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Mathematical Applications in the Measurement of Multidimensional Poverty in Colombia: An Approach Based on Aggregation Operators

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Abstract: The main objective of this study is to propose a new method to estimate multidimensional poverty in Colombia using aggregation operators that incorporate expert assessment into the calculation process. To this end, it proposes three operators: the poverty ordered weighted averaging (OWA) operator, the poverty-induced OWA operator, and the poverty penalty OWA operator. Each one integrates different types of subjective information, such as preferences, priorities, and levels of agreement regarding the need to improve access. Based on the structure of the official Multidimensional Poverty Index and responses from five experts, a mathematical model is developed to compare the results of the traditional method with measurements generated by the proposed operators. The findings show that expert judgment significantly influences the magnitude of the index and the severity attributed to deprivation, especially in dimensions such as Housing and Children and Youth. Likewise, it is shown that ordered operators allow for capturing variations in the relative importance of the indicators, reinforcing the usefulness of these methodologies for generating complementary and more flexible analyses in the evaluation of multidimensional poverty.

Keywords: multidimensional poverty, aggregation operators, measurement, complexity

1. Introduction

The measurement of poverty has been widely debated in the economic and statistical literature due to its conceptual and methodological complexity. Understanding poverty as a global and multidimensional phenomenon has been a process that has evolved from understanding poverty as a global and multidimensional phenomenon has been a process that has evolved from income-centered approaches toward comprehensive perspectives on human well-being; this transformation has been not only conceptual but also methodological [1], as evidenced by the diversity of definitions, measurement tools, and applications. The traditional approach, based on monetary income thresholds or gap indices, has been criticized for its inability to capture the full scope of poverty; recent studies have furthered this debate by showing that, although there is a correlation between monetary and multidimensional poverty, there are significant discrepancies in the identification of poor households and the severity of the deprivations they experience [2].

One of the main methodological challenges associated with measuring multidimensional poverty relates to the assignment of weights to the dimensions and indicators that make up the index. Here, "index" refers to the Multidimensional Poverty Index (MPI), which is used consistently throughout the manuscript to designate it. In the official Colombian formulation, each dimension is assigned uniform weights, and the aggregate structure is based on an arithmetic mean [3]. However, advances in applied mathematics and decision-making methods have made it possible to develop aggregation operators that explicitly represent the attitudes, preferences, and priorities of those participating in the evaluation process [4] and to develop multi-criteria

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decision-making models to address issues of inequality and polarization [5]. Among these developments, OWA aggregation operators stand out, as they allow for the combination of objective information with subjective criteria and capture different behaviors regarding risk, compensation, or the severity of deprivations. These operators are part of a broader family of aggregation techniques used in fuzzy systems to integrate uncertain or imprecise information in decision-making contexts [6]. From this base operator, relevant extensions have emerged, such as induced ordered weighted averaging (OWA), prioritized OWA, and operators based on penalty functions. Their applicability has been demonstrated in the construction of composite indicators, such as human development indices, where different dimensions are aggregated under varying decision attitudes [7].

In many countries around the world, poverty levels are extreme. The United Nations General Assembly adopted the Sustainable Development Goals (SDGs) for the 2030 Agenda, establishing goals for reducing this phenomenon. The MPI is a measure of poverty that reflects the multiple deprivations people face simultaneously. It goes beyond mere income and focuses on variables that affect development [8]. Traditionally, poverty has been assessed based on household income; however, this approach fails to capture the true magnitude of the phenomenon. For this reason, international organizations such as the United Nations Development Programme and the Oxford Poverty and Human Development Initiative developed the MPI based on a dichotomous classification model, in which households are categorized as either poor or non-poor [9].

In this sense, the objective of this paper is to propose a new methodological approach to estimate multidimensional poverty in Colombia through aggregation operators that integrate expert assessments from three perspectives: OWA, Induced OWA (IOWA), and P_OWA. Recent literature recognizes that poverty is a phenomenon sensitive to the heterogeneity of social judgments, and that flexible aggregation methods, including operators, can capture variations that rigid schemas do not reflect. It is important to say that poverty OWA operator (Pov_OWA), poverty-induced OWA operator (Pov_IOWA), and poverty penalty OWA operator (PPov_OWA) are adaptations of the classical OWA, IOWA, and penalty-based OWA operators, extended to the specific structure of the official MPI. Thus, the main contribution of this paper is methodological and applied since it embeds expert judgment into the MPI aggregation rule through these three extensions, rather than proposing a novel aggregation operator in the strict sense. Furthermore, for consistency, the notation P_OWA used here corresponds to the same penalty-based operator, while PPov_OWA is the extension of penalty poverty OWA, and this single notation (PPov_OWA) is adopted throughout the remainder of the manuscript. To this end, the theoretical properties of each operator are analyzed and adapted to the structure of the country's MPI, enabling their results to be compared with those of the traditional method based on simple averages. The proposal allows us to identify how the magnitude of the index, the severity of the deprivations, and the relative contribution of each dimension change when more flexible aggregation mechanisms are introduced and when expert judgment is involved. In a complementary way, the stability of the poverty diagnosis across different normative assumptions is examined, enabling visualization of alternative scenarios and evaluation of the robustness of classifying a household as poor or non-poor.

The article is structured as follows: Section 2 presents the conceptual foundations of the MPI and the elements that support the official measurement of multidimensional poverty in Colombia. Section 3 reviews the aggregation operators used in the proposal, highlighting their characteristics, properties, and relevance to measurement. Section 4 develops the mathematical properties of the operators Pov_OWA, Pov_IOWA, and PPov_OWA, showing how they transform the traditional MPI aggregation. Finally, Section 5 presents the findings and discusses the implications of incorporating these operators into the assessment and understanding of multidimensional poverty.

2. Literature Review

The most essential aspects of multidimensional poverty, decision-making, and aggregation operators are presented below.

2.1. Poverty

Poverty has been present throughout the history of humanity, manifesting itself from the first forms of social organization. Poverty based on the minimum income necessary to maintain an efficient physical existence, that is, with enough food, clothing and shelter to maintain health, this arose from the need to understand a historical phenomenon, since those who depended on the receipt of "charity" to survive due to their own condition of vulnerability, it was considered poor and supported by the Poor Law that sought to provide individuals with the basics to satisfy their needs [10].

In the nineteenth century, researchers such as Charles Booth and Seebohm Rowntree showed that poverty was not the result of personal failures, but rather of structural conditions, such as low wages, precarious employment, and limited access to basic services, which generate vulnerability and deprivation [11]. However, at the beginning of the twentieth century, a conception focused exclusively on the lack of income as the leading indicator of poverty persisted, ignoring other forms of deprivation that also affect human well-being. In Colombia, Although poverty has been a recurring concern, its impact intensified after the economic opening of the 1990s [12] caused by a significant economic contraction, accompanied by the stagnation of the industrial and agricultural sectors, accentuating that the changes in the markets derived from this transformation will affect poverty levels, sources of economic growth and income distribution, aggravated by the increase in unemployment and the reduction in national production [13].

The conceptualization of poverty has evolved from approaches focused on the satisfaction of minimum needs to perspectives that incorporate social, cultural, and capacity dimensions, that is, multidimensional dimensions. The literature recognizes classic definitions that are usually associated with an absolute level of deprivation, in which poverty is understood as the insufficiency of resources to cover what is strictly necessary for survival. Afterward, Townsend has broadened the concept by integrating the notion of relative poverty, understood as the lack of sufficient resources to participate fully in social, cultural, and economic life in accordance with the standards of society.

Indeed, understanding poverty in depth implies looking beyond the lack of income; it must also be understood as a multidimensional phenomenon, characterized by a deprivation of access to the rights necessary to enjoy an adequate and quality standard of living. This approach emerges as an alternative that conceives of poverty from a broader, more comprehensive perspective, unlike the dichotomous division between the poor and the non-poor, which is criticized for losing information and for eliminating the nuances between the well-being achieved and the material deprivations experienced. According to the Office of the United Nations High Commissioner for Human Rights, poverty is recognized as not only an economic problem but also a form of exclusion and domination that affects the dignity and agency of individuals.

In addition, Martha Nussbaum, the most important capabilities theorist alongside Sen, presents a detailed list of capacities that everyone should have access to live a decent life and argues that justice requires guaranteeing each person a minimum threshold in all capacities [14].

2.1.1. Evolution of measurement in Colombia

Measuring poverty is a complex challenge, where various studies have proposed multiple definitions and strategies for its measurement [1] (see Table 1). One of the main distinctions in measurement methods lies in the use of money as a unit of reference, a criterion that does not account for factors related to people's well-being and opportunities and that distinguishes one-dimensional from multidimensional approaches [15]. The diagnosis and quantification of the poor population are essential to guide social interventions and design effective policies, according to [16]. Poverty is measured to obtain essential data that guide decision-making and, at the same time, make it possible to verify whether progress has been made in reducing it.

Considering poverty from a subjective perspective, complemented by its objective dimension, offers a more complete vision that facilitates the monitoring and evaluation of public policies aimed at reducing it [17]. In addition, any measurement or conceptualization must be situated within a framework defined by place and time, since notions of the phenomenon are diffuse [18]. In this context, the country has had multidimensional indices of poverty, quality of life, and human development for several years. Its evaluation provides an overview of household well-being, which explains the interest of governments, statistical offices, and international organizations in developing and applying indicators [19]. Throughout the history of poverty measurement in Colombia, a significant evolution has been observed. It has been classified into indirect and direct measurements; the first is defined as the lack of resources. It has methodological versions such as monetary poverty. The second possibility is when it is stated that we are concerned about equality of results and the definition of deprivation is applied [20], which evaluates the satisfaction results that an individual has with respect to characteristics that are considered vital.

Although the measurements examine and record the poverty levels of the population, each one has a particular focus. In particular, the MPI draws on the methods of Alkire and Foster and the capabilities approach. It allows the identification of people who present simultaneous deprivations in different aspects valued as essential for the development of a life appreciated both individually and socially [19]. According to the methodology currently used by the National Planning Department of

Table 1
Evolution of poverty measurement in Colombia

Methodology	Description
Unsatisfied Basic Needs (NBI) (1987)	A composite indicator that assesses household living conditions through essential material and social dimensions, such as housing quality, access to public services, level of overcrowding, economic dependence, and school attendance. Its purpose is to identify shortcomings that affect the well-being of the population and, at the same time, to provide a tool for characterizing inequalities and guiding social policies [20].
Human Development Index (HDI) 1990	A synthetic indicator that measures the progress of countries, considering not only per capita income but also fundamental dimensions of well-being such as health, life expectancy, and education. Its purpose is to offer a measure that is easy to disseminate, comparable to per capita income, but with greater sensitivity to social aspects and the conditions of community life, which allows a more comprehensive vision of human development [21].
Gini coefficient (2002)	The most widely used measure to assess income inequality ranges from 0 to 1, where 0 represents perfect equality and one absolute inequality; it consists of two components that measure inequality within and between groups [22].
Homeless line	It corresponds to the minimum income necessary to acquire a basic food basket, sufficient only to cover subsistence nutritional needs [23]. It is estimated from the cost of this basket and marks the threshold below which a person is unable to meet even their essential food requirements.
Poverty line	Income threshold that determines whether a household or individual has sufficient resources to cover their basic needs, and represents the minimum level required to maintain an adequate standard of living [23].
Living Conditions Index (LCI)	A synthetic indicator prepared by the DNP's Social Mission that assesses the well-being of households, individuals, and territories integrates infrastructure and housing variables, access to public services, demographic and income characteristics, and dimensions of current and potential human capital, such as educational level and school attendance [24]
Multidimensional Poverty Index (MPI) (2011)	Methodology that assesses the incidence and intensity of deprivations affecting households in different dimensions of well-being [24]. For Colombia, the MPI proposal comprises 5 dimensions and 15 variables.

Colombia (DNP) and Departamento Administrativo Nacional de Estadística (DANE), if a person is deficient in at least 3 of the 15 weighted indicators, the MPI identifies them as poor [25].

2.1.2. MPI calculation

The calculation of the MPI is developed after the construction of a matrix of deprivations at the household level with values of 0 or 1 (deprivation and non-deprivation); in the matrix, the rows represent the number of households, and in the columns, the 15 indicators are evaluated, each with a different weight [26], and are grouped to define five dimensions: educational conditions, conditions of children and youth, work, health, and housing conditions and public services (see Figure 1). However, for Colombia, each dimension has the same weight (0.2). That is, in the dimension of educational conditions, with two indicators, the weight of each would be 0.1; for the dimension of housing conditions and public services, there are five indicators, each with a weight of 0.04. Finally, the MPI uses a poverty line where the sum of the pesos exceeds 0.33 (33%).

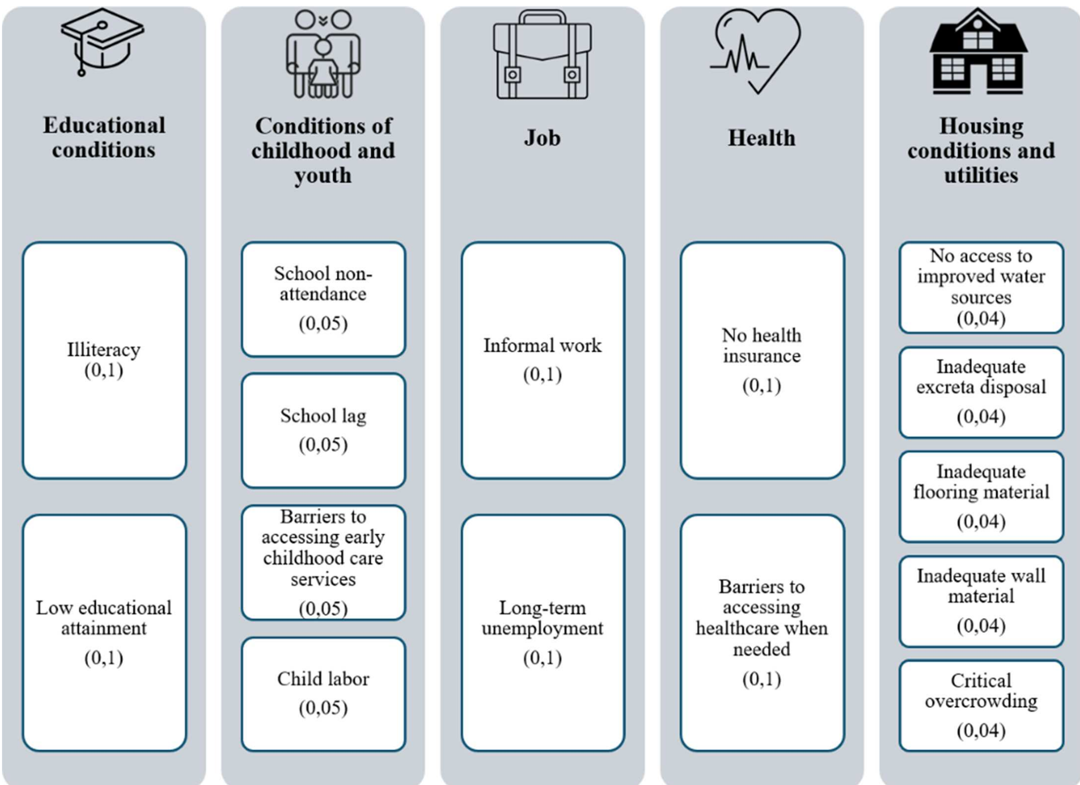
2.2. Decision-making in uncertainty

Decision-making is one of the most-studied fields in decision theory. In scenarios where information is incomplete, ambiguous, or contradictory, individuals, organizations, and/or entities must choose between alternatives without being able to anticipate their consequences with total precision. Deciding involves a process based on the analysis of available information and the evaluation of alternatives, which allows individuals to respond to different temporal scenarios, from the immediate to the strategic, particularly in contexts characterized by uncertainty [27]. In this context, risk and uncertainty are fundamental elements in decision-making processes, requiring individuals to evaluate different strategies depending on the level of available information and the degree of unpredictability involved [28]. The former occurs when there is no certainty about the outcome of the decision, although the probabilities of the different outcomes are known, and uncertainty occurs when we do not know the final result and cannot predict in terms of objective probabilities [29]. The latter redefines the framework of the possible, requiring decision models that recognize not only the variability inherent in systems but also the limited human capacity to process information, to foresee consequences, and to assign reliable probabilities.

Decision-making under conditions of uncertainty has been defined as a method that provides a unified framework for working with and dealing with uncertainty [30] and is characterized by the fact that the alternatives and potential results are not known and can only be evaluated by subjective estimates such as risk perception, previous experience, perception, intuition, sensation, blurred studies, etc., therefore, it is a process influenced by human subjectivity. Decision theory also states that people seek to minimize uncertainty through probabilistic tools, algorithms, or scenarios; however, choices are not solely governed by rational calculations.

According to this, fuzzy methods are presented as an essential tool in scenarios where the boundaries between the objective and the subjective are not clear; it also allows decisions to be considered from multiple perspectives and degrees of acceptance. Likewise, the level of information of uncertainty and chance does not correspond to the same thing; chance, randomness, and luck are words linked to the theory of probabilities, and then it is defined that chance is the measurable uncertainty with the help of the concept of probability. On the other hand, to ensure decision-making produces reliable solutions, it has been considered appropriate to adopt approaches that use learning from experience or heuristic reasoning (fuzzy), characterized by a desire to quantify uncertainty [31].

Figure 1
Dimensions of multidimensional poverty



2.2.1. Fuzzy logic and aggregation operators

In this framework, mathematics has enabled the development of models that provide tools for understanding and guiding decision-making in complex contexts. However, modeling real problems or systems that involve human and social components poses particular challenges, as such systems don´t lend themselves to simple binary representations and require broader scales of assessment [32]. Indeed, phenomena in the human sciences and reality are becoming less and less predictable and increasingly uncertain, diffuse, and difficult to grasp [33]. A theory that seeks to address the surrounding reality more adequately is fuzzy logic, considered a new trend for treating uncertainty with various applications across different fields and for real-world applications [34].

Fuzzy logic offers a practical framework for dealing with situations of uncertainty, allowing for the modeling of ambiguous phenomena or behaviors, which has driven the development of mathematical frameworks capable of operating with partial, linguistic, or intrinsically vague information, occupying a central role due to their ability to formalize degrees of truth and partial membership that elude bivalent logic [35]. Thus, it considers a logic of multiple values in which not all things must be of a certain type or cease to be so; there is an intermediate scale. It also includes multi-criteria analysis of the data, minimizing data modeling errors and allowing better incident detection [36]. Therefore, fuzzy logic offers multiple alternatives to conventional solutions [37] and has been used in relevant ways in decision-making analysis to bring them closer to the reality in which people, organizations, countries, and the world live.

To facilitate this process, techniques have been developed to deal with uncertain information using mathematical frameworks such as fuzzy set theory, Fuzzy relationships, and approximate reasoning [38]. These methods have focused on the treatment of decision-making problems to determine rankings, to evaluate the relative importance of multiple attributes, and to program fuzzy mathematical models. In addition, this fuzzy theory encompasses a variety of disciplines divided into five major branches: fuzzy mathematics extending the concepts of classical mathematics, fuzzy logic and artificial intelligence, fuzzy systems, uncertainty and information, and fuzzy decision-making.

The development of aggregation operators arises as a response to the limitations of classical combination schemes, such as the arithmetic mean or strictly compensatory operators, which impose strong assumptions regarding independence, linearity, and homogeneity among criteria. In this context, the OWA operator allows the importance of criteria to be decoupled from their ordinal position in the aggregation process, explicitly incorporating the notion of aggregative attitude ranging from optimistic to pessimistic behaviors [39]. Similarly, aggregation operators are essential to synthesize multiple judgments in a single assessment; one of the most influential families of aggregation operators is that of OWA and has given rise to extensions that incorporate meta-information about inputs such as induced (IOWA), prioritized [40], probability, and penalty functions [41].

In the case of extensions based on the OWA and IOWA operators, research has focused on everything from theoretical foundations to applications in artificial intelligence and environmental assessment. Theoretically, Jin et al. [42] provided a systematic framework for reconstructing IOWA operators from their weighted average expressions, establishing formal conditions under which both representations are equivalent and proposing a measure of orness based on inducing functions. Similarly, Ji et al. [43] introduced average-induced OWA operators, which overcome previous limitations by combining averaged ranking criteria with the classical IOWA structure, thereby improving the aggregation of expert opinions in group settings. Furthermore, He et al. [44] proposed an IOWA operator for processing extended comparative linguistic expressions with symbolic translation (ELICIT), advancing the integration of qualitative information into multi-criteria decision-making frameworks. In terms of applications, Pourshirazi et al. [45] developed a multi-classifier data-fusion scheme based on the OWA operator for detecting false data injection anomalies in smart grids, demonstrating the usefulness of the OWA operator when integrated with machine learning architectures under uncertainty. Meanwhile, Huang et al. [46] applied OWA operators to the fuzzy assessment of ecological vulnerability using the Sensitivity–Resilience–Pressure (SRP) model combined with the Social–Ecological System (SES) framework, modeling multiple decision-making attitudes toward environmental risks. In the field of efficiency analysis, Oukil and Amin [47] proposed a double-preference IOWA operator (2-IOWA) for the classification of decision units in the context of data envelopment analysis, resolving issues of non-uniqueness in the ranking of alternatives. Likewise, García-Zamora et al. [48] extended the OWA operators to the domain of fuzzy numbers under admissible orders, formalizing algebraic properties for settings where order relations are not total. Finally, Rathod et al. [41] introduced the penalty-based P-OWA and P-IOWA operators, applied to multi-criteria decision-making, while Zhang et al. [49] combined the IOWA operator with adaptive weighting to improve evidence combination rules in information fusion systems.

3. Methodology

3.1. Basic formulations

This section explains the different formulations that will be used in the document. The first to be explained will be the formula used by DANE [25], based on a weighted sum of deprivations, for measuring the MPI. The definition is as follows:

Definition 1. *The MPI is the weighted sum of deprivations as follows:*

$$w_i = \sum_{j_k=1}^{J_k} w_{ij_k}\,, \tag{1}$$

where i is the household identifier and j_k is the identifier of the household in the deprivation j of the dimension k.

The actual formulation of the MPI index was used as the basis for its recalculation using OWA operators, IOWA [50], and penalty function P-OWA [51]. The main idea is to incorporate experts' knowledge into the aggregation process, so that each deprivation's contribution can vary according to each operator's internal logic. The definitions are as follows:

Definition 2. *An OWA operator [39] of dimension n is a mapping $OWA: R^n \rightarrow R$ that has an associated weighing vector W of dimension n with $w_j \in [0, 1]$ and $\sum_{j=1}^{n} w_j = 1$, such that:*

$$OWA\,(a_1, a_2, \ldots, a_n) = \sum_{j=1}^{n} w_j b_j, \tag{2}$$

where b_i is the jth largest of the a_i.

The IOWA operator is an extension where the weighting vector is associated with the arguments based on an induced vector and not based on the maximum or minimum value of the argument. The formulation is as follows [50]:

Definition 3. *An IOWA operator of dimension* n *is an application* $IOWA: R^n \times R^n \to R$ *that has an associated weighting vector, W of dimension n, where the sum of the weights is* 1 *and* $w_j \in [0,1]$, *where an induced set of ordering variables is included:* (u_i) *such that the formula is:*

$$IOWA(\langle u_1, a_1\rangle, \langle u_2, a_2\rangle, \ldots, \langle u_n, a_n\rangle) = \sum_{j=1}^{n} w_j b_j, \quad (3)$$

where b_j *is the* a_i *value of the OWA pair*$<u_i, a_i>$*having the jth largest* u_i. u_i *is the order-inducing variable, and* a_i *is the argument variable.*

The penalty function is a measure of disagreement or dissimilarity that has been adjusted to account for penalty depreciation during the aggregation of expert opinions [51].

Definition 4: *The function* $P: X^{n+1} \to \overline{\mathbb{R}}_+ = [0, \infty)$ *is a penalty function if and only if it satisfies:*

1) $P(x_i, y) \geq 0$ *for all* $i = 1 \ldots n$;
2) $P(x_i, y) = 0$ *if* $x_i = y_i$ *and*
3) *For any fixed* $\mathbf{x}$, *the set of minimizers of* $P(\mathbf{x}, \mathrm{y})$ *is either an individual or an interval.*

Then the penalty-based function is:

$$f(x) = argmin_y P(\boldsymbol{x}, y), \quad (4)$$

where $\mathbf{x} = (x_1, x_2 \ldots x_n)$ is an input vector while y is an output value or a fused value. Additionally, there are two conditions: (1) negative penalties are not allowed; and (2) if there is full agreement, no penalty is imposed.

Thus, a penalty-based function was proposed by Rathod et al. [41] for aggregating symmetric inputs with penalty-based weights. Then, the formulation of penalty-weighted aggregation is as follows:

$$P(\boldsymbol{x}, y) = \sum_{i=1}^{n} p_i (x_i - y), \quad (5)$$

where p_i absorbed weights.

Thus,

$$P(\mathbf{x}, \mathrm{y}) = \sum_{i=1}^{n} w_i |x_i - y|^i, \quad (6)$$

where influences the variation of the contribution of the $|x_i - y|^i$ *i*-th input and the weights influence the relative importance of the *i*-th input.

3.2. Multidimensional Poverty Index by ordered weighted mean and extensions

The document presents a new extension of the OWA operator for the calculation of the MPI, which incorporates, within the same structure, a prior reordering of the deprivations before applying the weighting vector. In this operator, the weights are not assigned to each indicator in its original position, but rather to its position within the ordered set, allowing different aggregation patterns to be represented according to the decision-maker's attitude or criteria. Based on this integration, the Pov_OWA measure is defined, reflecting the ordering and weighting logic of the OWA operator. The formulation is as follows:

Proposition 1. *A Pov_OWA operator of dimension n is a mapping* $Pov\_OWA: R^n \to R$ *that has an associated weighing vector W of dimension n with* $w_i \in [0, 1]$ *and* $\sum_{j=1}^{n} w_i = 1$, *such that*

$$Pov_{OWA(a_1, a_2, \ldots, a_n)} = \sum_{j=1}^{n} w_i j_k, \quad (7)$$

where j_k *is the jth largest value of home in the deprivation j of the dimension k.*

Here, for a k-dimensional household, the deprivation values of the indicators belonging to that dimension are ranked from highest to lowest, and j_k *denotes the value occupying the jth position in that descending order (*j_1 *is the highest,* j_2 *the second highest, and so on); next, the weight vector W is applied to these ordered positions rather than to the original positions of the indicators. For example, if dimension k (Housing) has three indicators with deprivation values of* 0.8, 0.5, *and* 0.9 *for a given household, the ordered sequence is* $j_1 = 0.9$, $j_2 = 0.8$, $j_3 = 0.5$, *and* $w_1 = 0.4$, $w_2 = 0.3$, $w_3 = 0.3$. *Then, the operator aggregates* $(0.4 \times 0.9) + (0.3 \times 0.8) + (0.3 \times 0.5) = 0.75$.

The operator Pov_OWA exhibits monotonicity, boundedness, and idempotency.

Theorem 1 *(Monotonicity). Suppose f is the Pov_OWA operator. If* $a_i \geq e_i$ *for all, then i*

$$f(a_1, \ldots, a_n) \geq f(e_1, \ldots, e_n), \quad (8)$$

Theorem 2 *(Limits). Suppose that f is the operator Pov_OWA; then:*

$$min(a_i) \leq f(a_1, \ldots, a_n) \leq max(a_i), \quad (9)$$

Theorem 3 *(Idempotency). Suppose f is the operator Pov_OWA. If* $a_i = A$, *for all, then i*

$$f(a_1, \ldots, a_n) = A, \quad (10)$$

Proposition 2. *A Pov_IOWA operator of dimension n is a mapping* $Pov\_OWA: R^n \to R$ *that has an associated weighing vector W of dimension n with* $w_i \in [0, 1]$ *and* $\sum_{j=1}^{n} w_i = 1$ (u_i), *where an induced set of ordering variables is included such that the formula is, such that*

$$Pov\_IOWA\left(\left\langle u_1, j_{k_1}\right\rangle, \left\langle u_2, j_{k_2}\right\rangle, \ldots, \left\langle u_n, j_{k_n}\right\rangle\right) = \sum_{j=1}^{n} w_i j_k, \quad (11)$$

where j_k *is the jth largest value of home in the deprivation j of the dimension k.* j_k *value of the OWA pair*$<u_i, a_i>$*having the jth largest* u_i. u_i *is the order-inducing variable and* a_i *is the argument variable.*

Theorem 1 *(Monotonicity). Let's say f is the Pov_IOWA operator. If for two vectors of deprivation* a $= (a_1, \ldots, a_n)$ *and* $\mathbf{e} = (e_1, \ldots, e_n)$, *it is true that:*

$a_i \geq e_i$ *for everything i*

Then

$$f((u_1, a_1), \ldots, (u_n, a_n)) \geq f((u_1, e_1), \ldots, (u_n, e_n)), \quad (12)$$

Theorem 2 *(Limits). Suppose f it is the operator Pov_IOWA; then:*

$$min\,(a_i) \leq f((u_1, a_1), \ldots, (u_n, a_n)) \leq max(a_i), \quad (13)$$

Theorem 3 *(Idempotency). Let's say f is the Pov_IOWA operator. If for all of them i, it is true that:*

$$a_i = A,$$

Then

$$f((u_1, a_1), \ldots, (u_n, a_n)) = A, \quad (14)$$

Theorem 4 *(Induced). Let's assume that f is the Pov_IOWA operator and* $(u_1, \ldots, u_n)$ *that it is the set of variables that induce ordering. Then, the reordering of the set* $\{\langle u_i, a_i\rangle\}$ *is carried out according to the values* u_i*, so that:*

$$f((u_1, a_1), \ldots, (u_n, a_n)) = \sum_{j=1}^{n} w_i\, a_i, \quad (15)$$

where a_{i_j} *is the component associated with the j-th highest induction value* u_{i_j}*.*

Proposition 3. *The penalty function is a measure of disagreement or dissimilarity in poverty estimation*

$$PPov\_OWA\,(\mathbf{x}, y) = \sum_{i=1}^{n} w_i\, |j_k - y|^i, \quad (16)$$

where $|x_i - y|^i$ *influences the variation of the contribution of the i-th input and the weights influence the relative importance of the i-th input. y as mean (merged value).*

Theorem 1 *(Penalized Monotonicity). Let's say f is the PPov_OWA. If for two vectors* $\mathbf{x} = (x_1, \ldots, x_n)$ *and* $\mathbf{z} = (z_1, \ldots, z_n)$*, with respect to the same reference value y, it is true that:*

$|x_i - y|^i \geq |z_i - y|^i$, *for everything, i*

Then

$$f(x, y) \geq f(z, y), \quad (17)$$

Theorem 2 *(Penalized boundedness). Suppose f is the operator PPov_OWA. Then, for any vector* $\mathbf{x} = (x_1, \ldots, x_n)$ *and reference value y, it is verified that:*

$$\min_i |x_i - y|^i \leq f(x, y) \leq \max_i |x_i - y|^i \quad (18)$$

Theorem 3 *(Penalized Idempotency). Let's say f is the PPov_OWA. If:*

$$x_i = y \text{ for all } i$$

Then

$$f(x, y) = 0\,,\ \text{that is, when there is no discrepancy with the reference value, the penalty function is null} \quad (19)$$

Theorem 4 *(Penalized Ordering): Suppose f is the operator PPov_OWA, we define:*

$$di = |\, x_i - y^i,\ i = 1, \ldots, n,$$

and $\left(d_{1_k}, \ldots, d_{n_k}\right)$ *is the reordering of* d_i*, then*

$$d_{1_k} \geq d_{2_k} \geq \cdots \geq d_{n_k}$$

The penalized operator can then be expressed as:

$$f(x, y) = \sum_{i=1}^{n} w_i\, d_{1_k} = \sum_{i=1}^{n} w_i\, |x_{i_k} - y|^i, \quad (20)$$

where w_i is the weight associated with the i-th ordered position and $\sum_{i=1}^{n} w_i = 1$.

Finally, at the end of the document, all notations are in Appendix A.

3.3. Numerical example

To illustrate the practical application of the proposed methodological approach, a numerical example of the estimation of multidimensional poverty using the operators Pov_OWA, Pov_IOWA, and PPov_OWA is included. The purpose of this example is to illustrate the aggregation procedure associated with each operator and the differences it introduces compared to the traditional formulation of the index. The tables below summarize the final results of the aggregation process.

The weight of 0.20 assigned to Education, Children and Youth, Health, and Work corresponds to the official equal-weighting scheme described in Section 2.1.2 (five dimensions, each weighted 0.20). Here, for a numerical example, household H1 is constructed purely for illustrative purposes (see Table 2).

Table 2
Contribution of each dimension to the traditional MPI of household H1

Home	Education	Children and Youth	Bless You	Work	Housing
H1	0.20	0.20	0.20	0.20	0.12

Source: own elaboration, based on the dimensions of the official measurement of multidimensional poverty [25].

The formula for calculating the usual value of the MPI is to add the contributions from each dimension, each weighted by the corresponding value specified in the official method. The results for the H1 household are:

$$IPM_{Trad} = 0.20 + 0.20 + 0.20 + 0.20 + 0.12 = 0.92$$

Using the traditional formulation (Definition 1), when $IPM = 0.92$, the value exceeds the threshold of 0.33; H1 household is classified as poor according to the official methodology.

For the aggregation operators, we use the 15 indicators that make up the MPI. Table 3 summarizes, for household H1, the normalized deprivation values x_i and the associated weights w_i for each indicator.

For the Pov_OWA operator, a weighted sum of the normalized values of the 15 indicators is used. For household H1, the explicit calculation is:

$$\begin{aligned} Pov_{OWA_{H1}} &= 0.0621\,(1.0) + 0.0767\,(1.0) + 0.0627\,(0.7) \\ &+ 0.0614\,(0.8) + 0.0670\,(0.7) + 0.0696\,(0.7) \\ &+ 0.0637\,(0.9) + 0.0787\,(0.8) + 0.0457\,(0.5) \\ &+ 0.0544\,(0.8) + 0.0797(0.8) + 0.0797(0.8) \\ &+ 0.0674(0.8) + 0.0568(0.8) + 0.0742(0.9) \\ Pov_{OWA_{H1}} &= 0.8079 \end{aligned}$$

On the other hand, for the Pov_IOWA operator, the same normalized values x_i and weight vector W are used. Still, the

Table 3
Normalized values and weights per indicator for household H1

Indicator	x_i	w_i
E_LED	1.0	0.0621
E_ALF	1.0	0.0767
NJ_AVE	0.7	0.0627
NJ_ASI	0.8	0.0614
NJ_SPI	0.7	0.0670
NJ_TIN	0.7	0.0696
S_ASA	0.9	0.0637
S_ASN	0.8	0.0787
T_EFO	0.5	0.0457
T_DLD	0.8	0.0544
V_AFA	0.8	0.0797
V_AEE	0.8	0.0797
V_PID	0.8	0.0674
V_PAD	0.8	0.0568
V_HAC	0.9	0.0742

deprivations are reordered according to an induction vector U reflecting the experts' relative importance assigned to each indicator. Denoting by $x_{(1)}^{ind}, \ldots, x_{(15)}^{ind}$ the values reordered according to this criterion; the operator is calculated as:

$$\begin{aligned} Pov\_IOWA_{H1} &= 0.07974(1.0) + 0.07974(1.0) + 0.07867(0.9) \\ &\quad + 0.07666(0.9) + 0.07423(0.8) + \cdots \\ &\quad + 0.04572(0.5) \\ Pov\_IOWA_{H1} &= 0.8164 \end{aligned}$$

Finally, with the penalty function for household **H1**, and using the reference value y and the parameter i, we obtain for each indicator: $p_i = |x_i - y|^i$.

Applying this expression to the 15 indicators, the penalized values shown in Table 4 are obtained.

Table 4
Penalized values p_i for household H1

Indicator	$\lvert x_i \rvert$	$p_i = \lvert x_i - y \rvert^i$
E_LED	1.0	0.1687
E_ALF	1.0	0.2076
NJ_AVE	0.7	0.0844
NJ_ASI	0.8	0.0918
NJ_SPI	0.7	0.0818
NJ_TIN	0.7	0.0742
S_ASA	0.9	0.0880
S_ASN	0.8	0.1006
T_EFO	0.5	0.0352
T_DLD	0.8	0.0652
V_AFA	0.8	0.1045
V_AEE	0.8	0.1045
V_PID	0.8	0.0878
V_PAD	0.8	0.0660
V_HAC	0.9	0.0847

The OWA operator requires ordering the penalized values from highest to lowest, and for the H1 household, it is:

$$\begin{aligned} &p_{(1)} = 0.2076,\ p_{(2)} = 0.1687,\ p_{(3)} = 0.1045, \\ &p_{(4)} = 0.1006\ldots\ldots, \\ &PPov\_OWA_{(H1)} = w1 * 0.2076,\ p_{(2)} = 0.1687, \\ &\quad p_{(3)} = 0.1045,\ p_{(4)} = 0.1006\ldots\ldots, \end{aligned}$$

When evaluating this expression with the values of the H1 home, we obtain:

$$PPov\_OWA_{(H1)} = 0.6463$$

In summary, for household H1, the following index values are obtained:

$$\begin{aligned} IPM_{Trad} = 0.92,\ Pov\_OWA_{H1} = 0.8079, \\ Pov\_IOWA_{H1} = 0.8164,\ PPov\_OWA_{(H1)} = 0.6463, \end{aligned}$$

In all cases, the value exceeds the threshold of 0.33, so the H1 household is classified as poor under any operator. However, the magnitude of the index varies appreciably across aggregation schemes, indicating that the choice of operator can affect the estimated poverty intensity and, potentially, the relative ordering of households when analyzing the entire population. Additionally, we have introduced a new scenario considering several weighted vectors to observe robustness and sensitivity analyses of the expert opinions (see Appendix B).

4. Application of Operators in the Calculation of MPI

Once the conceptual foundations of the MPI have been exposed and the mathematical structure of the aggregation operators used—Pov_OWA, Pov_IOWA, and PPov_OWA—has been described, the application of the methodological proposal for its recalculation is presented. This section develops the implementation of the operators in the MPI estimation process, intending to evidence their aggregate behavior and the differences they introduce relative to the traditional formulation. To this end, an analytical sequence is established that integrates the ordering of deprivations, induction based on expert judgment, and the penalty component derived from the discrepancy relative to a reference value. The methodological steps and the results obtained from the operators' application of the database under consideration are described below:

Step 1. Initially, the information on the 15 deprivations that make up the MPI is considered. Based on these indicators, the experts who will participate in the construction of the necessary vectors for the aggregation operators are selected. At this stage, the experts were selected based on their experience in issues related to multidimensional poverty, social development, or well-being assessment, as well as their work in public or private institutions involved in poverty measurement or intervention, so that they have sufficient technical expertise to assess the relative importance of each indicator. This selection ensures that the assessments provided reflect specialist knowledge and a comprehensive understanding of how the MPI works. The panel was limited to five experts, given the exploratory and proof-of-concept nature of this first application of the Pov_OWA, Pov_IOWA, and PPov_OWA operators to the Colombian MPI; this sample size is generally in line with standard practice in OWA-based multi-expert decision-making studies, in which small panels are typically used because the weighting and induction vectors are obtained individually and then aggregated, rather than being statistically estimated from a large sample. At this stage, experts were selected based on their experience with issues related to multidimensional poverty, social development, or well-being assessment, as well as their work in public or private institutions dedicated to poverty measurement or poverty alleviation, ensuring they possessed sufficient technical expertise to evaluate the relative importance of each indicator. This selection process ensures that the assessments provided reflect specialized knowledge and a comprehensive understanding of how the MPI works.

Step 2. In this phase, the weighting vector, the induced order, and the values necessary for the penalty function are constructed. For the Pov_OWA operator, the weight vector is established based on the evaluations of the five experts, ensuring that the weights are non-negative and sum to 1. For the Pov_IOWA operator, in addition to the vector, a set of induced values is determined, which allows ordering the deprivations according to the expert criteria before performing the aggregation. Finally, for the PPov_OWA operator, a reference value is set, and the penalized distance is calculated, which will later be used in the reordering and weighting process. These inputs are essential for each operator to adequately reflect the structure of importance assigned by the experts and the differences in behavior between the methods. To determine the weighting vectors W, each expert was asked independently to rank each of the 15 indicators listed below on a scale of 1–10. The weights were obtained by averaging the normalized scores from the pairwise comparisons provided, which were subsequently rescaled so that their sum equaled 1 (see Appendix C).

$$
\begin{gathered}
W = 0.062;\ 0.077;\ 0.063;\ 0.061; \\
0.067;\ 0.070;\ 0.064;\ 0.079;\ 0.046; \\
0.054;\ 0.080;\ 0.080;\ 0.067;\ 0.057;\ 0.074
\end{gathered}
$$

Step 3. Calculations are made for each household using the process defined in Section 3.3. All these calculations were made using the previously defined structures and formulas, and the final results are presented in tabular form in the corresponding section.

Step 4. Once the final values for each operator have been obtained, the household is classified according to the MPI criteria, which indicate that a household is poor if it exceeds the threshold of 0.33. To facilitate the review, the results are presented in separate tables for each operator, allowing us to observe how household classification varies across aggregation methods. This organization allows the results to be directly compared, and the differences derived from the classical ordering, the induced order, and the penalty used in each operator to be analyzed.

The first analysis is carried out based on the first proposition, Pov_OWA, according to the assessments of the experts, who indicate which indicators they consider having the highest degree of access. Under this operator, household poverty is interpreted as a phenomenon strongly determined by housing deprivations and those related to childhood and adolescence. At the same time, the Work dimension obtains a comparatively low weight (see Table 5). At the severity level H5 > H4 > H1 > H3 > H2, H5 and H4 are the ones that denote the greatest preference and importance to deprivation, H2 is the most permissive, and H1–H3 are in intermediate positions (see Table 6).

Table 5
OWA weighted matrix

	Education		Children and Youth				Bless You		Work		Housing					Weighted Sum
	E_LED	E_ALF	NJ_AVE	NJ_ASI	NJ_SPI	NJ_TIN	S_ASA	S_ASN	T_EFO	T_DLD	V_AFA	V_AEE	V_PID	V_PAD	V_HAC	
H1	0.062	0.077	0.044	0.049	0.047	0.049	0.057	0.063	0.023	0.044	0.064	0.064	0.054	0.045	0.067	0.808
H2	0.031	0.077	0.006	0.061	0.067	0.035	0.006	0.039	0.005	0.005	0.040	0.040	0.067	0.028	0.074	0.583
H3	0.050	0.046	0.044	0.049	0.060	0.049	0.064	0.071	0.032	0.038	0.080	0.072	0.040	0.034	0.045	0.773
H4	0.037	0.077	0.050	0.061	0.067	0.070	0.051	0.063	0.032	0.044	0.080	0.080	0.054	0.051	0.067	0.883
H5	0.062	0.077	0.056	0.055	0.060	0.063	0.064	0.071	0.041	0.049	0.072	0.072	0.054	0.045	0.067	0.908

Table 6
Household poverty OWA operator

	Education	Children and Youth	Bless You	Work	Housing	Weighted Sum
H1	0.14	0.19	0.12	0.07	0.29	0.81
H2	0.11	0.17	0.05	0.01	0.25	0.58
H3	0.10	0.20	0.13	0.07	0.27	0.77
H4	0.11	0.25	0.11	0.08	0.33	0.88
H5	0.14	0.23	0.13	0.09	0.31	0.91

Continuing, when the Pov_IOWA is used, the nature of expert judgment changes, since the answers now correspond to the question about explicit priorities, enabling the use of an induction vector to order the indicators before adding them. In this case, the results show that the experts' priority continues to focus on the dimensions Housing and Children and Youth. Compared with the previous operator's calculation, the index value changes, but the tabulation of which dimensions weigh more does not.

When looking at H1, H2, H4, and H5, the index rises slightly when priority is introduced. At the same time, for H3, it decreases markedly, indicating that the H3 priority pattern reorders the indicators to reduce the joint contribution of the highest deprivations; that is, their way of prioritizing is less punishing than their original preference pattern. This result is observed only when using the induced operator, and it supports the idea that IOWA is especially useful for capturing conceptual differences between experts that, with a standard OWA, would appear similar (see Tables 7 and 8).

Finally, the most distinctive behavior is observed with the PPov_OWA operator, which incorporates the third dimension of expert judgment, being the degree of agreement on the need to improve the degree of access to the home. This operator uses a penalty function that amplifies or attenuates the influence of discrepancies (see Table 9) between the observed value and a reference value, the results obtained (0.646; 0.326; 0.593; 0.771; 0.823) show a significant decrease in the average index (0.632), allowing to identify that the penalized operator not only reduces the overall severity of the index, but also makes the differences of the experts more visible (see Table 10).

The case of the H2 expert shows that his index, which was 0.583 (OWA) and 0.607 (IOWA), is reduced to 0.326, remaining practically at the poverty line (0.33), reflecting a low level of agreement with the need to improve access, which substantially moderates the final magnitude of the indicator. In contrast, H5 and H4, whose level of agreement is high, maintain high indices even under penalty (0.823 and 0.771), and despite the reduction in magnitudes, the dimensional structure is maintained; that is, Housing continues to be the most decisive dimension, followed by Children and Youth, which reinforces the structural consistency of the proposed method (see Table 11).

In summary, when comparing the three operators, it is observed that the methodological proposal does not alter the internal logic of the Colombian MPI but rather enriches it by allowing different types of expert judgment to influence the aggregation of deprivations in a differentiated manner. With the operators, the dimensional structure is stable and consistent with the theory, but the severity and dispersion levels among experts change significantly, confirming the relevance of using aggregation operators to strengthen the measurement of multidimensional poverty.

Table 7
IOWA weighted matrix

T. 0.33	Education		Children and Youth				Health		Work		Housing					W
	E_LED	E_ALF	NJ_AVE	NJ_ASI	NJ_SPI	NJ_TIN	S_ASA	S_ASN	T_EFO	T_DLD	V_AFA	V_AEE	V_PID	V_PAD	V_HAC	sum
H1	0.080	0.080	0.038	0.059	0.043	0.040	0.069	0.051	0.023	0.050	0.056	0.054	0.054	0.050	0.071	0.816
H2	0.034	0.08	0.005	0.08	0.079	0.031	0.006	0.032	0.005	0.006	0.035	0.034	0.074	0.031	0.077	0.607
H3	0.043	0.022	0.045	0.031	0.064	0.033	0.080	0.060	0.030	0.031	0.062	0.080	0.020	0.016	0.020	0.637
H4	0.027	0.08	0.045	0.080	0.079	0.070	0.050	0.050	0.038	0.049	0.077	0.074	0.051	0.060	0.061	0.891
H5	0.080	0.080	0.055	0.069	0.067	0.056	0.079	0.057	0.051	0.056	0.061	0.06	0.044	0.037	0.063	0.914

Note: T: Threshold. W: Weighted

Table 8
Household poverty IOWA operator

	Education	Children and Youth	Health	Work	Housing	Weighted Sum
H1	0.16	0.18	0.12	0.07	0.28	0.82
H2	0.11	0.20	0.04	0.01	0.25	0.61
H3	0.07	0.17	0.14	0.06	0.20	0.64
H4	0.11	0.27	0.10	0.09	0.32	0.89
H5	0.16	0.25	0.14	0.11	0.26	0.91

Table 9
Merging value for penalty

C	Fused value
H1	0.800
H2	0.560
H3	0.767
H4	0.873
H5	0.907

To provide a better understanding of the results—as can be seen in Figure 2, the results vary depending on the method used—and based on expert opinion, only two households would fall below the poverty line (green).

Overall, the numerical and theoretical conclusions support that the use of OWA, IOWA, and P-OWA in the Colombian MPI allows (i) to make explicit the rule of compensation between deprivations, (ii) to incorporate expert priorities in a replicable way, and (iii) to reinforce sensitivity to severe profiles, all without violating the basic properties of the Alkire–Foster method or the comparability with the official measurement.

5. Conclusions

This paper examines the main methodological aspects of Colombia's MPI, including how its deprivations, dimensions, and indicators are synthesized into a household-weighted index. Based on the revision of the official formula [24] adopted by DANE, a recalculation of the MPI is proposed using order aggregation operators. This proposal did not modify the binary matrix of deprivations, the dimensions, the official weights, or the identification threshold k = 0.33; the change occurs in the aggregation rule that produces the weighted household index. Taking into account the above, the purpose of the research was to introduce expert knowledge in the weighting of deprivations and explore alternative compensation rules to the linear weighted sum of the official methodology, allowing the modification of the rule of compensation between deprivations, being consistent with the use of the MPI in Colombia as an indicator of incidence, gap, and severity tool for targeting and monitoring social policy. The application of these approaches demonstrates that it is possible to strengthen the measurement of the MPI through tools that better integrate expert knowledge, prioritize deprivations relative to one another, and penalize discrepancies between indicators in a differentiated manner, thus responding to the inherent complexity of the phenomenon of poverty.

First, the results show that the use of the Pov_OWA operator provides a more sensitive perspective on the internal distribution of deprivations within each MPI dimension. As stated in the seminal literature, OWA operators allow the aggregation of

Table 10
P-OWA weighted matrix

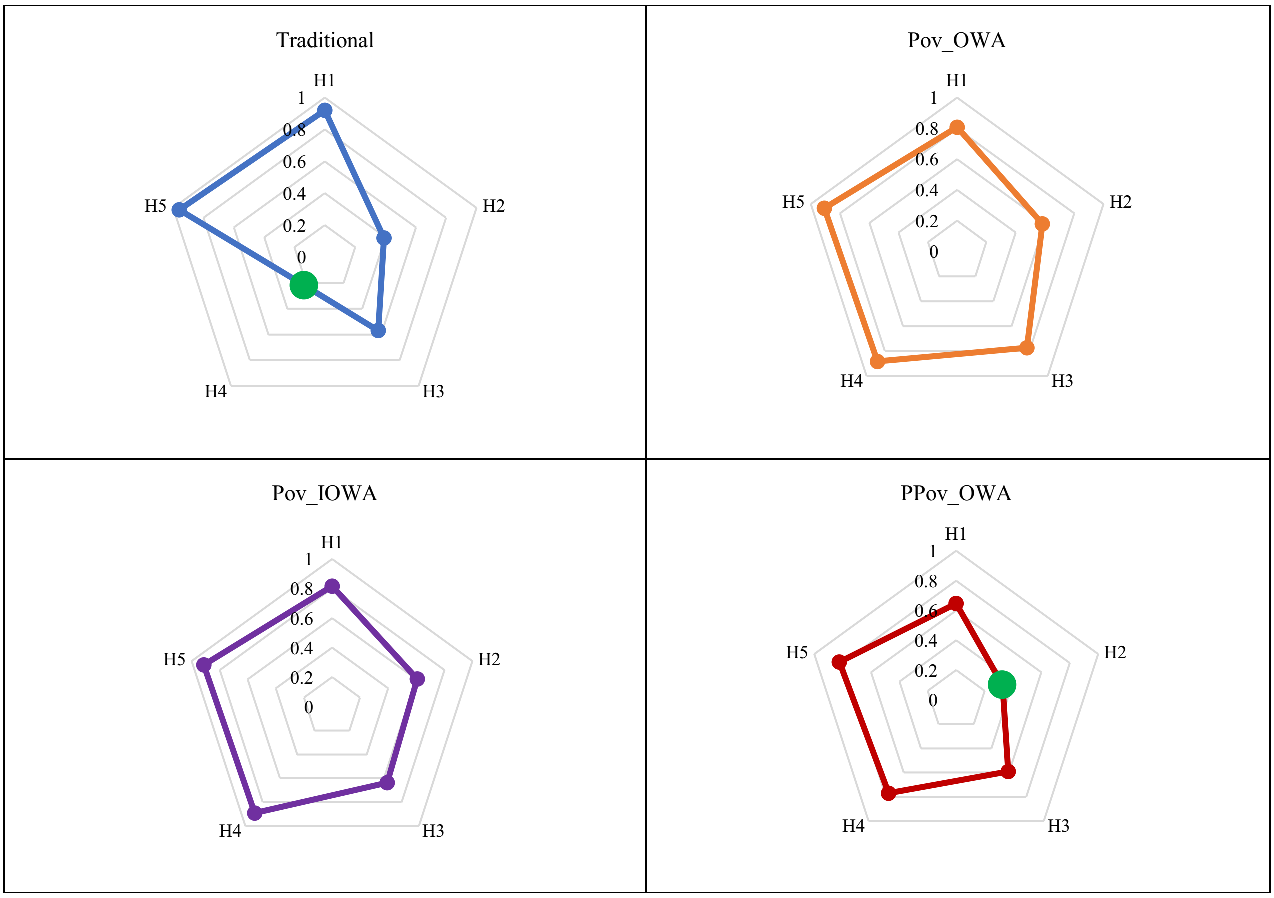
T. 0.33	Education		Children and Youth				Health		Work		Housing					W
	E_LED	E_ALF	NJ_AVE	NJ_ASI	NJ_SPI	NJ_TIN	S_ASA	S_ASN	T_EFO	T_DLD	V_AFA	V_AEE	V_PID	V_PAD	V_HAC	Sum
H1	0.05	0.061	0.035	0.039	0.038	0.039	0.046	0.05	0.018	0.035	0.051	0.051	0.043	0.036	0.053	0.646
H2	0.017	0.043	0.004	0.034	0.038	0.019	0.004	0.022	0.003	0.003	0.022	0.022	0.038	0.016	0.042	0.326
H3	0.038	0.035	0.034	0.038	0.046	0.037	0.049	0.054	0.025	0.029	0.061	0.055	0.031	0.026	0.034	0.593
H4	0.033	0.067	0.044	0.054	0.059	0.061	0.044	0.055	0.028	0.038	0.070	0.070	0.047	0.045	0.058	0.771
H5	0.056	0.07	0.051	0.05	0.055	0.057	0.058	0.064	0.037	0.044	0.065	0.065	0.049	0.041	0.061	0.823

Note: T: Threshold. W: Weighted

Table 11
Household poverty P-OWA operator

Condition	Education	Children and Youth	Health	Work	Housing	Weighted Sum
H1	0.11	0.15	0.10	0.05	0.24	0.65
H2	0.06	0.09	0.03	0.01	0.14	0.33
H3	0.07	0.15	0.10	0.05	0.21	0.59
H4	0.10	0.22	0.10	0.07	0.29	0.77
H5	0.13	0.21	0.12	0.08	0.28	0.82

Figure 2
Graphical representation of MPI



information to be adjusted by means of a weight vector that not only reflects relative importance but also the decision-maker's desired behavior in terms of compensation or demand. In this study, the operator identified apparent differences in the assessment of deprivation when weights derived from the expert analysis are incorporated. The results show that households with the same binary pattern of deprivation can obtain different aggregate values depending on how the information is reordered and weighted, thereby expanding the MPI's capacity to reflect heterogeneous vulnerability scenarios.

Second, the operator Pov_IOWA explicitly integrated the expert opinion into the ordering process before aggregation. As indicated by the literature associated with induced operators, this type of formulation allows the incorporation of additional variables, in this case, the preferences or weightings induced by the experts that determine the order in which the deprivations enter the aggregation process. The use of the induction vector showed that, for the same set of normalized values, the index structure changes when prioritizing indicators that, according to the expert's judgment, are more relevant to the territorial understanding of poverty. In this sense, the operator serves as a robust tool to strengthen the subjective and institutional components of measurement.

Third, the implementation of the PPov_OWA operator introduces an additional element of analysis: the penalty for discrepancies relative to a reference value. This approach is supported by the literature on penalty functions and P-OWA operators, which argues that the distance between observed values and a relevant reference point, such as an average, threshold, or desired value, can capture phenomena of internal inequality in the data or non-linear patterns of deprivation. In this study, the applied penalty function made it possible to reduce the relative weight of those indicators whose values were closer to the reference value, simultaneously increasing the impact of the most severe or distant deprivations, in the final result a significantly lower index was evidenced than that obtained in Pov_OWA and Pov_IOWA, which reveals that the penalization process counteracts the contribution of certain indicators, providing a more modulated reading of the poverty level.

Taken together, these findings confirm that Pov_OWA, Pov_IOWA, and PPov_OWA operators are solid and enriching methodological alternatives for the calculation of MPI, and their implementation facilitates a more nuanced reading of the phenomenon, incorporates expert judgment in a structured way, allows for the exploration of different prioritization criteria, and recognizes that not all deprivations have the same weight or degree of severity. In addition, there was significant variability in the identification and weighting of household access levels according to the methodology applied, and this is mainly due to the way in which each method structures the relationships between the indicators, as well as to the evaluations made by the experts consulted, reflecting the fact that there is no single way to measure multidimensional poverty but different approaches, that when combined, can provide a more complete view of the phenomenon. In conclusion, the heterogeneity of the indicators evaluated is particularly evident in diverse geographical and regional contexts, such as Colombia, requiring flexible, sensitive, and locally focused approaches. Measuring poverty is not only a technical exercise but also a strategic tool to highlight inequalities and advance the fulfillment of the SDGs. In addition, this study highlights the importance of considering multiple tools and perspectives when analyzing poverty, recognizing that it is a complex and dynamic phenomenon.

Other studies have explored poverty measurement by linking OWA operators using the dual decomposition of aggregation functions, which demonstrate that all classical rank-dependent measures of poverty, once normalized, an OWA operator provides a coherent breakdown of poverty into incidence, intensity, and inequality among the poor [52]. Here, the OWA operator with weights that are fixed analytical functions of the number of poor q and the population size n underlies each definition.

In considering the above, this study includes subjective information obtained from experts in its weighting vector. This made it possible to design Pov_OWA, Pov_IOWA, and PPov_OWA by defining the MPI's own aggregation rule directly on the household deprivation matrix, with weighting and induction vectors derived from experts' actual judgments rather than analytically, and with an explicit penalty component. In this regard, the mathematical contribution consists of formalizing this aggregation structure for the specific case of the MPI, while the applied contribution consists of putting it into practice using actual expert assessments of Colombia's official poverty index.

Finally, the research offers a complementary perspective on how to address the Colombian MPI without disrupting its statistical continuity. As a future line, it is suggested to extend the recalculation to national and regional microdata to evaluate its impact on departmental incidences, gaps, and severity. With this, the operators Pov_OWA, Pov_IOWA, and PPov-OWA can consolidate as a formal complement to DANE's MPI for robustness analysis, targeting, and evaluation of policies aimed at reducing multiple deprivations in Colombia. Furthermore, the incorporation of complex bipolar fuzzy sets makes it possible to simultaneously capture the magnitude and direction of deprivations, a fundamental feature in the multidimensional measurement of poverty [5]. Likewise, the adaptation of the Muirhead-multi-attribute decision-making (MADM) approach to assess social inequality, together with the Best–Worst Method (BWM) combined with Evaluation Based on Distance from Average Solution (EDAS) framework for prioritizing sustainable interventions, offers a set of transferable and formally rigorous tools [53]. Future work could explore its empirical validation using real-world data on multidimensional poverty indices in Latin American and Global South contexts.

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Ethical Statement

This study does not contain any studies with human or animal subjects performed by any of the authors.

Conflicts of Interest

The authors declare that they have no conflicts of interest to this work.

Data Availability Statement

Data are available from the corresponding author upon reasonable request.

Author Contribution Statement

Danna Suárez-Gomez: Conceptualization, Methodology, Validation, Investigation, Data curation, Writing – original draft. **Fabio Blanco-Mesa:** Conceptualization, Methodology, Writing – original draft, Writing – review & editing, Supervision, Project administration, Funding acquisition. **Ernesto León-Castro:** Software, Formal analysis, Resources, Writing – original draft, Visualization, Supervision. **Luis Asuncion Pérez-Domínguez:** Software, Formal analysis, Resources, Writing – review & editing. **Víctor G. Alfaro-García:** Validation, Investigation, Data curation, Visualization.

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Appendix A. Notation and Acronyms Used in the Manuscript

Notation	Description
MPI	Multidimensional Poverty Index
OWA	Ordered weighted averaging operator
IOWA	Induced ordered weighted averaging operator
Pov_OWA	Poverty OWA operator (adaptation of the OWA operator to the structure of the MPI)
Pov_IOWA	Poverty-induced OWA operator (adaptation of the IOWA operator to the structure of the MPI)
PPov_OWA (P_OWA)	Poverty penalty OWA operator (penalty-based adaptation of the OWA operator to the structure of the MPI)
IPM_Trad	Traditional MPI index value (official, non-operator-based calculation)
n	Dimension of the operator (number of arguments or indicators being aggregated)
i	Index identifying a household, an input, or the i-th ordered position
j	Index identifying a deprivation/indicator, or the j-th position in an ordered sequence
k	Index identifying a dimension of the MPI (e.g., Education, Health, Housing)
j_k	Number of indicators belonging to dimension k
w_i	Weight assigned to the i-th (or j-th) position in the weighting vector
W	Weighting vector, $W = (w_i, \ldots, w_n)$, with $w_i \in [0,1]$ and $\sum w_i = 1$
$a_i\,(a_1 \ldots a_n)$	Set of input arguments (e.g., deprivation values) to be aggregated
b_j	The j-th largest value among the arguments a_i (value after descending ordering)
u_i	Order-inducing variable (used in the IOWA/Pov_IOWA operator to determine ranking)
U	Induction vector; the set of order-inducing variables $(u_i, \ldots, u_n)$
$x = x_1, \ldots, x_n$x	Input vector of normalized deprivation values
x_i	Normalized deprivation value of indicator i for a given household
$x(1)\,ind, \ldots, x(15)\,ind$	Deprivation values reordered according to the induction (priority) criterion
a_{ij}	Argument component associated with the j-th highest induction value a_{ij} (Theorem of Induced Ordering)
$Y1$	Output, reference, or fused (merged) value used in the penalty function
$P(x, y)$	Penalty function measuring disagreement/dissimilarity between inputs x and reference y
$f(x)$	Penalty-based aggregation function, $f(x) = \text{argmin}\ P(x, y)$
p_i	Absorbed weight in the penalty formulation (Eq. 5); also used to denote the penalized value $p_i = \lvert x_i - y \rvert^i$ (Table 4)
d_i	Absolute deviation, $d_i = \lvert x_i - y_i \rvert$
$d(l_k), \ldots, d(n_k)$	Deviations d_i reordered in descending order
$e_i\,(e_1 \ldots e_n)$	Alternative deprivation vector used for comparison in the Monotonicity theorems
A	Constant value used in the Idempotency theorems (when all $a_i = A$)
H1–H5	Household identifiers used in the numerical example and application

Appendix B. Other Scenario with Weighted Vectors

Indicator	x_i	w_1	w_2	w_3	w_4	w_5
E_LED	1	0.042	0.112	0.094	0.006	0.091
E_ALF	1	0.036	0.118	0.103	0.083	0.046
NJ_AVE	0.7	0.097	0.037	0.094	0.069	0.114
NJ_ASI	0.8	0.042	0.018	0.009	0.084	0.033
NJ_SPI	0.7	0.104	0.018	0.067	0.105	0.09
NJ_TIN	0.7	0.068	0.092	0.065	0.122	0.171
S_ASA	0.9	0.085	0.064	0.105	0.119	0.038
S_ASN	0.8	0.016	0.115	0.015	0.01	0.019
T_EFO	0.5	0.094	0.026	0.037	0.035	0.081
T_DLD	0.8	0.096	0.099	0.012	0.008	0.1
V_AFA	0.8	0.077	0.087	0.103	0.062	0
V_AEE	0.8	0.092	0.071	0.105	0.097	0.001
V_PID	0.8	0.064	0.093	0.083	0.106	0.061
V_PAD	0.8	0.004	0.024	0.004	0.07	0.064
V_HAC	0.9	0.084	0.027	0.103	0.023	0.092
			Pov_OWA			
H1	Education	Children and Youth	Health	Work	Housing	Weighted Sum
Scenario 1	0.078	0.222	0.09	0.124	0.265	0.777
Scenario 2	0.230	0.117	0.15	0.092	0.244	0.832
Scenario 3	0.197	0.166	0.107	0.028	0.328	0.826
Scenario 4	0.090	0.274	0.115	0.024	0.288	0.792
Scenario 5	0.137	0.289	0.049	0.120	0.184	0.779
			Pov_IOWA			
H1	Education	Children and Youth	Health	Work	Housing	Weighted Sum
Scenario 1	0.123	0.222	0.090	0.124	0.265	0.823
Scenario 2	0.207	0.117	0.150	0.092	0.244	0.810
Scenario 3	0.195	0.166	0.107	0.028	0.328	0.825
Scenario 4	0.178	0.274	0.115	0.024	0.288	0.880
Scenario 5	0.129	0.289	0.049	0.12	0.184	0.77
			PPov_OWA			
H1	Education	Children and Youth	Health	Work	Housing	Weighted Sum
Scenario 1	0.062	0.178	0.072	0.099	0.212	0.622
Scenario 2	0.184	0.094	0.12	0.074	0.195	0.666
Scenario 3	0.157	0.133	0.086	0.023	0.263	0.661
Scenario 4	0.072	0.219	0.092	0.019	0.231	0.634
Scenario 5	0.109	0.231	0.039	0.096	0.147	0.623

Appendix C. Normalization

Normalized data

Dimensions MPI	Education		Children and Youth				Health		Work		Housing				
ID	E_LED	E_ALF	NJ_AVE	NJ_ASI	NJ_SPI	NJ_TIN	S_ASA	S_ASN	T_EFO	T_DLD	V_AFA	V_AEE	V_PID	V_PAD	V_HAC
Expert 1	1	1	0.7	0.8	0.7	0.7	0.9	0.8	0.5	0.8	0.8	0.8	0.8	0.8	0.9
Expert 2	0.5	1	0.1	1	1	0.5	0.1	0.5	0.1	0.1	0.5	0.5	1	0.5	1
Expert 3	0.8	0.6	0.7	0.8	0.9	0.7	1	0.9	0.7	0.7	1	0.9	0.6	0.6	0.6
Expert 4	0.6	1	0.8	1	1	1	0.8	0.8	0.7	0.8	1	1	0.8	0.9	0.9
Expert 5	1	1	0.9	0.9	0.9	0.9	1	0.9	0.9	0.9	0.9	0.9	0.8	0.8	0.9

Each expert is asked to rate the weight vector on a scale of 0–10 if it is normalized.

Dimensions MPI	Education		Children and Youth				Health		Work		Housing				
ID	E_LED	E_ALF	NJ_AVE	NJ_ASI	NJ_SPI	NJ_TIN	S_ASA	S_ASN	T_EFO	T_DLD	V_AFA	V_AEE	V_PID	V_PAD	V_HAC
Weight for each expert	0.373	0.46	0.376	0.369	0.402	0.418	0.382	0.472	0.274	0.327	0.478	0.478	0.404	0.341	0.445
Weight normalized	0.062	0.077	0.063	0.061	0.067	0.07	0.064	0.079	0.046	0.054	0.08	0.08	0.067	0.057	0.062

Based on expert opinions and standardized weights, the results for each dimension are obtained so they can be discussed with each operator.

Pov_OWA

Threshold 0.33	Education		Children and Youth				Health		Work		Housing				
	E_LED	E_ALF	NJ_AVE	NJ_ASI	NJ_SPI	NJ_TIN	S_ASA	S_ASN	T_EFO	T_DLD	V_AFA	V_AEE	V_PID	V_PAD	V_HAC
H1	0.062	0.077	0.044	0.049	0.047	0.049	0.057	0.063	0.023	0.044	0.064	0.064	0.054	0.045	0.067
H2	0.031	0.077	0.006	0.061	0.067	0.035	0.006	0.039	0.005	0.005	0.04	0.04	0.067	0.028	0.074
H3	0.05	0.046	0.044	0.049	0.06	0.049	0.064	0.071	0.032	0.038	0.08	0.072	0.04	0.034	0.045
H4	0.037	0.077	0.05	0.061	0.067	0.07	0.051	0.063	0.032	0.044	0.08	0.08	0.054	0.051	0.067
H5	0.062	0.077	0.056	0.055	0.06	0.063	0.064	0.071	0.041	0.049	0.072	0.072	0.054	0.045	0.067

Induced variables are obtained from experts, which are used to obtain Pov_IOWA

Induced variable

	Education		Children and Youth				Health		Work		Housing				
ID	E_LED	E_ALF	NJ_AVE	NJ_ASI	NJ_SPI	NJ_TIN	S_ASA	S_ASN	T_EFO	T_DLD	V_AFA	V_AEE	V_PID	V_PAD	V_HAC
Induced variable	9.3	8.8	8.7	8.7	8.3	8.2	8.2	8	7.8	7.8	7.5	7.3	6.8	6.5	5.7

Pov_IOWA

Threshold 0.33	Education		Children and Youth				Health		Work		Housing				
	E_LED	E_ALF	NJ_AVE	NJ_ASI	NJ_SPI	NJ_TIN	S_ASA	S_ASN	T_EFO	T_DLD	V_AFA	V_AEE	V_PID	V_PAD	V_HAC
H1	0.08	0.08	0.038	0.059	0.043	0.04	0.069	0.051	0.023	0.05	0.056	0.054	0.054	0.05	0.071
H2	0.034	0.08	0.005	0.08	0.079	0.031	0.006	0.032	0.005	0.006	0.035	0.034	0.074	0.031	0.077
H3	0.043	0.022	0.045	0.031	0.064	0.033	0.08	0.06	0.03	0.031	0.062	0.08	0.02	0.016	0.02
H4	0.027	0.08	0.045	0.08	0.079	0.07	0.05	0.05	0.038	0.049	0.077	0.074	0.051	0.06	0.061
H5	0.08	0.08	0.055	0.069	0.067	0.056	0.079	0.057	0.051	0.056	0.061	0.06	0.044	0.037	0.063

Fused values obtained from a weighted vector with penalization.

Dimensions MPI	Education		Children and Youth				Health		Work		Housing				
ID	E_LED	E_ALF	NJ_AVE	NJ_ASI	NJ_SPI	NJ_TIN	S_ASA	S_ASN	T_EFO	T_DLD	V_AFA	V_AEE	V_PID	V_PAD	V_HAC
H1	0.8	0.8	0.56	0.64	0.56	0.56	0.72	0.64	0.4	0.64	0.64	0.64	0.64	0.64	0.72
H2	0.28	0.56	0.056	0.56	0.56	0.28	0.056	0.28	0.056	0.056	0.28	0.28	0.56	0.28	0.56
H3	0.613	0.46	0.537	0.613	0.69	0.537	0.767	0.69	0.537	0.537	0.767	0.69	0.46	0.46	0.46
H4	0.524	0.873	0.699	0.873	0.873	0.873	0.699	0.699	0.611	0.699	0.873	0.873	0.699	0.786	0.786
H5	0.907	0.907	0.816	0.816	0.816	0.816	0.907	0.816	0.816	0.816	0.816	0.816	0.725	0.725	0.816

PPov_OWA

Threshold 0.33	Education		Children and Youth				Health		Work		Housing				
	E_LED	E_ALF	NJ_AVE	NJ_ASI	NJ_SPI	NJ_TIN	S_ASA	S_ASN	T_EFO	T_DLD	V_AFA	V_AEE	V_PID	V_PAD	V_HAC
H1	0.05	0.061	0.035	0.039	0.038	0.039	0.046	0.05	0.018	0.035	0.051	0.051	0.043	0.036	0.053
H2	0.017	0.043	0.004	0.034	0.038	0.019	0.004	0.022	0.003	0.003	0.022	0.022	0.038	0.016	0.042
H3	0.038	0.035	0.034	0.038	0.046	0.037	0.049	0.054	0.025	0.029	0.061	0.055	0.031	0.026	0.034
H4	0.033	0.067	0.044	0.054	0.059	0.061	0.044	0.055	0.028	0.038	0.07	0.07	0.047	0.045	0.058
H5	0.056	0.07	0.051	0.05	0.055	0.057	0.058	0.064	0.037	0.044	0.065	0.065	0.049	0.041	0.061