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Fuzzy Hidden Markov Model Using Aggregation Operators

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Abstract: The Fuzzy Markov Model is a fascinating domain for dealing with ambiguity in real-world scenarios. In type-2 fuzzy set (T2F), it has an uncertainty footprint, and the region circumscribed by the lower and upper interval membership functions is uncertain. In fuzzy sets, triangular norms (t-norms) are a valuable tool for understanding the conjunction in fuzzy logic and, as a result, determining where fuzzy sets intersect. Norms and conforms in triangular operations that generalise logical conjunction and disjunction. They also provide a natural explanation for the conjunction and disjunction in mathematical fuzzy logic semantics. Fuzzy Frank t-norms have been used to verify this t-norm as there are many of the aggregation qualities of trapezoidal interval type-2 numbers (TpIT2FNs) because triangular norm meets the compatibility with Frank norms. Frank t-norms provide more flexibility and robustness; this requires more justification in the information fusion process than other t-norms. Previous works on not concentrate on Frank's norms. Other aggregation works on norms that are not flexible to get the solution. Because of that, the Frank norms are used for the hidden Markov model. We have also used them in the Viterbi method with TpIT2FNs for Fuzzy hidden Markov model in the staff selection process.

Keywords: triangular norms, aggregation operators, trapezoidal internal type-2 fuzzy number mathematical classification number: 60

1. Introduction

A logic that only accepts the binary values with ambiguous ideas governs crisp set theory. In order to solve this issue, fuzzy sets were developed. Fuzzy logic, a branch of fuzzy set theory, is used to define uncertainty and characterise fuzziness. The foundation of fuzzy logic is the idea that everything has a degree and imitates human thought. This logic runs experiments to simulate human judgement, word sense, and common sense, leading to the development of intelligent systems. Fuzzy set theory has several issues that can be resolved by resolving various kinds of functional equations. These equations typically employ unary and binary operators like negations, t-norms, t-conorms, etc. It is commonly known that Frank and Alsina formulated and solved two functional equations. The last two unknowns are a t-norm and a t-conorm. The same is true for the second one, which likewise has an unnamed significant negative. Uninorms were introduced by Yager and Rybalov as a generalisation of t-norms and t-conorms. Instead of being limited to either 0 or 1, the neutral element for uninorms can be any value inside the unit interval for a list of all the abnormal classes that have been found. Uninorms are generalisations of t-norms and t-conorms; hence, it only makes sense to look at the Frank equation if one of the unidentified binary operators is an uninorm U with a known neutral element, e [0; 1]. Given the significance of Frank t-norms in preference modelling, the scenario of a conjunctive uninorm piques our curiosity in particular.

Zadeh invented type-1 fuzzy sets (T1F) to cope with uncertainty in real-world scenarios (Zadeh, 1965; Zimmermann, 2010). Type-2 fuzzy set (T2F) is a significant advancement in fuzzy logic. Crisp sets 0 and 1 have values that belong fully or partially to the component, but a fuzzy set determined by a characteristic function has values that fall somewhere in between (Werro, 2015; Zadeh, 1975). The T1F membership function (MF) will be implemented. In the second case, the T1F MF will be sharp, whereas the T2F MF will be T1F, resulting in higher ambiguity. A T2F may effectively handle verbal and numerical ambiguity (Wu & Mendel, 2007; Gong et al., 2014; Nagarajan et al., 2022). As a result, type-1 fuzzy mentioned by John (1998) helps knowledge representation hidden Markov model enhancement (Nagarajan et al., 2020). Due to constraints for working with T2F, interval-based T2F is desirable in many real-world applications (Aminifar & Marzuki, 2013; Nasseri, 2008; Qin & Liu, 2014). Set-theoretic operations are widely used and endorsed in the applications since they handle more uncertainty than T1F. It may be used to model and deal with uncertainty in a variety of fields (Beliakov, 2003). In aggregation operators, fuzzy numbers are a development of real numbers mentioned by Karaaslan & Al-Husseinawi (2022) and Aisbett et al. (2011) can be used to deal with the data's obscurity. Operators are scattered over many domains with T1F. In an interval-based fuzzy set (Mendel et al., 2006), this work is made easier. The fuzzy weighted average (FWA) provides some strong mathematical analytical characteristics that can be applied to the

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interval type-2 aggregation process, according to (Zhou & Ying, 2012). To handle real-world scenarios, Abbadi et al (2013) note that arithmetic and geometric operators were designed as aggregation operators (Akay et al., 2011; Choi & Rhee, 2009; Deschrijver et al., 2004; Hwang et al., 2011; Liu, 2011; Liu & Jin, 2012; Sujatha et al., 2020; Maldonado et al., 2013; Wang et al., 2005) The alpha-cut decomposition theorem noted by Zadeh (1975) is used to create the aggregation operators. In operators, the fuzzy extension concept has been exploited, and triangle norms have been used to produce fuzzy aggregation operators (Deschrijver et al., 2004; Xia et al., 2012). Triangular norms (t-norms) have been used to construct a variety of fuzzy aggregation operators, and they are crucial in information fusion (Beliakov et al., 2007; Liu, 2013; Wang & Liu, 2012). Because they give more flexibility and robustness in the information fusion process than other triangle norms, Frank triangle norms are superior to other triangle norms for representing practical decision-making situations (Calvo et al., 2001). For commutative, associative, and rising qualities, binary operators were utilised to analyse Frank and Alsina's operating equations (Yager, 2004). The Yager and Frank operators define the conditions that must be met in order to construct the issue's whole structure (Sarkoci, 2005). HMM demonstrated through a web application. HMMs are graphical models that may be employed in a range of scenarios requiring unobservable (hidden) data sequences, such as in ecology, social dynamics, and epidemic emergence (Singh et al., 2022). HMMs can assist in reaching difficult-to-arrive-at conclusions concerning complicated system state dynamics. By using the existence theorem on possibility, a versatile method for dealing with missing data (McClintock et al., 2020). In this scenario, the beginning and values of probabilities were translated. To increase numerical power and simplify mathematical operations, we employed logarithmic probability. Exploiters use links and browsers to travel the board web, which is a large, allocated hypertext data monument. These links will need to be updated again. Exploiters have access to details about their problems. The exploiters' visits to the website are recorded in the weblog files. The information is carried via the files' origins, which are analysed using a web analyser. We cannot allow the probability measurements to work because some of the paths cause them to break down. As a result, in this regard, our strategy is appropriate. The result of the valuation study reveals that the exploiters have a lot of familiarity and likelihood value for an afforded watching in sequence. The Frank aggregation operation for TpIT2FN has been used in this paper. The fundamental notions of Frank operators for arithmetic operations are presented, and the Frank sum and product for TpIT2FN is explained. We used the TpIT2FFWA and TpIT2FFWG operators to construct theorems. Finally, we used the Viterbi algorithm to discover the best hidden Markov model path utilising TpIT2FN in the staff selection process. In bioinformatics, it is possible to determine if new sequences belong to the family by summarising the diversity of information in an existing alignment of sequences. By rescoring their profiles, the fuzzy profile HMMs can be utilised in bioinformatics to restructure protein domains. The new scores produced by the definition of the fuzzy HMM enable the discovery of novel motifs that may exist in protein sequences, which can result in a different representation of protein data in terms of motifs as a result of this procedure. Using various motif databases, it is possible to predict the protein function as it is indicated by its motifs. The bioinformatician can then do motif-based protein classification, offering a higher level of quality assessment of the classification outcomes.

As a result, it aids in the representation of knowledge and the development of type-1 fuzzy logic reasoning. To express the likelihood of an event, C2 has two MFs: primary and secondary. The membership values of T1F and T2F have been the subject of some debate, so interval T2Fs have found extensive use in a number of real-world applications. Fuzzy logic uses membership values to mimic Boolean logic, and fuzzy operators can be used in place of the fundamental operators AND, OR, and NOT. Set-theoretic procedures can be used to model and manage uncertainties in a variety of contexts. The interval-based fuzzy set's cogent explanation method for FWA, which has some good mathematical analysis properties and can be used for interval type-2 aggregation process, makes it easier to synthesise the information aggregation operators that are dispersed across various domains with C1. T-norms are a key component of information fusion and have been effectively used to create a number of fuzzy aggregation operators. Due to the imprecise character of many real-life situations, only fuzzy logic environments—not the traditional approaches—can be used to resolve them. Since the parameter is a part of the Frank t-norms, information synthesis may be more adaptable and robust as a result. By applying binary operators, the operative equations of Frank and Alsina meet the commutative, associative, and increasing characteristics. Operators offer necessary and basic preconditions for constructing the entire structure of uncertainty-containing real-world issues. Fuzzy modelling and optimisation are tasks used to comprehend and solve a challenging problem under fuzzy environment. Based on the problem's perspective and research of the fuzzy data, a suitable model can be created via fuzzy modelling. Using tools and methods for optimisation that are based on the design of the fuzzy information in terms of their MFs, fuzzy models can be solved to the best of their ability. A Markov model is a stochastic model that is used to represent a system where it is conceivable for random changes to occur. Future states in this method depend solely on the current state, making modelling and computation tractable.

Fuzzy operators will be particularly helpful in dealing with fuzzy data because uncertainty exists by nature in HMM. By using the existence theorem on the possibility, HMM with possibility space has a flexible method of handling missing information. Here, the HMM adaptable relationship between the data on the observation and the latent state sequence is used to modify the start and transition probabilities into a TpIT2FN. With C1 and C2 FSs, the components of the HMM and Viterbi algorithms can be designed. In order to boost the numerical power and make mathematical operations easier to complete, we have used logarithmic probabilities in logarithmic space in place of the conventional interval [0, 1] in our work. Utilising the monotonously growing property of bounding functions, it is possible to overcome the challenges associated with the typical adaptive backstepping control architecture. An observer-based fuzzy adaptive event-triggered control for pure feedback nonlinear systems with recommended performance can use a fuzzy state observer to estimate the unmeasured states availability of the board The internet is a free hypertext data monument, and links and browsers are its exploiters. These links will introduce fresh links as you browse. Exploiters get the information they need. Web log files contain information about website visitors who are exploiters. Information is carried in the files' origin, and web analyser is used. Since some of the paths cause them to falter, we cannot allow the probability measurements to operate. introduced together with the concepts of double, single discrete T2FSs, and single continuous T2FSs (Xu & Qin, 2023). The fundamentals of interval T2F and T1F and systems, especially it is useful for beginners (Wu et al., 2023). To estimation error of the type-2 fuzzy approach is also considered in the stability analysis (Firouzi et al., 2022). For the hierarchical ACFC system, a comprehensive control technique is proposed that incorporates type-2 fuzzy control and the variable time headway model (Mo et al., 2022).

1.1. The main points of the contribution

- Fuzzy Frank t-norms have been used to verify this t-norm as there are many of the aggregation qualities of trapezoidal interval type-2 numbers (TpIT2FNs) because triangular norm meets the compatibility with Frank norms.
- Frank t-norms provide more flexibility and robustness; this requires more justification in the information fusion process than other t-norms.
- Previous works on not concentrate on Frank's norms.
- Other aggregation works on norms that are not flexible to get the solution. Because of that, the Frank norms are used for the hidden Markov model.
- We have also used them in the Viterbi method with TpIT2FNs for Fuzzy hidden Markov model in the staff selection process.

2. Basic Definitions

In this section, we define the basic operation on triangular interval type-2 fuzzy numbers. The graphical interpretation of trapezoidal interval type-2 in Fig. 1.

Definition 2.1: Aggregation Operator (AO) (12)

AO on fuzzy set functions by which various sets are aggregated in a suitable direction to develop a single fuzzy set. Let $(A_\alpha)_{\alpha\in[0,1]}$ be a family of AOs which is non-decreasing and B be any aggregation operator. Then, $A_B : \underset{n\in N}{\cup}[0,1]^n \rightarrow [0,1]$ defined by $A_B(\tau_1,\tau_2,\ldots,\tau_n)$ $= A_{B(\tau_1,\tau_2,\ldots,\tau_n)}(\tau_1,\tau_2,\ldots,\tau_n)$ is an AO.

Definition 2.2: TpIT2FS

$$\overline{M} = \left(\left[Trap1l_{\underline{M}}, Trap2l_{\underline{M}}, Trap1\overline{l_M}, Trap2\overline{l_M}\right], a_M, b_M, \left[Trap2r_{\underline{M}}, Trap2\overline{r_M}, Trap1r_{\underline{M}}, Trap1\overline{r_M}\right]\right) \tag{1}$$

where $0 \le Trap1l_{\underline{M}} Trap2l_{\underline{M}} \le Trap1\overline{l_M} \le Trap2\overline{l_M}$ $a_M \le b_M \le Trap2r_{\underline{M}} \le Trap2\overline{r_M} \le Trap1r_{\underline{M}} \le Trap1\overline{r_M} \le 1$ a_M and b_M denotes membership values of $h(Trap2) = a_M$ and $h(Trap1) = b_M$.

Definition 2.3: Frank sum operator (35)

$$u \underset{F}{\oplus} v = 1 - log_\xi\left(1 + \frac{(\xi^{1-u}-1)(\xi^{1-v}-1)}{\xi-1}\right), \xi \succ 1, \forall(u, v()[0,1]^2) \tag{2}$$

Definition 2.4: Frank product operator (35)

$$u \underset{F}{\otimes} v = log_\lambda\left(1 + \frac{(\xi^{u}-1)(\xi^{v}-1)}{\xi-1}\right), \xi \succ 1, \forall(u, v()[0,1]^2) \tag{3}$$

2.1. Frank operational laws for TpIT2FN

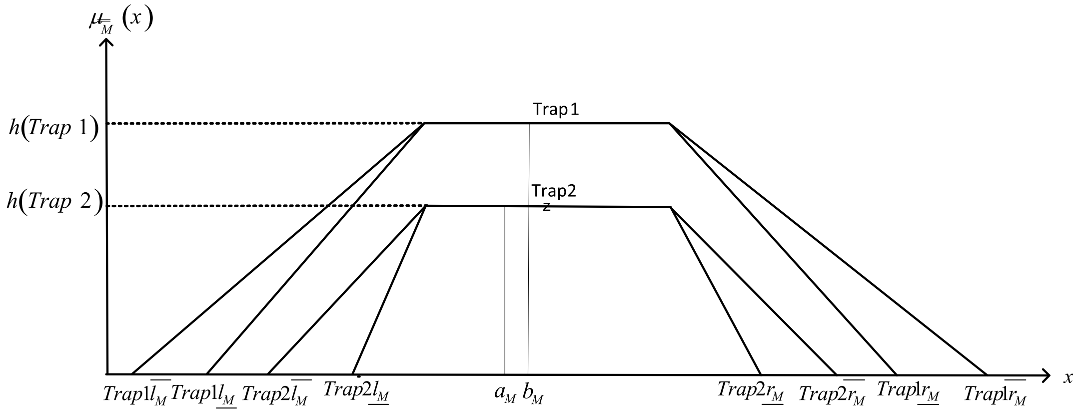
Let the TpIT2FN be $\overline{M} = \left(\left[Trap1l_{\underline{M}}, Trap2l_{\underline{M}}, Trap1\overline{l_M}, Trap2\overline{l_M}\right]., a_M, b_M, , a_M, b_M, \left[Trap2r_{\underline{M}}, Trap2\overline{r_M}, Trap1r_{\underline{M}}, Trap1\overline{r_M}\right]\right)$. Let $\overline{M}_1$ and $\overline{M}_2$ be two TpIT2FNs, $\xi \succ 1$.

Derived addition operation, Product Operation, Product by an arbitrary number Operation, Power Operation and also here we prove aggregation properties for TpIT2FN using TpIT2FFWA and TpIT2FFWG Operators. These operators and theorems are used to describe Idempotency, Boundedness, and the operator is monotonically decreasing.

2.1.1. Law 2.2 addition

$$\overline{M_1} \underset{F}{\oplus} \overline{M_2} = \left\{\left[1 - log_\xi\left(1 + \frac{POT\left(\xi^{1-\left(T1\left(\underline{l_{M_i}}\right)+T2\left(\underline{l_{M_i}}\right)\right)} - 1\right)}{\xi - 1}\right),\right.\right.$$
$$\left. 1 - log_\xi\left(1 + \frac{POT\left(\xi^{1-\left(T1\left(\overline{l_{M_i}}\right)+T2\left(\overline{l_{M_i}}\right)\right)} - 1\right)}{\xi - 1}\right)\right],$$

Figure 1
Trapezoidal interval type-2 fuzzy set



$$1-log_{\xi}\left(1+\frac{POT\left(\xi^{1-a_{M_i}}-1\right)}{\xi-1}\right),1-log_{\xi}\left(1+\frac{POT\left(\xi^{1-b_{M_i}}-1\right)}{\xi-1}\right),$$

$$\left[1-log_{\xi}\left(1+\frac{POT\left(\xi^{1-\left(T1\left(\underline{r_{M_i}}\right)+T2\left(\underline{r_{M_i}}\right)\right)}-1\right)}{\xi-1}\right),\right.$$
$$\left.\left.1-log_{\xi}\left(1+\frac{POT\left(\xi^{1-\left(T1\left(\overline{r_{M_i}}\right)+T2\left(\overline{r_{M_i}}\right)\right)}-1\right)}{\xi-1}\right)\right]\right\}, \tag{4}$$

2.1.2. Law 2.3 product

$$\overline{M}_1\underset{F}{\otimes}\overline{M}_2=\left\{\left[log_{\xi}\left(1+\frac{POT\left(\xi^{1-\left(T1\left(\underline{l_{M_i}}\right)+T2\left(\underline{l_{M_i}}\right)\right)}-1\right)}{\xi-1}\right),\right.\right.$$
$$\left.log_{\xi}\left(1+\frac{POT\left(\xi^{1-\left(T1\left(\overline{l_{M_i}}\right)+T2\left(\overline{l_{M_i}}\right)\right)}-1\right)}{\xi-1}\right)\right],$$

$$log_{\xi}\left(1+\frac{POT\left(\xi^{1-a_{M_i}}\right)}{\xi-1}\right),log_{\xi}\left(1+\frac{POT\left(\xi^{1-b_{M_i}}\right)}{\xi-1}\right),$$

$$\left[log_{\xi}\left(1+\frac{POT\left(\xi^{1-\left(T1\left(\underline{r_{M_i}}\right)+T2\left(\underline{r_{M_i}}\right)\right)}-1\right)}{\xi-1}\right),\right.$$
$$\left.\left.log_{\xi}\left(1+\frac{POT\left(\xi^{1-\left(T1\left(\overline{r_{M_i}}\right)+T2\left(\overline{r_{M_i}}\right)\right)}-1\right)}{\xi-1}\right)\right]\right\} \tag{5}$$

2.2. Viterbi algorithm

In uncertain situations, the fuzzy hidden Markov model is a handy tool for determining the optimal course of action. In a hazy environment, aggregation operators have surpassed conventional operators in uncertainty. Doubled-enclosed with a concealed process that is not visible but can be seen through a different set of Special paths (Sujatha et al., 2013). The ideal approach for decision-making difficulties is to use a fuzzy hidden Markov chain with a Frank norm.

The Viterbi technique is used to discover the best path for a decision-making. The Viterbi algorithm is divided into four steps: The four steps are Initialisation, Induction, Termination, and Path Backtracking. The values of Viterbi are derived in the first three phases. In the final stage, these values are reversed to determine the optimum possible state sequence using *t*-norms.

Initialisation

$$\chi_0(i)=T-Norm\{p_i^0,h_i(o_0)\},1\leq i\leq s \tag{6}$$

t-norms play a vital role in fuzzy logic, and it is useful to determine the solution to multicriteria problems and best path decision-making problems.

Recursion

$$\chi_{n+1}(j)=T-norm\left\{\left[S-norm\left[T-norm\left(\chi_n(i),p_{ij}\right)\right]\right],\right.$$
$$\left.h_j(o_{n+1})\right\},0\leq n\leq N-2,1\leq j\leq s \tag{7}$$

$$\gamma_{n+1}(j)=arg\,S\underset{1\leq i\leq s}{-norm}\left[T-norm\left(\chi_n(i),p_{ij}\right)\right],$$
$$0\leq n\leq N-1,1\leq j\leq s. \tag{8}$$

Termination

$$\Omega^*=S-norm[\chi_{N-1}(i)]$$

$$S_N^*=arg\,S\underset{1\leq i\leq s}{-norm}[\chi_{N-1}(i)]$$

Path backtracking

$$S_N^*=\gamma_{+1}\left(S_{n+1}^*\right),n==N-2,N-3,\ldots,0.$$

3. Experiment for the Proposed t-norms Using Viterbi Algorithm

Assume the state space as $S=\{Knowledge,Ability\}$ and the outcomes are Work (W), News (N). $E=\{e_1,e_2\}=\{W,N\}$. Here we are involved in finding the job opportunities of the persons and the most suitable state sequence. Therefore, we need to figure out the odds of using each state in a selection activity and the conditional odds of hitting Work and News in each state. The following is a description of the technique described above. States: S= {Knowledge, Ability} = {1, 2}.

$$E=\{e_1,e_2\}=\{W,N\}$$

- The activity sequence: $S=(s_1,s_2)$ and $s_1,s_2\in\mathbb{S}$

  Since we have 2 states, there will be $2^2=4$ a possible sequence.

$$S_1=(1,1);S_2=(1,2);S_3=(2,1);S_4=(2,2)$$

- The outcome sequence$O=(o_0,o_1)$. And $o_n\in E(o_n=WorN)$, $n=0,1$
- General form of the sequence is given by

$$P=\begin{bmatrix}p_{11}&p_{12}\\p_{21}&p_{22}\end{bmatrix}$$

  where p_{ij} is the probability of the states.

- $h_1(W)=\varphi_1$ and $h_1(N)=\varphi_2$,

  for Ability: $h_2(W)=\varphi_3$ and $h_2(N)=\varphi_4$.

$$H=[\,h_1(W)\quad h_2(N)\quad h_3(W)\quad h_4(N)\,]=[\varphi_1\quad\varphi_2\quad\varphi_3\quad\varphi_4].$$

- The possibilities of Knowledge and Ability being used at initial probability: $p^{(0)}=\left[p_1^{(0)},p_2^{(0)}\right]$
- The parameters of the model denoted by $\eta=\left\{P,H,p^{(0)}\right\}$ (9)

$$\chi(O,S_1|)=T-Norm\{\chi(O|S_1,\eta),\chi(S_1|\eta)\}$$
$$=T-Norm\left\{p_1^{(0)},\varphi_1,p_{11},\varphi_2\right\}$$

$$\begin{aligned}\chi(O,S_2|\eta) &= T-Norm\{\chi(O|S_2,\eta),\chi(S_2|\eta)\}\\ &= T-Norm\left\{p_1^{(0)},\varphi_2,p_{12},\varphi_2\right\}\end{aligned}$$

$$\begin{aligned}\chi(O,S_3|\eta) &= T-Norm\{\chi(O|S_3,\eta),\chi(S_3|\eta)\}\\ &= T-Norm\left\{p_2^{(0)},\varphi_4,p_{21},\varphi_1\right\}\end{aligned}$$

$$\begin{aligned}\chi(O,S_4|\eta) &= T-Norm\{\chi(O|S_4,\eta),\chi(S_4|\eta)\}\\ &= T-Norm\left\{p_2^{(0)},\varphi_4,p_{22},\varphi_3\right\}\end{aligned}$$

$$\begin{aligned}\chi(O|\eta) &= S-Norm\{T-Norm[\chi(O|S_i,\eta),\chi(S_i|\eta)]\}\\ &= S-Norm\{\chi(O,S_i|\eta)\}\end{aligned} \tag{10}$$

4. Application of Proposed t-norms Using Viterbi Algorithm

Assume the initial probability vector is defined as non-zero direction vector from the state Knowledge and Ability

$$\begin{aligned}p^{(0)} = \left[p_1^{(0)},p_2^{(0)}\right] &= [[(1,1,1,1),1,1,(1,1,1,1)],\\ &[(0.4,0.5,0.55,0.6),0.8,1,(0.35,0.45,0.55,0.65)]]\end{aligned}$$

Transition Probability Matrix:

$$p_{ij} = \begin{bmatrix} [(1,1,1,1),1,1,(1,1,1,1)] & [(0.3,0.35,0.4,0.45),0.8,1,(0.35,0.45,0.55,0.65)] \\ [(0.3,0.35,0.4,0.45),0.8,1,(0.6,0.65,0.7,0.75)] & [(0.5,0.55,0.6,0.7),0.8,1,(0.6,0.7,0.79,0.8)] \end{bmatrix}$$

Probability of hitting F and N

$$p_1(F) = \varphi_1 = [(0.3,0.4,0.5,0.6),0.8,1,(0.4,0.5,0.65,0.7)]$$

$$p_1(N) = \varphi_2 = [(1,1,1,1),1,1,(1,1,1,1)]$$

$$p_2(F) = \varphi_3 = [(0.5,0.55,0.6,0.7),0.8,1,(0.6,0.7,0.79,0.8)]$$

$$p_2(N) = \varphi_4 = [(1,1,1,1),1,1,(1,1,1,1)]$$

Output symbol observation probability is given by $[\varphi_1,\varphi_2,\varphi_3,\varphi_4]$

$$\chi(O,S_1|\eta) = T-Norm\left\{p_1^{(0)},\varphi_1,p_{11},\varphi_2\right\}$$

$$= T-Norm\left\{\begin{matrix}[(1,1,1,1;1),(1,1,1,1;1)],[(0.3,0.4,0.5,0.6),0.8,1,(0.4,0.5,0.65,0.7)]\\ [(1,1,1,1),1,1,(1,1,1,1)],[(1,1,1,1),1,1,(1,1,1,1)]\end{matrix}\right\}$$

$$\begin{aligned}=\{&[log_2(1+\Pi_{i=1}^4(2^{1-\left(Trap1\left(\underline{l_{M_i}}\right)+Trap2\left(\underline{l_{M_i}}\right)\right)}-1)),\\ &log_2(1+\Pi_{i=1}^4(2^{1-\left(Trap1\left(\overline{l_{M_i}}\right)+Trap2\left(\overline{l_{M_i}}\right)\right)}-1))],\end{aligned}$$

$$log_2(1+\Pi_{i=1}^4(2^{1-a_{M_i}})),log_2(1+\Pi_{i=1}^4(2^{1-b_{M_i}})),$$

$$\left[log_2\left(1+\Pi_{i=1}^4\left(2^{1-\left(Trap1\left(\underline{r_{M_i}}\right)+Trap2\left(\underline{r_{M_i}}\right)\right)}-1\right)\right),log_2\left(1+\Pi_{i=1}^4\left(2^{1-\left(Trap1\left(\overline{r_{M_i}}\right)+Trap2\left(\overline{r_{M_i}}\right)\right)}-1\right)\right)\right]\}$$

$$= [log_2(16.74),log_2(31.78),log_2(1.74),log_2(2),log_2(24.49),log_2(42.85)]$$

$$= [4.06,4.99,0.799,1,4.6141,5.4212]$$

$$= T_1$$

$$\chi(O,S_2|\eta) = T-Norm\left\{p_1^{(0)},\varphi_2,p_{12},\varphi_3\right\}$$

$$= T-Norm\left\{\begin{matrix}[(1,1,1,1;1),(1,1,1,1;1)],[(1,1,1,1),1,1,(1,1,1,1)]\\ [(0.3,0.35,0.4,0.45),0.8,1,(0.35,0.45,0.55,0.65)],[(1,1,1,1),1,1,(1,1,1,1)]\end{matrix}\right\}$$

$$\begin{aligned}&= [log_2(6.489),log_2(11.512),log_2(1.547),log_2(2),log_2(2.0804),log_2(24.336)]\\ &= [2.6979,3.525,0.6294,1,1.0568,4.6050]\end{aligned}$$

$$= T_2$$

$$\begin{aligned}\chi(O,S_3|\eta) &= T-Norm\left\{p_2^{(0)},\varphi_4,p_{21},\varphi_1\right\}\\ &= T-Norm\left\{\begin{matrix}[(0.4,0.5,0.55,0.6),0.8,1,(0.35,0.45,0.55,0.65)],[(1,1,1,1),1,1,(1,1,1,1)]\\ [(0.3,0.35,0.4,0.45),0.8,1,(0.6,0.65,0.7,0.75)],[(0.3,0.4,0.5,0.6),0.8,1,(0.4,0.5,0.65,0.7)]\end{matrix}\right\}\end{aligned}$$

$$\begin{aligned}=&[log_2(1.9165),log_2(4.335),log_2(1.407),\\ &log_2(2),log_2(3.652),log_2(11.3707)]\end{aligned}$$

$$= [0.9384,2.1160,0.4926,1,1.8686,3.5072]$$

$$= T_3$$

$$\begin{aligned}\chi(O,S_4|\eta) &= T-Norm\left\{p_2^{(0)},\varphi_4,p_{22},\varphi_3\right\}\\ &= T-Norm\left\{\begin{matrix}[(0.4,0.5,0.55,0.6),0.8,1,(0.35,0.45,0.55,0.65)],[(1,1,1,1),1,1,(1,1,1,1)]\\ [(0.5,0.55,0.6,0.7),0.8,1,(0.6,0.7,0.79,0.8)],[(0.5,0.55,0.6,0.7),0.8,1,(0.6,0.7,0.79,0.8)]\end{matrix}\right\}\end{aligned}$$

$$\begin{aligned}=&[log_2(9.3434),log_2(17.017),log_2(1.5490),log_2(2),\\ &log_2(10.7367),log_2(24.40)]\end{aligned}$$

$$= [3.2239,4.0889,0.6313,1,3.4244,4.6088]$$

$$= T_4$$

$$\chi(O|\eta) = S-Norm\{T_1,T_2,T_3,T_4\}$$

$$\begin{aligned}= \Bigg\{&\left[1-log_2\left(1+\Pi_{i=1}^4\left(2^{1-\left(Trap1\left(\underline{l_{M_i}}\right)+Trap2\left(\underline{l_{M_i}}\right)\right)}-1\right)\right),\right.\\ &\left.1-log_2\left(1+\Pi_{i=1}^4\left(2^{1-\left(Trap1\left(\overline{l_{M_i}}\right)+Trap2\left(\overline{l_{M_i}}\right)\right)}-1\right)\right)\right],\end{aligned}$$

$$1-log_2\left(1+\Pi_{i=1}^4\left(2^{1-a_{M_i}}-1\right)\right),1-log_2\left(1+\Pi_{i=1}^4\left(2^{1-b_{M_i}}-1\right)\right),$$

$$\begin{aligned}&\left[1-log_2\left(1+\Pi_{i=1}^4\left(2^{1-\left(Trap1\left(\underline{r_{M_i}}\right)+Trap2\left(\underline{r_{M_i}}\right)\right)}-1\right)\right),\right.\\ &\left.1-log_2\left(1+\Pi_{i=1}^4\left(2^{1-\left(Trap1\left(\overline{r_{M_i}}\right)+Trap2\left(\overline{r_{M_i}}\right)\right)}-1\right)\right)\right]\Bigg\}\end{aligned}$$

$$= [0,0,0.8,1,0,0]$$

Path Backtracking

$$S_n^* = arg\{S-Norm(\varphi)\} = [0,0.8754]$$

As a result, Ability-Ability is the optimal path, and these states also declare that they are meant to help people find employment prospects. According to the decomposition theorem, smaller variations are frequently utilised in applications due to the difficulty of operations on interval T2Fs. Due to the complexity of operations on interval T2Fs, smaller variants are often used in applications, according to the decomposition theorem. T-norms were employed to solve the Viterbi algorithm's problems to get quickly in path backtracking.

5. Conclusions

Frank operational laws for TpIT2FS have been derived in this research. We also used the TpIT2FFWA and TpIT2FFWG operators to show some aggregation operator features for TpIT2FN. We also employed the fuzzy hidden Markov model concept for TpIT2FN, replacing the probability operators with Frank t-norms, and applied it to the staff selection process using the Viterbi algorithm to find the optimal path. The Viterbi technique was used to apply a fuzzy hidden Markov model in an interval type-2 fuzzy environment to deal with added uncertainty in the decision-making process. The advantage of type 2 fuzzy is a rule-based strategy that fully acknowledges uncertainty, adaptability (fixed type-1 fuzzy sets are used to calculate the bounds of the type-reduced interval change as input changes), and the novel (the upper and lower MFs may be used concurrently in calculating every bound of the type-reduced interval). The limitation of the type is given the fuzziness of the MFs themselves, computational complexity is significant. After assessing the optimised path with proposed aggregation operators and the Viterbi algorithm, the norms will be based on a neutrosophic hidden Markov model in the future.

Conflicts of Interest

The authors declare that they have no conflicts of interest to this work.

Ethical Approval

The article does not contain any studies with human participants or animals performed by any authors.

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