (2018). Geometric control of quadrotor UAVs transporting a cable-suspended rigid body. *IEEE Transactions on Control Systems Technology*, 26(1), 255–264. <https://doi.org/10.1109/TCST.2017.2656060>
- [7] Ren, Y., Yu, Y., Han, Z., & Liu, L. (2026). Modeling and unilateral adaptive control of a flexible slung load system for multirotor UAV with actuator constraints and faults. *IEEE Transactions on Aerospace and Electronic Systems*, 62, 7054–7069. <https://doi.org/10.1109/TAES.2026.3668455>
- [8] Kang, J., & Shan, J. (2026). Robust control of aerial cable-suspended payload transportation via fully actuated system approach. *Control Engineering Practice*, 170, 106837. <https://doi.org/10.1016/j.conengprac.2026.106837>
- [9] Liang, J., Wang, Y., Zhong, H., Chen, Y., Li, H., Hua, H., & Wang, W. (2025). Robust adaptive tracking control for aerial transporting a cable-suspended payload using backstepping sliding mode techniques. *IEEE Transactions on Automation Science and Engineering*, 22, 4490–4500. <https://doi.org/10.1109/TASE.2024.3355225>
- [10] Qiu, Q., Yang, Y., & Chen, Y. (2026). Nonlinear H_∞ control strategy for a suspended transport system of dual quadrotor UAVs. *International Journal of Robust and Nonlinear Control*, 36(4), 2011–2025. <https://doi.org/10.1002/rnc.70252>
- [11] Liu, W., Chen, M., & Shi, P. (2022). Fixed-time disturbance observer-based control for quadcopter suspension transportation system. *IEEE Transactions on Circuits and Systems I: Regular Papers*, 69(11), 4632–4642. <https://doi.org/10.1109/TCSI.2022.3193878>
- [12] Fan, Y., Chen, X., Zhao, Y., & Song, B. (2023). Nonlinear control of quadrotor suspension system based on extended state observer. *Acta Automatica Sinica*, 49(8), 1758–1770.
- [13] Liu, W., Chen, M., & Zhang, P. (2025). Appointed-time disturbance observer-based control for multiquadrotor suspended transport system within virtual trajectory. *IEEE Transactions on Aerospace and Electronic Systems*, 61(3), 7320–7336. <https://doi.org/10.1109/TAES.2025.3534741>
- [14] Lee, T., Sreenath, K., & Kumar, V. (2013). Geometric control of cooperating multiple quadrotor UAVs with a suspended payload. In *52nd IEEE Conference on Decision and Control*, 5510–5515. <https://doi.org/10.1109/CDC.2013.6760757>

- [15] Dong, X., & Hu, G. (2017). Time-varying formation tracking for linear multiagent systems with multiple leaders. *IEEE Transactions on Automatic Control*, 62(7), 3658–3664. <https://doi.org/10.1109/TAC.2017.2673411>
- [16] Hua, Y., Dong, X., Hu, G., Li, Q., & Ren, Z. (2019). Distributed time-varying output formation tracking for heterogeneous linear multiagent systems with a nonautonomous leader of unknown input. *IEEE Transactions on Automatic Control*, 64(10), 4292–4299. <https://doi.org/10.1109/TAC.2019.2893978>
- [17] Hu, J., Bhowmick, P., & Lanzon, A. (2020). Distributed adaptive time-varying group formation tracking for multiagent systems with multiple leaders on directed graphs. *IEEE Transactions on Control of Network Systems*, 7(1), 140–150. <https://doi.org/10.1109/TCNS.2019.2913619>
- [18] Yang, S., & Xian, B. (2020). Energy-based nonlinear adaptive control design for the quadrotor UAV system with a suspended payload. *IEEE Transactions on Industrial Electronics*, 67(3), 2054–2064. <https://doi.org/10.1109/TIE.2019.2902834>
- [19] Dierks, T., & Jagannathan, S. (2010). Output feedback control of a quadrotor UAV using neural networks. *IEEE Transactions on Neural Networks*, 21(1), 50–66. <https://doi.org/10.1109/TNN.2009.2034145>
- [20] Wang, Y., Yu, G., Xie, W., Zhang, W., & Silvestre, C. (2024). Robust cooperative transportation of a cable-suspended payload by multiple quadrotors featuring cable-reconfiguration capabilities. *IEEE Transactions on Intelligent Transportation Systems*, 25(9), 11833–11843. <https://doi.org/10.1109/TITS.2024.3380668>

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