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A Cognitive-Based Similarity Measure for Decision-Making with Spherical Fuzzy Information

Iman Mohamad Sharaf[1,*]

[1]*Department of Basic Sciences, Higher Technological Institute, Egypt*

Abstract: This study aims to develop a new perspective of similarity measures (SMs) for the recently introduced spherical fuzzy sets (SFSs). SFSs have several favorable properties making them superior to other types of fuzzy sets (FSs). As a consequence, SFSs are currently subject to extensive study to establish robust measures. SMs are one of the known measures of FSs. In the spherical fuzzy environment, some of the extant SMs cannot satisfy the axioms of similarity and provide counter-intuitive cases. Moreover, these conventional SMs are generalizations of SMs for intuitionistic and Pythagorean fuzzy information. None of them reflects the cognitive dimension of a SFS. Hence, the concept of the cognitive impact of a SFS is introduced. The cognitive impact is the logical implications for what human perception ought to ensue. Based on this concept, a new SM is introduced. While the conventional SMs are based on the position of the SFSs relative to each other, the novel SM is based on the effect of each evaluation on decision-making. First, an extensive review of the SMs for SFSs is presented. Second, the new concept of cognitive impact is introduced in the spherical fuzzy environment. Then, the novel SM is developed. A comparative analysis between the novel SM and the extant SMs is conducted. Finally, a multi-criteria decision-making problem is solved, namely green supplier selection using the proposed cognitive-based SM to check its applicability and its validity.

Keywords: similarity measures, spherical fuzzy sets, multi-criteria decision-making, green supplier selection

1. Introduction

The measures of similarity are very important tools that estimate the degree of similarity between two things. A measure of similarity proved to be an important aspect in various applications, for example, image processing, disease diagnosis, texture analysis, pattern recognition, and multi-criteria decision-making (Mishra et al., 2022). Since the development of fuzzy sets (FSs), the well-known conventional similarity measures (SMs) have been modified to handle different types of fuzzy information.

The notion of FSs was initially introduced by Zadeh (1965), later termed type-1 fuzzy sets (T1FSs) or ordinary FSs. T1FSs are widely employed to solve diverse problems in various applications. Nevertheless, with the continuous evolution of technologies, real-world problems became more complicated and T1FSs are incapable of modeling human judgment, evaluation, and reasoning when uncertain and ambiguous information is encountered. Subsequently, more elaborate types of FSs were developed and proposed.

Atanassov (1986) proposed intuitionistic fuzzy sets (IFSs), which is a general form of T1FSs. IFSs handle imprecision and indeterminacy from a different perspective. While an ordinary FS is composed of only one degree, an IFS is composed of a duplet. The two degrees are the degree of membership (MD) and the degree of non-membership (NMD). The sum of these degrees is less than or equal to one. The hesitation degree (HD) depends on these two degrees so that the totality of the three degrees is equal to one.

Samarandache (1998) developed neutrosophic fuzzy sets (NFSs) as an extension of IFSs. A NFS is composed of the independent triplet, namely truthiness, falsity, and indeterminacy degrees. Each degree is less than or equal to one, and the totality of these degrees is less than or equal to three.

Yager and Abbasov (2013) and Yager (2014) made another development of IFSs and introduced Pythagorean fuzzy sets (PFSs). PFSs are composed of the same duplet of the IFSs. This time, the sum of squares of the duplet is less than or equal to one to provide a larger domain. Still, the HD depends on the MD and the NMD such that the sum of the squares of the three degrees is equal to one.

Cuong and Kreiovich (2013) proposed picture fuzzy sets (PcFSs). A PcFS is composed of the independent triplet, namely the positive membership degree, the negative membership degree, and the neutral membership degree. Each degree is less than or equal to one, and the sum of the three degrees together with the refusal degree (RD) is equal to one.

Spherical fuzzy sets (SFSs) are new additions to the family of FSs. SFSs are generalizations of the previous extensions of IFSs, which is PcFSs, and NFSs. Hence, they encompass the merits of the previous FSs. SFSs allow decision-makers to express their doubts about the definition of the MD and the NMD. The three independent membership parameters are related to their squared summation. SFSs define the membership parameters on a unit sphere, which allows the expression of human cognition on a larger domain. Consequently, SFSs are more capable of handling

*__Corresponding author:__ Iman Mohamad Sharaf, Department of Basic Sciences, Higher Technological Institute, Egypt. Email: iman_sharaf@hotmail.com

imprecise and uncertain information faced in various real-world problems.

Mahmood et al. (2019), Kutlu Gündoğdu and Kahraman (2019), and Ashraf et al. (2019) introduced SFSs. Mahmood et al. (2019) proposed the concept of SFSs and T-SFSs along with spherical fuzzy relations. They also defined some of their operations and operators. Kutlu Gündoğdu and Kahraman (2019) introduced the generalized three-dimensional SFSs and defined the necessary arithmetic operations and aggregation operators. Ashraf et al. (2019) introduced SFSs with their basic operations. Based on these basic operations, they developed the SF-weighted averaging aggregation operator and the SF-weighted geometric aggregation operator.

In the SF-environment, almost all the SMs are extensions of SMs for IFSs and PFSs. Furthermore, the SMs developed in the literature have some limitations that might affect the validity of results in the SF-environment (Shishavan et al., 2020). Some of the extant SMs cannot satisfy the axioms of similarity and provide counter-intuitive cases (Khan et al., 2020). The extant SMs focus on the position of the SFSs in the three-dimensional space ignoring the implications of the SF-information that has a decisive influence on choice.

This study develops a new perspective of SMs for SFSs. SMs are one of the known measures of FSs. In the SF-environment, some of the extant SMs have several drawbacks. Furthermore, none of the extant SMs reflects the cognitive dimension of a SFS. They are simply generalizations of SMs for IFSs and PFSs. Therefore, the concept of the cognitive impact of a SFS is introduced. The cognitive impact is the logical implications for what human perception ought to ensue. Based on this concept, a new SM is proposed. The proposed SM is based on the effect of each evaluation on decision-making, in contrast to the conventional SMs that are based on the position of the SFSs relative to each other. First, an extended review of SMs of SFSs is presented. Second, the new concept of cognitive impact is introduced in the SF-environment. Then, the cognitive-based SM (CBSM) is developed. A comparative analysis between the proposed SM and some of the extant SMs is conducted. Finally, an application in multi-criteria decision-making (MCDM) is solved, namely green supplier selection (GSS) using the proposed CBSM to check its applicability and validity.

From the previous, the study serves the following four purposes:

(i) A review of the extant SMs for SFSs.
(ii) The concept of the cognitive impact of a SFS is introduced.
(iii) A new SM is proposed based on the concept of cognitive impact.
(iv) The CBSM is utilized to increase the robustness and accuracy of decision-making.

The article is organized as follows. The basic preliminaries of SFSs are given in Section 2. An extensive literature review of SMs is presented in Section 3. In Section 4, the concept of the cognitive impact of a SFS and the novel SM is introduced. A comparative analysis is conducted in Section 5. The CBSM is utilized in MCDM in Section 6. The conclusion is presented in Section 7.

The abbreviations and acronyms used in the article are listed in Table 1.

Table 1
List of abbreviation and acronyms used

Abbreviation	Definition	Abbreviation	Definition
CBSM	Cognitive-based similarity measure	NFS	Neutrosophic fuzzy set
CI	Choquet integral	NMD	Non-membership degree
CoI	Cognitive impact	PcFS	Picture fuzzy set
CSFSM	Cosine formula similarity measure	PFS	Pythagorean fuzzy set
CSSM	Cosine similarity measure	SFS	Spherical fuzzy set
CTFSM	Cotangent formula similarity measure	SM	Similarity measure
DBSM	Distance-based similarity measure	SQCSSM	Square root cosine similarity measure
DSM	Dice similarity measure	STSM	Set-theoretic similarity measure
ESM	Exponential similarity measure	T1FS	Type-1 fuzzy set
FM	Fuzzy measure	VBSM	Vector-based similarity measure
FS	Fuzzy set	WCSFSM	Weighted cosine function similarity measure
GSCM	Green supplier chain management	WCSSM	Weighted cosine similarity measure
GSM	Grey similarity measure	WCTFSM	Weighted cotangent function similarity measure
GSS	Green supplier selection	WDSM	Weighted Dice similarity measure
HD	Hesitancy degree	WESM	Weighted exponential similarity measure
IFS	Intuitionistic fuzzy set	WGSM	Weighted grey similarity measure
JSM	Jaccard similarity measure	WJSM	Weighted Jaccard similarity measure
MCDM	Multi-criteria decision-making	WSCQSM	Weighted square root cosine similarity measure
MD	Membership degree	WSTSM	Weighted set-theoretic similarity measure

2. Preliminaries

In this section, some fundamental concepts of SFSs with their basic operations and distance measures are presented.

Definition 2.1. (Kutlu Gündoğdu & Kahraman, 2019). A SFS on a universe of discourse U is composed of the triplet $\mu_{\tilde{A}_s}(MD) : U \to [0,1]$, $\upsilon_{\tilde{A}_s}(NMD) : U \to [0,1]$, and $\pi_{\tilde{A}_s}(HD) : U \to [0,1]$ and is denoted by

$$\widetilde{A}_s = \left\{ \left\langle u, \left(\mu_{\tilde{A}_s}(u), \upsilon_{\tilde{A}_s}(u), \pi_{\tilde{A}_s}(u) \right) \middle| u \in U \right\rangle \right\} \quad (1)$$

satisfying

$$0 \le \mu^2_{\tilde{A}_s}(u) + \upsilon^2_{\tilde{A}_s}(u) + \pi^2_{\tilde{A}_s}(u) \le 1, \forall u \in U. \quad (2)$$

The refusal degree (RD) can be computed by

$$r_{\tilde{A}_s}(u) = \sqrt{1 - \left(\mu^2_{\tilde{A}_s}(u) + \upsilon^2_{\tilde{A}_s}(u) + \pi^2_{\tilde{A}_s}(u) \right)}, \forall u \in U \quad (3)$$

The RD is the possibility of refusing the three previous degrees about the event.

Hence, a SFS can be expressed using the four degrees by

$$\widetilde{A}_s = \left\{ \left\langle u, \left(\mu_{\tilde{A}_s}(u), \upsilon_{\tilde{A}_s}(u), \pi_{\tilde{A}_s}(u), r_{\tilde{A}_s}(u) \right) \middle| u \in U \right\rangle \right\} \quad (4)$$

satisfying

$$0 \le \mu^2_{\tilde{A}_s}(u) + \upsilon^2_{\tilde{A}_s}(u) + \pi^2_{\tilde{A}_s}(u) + r^2_{\tilde{A}_s} = 1, \forall u \in U. \quad (5)$$

The score and accuracy functions for SFSs are given by

$$\text{Score}(\widetilde{A}_s) = (\mu_{\tilde{A}_s} - \pi_{\tilde{A}_s})^2 - (\upsilon_{\tilde{A}_s} - \pi_{\tilde{A}_s})^2 \quad (6)$$

and

$$\text{Accuracy}(\widetilde{A}_s) = \mu^2_{\tilde{A}_s} + \upsilon^2_{\tilde{A}_s} + \pi^2_{\tilde{A}_s} \quad (7)$$

$$\widetilde{A}_s < \widetilde{B}_s \quad \text{iff} \quad \text{Score}(\widetilde{A}_s) < \text{Score}(\widetilde{B}_s),$$

$$or\ \text{Score}(\widetilde{A}_s) = \text{Score}(\widetilde{B}_s) \text{ and } \text{Accuracy}(\widetilde{A}_s) < \text{Accuracy}(\widetilde{B}_s).$$

For two SFSs $\widetilde{\mathbf{A}} = (\widetilde{A}_1, \widetilde{A}_2, \ldots, \widetilde{A}_n)$ and $\widetilde{\mathbf{B}} = (\widetilde{B}_1, \widetilde{B}_2, \ldots, \widetilde{B}_n)$, the following distance measures are defined.

Definition 2.2. (Shishavan et al., 2020). The normalized Hamming distance can be calculated by using either the three or the four membership grades as follows:

$$d_H(\widetilde{\mathbf{A}}, \widetilde{\mathbf{B}}) = \frac{1}{n} \sum_{i=1}^{n} \left(|\mu_{\tilde{A}_i} - \mu_{\tilde{B}_i}| + |\upsilon_{\tilde{A}_i} - \upsilon_{\tilde{B}_i}| + |\pi_{\tilde{A}_i} - \pi_{\tilde{B}_i}| \right), \quad (8)$$

$$d_{H'}(\widetilde{\mathbf{A}}, \widetilde{\mathbf{B}}) = \frac{1}{n} \sum_{i=1}^{n} \left(|\mu_{\tilde{A}_i} - \mu_{\tilde{B}_i}| + |\upsilon_{\tilde{A}_i} - \upsilon_{\tilde{B}_i}| + |\pi_{\tilde{A}_i} - \pi_{\tilde{B}_i}| + |r_{\tilde{A}_i} - r_{\tilde{B}_i}| \right). \quad (9)$$

Definition 2.3. (Shishavan et al., 2020). The normalized Euclidean distance can be calculated using either the three or the four membership grades as follows:

$$d_E(\widetilde{\mathbf{A}}, \widetilde{\mathbf{B}}) = \sqrt{\frac{1}{n} \sum_{i=1}^{n} \left((\mu_{\tilde{A}_i} - \mu_{\tilde{B}_i})^2 + (\upsilon_{\tilde{A}_i} - \upsilon_{\tilde{B}_i})^2 + (\pi_{\tilde{A}_i} - \pi_{\tilde{B}_i})^2 \right)}, \quad (10)$$

$$d_{E'}(\widetilde{\mathbf{A}}, \widetilde{\mathbf{B}}) = \sqrt{\frac{1}{n} \sum_{i=1}^{n} \left((\mu_{\tilde{A}_i} - \mu_{\tilde{B}_i})^2 + (\upsilon_{\tilde{A}_i} - \upsilon_{\tilde{B}_i})^2 + (\pi_{\tilde{A}_i} - \pi_{\tilde{B}_i})^2 + (r_{\tilde{A}_i} - r_{\tilde{B}_i})^2 \right)} \quad (11)$$

Definition 2.4. (Donyatalab et al., 2022). The normalized Minkowski distance, also known as the generalized distance, is given as

$$d_M(\widetilde{\mathbf{A}}, \widetilde{\mathbf{B}}) = \sqrt[\alpha]{\frac{1}{2n} \sum_{i=1}^{n} \left[\left| \mu^2_{\tilde{A}_i} - \mu^2_{\tilde{B}_i} \right|^\alpha + \left| \upsilon^2_{\tilde{A}_i} - \upsilon^2_{\tilde{B}_i} \right|^\alpha + \left| \pi^2_{\tilde{A}_i} - \pi^2_{\tilde{B}_i} \right|^\alpha \right]}, \alpha \ge 1. \quad (12)$$

When the sets under observations are not of equal importance, weighted distance measures are defined employing the weights of the sets to signify their importance. If the weight vector is considered, it is given by

$$w_i = (w_1, w_2, \ldots w_n), \text{ where } w_i \in [0,1], \text{ and } \sum_{i=1}^{n} w_i = 1.$$

The weighted Minkowski distance is given as

$$d_{wM}(\widetilde{\mathbf{A}}, \widetilde{\mathbf{B}}) = \sqrt[\alpha]{\frac{1}{2} \sum_{i=1}^{n} w_i \left[\left| \mu^2_{\tilde{A}_i} - \mu^2_{\tilde{B}_i} \right|^\alpha + \left| \upsilon^2_{\tilde{A}_i} - \upsilon^2_{\tilde{B}_i} \right|^\alpha + \left| \pi^2_{\tilde{A}_i} - \pi^2_{\tilde{B}_i} \right|^\alpha \right]}, \alpha \ge 1. \quad (13)$$

When $\alpha = 1$, the normalized Minkowski distance will reduce to the SF-Hamming distance.

$$d_{HM}(\widetilde{\mathbf{A}}, \widetilde{\mathbf{B}}) = \frac{1}{2n} \sum_{i=1}^{n} \left[\left| \mu^2_{\tilde{A}_i} - \mu^2_{\tilde{B}_i} \right| + \left| \upsilon^2_{\tilde{A}_i} - \upsilon^2_{\tilde{B}_i} \right| + \left| \pi^2_{\tilde{A}_i} - \pi^2_{\tilde{B}_i} \right| \right]. \quad (14)$$

When $\alpha = 2$, the normalized Minkowski distance will reduce to the SF-Euclidean distance.

$$d_{EM}(\widetilde{\mathbf{A}}, \widetilde{\mathbf{B}}) = \sqrt[2]{\frac{1}{2n} \sum_{i=1}^{n} \left[\left| \mu^2_{\tilde{A}_i} - \mu^2_{\tilde{B}_i} \right|^2 + \left| \upsilon^2_{\tilde{A}_i} - \upsilon^2_{\tilde{B}_i} \right|^2 + \left| \pi^2_{\tilde{A}_i} - \pi^2_{\tilde{B}_i} \right|^2 \right]}. \quad (15)$$

Definition 2.5. (Donyatalab et al., 2022). The normalized Minkowski–Hausdorff is given as

$$d_{MH}(\widetilde{\mathbf{A}}, \widetilde{\mathbf{B}}) = \sqrt[\alpha]{\frac{1}{2n} \sum_{i=1}^{n} max_i \left[\left| \mu^2_{\tilde{A}_i} - \mu^2_{\tilde{B}_i} \right|^\alpha, \left| \upsilon^2_{\tilde{A}_i} - \upsilon^2_{\tilde{B}_i} \right|^\alpha, \left| \pi^2_{\tilde{A}_i} - \pi^2_{\tilde{B}_i} \right|^\alpha \right]}, \alpha \ge 1. \quad (16)$$

The weighted Minkowski–Hausdorff distance is given as

$$d_{wMH}(\widetilde{\mathbf{A}}, \widetilde{\mathbf{B}}) = \sqrt[\alpha]{\frac{1}{2} \sum_{i=1}^{n} w_i\, max_i \left[\left| \mu^2_{\tilde{A}_i} - \mu^2_{\tilde{B}_i} \right|^\alpha, \left| \upsilon^2_{\tilde{A}_i} - \upsilon^2_{\tilde{B}_i} \right|^\alpha, \left| \pi^2_{\tilde{A}_i} - \pi^2_{\tilde{B}_i} \right|^\alpha \right]}, \alpha \ge 1. \quad (17)$$

When $\alpha = 1$, the normalized Minkowski distance will reduce to the SF-Hamming–Hausdorff distance.

$$d_{HH}(\widetilde{\mathbf{A}}, \widetilde{\mathbf{B}}) = \frac{1}{2n} \sum_{i=1}^{n} max_i \left[\left| \mu^2_{\tilde{A}_i} - \mu^2_{\tilde{B}_i} \right|, \left| \upsilon^2_{\tilde{A}_i} - \upsilon^2_{\tilde{B}_i} \right|, \left| \pi^2_{\tilde{A}_i} - \pi^2_{\tilde{B}_i} \right| \right]. \quad (18)$$

When $\alpha = 2$, the normalized Minkowski distance will reduce to the SF-Euclidean–Hausdorff distance.

$$d_{EH}(\tilde{\mathbf{A}},\tilde{\mathbf{B}}) = \sqrt[2]{\frac{1}{2n}\sum_{i=1}^{n} max_i\left[\left|\mu_{\tilde{A}_i}^2 - \mu_{\tilde{B}_i}^2\right|^2, \left|\upsilon_{\tilde{A}_i}^2 - \upsilon_{\tilde{B}_i}^2\right|^2, \left|\pi_{\tilde{A}_i}^2 - \pi_{\tilde{B}_i}^2\right|^2\right]}. \quad (19)$$

3. A Literature Review

SMs and distance measures are widely used to determine the relationship between two objects in various domains, for example, network comparison, machine learning, and data mining. In the framework of SFSs, some different distance and SMs of SFSs have been proposed. However, they are limited through the literature. The existing SMs can be classified into two types: vector-based similarity measures (VBSMs) and distance-based similarity measures (DBSMs). VBSMs rely on the angle between SFSs. DBSMs rely on the distance between SFSs. Almost all of the SF-SMs are generalizations of existing SMs for IFSs (Wei, 2018; Xu & Cai, 2012; Ye, 2011) and PFSs. The most common SMs include the Dice similarity measure (DSM), the Jaccard similarity measure (JSM), the cosine similarity measure (CSSM), the cosine formula similarity measure (CSFSM), and the cotangent formula similarity measure (CTFSM). The novel SMs between SFSs were investigated by considering the positive, neutral, negative, and refusal grades.

Ullah et al. (2018) proposed a CSSM and a weighted cosine similarity measure (WCSSM). Wei et al. (2019) and Rafiq et al. (2019) also proposed a CSSM and a WCSSM. Moreover, they concurrently defined 10 SMs between SFSs based on the cosine function. Wei et al. (2019) applied these SMs to pattern recognition and medical diagnosis. Meanwhile, Rafiq et al. (2019) applied the proposed SMs to decision-making problems. These SMs can be defined as follows.

Definition 3.1. (Rafiq et al., 2019; Ullah et al., 2018; Wei et al., 2019). A CSSM and a WCSSM between two SFSs $\tilde{\mathbf{A}}$ and $\tilde{\mathbf{B}}$ using (1) are defined by

$$S_{c_1}(\tilde{\mathbf{A}},\tilde{\mathbf{B}}) = \frac{1}{n}\sum_{i=1}^{n}\frac{\mu_{\tilde{A}_i}^2\cdot\mu_{\tilde{B}_i}^2 + \upsilon_{\tilde{A}_i}^2\cdot\upsilon_{\tilde{B}_i}^2 + \pi_{\tilde{A}_i}^2\cdot\pi_{\tilde{B}_i}^2}{\sqrt{\mu_{\tilde{A}_i}^4 + \upsilon_{\tilde{A}_i}^4 + \pi_{\tilde{A}_i}^4}\sqrt{\mu_{\tilde{B}_i}^4 + \upsilon_{\tilde{B}_i}^4 + \pi_{\tilde{B}_i}^4}}, \quad (20)$$

$$S_{wc_1}(\tilde{\mathbf{A}},\tilde{\mathbf{B}}) = \sum_{i=1}^{n} w_i\frac{\mu_{\tilde{A}_i}^2\cdot\mu_{\tilde{B}_i}^2 + \upsilon_{\tilde{A}_i}^2\cdot\upsilon_{\tilde{B}_i}^2 + \pi_{\tilde{A}_i}^2\cdot\pi_{\tilde{B}_i}^2}{\sqrt{\mu_{\tilde{A}_i}^4 + \upsilon_{\tilde{A}_i}^4 + \pi_{\tilde{A}_i}^4}\sqrt{\mu_{\tilde{B}_i}^4 + \upsilon_{\tilde{B}_i}^4 + \pi_{\tilde{B}_i}^4}}. \quad (21)$$

Definition 3.2. (Rafiq et al., 2019; Wei et al., 2019). A CSSM and a WCSSM between two SFSs $\tilde{\mathbf{A}}$ and $\tilde{\mathbf{B}}$ using (4) are defined by

$$S_{c_2}(\tilde{\mathbf{A}},\tilde{\mathbf{B}}) = \frac{1}{n}\sum_{i=1}^{n}\frac{\mu_{\tilde{A}_i}^2\cdot\mu_{\tilde{B}_i}^2 + \upsilon_{\tilde{A}_i}^2\cdot\upsilon_{\tilde{B}_i}^2 + \pi_{\tilde{A}_i}^2\cdot\pi_{\tilde{B}_i}^2 + r_{\tilde{A}_i}^2\cdot r_{\tilde{B}_i}^2}{\sqrt{\mu_{\tilde{A}_i}^4 + \upsilon_{\tilde{A}_i}^4 + \pi_{\tilde{A}_i}^4 + r_{\tilde{A}_i}^4}\sqrt{\mu_{\tilde{B}_i}^4 + \upsilon_{\tilde{B}_i}^4 + \pi_{\tilde{B}_i}^4 + r_{\tilde{B}_i}^4}}, \quad (22)$$

$$S_{wc_2}(\tilde{\mathbf{A}},\tilde{\mathbf{B}}) = \sum_{i=1}^{n} w_i\frac{\mu_{\tilde{A}_i}^2\cdot\mu_{\tilde{B}_i}^2 + \upsilon_{\tilde{A}_i}^2\cdot\upsilon_{\tilde{B}_i}^2 + \pi_{\tilde{A}_i}^2\cdot\pi_{\tilde{B}_i}^2 + r_{\tilde{A}_i}^2\cdot r_{\tilde{B}_i}^2}{\sqrt{\mu_{\tilde{A}_i}^4 + \upsilon_{\tilde{A}_i}^4 + \pi_{\tilde{A}_i}^4 + r_{\tilde{A}_i}^4}\sqrt{\mu_{\tilde{B}_i}^4 + \upsilon_{\tilde{B}_i}^4 + \pi_{\tilde{B}_i}^4 + r_{\tilde{B}_i}^4}}. \quad (23)$$

Definition 3.3. (Rafiq et al., 2019; Wei et al., 2019). A CSFSM and a WCSFSM between two SFSs $\tilde{\mathbf{A}}$ and $\tilde{\mathbf{B}}$ using (1) are given by

$$S_{cf_1}(\tilde{\mathbf{A}},\tilde{\mathbf{B}}) = \frac{1}{n}\sum_{i=1}^{n} cos\left[\frac{\pi}{2}max\left(\left|\mu_{\tilde{A}_i}^2 - \mu_{\tilde{B}_i}^2\right|, \left|\upsilon_{\tilde{A}_i}^2 - \upsilon_{\tilde{B}_i}^2\right|, \left|\pi_{\tilde{A}_i}^2 - \pi_{\tilde{B}_i}^2\right|\right)\right], \quad (24)$$

$$S_{wcf_1}(\tilde{\mathbf{A}},\tilde{\mathbf{B}}) = \sum_{i=1}^{n} w_i cos\left[\frac{\pi}{2}max\left(\left|\mu_{\tilde{A}_i}^2 - \mu_{\tilde{B}_i}^2\right|, \left|\upsilon_{\tilde{A}_i}^2 - \upsilon_{\tilde{B}_i}^2\right|, \left|\pi_{\tilde{A}_i}^2 - \pi_{\tilde{B}_i}^2\right|\right)\right], \quad (25)$$

$$S_{cf_2}(\tilde{\mathbf{A}},\tilde{\mathbf{B}}) = \frac{1}{n}\sum_{i=1}^{n} cos\left[\frac{\pi}{4}\left(\left|\mu_{\tilde{A}_i}^2 - \mu_{\tilde{B}_i}^2\right| + \left|\upsilon_{\tilde{A}_i}^2 - \upsilon_{\tilde{B}_i}^2\right| + \left|\pi_{\tilde{A}_i}^2 - \pi_{\tilde{B}_i}^2\right|\right)\right], \quad (26)$$

$$S_{wcf_2}(\tilde{\mathbf{A}},\tilde{\mathbf{B}}) = \sum_{i=1}^{n} w_i cos\left[\frac{\pi}{4}\left(\left|\mu_{\tilde{A}_i}^2 - \mu_{\tilde{B}_i}^2\right| + \left|\upsilon_{\tilde{A}_i}^2 - \upsilon_{\tilde{B}_i}^2\right| + \left|\pi_{\tilde{A}_i}^2 - \pi_{\tilde{B}_i}^2\right|\right)\right]. \quad (27)$$

Definition 3.4. (Rafiq et al., 2019; Wei et al., 2019). A CSFSM and a WCSFSM between two SFSs $\tilde{\mathbf{A}}$ and $\tilde{\mathbf{B}}$ using (4) are given by

$$S_{cf_3}(\tilde{\mathbf{A}},\tilde{\mathbf{B}}) = \frac{1}{n}\sum_{i=1}^{n} cos\left[\frac{\pi}{2}max\left(\left|\mu_{\tilde{A}_i}^2 - \mu_{\tilde{B}_i}^2\right|, \left|\upsilon_{\tilde{A}_i}^2 - \upsilon_{\tilde{B}_i}^2\right|, \left|\pi_{\tilde{A}_i}^2 - \pi_{\tilde{B}_i}^2\right|, \left|r_{\tilde{A}_i}^2 - r_{\tilde{B}_i}^2\right|\right)\right] \quad (28)$$

$$S_{wcf_3}(\tilde{\mathbf{A}},\tilde{\mathbf{B}}) = \sum_{i=1}^{n} w_i cos\left[\frac{\pi}{2}max\left(\left|\mu_{\tilde{A}_i}^2 - \mu_{\tilde{B}_i}^2\right|, \left|\upsilon_{\tilde{A}_i}^2 - \upsilon_{\tilde{B}_i}^2\right|, \left|\pi_{\tilde{A}_i}^2 - \pi_{\tilde{B}_i}^2\right|, \left|r_{\tilde{A}_i}^2 - r_{\tilde{B}_i}^2\right|\right)\right], \quad (29)$$

$$S_{cf_4}(\tilde{\mathbf{A}},\tilde{\mathbf{B}}) = \frac{1}{n}\sum_{i=1}^{n} cos\left[\frac{\pi}{4}\left(\left|\mu_{\tilde{A}_i}^2 - \mu_{\tilde{B}_i}^2\right| + \left|\upsilon_{\tilde{A}_i}^2 - \upsilon_{\tilde{B}_i}^2\right| + \left|\pi_{\tilde{A}_i}^2 - \pi_{\tilde{B}_i}^2\right| + \left|r_{\tilde{A}_i}^2 - r_{\tilde{B}_i}^2\right|\right)\right], \quad (30)$$

$$S_{wcf_4}(\tilde{\mathbf{A}},\tilde{\mathbf{B}}) = \sum_{i=1}^{n} w_i cos\left[\frac{\pi}{4}\left(\left|\mu_{\tilde{A}_i}^2 - \mu_{\tilde{B}_i}^2\right| + \left|\upsilon_{\tilde{A}_i}^2 - \upsilon_{\tilde{B}_i}^2\right| + \left|\pi_{\tilde{A}_i}^2 - \pi_{\tilde{B}_i}^2\right| + \left|r_{\tilde{A}_i}^2 - r_{\tilde{B}_i}^2\right|\right)\right]. \quad (31)$$

Definition 3.5. (Rafiq et al., 2019; G. Wei et al., 2019). A CTFSM and a WCTFSM between two SFSs $\tilde{\mathbf{A}}$ and $\tilde{\mathbf{B}}$ using (1) are given by

$$S_{ctf_1}(\tilde{\mathbf{A}},\tilde{\mathbf{B}}) = \frac{1}{n}\sum_{i=1}^{n} cot\left(\frac{\pi}{4} + \left[\frac{\pi}{4}max\left(\left|\mu_{\tilde{A}_i}^2 - \mu_{\tilde{B}_i}^2\right|, \left|\upsilon_{\tilde{A}_i}^2 - \upsilon_{\tilde{B}_i}^2\right|, \left|\pi_{\tilde{A}_i}^2 - \pi_{\tilde{B}_i}^2\right|\right)\right]\right), \quad (32)$$

$$S_{wctf_1}(\tilde{\mathbf{A}},\tilde{\mathbf{B}}) = \sum_{i=1}^{n} w_i cot\left(\frac{\pi}{4} + \left[\frac{\pi}{4}max\left(\left|\mu_{\tilde{A}_i}^2 - \mu_{\tilde{B}_i}^2\right|, \left|\upsilon_{\tilde{A}_i}^2 - \upsilon_{\tilde{B}_i}^2\right|, \left|\pi_{\tilde{A}_i}^2 - \pi_{\tilde{B}_i}^2\right|\right)\right]\right), \quad (33)$$

$$S_{ctf_2}(\tilde{\mathbf{A}},\tilde{\mathbf{B}}) = \frac{1}{n}\sum_{i=1}^{n} cot\left(\frac{\pi}{4} + \left[\frac{\pi}{8}\left(\left|\mu_{\tilde{A}_i}^2 - \mu_{\tilde{B}_i}^2\right| + \left|\upsilon_{\tilde{A}_i}^2 - \upsilon_{\tilde{B}_i}^2\right| + \left|\pi_{\tilde{A}_i}^2 - \pi_{\tilde{B}_i}^2\right|\right)\right]\right) \quad (34)$$

$$S_{wctf_2}(\tilde{\mathbf{A}},\tilde{\mathbf{B}}) = \sum_{i=1}^{n} w_i cot\left(\frac{\pi}{4} + \left[\frac{\pi}{8}\left(\left|\mu_{\tilde{A}_i}^2 - \mu_{\tilde{B}_i}^2\right| + \left|\upsilon_{\tilde{A}_i}^2 - \upsilon_{\tilde{B}_i}^2\right| + \left|\pi_{\tilde{A}_i}^2 - \pi_{\tilde{B}_i}^2\right|\right)\right]\right). \quad (35)$$

Definition 3.6. (Rafiq et al., 2019; G. Wei et al., 2019). A CTFSM and a WCTFSM between two SFSs $\tilde{\mathbf{A}}$ and $\tilde{\mathbf{B}}$ using (4) are given by

$$S_{ctf_3}(\tilde{\mathbf{A}},\tilde{\mathbf{B}}) = \frac{1}{n}\sum_{i=1}^{n} cot\left(\frac{\pi}{4} + \left[\frac{\pi}{4}max\left(\left|\mu_{\tilde{A}_i}^2 - \mu_{\tilde{B}_i}^2\right|, \left|\upsilon_{\tilde{A}_i}^2 - \upsilon_{\tilde{B}_i}^2\right|, \left|\pi_{\tilde{A}_i}^2 - \pi_{\tilde{B}_i}^2\right|, \left|r_{\tilde{A}_i}^2 - r_{\tilde{B}_i}^2\right|\right)\right]\right), \quad (36)$$

$$S_{wctf_3}(\tilde{\mathbf{A}},\tilde{\mathbf{B}}) = \sum_{i=1}^{n} w_i cot\left(\frac{\pi}{4} + \left[\frac{\pi}{4}max\left(\left|\mu_{\tilde{A}_i}^2 - \mu_{\tilde{B}_i}^2\right|, \left|\upsilon_{\tilde{A}_i}^2 - \upsilon_{\tilde{B}_i}^2\right|, \left|\pi_{\tilde{A}_i}^2 - \pi_{\tilde{B}_i}^2\right|, \left|r_{\tilde{A}_i}^2 - r_{\tilde{B}_i}^2\right|\right)\right]\right), \quad (37)$$

$$S_{ctf_4}\left(\widetilde{\mathbf{A}},\widetilde{\mathbf{B}}\right)=\frac{1}{n}\sum_{i=1}^{n}cot\left(\frac{\pi}{4}+\left[\frac{\pi}{8}\left(\left|\mu_{\tilde{A}_i}^2-\mu_{\tilde{B}_i}^2\right|+\left|\upsilon_{\tilde{A}_i}^2-\upsilon_{\tilde{B}_i}^2\right|+\left|\pi_{\tilde{A}_i}^2-\pi_{\tilde{B}_i}^2\right|+\left|r_{\tilde{A}_i}^2-r_{\tilde{B}_i}^2\right|\right)\right]\right), \tag{38}$$

$$S_{wctf_4}\left(\widetilde{\mathbf{A}},\widetilde{\mathbf{B}}\right)=\sum_{i=1}^{n}w_i cot\left(\frac{\pi}{4}+\left[\frac{\pi}{8}\left(\left|\mu_{\tilde{A}_i}^2-\mu_{\tilde{B}_i}^2\right|+\left|\upsilon_{\tilde{A}_i}^2-\upsilon_{\tilde{B}_i}^2\right|+\left|\pi_{\tilde{A}_i}^2-\pi_{\tilde{B}_i}^2\right|+\left|r_{\tilde{A}_i}^2-r_{\tilde{B}_i}^2\right|\right)\right]\right). \tag{39}$$

In addition to the CSSM and the WCSSM, Ullah et al. (2018) proposed grey similarity measures (GSMs) and set-theoretic similarity measures (STSMs). The newly defined SMs were applied to the well-known problem of building material recognition.

Definition 3.7. (Ullah et al., 2018). A STSM and a WSTSM between two SFSs $\widetilde{\mathbf{A}}$ and $\widetilde{\mathbf{B}}$ are defined by

$$S_{st1}\left(\widetilde{\mathbf{A}},\widetilde{\mathbf{B}}\right)=\frac{1}{n}\sum_{i=1}^{n}\frac{\mu_{\tilde{A}_i}^2.\mu_{\tilde{B}_i}^2+\upsilon_{\tilde{A}_i}^2.\upsilon_{\tilde{B}_i}^2+\pi_{\tilde{A}_i}^2.\pi_{\tilde{B}_i}^2}{max\left(\left(\mu_{\tilde{A}_i}^4+\upsilon_{\tilde{A}_i}^4+\pi_{\tilde{A}_i}^4\right),\left(\mu_{\tilde{B}_i}^4+\upsilon_{\tilde{B}_i}^4+\pi_{\tilde{B}_i}^4\right)\right)}, \tag{40}$$

$$S_{wst1}\left(\widetilde{\mathbf{A}},\widetilde{\mathbf{B}}\right)=\sum_{i=1}^{n}w_i\frac{\mu_{\tilde{A}_i}^2.\mu_{\tilde{B}_i}^2+\upsilon_{\tilde{A}_i}^2.\upsilon_{\tilde{B}_i}^2+\pi_{\tilde{A}_i}^2.\pi_{\tilde{B}_i}^2}{max\left(\left(\mu_{\tilde{A}_i}^4+\upsilon_{\tilde{A}_i}^4+\pi_{\tilde{A}_i}^4\right),\left(\mu_{\tilde{B}_i}^4+\upsilon_{\tilde{B}_i}^4+\pi_{\tilde{B}_i}^4\right)\right)}. \tag{41}$$

Definition 3.8. (Ullah et al., 2018). A GSM and a WGSM between two SFSs $\widetilde{\mathbf{A}}$ and $\widetilde{\mathbf{B}}$ are defined by

$$S_g\left(\widetilde{\mathbf{A}},\widetilde{\mathbf{B}}\right)=\frac{1}{3n}\sum_{i=1}^{n}\left(\frac{\Delta\mu_{min}+\Delta\mu_{max}}{\Delta\mu_i+\Delta\mu_{max}}+\frac{\Delta\upsilon_{min}+\Delta\upsilon_{max}}{\Delta\upsilon_i+\Delta\upsilon_{max}}+\frac{\Delta\pi_{min}+\Delta\pi_{max}}{\Delta\pi_i+\Delta\pi_{max}}\right), \tag{42}$$

$$S_{wg}\left(\widetilde{\mathbf{A}},\widetilde{\mathbf{B}}\right)=\frac{1}{3}\sum_{i=1}^{n}w_i\left(\frac{\Delta\mu_{min}+\Delta\mu_{max}}{\Delta\mu_i+\Delta\mu_{max}}+\frac{\Delta\upsilon_{min}+\Delta\upsilon_{max}}{\Delta\upsilon_i+\Delta\upsilon_{max}}+\frac{\Delta\pi_{min}+\Delta\pi_{max}}{\Delta\pi_i+\Delta\pi_{max}}\right), \tag{43}$$

where $\Delta u_i=\left|\mu_{\tilde{A}_i}^2-\mu_{\tilde{B}_i}^2\right|, \Delta\upsilon_i=\left|\upsilon_{\tilde{A}_i}^2-\upsilon_{\tilde{B}_i}^2\right|, \Delta\pi_i=\left|\pi_{\tilde{A}_i}^2-\pi_{\tilde{B}_i}^2\right|,$

$\Delta\mu_{min}=min\left\{\left|\mu_{\tilde{A}_i}^2-\mu_{\tilde{B}_i}^2\right|\right\}, \Delta\upsilon_{min}=min\left\{\left|\upsilon_{\tilde{A}_i}^2-\upsilon_{\tilde{B}_i}^2\right|\right\}, \Delta\pi_{min}=min\left\{\left|\pi_{\tilde{A}_i}^2-\pi_{\tilde{B}_i}^2\right|\right\},$

$$\Delta\mu_{max}=max\left\{\left|\mu_{\tilde{A}_i}^2-\mu_{\tilde{B}_i}^2\right|\right\}, \Delta\upsilon_{max}=max\left\{\left|\upsilon_{\tilde{A}_i}^2-\upsilon_{\tilde{B}_i}^2\right|\right\}, \text{and } \Delta\pi_{max}=max\left\{\left|\pi_{\tilde{A}_i}^2-\pi_{\tilde{B}_i}^2\right|\right\}. \tag{44}$$

Khan et al. (2020) proposed STSMs and distance measures. They applied the proposed measures for selecting mega projects in developed countries.

Definition 3.9. (Khan et al., 2020). A STSM and a WSTSM between two SFSs $\widetilde{\mathbf{A}}$ and $\widetilde{\mathbf{B}}$ are defined by

$$S_{st2}\left(\widetilde{\mathbf{A}},\widetilde{\mathbf{B}}\right)=\frac{\sum_{i=1}^{n}\left(\mu_{\tilde{A}_i}^2\cdot\mu_{\tilde{B}_i}^2+\upsilon_{\tilde{A}_i}^2\cdot\upsilon_{\tilde{B}_i}^2+\pi_{\tilde{A}_i}^2\cdot\pi_{\tilde{B}_i}^2\right)}{\sum_{i=1}^{n}\left(max\left(\mu_{\tilde{A}_i}^4,\mu_{\tilde{B}_i}^4\right)+max\left(\upsilon_{\tilde{A}_i}^4,\upsilon_{\tilde{B}_i}^4\right)+max\left(\pi_{\tilde{A}_i}^4,\pi_{\tilde{B}_i}^4\right)\right)}, \tag{45}$$

$$S_{wst2}\left(\widetilde{\mathbf{A}},\widetilde{\mathbf{B}}\right)=\frac{\sum_{i=1}^{n}w_i\left(\mu_{\tilde{A}_i}^2\cdot\mu_{\tilde{B}_i}^2+\upsilon_{\tilde{A}_i}^2\cdot\upsilon_{\tilde{B}_i}^2+\pi_{\tilde{A}_i}^2\cdot\pi_{\tilde{B}_i}^2\right)}{\sum_{i=1}^{n}\left(max\left(\mu_{\tilde{A}_i}^4,\mu_{\tilde{B}_i}^4\right)+max\left(\upsilon_{\tilde{A}_i}^4,\upsilon_{\tilde{B}_i}^4\right)+max\left(\pi_{\tilde{A}_i}^4,\pi_{\tilde{B}_i}^4\right)\right)}. \tag{46}$$

Shishavan et al. (2020) proposed some similarity measuring tools including the JSMs, the exponential similarity measures (ESMs) based on the Hamming and the Euclidean distances, and a square root cosine similarity measure (SQCSSM). The proposed SMs were applied to medical diagnosis and GSS problems.

Definition 3.10. (Shishavan et al., 2020). A JSM and a WJSM between two SFSs $\widetilde{\mathbf{A}}$ and $\widetilde{\mathbf{B}}$ considering (1) and (4) are defined by

$$S_{J1}\left(\widetilde{\mathbf{A}},\widetilde{\mathbf{B}}\right)=\frac{1}{n}\sum_{i=1}^{n}\frac{\mu_{\tilde{A}_i}^2.\mu_{\tilde{B}_i}^2+\upsilon_{\tilde{A}_i}^2.\upsilon_{\tilde{B}_i}^2+\pi_{\tilde{A}_i}^2.\pi_{\tilde{B}_i}^2}{\left(\mu_{\tilde{A}_i}^4+\upsilon_{\tilde{A}_i}^4+\pi_{\tilde{A}_i}^4\right)+\left(\mu_{\tilde{B}_i}^4+\upsilon_{\tilde{B}_i}^4+\pi_{\tilde{B}_i}^4\right)-\left(\mu_{\tilde{A}_i}^2\mu_{\tilde{B}_i}^2+\upsilon_{\tilde{A}_i}^2\upsilon_{\tilde{B}_i}^2+\pi_{\tilde{A}_i}^2\pi_{\tilde{B}_i}^2\right)}, \tag{47}$$

$$S_{wJ1}\left(\widetilde{\mathbf{A}},\widetilde{\mathbf{B}}\right)=\sum_{i=1}^{n}w_i\frac{\mu_{\tilde{A}_i}^2.\mu_{\tilde{B}_i}^2+\upsilon_{\tilde{A}_i}^2.\upsilon_{\tilde{B}_i}^2+\pi_{\tilde{A}_i}^2.\pi_{\tilde{B}_i}^2}{\left(\mu_{\tilde{A}_i}^4+\upsilon_{\tilde{A}_i}^4+\pi_{\tilde{A}_i}^4\right)+\left(\mu_{\tilde{B}_i}^4+\upsilon_{\tilde{B}_i}^4+\pi_{\tilde{B}_i}^4\right)-\left(\mu_{\tilde{A}_i}^2\mu_{\tilde{B}_i}^2+\upsilon_{\tilde{A}_i}^2\upsilon_{\tilde{B}_i}^2+\pi_{\tilde{A}_i}^2\pi_{\tilde{B}_i}^2\right)}. \tag{48}$$

$$S_{J2}\left(\widetilde{\mathbf{A}},\widetilde{\mathbf{B}}\right)=\frac{1}{n}\sum_{i=1}^{n}\frac{\mu_{\tilde{A}_i}^2.\mu_{\tilde{B}_i}^2+\upsilon_{\tilde{A}_i}^2.\upsilon_{\tilde{B}_i}^2+\pi_{\tilde{A}_i}^2.\pi_{\tilde{B}_i}^2+r_{\tilde{A}_i}^2.r_{\tilde{B}_i}^2}{\left(\mu_{\tilde{A}_i}^4+\upsilon_{\tilde{A}_i}^4+\pi_{\tilde{A}_i}^4+r_{\tilde{A}_i}^4\right)+\left(\mu_{\tilde{B}_i}^4+\upsilon_{\tilde{B}_i}^4+\pi_{\tilde{B}_i}^4+r_{\tilde{B}_i}^4\right)-\left(\mu_{\tilde{A}_i}^2\mu_{\tilde{B}_i}^2+\upsilon_{\tilde{A}_i}^2\upsilon_{\tilde{B}_i}^2+\pi_{\tilde{A}_i}^2\pi_{\tilde{B}_i}^2+r_{\tilde{A}_i}^2r_{\tilde{B}_i}^2\right)}, \tag{49}$$

$$S_{wJ2}\left(\widetilde{\mathbf{A}},\widetilde{\mathbf{B}}\right)=\sum_{i=1}^{n}w_i\frac{\mu_{\tilde{A}_i}^2.\mu_{\tilde{B}_i}^2+\upsilon_{\tilde{A}_i}^2.\upsilon_{\tilde{B}_i}^2+\pi_{\tilde{A}_i}^2.\pi_{\tilde{B}_i}^2+r_{\tilde{A}_i}^2.r_{\tilde{B}_i}^2}{\left(\mu_{\tilde{A}_i}^4+\upsilon_{\tilde{A}_i}^4+\pi_{\tilde{A}_i}^4+r_{\tilde{A}_i}^4\right)+\left(\mu_{\tilde{B}_i}^4+\upsilon_{\tilde{B}_i}^4+\pi_{\tilde{B}_i}^4+r_{\tilde{B}_i}^4\right)-\left(\mu_{\tilde{A}_i}^2\mu_{\tilde{B}_i}^2+\upsilon_{\tilde{A}_i}^2\upsilon_{\tilde{B}_i}^2+\pi_{\tilde{A}_i}^2\pi_{\tilde{B}_i}^2+r_{\tilde{A}_i}^2r_{\tilde{B}_i}^2\right)}. \tag{50}$$

Definition 3.11. (Shishavan et al., 2020). An ESM and a WESM between two SFSs $\widetilde{\mathbf{A}}$ and $\widetilde{\mathbf{B}}$ considering (1) and (4) based on distance measures (8), (9), (10), and (11) are defined by

$$S_{ed}\left(\widetilde{\mathbf{A}},\widetilde{\mathbf{B}}\right)=e^{-d\left(\widetilde{\mathbf{A}},\widetilde{\mathbf{B}}\right)}, \tag{51}$$

$$S_{wed}\left(\widetilde{\mathbf{A}},\widetilde{\mathbf{B}}\right)=\sum_{i=1}^{n}w_i e^{-d\left(\widetilde{A}_i,\widetilde{B}_i\right)}. \tag{52}$$

Definition 3.12. (Shishavan et al., 2020). A SQCSSM and a WSQCSSM between two SFSs $\widetilde{\mathbf{A}}$ and $\widetilde{\mathbf{B}}$ considering (1) and (4) are defined by

$$S_{sqc_1}\left(\widetilde{\mathbf{A}},\widetilde{\mathbf{B}}\right)=\frac{1}{n}\sum_{i=1}^{n}\frac{\sqrt{\mu_{\tilde{A}_i}^2.\mu_{\tilde{B}_i}^2+\upsilon_{\tilde{A}_i}^2.\upsilon_{\tilde{B}_i}^2+\pi_{\tilde{A}_i}^2.\pi_{\tilde{B}_i}^2}}{\sqrt{\mu_{\tilde{A}_i}^4+\upsilon_{\tilde{A}_i}^4+\pi_{\tilde{A}_i}^4}\sqrt{\mu_{\tilde{B}_i}^4+\upsilon_{\tilde{B}_i}^4+\pi_{\tilde{B}_i}^4}}, \tag{53}$$

$$S_{wsqc_1}\left(\widetilde{\mathbf{A}},\widetilde{\mathbf{B}}\right)=\sum_{i=1}^{n}w_i\frac{\sqrt{\mu_{\tilde{A}_i}^2.\mu_{\tilde{B}_i}^2+\upsilon_{\tilde{A}_i}^2.\upsilon_{\tilde{B}_i}^2+\pi_{\tilde{A}_i}^2.\pi_{\tilde{B}_i}^2}}{\sqrt{\mu_{\tilde{A}_i}^4+\upsilon_{\tilde{A}_i}^4+\pi_{\tilde{A}_i}^4}\sqrt{\mu_{\tilde{B}_i}^4+\upsilon_{\tilde{B}_i}^4+\pi_{\tilde{B}_i}^4}}, \tag{54}$$

$$S_{sqc_2}\left(\widetilde{\mathbf{A}},\widetilde{\mathbf{B}}\right)=\frac{1}{n}\sum_{i=1}^{n}\frac{\sqrt{\mu_{\tilde{A}_i}^2.\mu_{\tilde{B}_i}^2+\upsilon_{\tilde{A}_i}^2.\upsilon_{\tilde{B}_i}^2+\pi_{\tilde{A}_i}^2.\pi_{\tilde{B}_i}^2+r_{\tilde{A}_i}^2.r_{\tilde{B}_i}^2}}{\sqrt{\mu_{\tilde{A}_i}^4+\upsilon_{\tilde{A}_i}^4+\pi_{\tilde{A}_i}^4+r_{\tilde{A}_i}^2}\sqrt{\mu_{\tilde{B}_i}^4+\upsilon_{\tilde{B}_i}^4+\pi_{\tilde{B}_i}^4+r_{\tilde{B}_i}^2}}, \tag{55}$$

$$S_{wsqc_2}\left(\tilde{\mathbf{A}},\tilde{\mathbf{B}}\right) = \sum_{i=1}^{n} w_i \frac{\sqrt{\mu_{\tilde{A}_i}^2.\mu_{\tilde{B}_i}^2 + \upsilon_{\tilde{A}_i}^2.\upsilon_{\tilde{B}_i}^2 + \pi_{\tilde{A}_i}^2.\pi_{\tilde{B}_i}^2 + r_{\tilde{A}_i}^2.r_{\tilde{B}_i}^2}}{\sqrt{\mu_{\tilde{A}_i}^4 + \upsilon_{\tilde{A}_i}^4 + \pi_{\tilde{A}_i}^4 + r_{\tilde{A}_i}^2}\sqrt{\mu_{\tilde{B}_i}^4 + \upsilon_{\tilde{B}_i}^4 + \pi_{\tilde{B}_i}^4 + r_{\tilde{B}_i}^2}}, \tag{56}$$

Wang et al. (2021) presented some novel DSMs and the generalized DSMs of SFSs. The proposed SMs were utilized to select a desirable enterprise resource planning system.

Definition 3.13. (Wang et al., 2021). A DSM and a WDSM between two SFSs $\tilde{\mathbf{A}}$ and $\tilde{\mathbf{B}}$ considering (1) and (4) are defined by

$$S_{D1}\left(\tilde{\mathbf{A}},\tilde{\mathbf{B}}\right) = \frac{1}{n}\sum_{i=1}^{n} \frac{2\left(\mu_{\tilde{A}_i}^2\cdot\mu_{\tilde{B}_i}^2 + \upsilon_{\tilde{A}_i}^2\cdot\upsilon_{\tilde{B}_i}^2 + \pi_{\tilde{A}_i}^2\cdot\pi_{\tilde{B}_i}^2\right)}{\left(\left(\mu_{\tilde{A}_i}^4 + \upsilon_{\tilde{A}_i}^4 + \pi_{\tilde{A}_i}^4\right) + \left(\mu_{\tilde{B}_i}^4 + \upsilon_{\tilde{B}_i}^4 + \pi_{\tilde{B}_i}^4\right)\right)}, \tag{57}$$

$$S_{wD1}\left(\tilde{\mathbf{A}},\tilde{\mathbf{B}}\right) = \sum_{i=1}^{n} w_i \frac{2\left(\mu_{\tilde{A}_i}^2\cdot\mu_{\tilde{B}_i}^2 + \upsilon_{\tilde{A}_i}^2\cdot\upsilon_{\tilde{B}_i}^2 + \pi_{\tilde{A}_i}^2\cdot\pi_{\tilde{B}_i}^2\right)}{\left(\left(\mu_{\tilde{A}_i}^4 + \upsilon_{\tilde{A}_i}^4 + \pi_{\tilde{A}_i}^4\right) + \left(\mu_{\tilde{B}_i}^4 + \upsilon_{\tilde{B}_i}^4 + \pi_{\tilde{B}_i}^4\right)\right)}, \tag{58}$$

$$S_{D2}\left(\tilde{\mathbf{A}},\tilde{\mathbf{B}}\right) = \frac{1}{n}\frac{\sum_{i=1}^{n} 2\left(\mu_{\tilde{A}_i}^2\cdot\mu_{\tilde{B}_i}^2 + \upsilon_{\tilde{A}_i}^2\cdot\upsilon_{\tilde{B}_i}^2 + \pi_{\tilde{A}_i}^2\cdot\pi_{\tilde{B}_i}^2\right)}{\sum_{i=1}^{n}\left(\mu_{\tilde{A}_i}^4 + \upsilon_{\tilde{A}_i}^4 + \pi_{\tilde{A}_i}^4\right) + \sum_{i=1}^{n}\left(\mu_{\tilde{B}_i}^4 + \upsilon_{\tilde{B}_i}^4 + \pi_{\tilde{B}_i}^4\right)}, \tag{59}$$

$$S_{wD2}\left(\tilde{\mathbf{A}},\tilde{\mathbf{B}}\right) = \frac{\sum_{i=1}^{n} 2w_i^4\left(\mu_{\tilde{A}_i}^2\cdot\mu_{\tilde{B}_i}^2 + \upsilon_{\tilde{A}_i}^2\cdot\upsilon_{\tilde{B}_i}^2 + \pi_{\tilde{A}_i}^2\cdot\pi_{\tilde{B}_i}^2\right)}{\sum_{i=1}^{n} w_i^4\left(\mu_{\tilde{A}_i}^4 + \upsilon_{\tilde{A}_i}^4 + \pi_{\tilde{A}_i}^4\right) + \sum_{i=1}^{n} w_i^4\left(\mu_{\tilde{B}_i}^4 + \upsilon_{\tilde{B}_i}^4 + \pi_{\tilde{B}_i}^4\right)} \tag{60}$$

$$S_{D3}\left(\tilde{\mathbf{A}},\tilde{\mathbf{B}}\right) = \frac{1}{n}\sum_{i=1}^{n} \frac{2\left(\mu_{\tilde{A}_i}^2\cdot\mu_{\tilde{B}_i}^2 + \upsilon_{\tilde{A}_i}^2\cdot\upsilon_{\tilde{B}_i}^2 + \pi_{\tilde{A}_i}^2\cdot\pi_{\tilde{B}_i}^2 + r_{\tilde{A}_i}^2\cdot r_{\tilde{B}_i}^2\right)}{\left(\left(\mu_{\tilde{A}_i}^4 + \upsilon_{\tilde{A}_i}^4 + \pi_{\tilde{A}_i}^4 + r_{\tilde{A}_i}^4\right) + \left(\mu_{\tilde{B}_i}^4 + \upsilon_{\tilde{B}_i}^4 + \pi_{\tilde{B}_i}^4 + r_{\tilde{B}_i}^4\right)\right)}, \tag{61}$$

$$S_{wD3}\left(\tilde{\mathbf{A}},\tilde{\mathbf{B}}\right) = \sum_{i=1}^{n} w_i \frac{2\left(\mu_{\tilde{A}_i}^2\cdot\mu_{\tilde{B}_i}^2 + \upsilon_{\tilde{A}_i}^2\cdot\upsilon_{\tilde{B}_i}^2 + \pi_{\tilde{A}_i}^2\cdot\pi_{\tilde{B}_i}^2 + r_{\tilde{A}_i}^2\cdot r_{\tilde{B}_i}^2\right)}{\left(\left(\mu_{\tilde{A}_i}^4 + \upsilon_{\tilde{A}_i}^4 + \pi_{\tilde{A}_i}^4 + r_{\tilde{A}_i}^4\right) + \left(\mu_{\tilde{B}_i}^4 + \upsilon_{\tilde{B}_i}^4 + \pi_{\tilde{B}_i}^4 + r_{\tilde{B}_i}^4\right)\right)}, \tag{62}$$

$$S_{D4}\left(\tilde{\mathbf{A}},\tilde{\mathbf{B}}\right) = \frac{1}{n}\frac{\sum_{i=1}^{n} 2\left(\mu_{\tilde{A}_i}^2\cdot\mu_{\tilde{B}_i}^2 + \upsilon_{\tilde{A}_i}^2\cdot\upsilon_{\tilde{B}_i}^2 + \pi_{\tilde{A}_i}^2\cdot\pi_{\tilde{B}_i}^2 + r_{\tilde{A}_i}^2\cdot r_{\tilde{B}_i}^2\right)}{\sum_{i=1}^{n}\left(\mu_{\tilde{A}_i}^4 + \upsilon_{\tilde{A}_i}^4 + \pi_{\tilde{A}_i}^4 + r_{\tilde{A}_i}^4\right) + \sum_{i=1}^{n}\left(\mu_{\tilde{B}_i}^4 + \upsilon_{\tilde{B}_i}^4 + \pi_{\tilde{B}_i}^4 + r_{\tilde{B}_i}^4\right)}, \tag{63}$$

$$S_{wD4}\left(\tilde{\mathbf{A}},\tilde{\mathbf{B}}\right) = \frac{\sum_{i=1}^{n} 2w_i^4\left(\mu_{\tilde{A}_i}^2\cdot\mu_{\tilde{B}_i}^2 + \upsilon_{\tilde{A}_i}^2\cdot\upsilon_{\tilde{B}_i}^2 + \pi_{\tilde{A}_i}^2\cdot\pi_{\tilde{B}_i}^2 + r_{\tilde{A}_i}^2\cdot r_{\tilde{B}_i}^2\right)}{\sum_{i=1}^{n} w_i^4\left(\mu_{\tilde{A}_i}^4 + \upsilon_{\tilde{A}_i}^4 + \pi_{\tilde{A}_i}^4 + r_{\tilde{A}_i}^4\right) + \sum_{i=1}^{n} w_i^4\left(\mu_{\tilde{B}_i}^4 + \upsilon_{\tilde{B}_i}^4 + \pi_{\tilde{B}_i}^4 + r_{\tilde{B}_i}^4\right)}. \tag{64}$$

Donyatalab et al. (2022) proposed the Minkowski, the Minkowski–Hausdorff, the weighted Minkowski, and the weighted Minkowski–Hausdorff distance measures for SFSs. They also developed trigonometric and *f*-SMs based on the proposed distance measures. They applied the proposed SMs to a medical diagnosis problem for the COVID-19 virus.

Definition 3.14. (Donyatalab et al., 2022). A spherical fuzzy distance-based f-SM can be defined as follows:

$$S_{db}\left(\tilde{\mathbf{A}},\tilde{\mathbf{B}}\right) = \frac{f\left(d\left(\tilde{\mathbf{A}},\tilde{\mathbf{B}}\right)\right) - f\left(d_{max}\right)}{f\left(d_{min}\right) - f\left(d_{max}\right)}. \tag{65}$$

Since the distance between two SFSs $\tilde{\mathbf{A}}$ and $\tilde{\mathbf{B}}$ using any of the distance measures (12)–(19) satisfies $0 \le d\left(\tilde{\mathbf{A}},\tilde{\mathbf{B}}\right) \le 1$, we have $d_{min} = 0$ and $d_{max} = 1$. Then, (65) can be rewritten as

$$S_{db}\left(\tilde{\mathbf{A}},\tilde{\mathbf{B}}\right) = \frac{f\left(d\left(\tilde{\mathbf{A}},\tilde{\mathbf{B}}\right)\right) - f\left(d_{max}\right)}{f(0) - f(1)}. \tag{66}$$

The simplest function that can be employed in (66) is the linear function $f(x) = 1 - x$. This yields the most popular SF-DBSM

$$S_{dbL}\left(\tilde{\mathbf{A}},\tilde{\mathbf{B}}\right) = 1 - d\left(\tilde{\mathbf{A}},\tilde{\mathbf{B}}\right). \tag{67}$$

The exponential function $f(x) = e^{-x}$ can also be employed yielding

$$S_{dbe}\left(\tilde{\mathbf{A}},\tilde{\mathbf{B}}\right) = \frac{e^{-d\left(\tilde{\mathbf{A}},\tilde{\mathbf{B}}\right)} - e^{-1}}{e^{0} - e^{-1}}. \tag{68}$$

Another simple function is the logarithmic function $f(x) = log_2(2 - x)$ that gives

$$S_{dbe}\left(\tilde{\mathbf{A}},\tilde{\mathbf{B}}\right) = \frac{log_2\left(2 - d\left(\tilde{\mathbf{A}},\tilde{\mathbf{B}}\right)\right) - log_2(1)}{log_2(2) - log_2(1)}. \tag{69}$$

The simple rational function $f(x) = 1/(1 + x)$ can also be utilized to give

$$S_{dbe}\left(\tilde{\mathbf{A}},\tilde{\mathbf{B}}\right) = \frac{1 - d\left(\tilde{\mathbf{A}},\tilde{\mathbf{B}}\right)}{1 + d\left(\tilde{\mathbf{A}},\tilde{\mathbf{B}}\right)}. \tag{70}$$

SMs (67), (68), (69), and (70) can utilize any of the distance measures (12)–(19).

Definition 3.15. (Donyatalab et al., 2022). A SF-cosine and a weighted cosine Minkowski and Minkowski–Hausdorff SMs can be defined as follows:

$$S_{CM}\left(\tilde{\mathbf{A}},\tilde{\mathbf{B}}\right) = \sqrt[\alpha]{\frac{1}{n}\sum_{i=1}^{n} cos\frac{\pi}{4}\left[\left|\mu_{\tilde{A}_i}^2 - \mu_{\tilde{B}_i}^2\right|^{\alpha} + \left|\upsilon_{\tilde{A}_i}^2 - \upsilon_{\tilde{B}_i}^2\right|^{\alpha} + \left|\pi_{\tilde{A}_i}^2 - \pi_{\tilde{B}_i}^2\right|^{\alpha}\right]}, \alpha \ge 1, \tag{71}$$

$$S_{WCM}\left(\tilde{\mathbf{A}},\tilde{\mathbf{B}}\right) = \sqrt[\alpha]{\sum_{i=1}^{n} w_i cos\frac{\pi}{4}\left[\left|\mu_{\tilde{A}_i}^2 - \mu_{\tilde{B}_i}^2\right|^{\alpha} + \left|\upsilon_{\tilde{A}_i}^2 - \upsilon_{\tilde{B}_i}^2\right|^{\alpha} + \left|\pi_{\tilde{A}_i}^2 - \pi_{\tilde{B}_i}^2\right|^{\alpha}\right]}, \alpha \ge 1, \tag{72}$$

$$S_{CMH}\left(\tilde{\mathbf{A}},\tilde{\mathbf{B}}\right) = \sqrt[\alpha]{\frac{1}{n}\sum_{i=1}^{n} cos\frac{\pi}{2} \max_i\left[\left|\mu_{\tilde{A}_i}^2 - \mu_{\tilde{B}_i}^2\right|^{\alpha}, \left|\upsilon_{\tilde{A}_i}^2 - \upsilon_{\tilde{B}_i}^2\right|^{\alpha}, \left|\pi_{\tilde{A}_i}^2 - \pi_{\tilde{B}_i}^2\right|^{\alpha}\right]}, \alpha \ge 1, \tag{73}$$

$$S_{WCMH}\left(\tilde{\mathbf{A}},\tilde{\mathbf{B}}\right) = \sqrt[\alpha]{\sum_{i=1}^{n} w_i cos\frac{\pi}{2} \max_i\left[\left|\mu_{\tilde{A}_i}^2 - \mu_{\tilde{B}_i}^2\right|^{\alpha}, \left|\upsilon_{\tilde{A}_i}^2 - \upsilon_{\tilde{B}_i}^2\right|^{\alpha}, \left|\pi_{\tilde{A}_i}^2 - \pi_{\tilde{B}_i}^2\right|^{\alpha}\right]}, \alpha \ge 1. \tag{74}$$

Definition 3.16. (Donyatalab et al., 2022). A SF-sine and weighted sine Minkowski and Minkowski–Hausdorff SMs can be defined as follows:

$$S_{SM}(\widetilde{\mathbf{A}},\widetilde{\mathbf{B}}) = 1 - \sqrt[\alpha]{\frac{1}{n}\sum_{i=1}^{n} sin\frac{\pi}{4}\left[\left|\mu_{\tilde{A}_i}^2 - \mu_{\tilde{B}_i}^2\right|^\alpha + \left|\upsilon_{\tilde{A}_i}^2 - \upsilon_{\tilde{B}_i}^2\right|^\alpha + \left|\pi_{\tilde{A}_i}^2 - \pi_{\tilde{B}_i}^2\right|^\alpha\right]}, \alpha \geq 1. \tag{75}$$

$$S_{WSM}(\widetilde{\mathbf{A}},\widetilde{\mathbf{B}}) = 1 - \sqrt[\alpha]{\sum_{i=1}^{n} w_i sin\frac{\pi}{4}\left[\left|\mu_{\tilde{A}_i}^2 - \mu_{\tilde{B}_i}^2\right|^\alpha + \left|\upsilon_{\tilde{A}_i}^2 - \upsilon_{\tilde{B}_i}^2\right|^\alpha + \left|\pi_{\tilde{A}_i}^2 - \pi_{\tilde{B}_i}^2\right|^\alpha\right]}, \alpha \geq 1. \tag{76}$$

$$S_{SMH}(\widetilde{\mathbf{A}},\widetilde{\mathbf{B}}) = 1 - \sqrt[\alpha]{\frac{1}{n}\sum_{i=1}^{n} sin\frac{\pi}{2} \max_i\left[\left|\mu_{\tilde{A}_i}^2 - \mu_{\tilde{B}_i}^2\right|^\alpha, \left|\upsilon_{\tilde{A}_i}^2 - \upsilon_{\tilde{B}_i}^2\right|^\alpha, \left|\pi_{\tilde{A}_i}^2 - \pi_{\tilde{B}_i}^2\right|^\alpha\right]}, \alpha \geq 1, \tag{77}$$

$$S_{WSMH}(\widetilde{\mathbf{A}},\widetilde{\mathbf{B}}) = 1 - \sqrt[\alpha]{\sum_{i=1}^{n} w_i sin\frac{\pi}{2} \max_i\left[\left|\mu_{\tilde{A}_i}^2 - \mu_{\tilde{B}_i}^2\right|^\alpha, \left|\upsilon_{\tilde{A}_i}^2 - \upsilon_{\tilde{B}_i}^2\right|^\alpha, \left|\pi_{\tilde{A}_i}^2 - \pi_{\tilde{B}_i}^2\right|^\alpha\right]}, \alpha \geq 1. \tag{78}$$

Ünver et al. (2022) redefined the trigonometric SMs developed by Rafiq et al. (2019) and Wei et al. (2019) by utilizing fuzzy measures (FM) and Choquet integral (CI). The proposed trigonometric SMs satisfy the axiomatic definition of the classical SMs. These SMs were applied to pattern recognition problems.

Definition 3.17. (Ünver et al., 2022). Let $X = \{\tilde{x}_1, \ldots, \tilde{x}_n\}$ be a finite set, let $\widetilde{\mathbf{A}}$ and $\widetilde{\mathbf{B}}$ be two SFSs in X, and let σ be a FM on X.

A trigonometric SM based on CI is given by

$$S_{trig}^{(C,\sigma)}(\widetilde{\mathbf{A}},\widetilde{\mathbf{B}}) = (C)\int_X F_{\tilde{A}_i,\tilde{B}_i}(x)d\sigma \tag{79}$$

where $F_{\tilde{A}_i,\tilde{B}_i}$ is the cosine formula employed in (20) and (22); the cosine function formulas employed in (24), (26), (28), and (30); the cotangent function formulas employed in (32), (34), (36), and (38).

If σ is 2-additive, then (79) is given as

$$S_{trig}^{(C_{2-add},\sigma)}(\widetilde{\mathbf{A}},\widetilde{\mathbf{B}}) = (C_{2-add})\int_X F_{\tilde{A}_i,\tilde{B}_i}(x)d\sigma. \tag{80}$$

The SMs in the SF-environment developed in literature have some limitations that might affect the validity of results (Shishavan et al., 2020). Some of the extant SMs provide counter-intuitive cases (Khan et al., 2020). The reason for these drawbacks is working with three independent degrees. VBSMs handle SFSs as vectors in space and consider the angle between them as an indication of closeness, the smaller the angle the greater the similarity. Here, a SFS has the same degree of similarity with all the SFSs making the same angle with it. In the same way, DBSMs handle SFSs regarding their position in the three-dimensional space and consider the distance between them as an indication of closeness, the smaller the distance the greater the similarity. Subsequently, a SFS is similar to all the SFSs equidistant from it.

From the previous review, it can be noted that the existing SMs handle the parameters of a SFS according to their position in the three-dimensional space. Thereby, they neglect the cognitive impact of each position regarding acceptance, rejection, and doubtfulness in defining these evaluations. This explains why the existing SMs have some limitations, and in some cases, they cannot be properly applied to problems with SF-information (Shishavan et al., 2020). Therefore, developing a CBSM that reflects the different impacts of the parameters of a SFS in evaluation is essential to avoid unreliable results.

4. The Proposed SM

As previously mentioned, a SFS is composed of three parameters. The MD is the possibility of having an event; the NMD is the possibility of not having that event. The third degree, the degree of hesitation, indicates how much the expert is in doubt about defining the MD and the NMD (Donyatalab et al., 2022).

Definition 4.1. The cognitive impact measures the amount of information inferred from the human perception that motivates the selection of one option over the other. This is accomplished by measuring the closeness from the acceptance, the remoteness from rejection, and the confidence in these evaluations.

From **Definition 4.1**, for a SFS $\tilde{A} = (\mu_{\tilde{A}}, \upsilon_{\tilde{A}}, \pi_{\tilde{A}})$, the closeness from absolute acceptance is measured by $\mu_{\tilde{A}}$. The remoteness from complete rejection is measured by $(1 - \upsilon_{\tilde{A}})$. The degree of confidence is measured by $(1 - \pi_{\tilde{A}})$. Therefore, the cognitive impact can be measured as follows.

Definition 4.2. For any SFS $\tilde{A} = (\mu_{\tilde{A}}, \upsilon_{\tilde{A}}, \pi_{\tilde{A}})$, the cognitive impact denoted as $CoI(\tilde{A})$ is defined by

$$CoI(\tilde{A}) = \left(\sqrt{\frac{1}{2}\left(\mu_{\tilde{A}}^2 + (1-\upsilon_{\tilde{A}})^2\right)}\right)(1-\pi_{\tilde{A}}), \tag{81}$$

where $CoI(\tilde{A}) \in [0,1]$.

Before proceeding with the proposed SMs, first, it is proved that $CoI(\tilde{A}) \leq 1$.

Proposition 4.1. The cognitive impact of a SFS is less than or equal to one.

***Proof*.** We have $\mu_{\tilde{A}}^2 + \upsilon_{\tilde{A}}^2 \leq 1$.
Hence, $\mu_{\tilde{A}}^2 + \upsilon_{\tilde{A}}^2 - 2\upsilon_{\tilde{A}} \leq 1$,
$\mu_{\tilde{A}}^2 + \upsilon_{\tilde{A}}^2 - 2\upsilon_{\tilde{A}} + 1 \leq 2$
$\mu_{\tilde{A}}^2 + (1-\upsilon_{\tilde{A}})^2 \leq 2$
$\frac{1}{2}\left(\mu_{\tilde{A}}^2 + (1-\upsilon_{\tilde{A}})^2\right) \leq 1 \Longleftrightarrow \sqrt{\frac{1}{2}\left(\mu_{\tilde{A}}^2 + (1-\upsilon_{\tilde{A}})^2\right)} \leq 1.$

Since $0 \leq \pi_{\tilde{A}} \leq 1 \Rightarrow (1-\pi_{\tilde{A}}) \leq 1$, then $\sqrt{\frac{1}{2}\left(\mu_{\tilde{A}}^2 + (1-\upsilon_{\tilde{A}})^2\right)}(1-\pi_{\tilde{A}}) \leq 1$. ■

Definition 4.3. A SM between two SFSs $\widetilde{\mathbf{A}}$ and $\widetilde{\mathbf{B}}$ based on the cognitive impact can be computed by

$$S_{CoI}(\widetilde{\mathbf{A}},\widetilde{\mathbf{B}}) = 1 - \frac{1}{n}\sum_{i=1}^{n}\left|CoI(\tilde{A}_i) - CoI(\tilde{B}_i)\right|. \tag{82}$$

Definition 4.4. If the sets are not of equal importance, a weighted SM is defined utilizing the weights of the sets to signify their importance. If the weight vector is given by $w_i = (w_1, w_2, \ldots w_n)$, with $w_i \in [0,1]$, and $\sum_{i=1}^{n} w_i = 1$, the weighted CBSM is defined as

$$S_{CoI}(\widetilde{\mathbf{A}},\widetilde{\mathbf{B}}) = 1 - \sum_{i=1}^{n} w_i\left|CoI(\tilde{A}_i) - CoI(\tilde{B}_i)\right|. \tag{83}$$

The following examples illustrate the concept of cognitive impact and the associated SM.

Example 4.1. Consider the SFSs $\widetilde{A} = (1, 0, 0)$ and $\widetilde{B} = (0, 1, 0)$. The SFS, $\widetilde{A}$, has a cognitive impact $CoI(\widetilde{A}) = 1$, indicating 100% approval. Meanwhile, $CoI(\widetilde{B}) = 0$, indicating 0% approval. Hence, the SM $S_{CoI}(\widetilde{A}, \widetilde{B}) = 0$.

Example 4.2. Consider the SFSs $\widetilde{A} = (1, 0, 0)$ and $\widetilde{C} = (0, 0, 1)$. The cognitive impacts of these SFSs are $CoI(\widetilde{A}) = 1$, $CoI(\widetilde{C}) = 0$, and the SM $S_{CoI}(\widetilde{A}, \widetilde{C}) = 0$.

From the previous examples, although the SFSs $\widetilde{B}$ and $\widetilde{C}$ have different implications, they have the same cognitive impact, a "zero" approval. When the evaluation of an alternative for a criterion is $(0, 1, 0)$, this points in the direction of absolute rejection. In contrast, the evaluation $(0, 0, 1)$ does not point in any direction due to hesitation. A decision-maker evaluation $\widetilde{B}$ indicates that this expert is not satisfied with the performance of an alternative for one of the evaluation criteria. On the contrary, an expert evaluation $\widetilde{C}$ indicates that an expert is unable to decide on the performance of that alternative. While the first evaluation reflects the incompetence of the alternative, the second evaluation reflects the indecision of the expert. In both cases, an option is not selected.

Example 4.3. Consider the SFSs $\widetilde{A} = (0.5, 0.7, 0.1)$ and $\widetilde{B} = (0.3, 0.5, 0.1)$. These SFSs have the same cognitive impact $CoI(\widetilde{A}) = CoI(\widetilde{B}) = 0.3711$, that is, percentage of approval 37.11%, implying $S_{CoI}(\widetilde{A}, \widetilde{B}) = 1$. Although $\widetilde{A}$ is closer than $\widetilde{B}$ to acceptance, it is also closer to rejection. The closeness of $\widetilde{A}$ from acceptance equals the remoteness of $\widetilde{B}$ from rejection, and the remoteness of $\widetilde{A}$ from rejection equals the closeness of $\widetilde{B}$ from acceptance giving the same net impact.

Example 4.4. Consider the SFSs $\widetilde{A} = (0.5, 0.7, 0.1)$ and $\widetilde{A}^c = (0.7, 0.5, 0.1)$, $CoI(\widetilde{A}^c) = 0.5474 > CoI(\widetilde{A})$, since the information of $\widetilde{A}^c$ has a greater percentage of approval.

Example 4.5. Consider the two SFSs $\widetilde{A} = (0.4, 0.4, 0.1)$ and $\widetilde{B} = (0.6, 0.6, 0.1)$. The closeness of $\widetilde{A}$ from acceptance equals the remoteness of $\widetilde{B}$ from rejection, and vice versa. They have the same cognitive impact, $CoI(\widetilde{A}) = CoI(\widetilde{B}) = 0.4589$, implying $S_{CoI}(\widetilde{A}, \widetilde{B}) = 1$.

In the literature on SMs for SFSs, researchers mentioned the axioms of SMs (Donyatalab et al., 2022). Let $\widetilde{A}, \widetilde{B}$, and $\widetilde{C}$ be three SFSs; the following properties should hold for SMs

$Ax(1)$. $0 \leq S(\widetilde{A}, \widetilde{B}) \leq 1$

$Ax(2)$. $S(\widetilde{A}, \widetilde{B}) = S(\widetilde{B}, \widetilde{A})$

$Ax(3)$. $\widetilde{A} = \widetilde{B} \Longleftrightarrow S(\widetilde{A}, \widetilde{B}) = 1$

All the proposed SMs for SFSs satisfy the first two axioms. However, some of them satisfy the third axiom partially, $\widetilde{A} = \widetilde{B} \Rightarrow S(\widetilde{A}, \widetilde{B}) = 1$. For example, the CSSM (20) can give $S(\widetilde{A}, \widetilde{B}) = 1$ although $\widetilde{A} \neq \widetilde{B}$, for example, $\widetilde{A} = (0.1, 0.1, 0.1)$, $\widetilde{B} = (0.3, 0.3, 0.3)$.

The proposed CBSM satisfies the first two axioms, and the proof is trivial and follows from the definition of the cognitive impact $CoI(\widetilde{A}) \in [0, 1]$ and the SM (82). As for the third axiom, it is also partially satisfied, $\widetilde{A} = \widetilde{B} \Rightarrow S(\widetilde{A}, \widetilde{B}) = 1$, the converse is not true. As mentioned earlier, SFSs are characterized by three independent degrees, MD, NMD, and HD. The MD positively affects the cognitive impact since it motivates the acceptance of an option. In other words, the MD points in the "in favor of" direction. On the other hand, both the NMD and the HD negatively affect the cognitive impact. Here, one of the issues of SF-information arises having two independent degrees giving a negative influence. The NMD points in the direction of "not in favor of." The HD is subtle, complex, and paradoxical (Jin, 2019). It affects the whole judgment of the MD and the NMD. For this reason, hesitation is critical and must be handled differently.

On the other hand, the proposed CBSM satisfies a new property.

Proposition 4.2. A SM between the SFSs $\widetilde{A}, \widetilde{B}$, and $\widetilde{C}$ based on the cognitive impact satisfies the following property:

$$S(\widetilde{A}, \widetilde{B}) = S(\widetilde{A}, \widetilde{C}) \Rightarrow S(\widetilde{B}, \widetilde{C}) = 1$$

Proof. From **Definition 4.3.**

$$S_{CoI}(\widetilde{A}, \widetilde{B}) = S_{CoI}(\widetilde{A}, \widetilde{C}),$$

$$1 - |CoI(\widetilde{A}) - CoI(\widetilde{B})| = 1 - |CoI(\widetilde{A}) - CoI(\widetilde{C})|,$$

$$|CoI(\widetilde{A}) - CoI(\widetilde{B})| = |CoI(\widetilde{A}) - CoI(\widetilde{C})|,$$

$$CoI(\widetilde{B}) = CoI(\widetilde{C}),$$

$$S_{CoI}(\widetilde{B}, \widetilde{C}) = 1 - |CoI(\widetilde{B}) - CoI(\widetilde{C})| = 1. \quad \blacksquare$$

Whenever $S(\widetilde{A}, \widetilde{B}) = S(\widetilde{A}, \widetilde{C})$, this implies that the SFSs $\widetilde{B}$ and $\widetilde{C}$ have the same cognitive impact, that is, the SF-information of $\widetilde{B}$ and $\widetilde{C}$ point in the same direction with the same percentage of approval.

5. Comparative Analysis

In this section, the performance of the CBSM is compared to some extant methods. The proposed CBSM uses the three independent degrees only since the fourth degree is not independent and adds no information. Thus, the SMs selected for comparison are those that utilize the three independent degrees. Seven vector-based methods are employed in comparison, the CSSM (20), the CSFSMs (24) and (26), the CTFSMs (32) and (34), the JSM (47), and the DSM (57). The most popular SF-DBSM (67) is also employed using four distance measures, namely the SF-Hamming distance (14), the SF-Euclidean distance (15), the SF-Hamming–Hausdorff distance (18), and the SF-Euclidean–Hausdorff distance (19). The SFSs to be compared are given in Table 2.

Table 2
The SFSs used in comparison

SFSs 1	$\mathbf{\widetilde{A}} = \{(0.50, 0.50, 0.50), (0.30, 0.30, 0.30), (0.40, 0.40, 0.40)\}$
	$\mathbf{\widetilde{B}} = \{(0.41, 0.41, 0.41), (0.27, 0.27, 0.27), (0.33, 0.33, 0.33)\}$
SFSs 2	$\mathbf{\widetilde{A}} = \{(0.70, 0.50, 0.30), (0.40, 0.30, 0.50), (0.60, 0.40, 0.30)\}$
	$\mathbf{\widetilde{B}} = \{(0.50, 0.70, 0.30), (0.30, 0.40, 0.50), (0.40, 0.60, 0.30)\}$
	$\mathbf{\widetilde{C}} = \{(0.50, 0.40, 0.40), (0.40, 0.60, 0.50), (0.60, 0.70, 0.30)\}$
SFSs 3	$\mathbf{\widetilde{A}} = \{(0.50, 0.70, 0.30), (0.30, 0.80, 0.40), (0.60, 0.30, 0.10)\}$
	$\mathbf{\widetilde{B}} = \{(0.60, 0.70, 0.30), (0.80, 0.40, 0.40), (0.60, 0.40, 0.20)\}$
	$\mathbf{\widetilde{C}} = \{(0.60, 0.40, 0.30), (0.50, 0.50, 0.40), (0.70, 0.40, 0.20)\}$
SFSs 4	$\mathbf{\widetilde{A}} = \{(0.70, 0.30, 0.10), (0.60, 0.50, 0.10), (0.30, 0.40, 0.10)\}$
	$\mathbf{\widetilde{B}} = \{(0.30, 0.70, 0.10), (0.50, 0.60, 0.10), (0.40, 0.30, 0.10)\}$
	$\mathbf{\widetilde{C}} = \{(0.40, 0.40, 0.40), (0.40, 0.40, 0.40), (0.40, 0.40, 0.40)\}$

Table 3
The results of the comparison

Method		SFSs 1	SFSs 2		SFSs 3		SFSs 4	
		$(\tilde{\mathbf{A}},\tilde{\mathbf{B}})$	$(\tilde{\mathbf{A}},\tilde{\mathbf{C}})$	$(\tilde{\mathbf{B}},\tilde{\mathbf{C}})$	$(\tilde{\mathbf{A}},\tilde{\mathbf{C}})$	$(\tilde{\mathbf{B}},\tilde{\mathbf{C}})$	$(\tilde{\mathbf{A}},\tilde{\mathbf{C}})$	$(\tilde{\mathbf{B}},\tilde{\mathbf{C}})$
S_{c_1}	(20)	1	0.8813	0.9101	0.8617	0.9070	0.5321	0.7720
S_{cf_1}	(24)	0.9961	0.9033	0.9236	0.8887	0.8887	0.9307	0.9307
S_{cf_2}	(26)	0.9911	0.9651	0.9651	0.9443	0.9637	0.9447	0.9447
S_{ctf_1}	(32)	0.9251	0.6354	0.6780	0.6381	0.6381	0.6986	0.6986
S_{ctf_2}	(34)	0.89011	0.7679	0.7679	0.7256	0.7839	0.7278	0.7278
S_{J1}	(47)	0.8965	0.6676	0.7122	0.6755	0.7243	0.5321	0.5321
S_{D1}	(57)	0.9449	0.8001	0.8279	0.7965	0.8323	0.6902	0.6902
S_{HM}	(67), (14)	0.9249	0.8333	0.8333	0.7967	0.8433	0.7983	0.7983
S_{EM}	(67), (15)	0.9307	0.7949	0.8104	0.7686	0.7817	0.8024	0.8024
S_{HH}	(67), (18)	0.9750	0.8600	0.8783	0.8583	0.8583	0.8867	0.8867
S_{EH}	(67), (19)	0.9961	0.9023	0.9229	0.8865	0.8865	0.9300	0.9300
S_{CoI}	(82)	0.9592	0.9109	0.9560	0.8855	0.9154	0.7880	0.8851

The results of the comparison are given in Table 3. From Table 3, it is clear that the CSSM (20) gives the same degree of similarity for $\tilde{\mathbf{A}}$ and $\tilde{\mathbf{B}}$ for SFSs 1, although they are completely different, even in their cognitive impact.

For SFSs 2, it was found that $\tilde{\mathbf{B}}$ is more similar to $\tilde{\mathbf{C}}$ than $\tilde{\mathbf{A}}$ by most of the SMs used in comparison, except for the CSFSM (26), the CTFSM (34), and the linear function SM (67) based on the SF-Hamming distance (14), could not determine which is more similar to $\tilde{\mathbf{C}}$, $\tilde{\mathbf{A}}$ or $\tilde{\mathbf{B}}$.

For SFSs 3, the CSFSM (24), the CTFSM (32), and the linear function SM (67) based on the SF-Hamming-Hausdorff (18) and the SF-Euclidean-Hausdorff (19) distances could not decide which is more similar to $\tilde{\mathbf{C}}$. The other SMs classified $\tilde{\mathbf{B}}$ to be more similar to $\tilde{\mathbf{C}}$ than $\tilde{\mathbf{A}}$.

Finally, the used SMs could not discriminate $\tilde{\mathbf{A}}\tilde{\mathbf{B}}$, and $\tilde{\mathbf{C}}$ for SFSs 4, except for the CSSM (20) and the proposed CBSM (82).

The previous results do not mean that the CBSM can give equal similarity degrees for different SFSs. In this case, equal similarity indicates that the information obtained from these SFSs has a similar influence on decision.

6. Applications

Green supply chain management (GSCM) is a new management direction that is concerned with environmental issues. Green supply chain practices help to sustain business market competition, achieve customer loyalty, improve brand image, and minimize negative environmental impacts (Kaur et al., 2019). GSCM includes several activities, for example, green supplier evaluation, green production, green packaging, and green marketing. In sustainable development, the selection of the appropriate supplier regarding environmental and social aspects has a vital role (Nourmohamadi Shalke et al., 2018). GSS is the basic step in GSCM that directly impacts the protection of the environment. GSS is a MCDM problem that comprises many contradictory assessment criteria. Supplier management professionals recognized the challenges associated with GSS. The development and implementation of practical decision-making tools to handle these challenges evolve rapidly (Banaeian et al., 2018). MCDM is one of the major applications of SMs. In this section, the CBSM is applied to solve a GSS problem. The following example is adapted from Shishavan et al. (2020).

A company in the agri-food industry sector manufactures eatable vegetable oils and detergents. The company is ISO 14000 certified. It uses the related guidelines to manage its environmental responsibilities. This includes encouraging its suppliers to constantly improve their environmental performance and practices (Banaeian et al., 2018). Olive oil, palm oil, sunflower oil, and soybean oil are the prime raw oils that the company uses. It is required to evaluate and select the suppliers of each raw material. Thirteen suppliers are available, four suppliers for olive oil $\{S_1^o, S_2^o, S_3^o, S_4^o\}$, three suppliers for palm oil $\{S_1^p, S_2^p, S_3^p\}$, three suppliers for sunflower oil $\{S_1^n, S_2^n, S_3^n\}$, and three suppliers for soybean oil $\{S_1^y, S_2^y, S_3^y\}$. The assessment criteria for a green supplier are the same as a traditional supplier adding the environmental criterion into consideration. The criteria are service level (C_1), quality (C_2), price (C_3), delivery time (C_4), and environmental management system (C_5). Three experts were assigned to evaluate the ratings of the suppliers for the assessment criteria.

The steps of the solution are summarized as follows.

Step 1. Construct the SF decision matrices and the weights of the criteria based on the evaluation of the experts.

Step 2. Aggregate the evaluations of the experts to get the overall decision matrix, and the average weights of the criteria.

Step 3. A reference index (RI) is chosen for each criterion.

Step 4. For each supplier, calculate the similarity degree between the ratings of the supplier for the assessment criteria with the RI using the CBSM (83).

$$S_{CoI}\left(\tilde{\mathbf{S}}_i,\tilde{\mathbf{RI}}\right) = 1 - \sum_{j=1}^{m} w_j \left|CoI\left(\tilde{S}_{ij}\right) - CoI\left(\tilde{RI}_j\right)\right|,$$

where $\tilde{S}_{ij}$ is the rating of the i^{th} supplier for the j^{th} assessment criterion, and $\tilde{RI}_j$ is the RI for the j^{th}criterion.

Step 5. Rank the suppliers regarding the obtained SMs in descending order. A supplier with the highest degree of similarity is the best.

Table 4
The ratings of the suppliers, the weights of the criteria, and the reference index

Type of oil	Supplier	C_1	C_2	C_3	C_4	C_5
Olive oil	S_1^o	0.05 (0.41, 0.59, 0.38)	0.15 (0.56, 0.45, 0.35)	0.15 (0.54, 0.48, 0.26)	0.35 (0.74, 0.27, 0.28)	0.3 (0.72, 0.3, 0.17)
	S_2^o	0.4 (0.76, 0.25, 0.25)	0.1 (0.54, 0.47, 0.34)	0.2 (0.61, 0.39, 0.35)	0.2 (0.61, 0.39, 0.35)	0.1 (0.54, 0.47, 0.34)
	S_3^o	0.15 (0.72, 0.3, 0.17)	0.2 (0.75, 0.26, 0.17)	0.3 (0.78, 0.23, 0.17)	0.3 (0.8, 0.21, 0.14)	0.05 (0.67, 0.33, 0.35)
	S_4^o	0.05 (0.41, 0.59, 0.38)	0.05 (0.41, 0.63, 0.23)	0.3 (0.54, 0.47, 0.34)	0.35 (0.59, 0.42, 0.27)	0.25 (0.47, 0.56, 0.36)
Palm oil	S_1^p	0.35 (0.8, 0.21, 0.41)	0.3 (0.76, 0.25, 0.2)	0.05 (0.64, 0.36, 0.27)	0.1 (0.67, 0.33, 0.23)	0.2 (0.69, 0.35, 0.16)
	S_2^p	0.15 (0.44, 0.58, 0.25)	0.1 (0.41, 0.59, 0.38)	0.3 (0.46, 0.55, 0.28)	0.1 (0.34, 0.66, 0.24)	0.35 (0.61, 0.39, 0.35)
	S_3^p	0.15 (0.57, 0.43, 0.38)	0.1 (0.5, 0.5, 0.5)	0.3 (0.65, 0.36, 0.32)	0.1 (0.51, 0.49, 0.38)	0.35 (0.71, 0.31, 0.28)
Sunflower oil	S_1^n	0.3 (0.8, 0.21, 0.14)	0.15 (0.56, 0.45, 0.35)	0.25 (0.78, 0.23, 0.17)	0.15 (0.54, 0.46, 0.44)	0.15 (0.38, 0.65, 0.37)
	S_2^n	0.15 (0.55, 0.46, 0.3)	0.2 (0.51, 0.49, 0.38)	0.3 (0.47, 0.56, 0.36)	0.3 (0.47, 0.56, 0.36)	0.05 (0.48, 0.57, 0.18)
	S_3^n	0.1 (0.5, 0.53, 0.2)	0.3 (0.69, 0.35, 0.16)	0.1 (0.51, 0.49, 0.38)	0.15 (0.54, 0.46, 0.44)	0.35 (0.72, 0.3, 0.17)
Soybean oil	S_1^y	0.1 (0.44, 0.56, 0.39)	0.1 (0.54, 0.47, 0.34)	0.4 (0.8, 0.21, 0.14)	0.2 (0.63, 0.38, 0.23)	0.2 (0.76, 0.25, 0.2)
	S_2^y	0.05 (0.76, 0.25, 0.25)	0.05 (0.58, 0.44, 0.24)	0.3 (0.46, 0.55, 0.28)	0.35 (0.51, 0.49, 0.38)	0.25 (0.5, 0.53, 0.2)
	S_3^y	0.35 (0.76, 0.25, 0.2)	0.3 (0.38, 0.63, 0.36)	0.05 (0.6, 0.4, 0.3)	0.1 (0.74, 0.28, 0.14)	0.2 (0.54, 0.46, 0.44)
	RI	(0.9, 0.1, 0)	(0.9, 0.1, 0)	(0.9, 0.1, 0)	(0.9, 0.1, 0)	(0.9, 0.1, 0)

Table 5
Results of cognitive-based similarity measure

Supplier	Degree of similarity	Rank	Supplier	Degree of similarity	Rank
S_1^o	0.5828	2	S_1^p	0.7081	1
S_2^o	0.5557	3	S_2^p	0.4362	3
S_3^o	0.7360	1	S_3^p	0.5176	2
S_4^o	0.4560	4			
S_1^n	0.5999	1	S_1^y	0.6527	1
S_2^n	0.4140	3	S_2^y	0.4560	3
S_3^n	0.5910	2	S_3^y	0.5277	2

The overall evaluation matrix, the weights of the criteria, and the RI of the problem are given in Table 4. More details on the problem and its data can be found in Shishavan et al. (2020). The obtained SMs are given in Table 5.

From Table 5, four suppliers were selected, one for each type. The third supplier S_3^o is the best for olive oil; the first supplier S_1^p is the best for palm oil; the first supplier S_1^n is the best for sunflower oil; and finally, the first supplier S_1^y is the best for soybean oil.

The results obtained using the CBSM are compared with the results obtained with some extant SMs and two MCDM methods. The SMs used are the JSM (47), the ESM (51) based on the Hamming distance (8) and the Euclidean distance (10), and the SQCSSM (53). The MCDM methods are the Technique of Order Preference by Similarity to an Ideal Solution (TOPSIS) and VIsekriterijumska optimizacija i KOmpromisno Resenje (VIKOR). TOPSIS and VIKOR are distance-based methods that utilize a distance metric to know how far a solution is from optimality. Banaeian et al. (2018), who originally introduced this problem, converted the experts' evaluations expressed in linguistic terms into triangular fuzzy numbers. Then, fuzzy TOPSIS and fuzzy VIKOR were employed to select the optimal supplier. Meanwhile, Shishavan et al. (2020) converted linguistic terms into SFSs and employed SMs of SFSs to select the best supplier. Table 6 demonstrates the final results obtained by the methods previously mentioned (Shishavan et al., 2020).

It can be observed from Table 6 that the ranking obtained by the CBSM coincides with the ranking list of the F-TOPSIS, F-VIKOR, and the JSM for the supplier of olive oil. All the methods agreed on the third supplier S_3^o as the best supplier and the fourth supplier S_4^o as the worst supplier. For palm oil, all the methods gave the same rank-

Table 6
The results obtained using different methods

Method	Different supplier rank			
	Olive oil	Palm oil	Sunflower oil	Soybean oil
Fuzzy TOPSIS	$S_3^o \succ S_2^o \succ S_1^o \succ S_4^o$	$S_1^p \succ S_3^p \succ S_2^p$	$S_3^n \succ S_1^n \succ S_2^n$	$S_1^y \succ S_3^y \succ S_2^y$
Fuzzy VIKOR	$S_3^o \succ S_2^o \succ S_1^o \succ S_4^o$	$S_1^p \succ S_3^p \succ S_2^p$	$S_3^n \succ S_1^n \succ S_2^n$	$S_1^y \succ S_3^y \succ S_2^y$
$S_{J1}(\widetilde{\mathbf{A}},\widetilde{\mathbf{B}})$ (47)	$S_3^o \succ S_2^o \succ S_1^o \succ S_4^o$	$S_1^p \succ S_3^p \succ S_2^p$	$S_3^n \succ S_1^n \succ S_2^n$	$S_1^y \succ S_2^y \succ S_3^y$
$S_{ed}(\widetilde{\mathbf{A}},\widetilde{\mathbf{B}})$ (51) based on (8)	$S_3^o \succ S_1^o \succ S_2^o \succ S_4^o$	$S_1^p \succ S_3^p \succ S_2^p$	$S_3^n \succ S_1^n \succ S_2^n$	$S_1^y \succ S_3^y \succ S_2^y$
$S_{ed}(\widetilde{\mathbf{A}},\widetilde{\mathbf{B}})$ (51) based on (10)	$S_3^o \succ S_1^o \succ S_2^o \succ S_4^o$	$S_1^p \succ S_3^p \succ S_2^p$	$S_3^n \succ S_1^n \succ S_2^n$	$S_1^y \succ S_3^y \succ S_2^y$
$S_{sqc_1}(\widetilde{\mathbf{A}},\widetilde{\mathbf{B}})$ (53)	$S_3^o \succ S_1^o \succ S_2^o \succ S_4^o$	$S_1^p \succ S_3^p \succ S_2^p$	$S_3^n \succ S_1^n \succ S_2^n$	$S_1^y \succ S_2^y \succ S_3^y$
$S_{CoI}(\widetilde{\mathbf{A}},\widetilde{\mathbf{B}})$ (83)	$S_3^o \succ S_2^o \succ S_1^o \succ S_4^o$	$S_1^p \succ S_3^p \succ S_2^p$	$S_1^n \succ S_3^n \succ S_2^n$	$S_1^y \succ S_3^y \succ S_2^y$

ing list. For soybean, all the methods agreed on the first supplier S_1^y as the best. F-TOPSIS, F-VIKOR, the ESM, and the CBSM agreed on the second supplier S_2^y as the worst. The JSM and the SCSSM agreed on the third supplier S_3^y as the worst. Finally, for the sunflower oil, all the methods gave the same ranking list, and the third supplier S_3^n was ranked first, except for the CDSM which ranked the first supplier S_1^n in the first place. This result coincides with the result obtained by Banaeian et al. (2018) using triangular fuzzy data. From Table 5, it is clear that the degree of similarity between the first and third suppliers is quite equivalent using SFSs. According to Banaeian et al. (2018), sunflower and soybean share the same supplier; hence, getting the same rank for sunflower and soybean is more acceptable.

7. Conclusion

This study developed a new perspective of SMs for SFSs. In the SF-environment, the extant SMs handle the SFSs according to their positions in the three-dimensional space neglecting the implications of the different parameters of a SFS. For this reason, these conventional SMs have some limitations that provide counter-intuitive results in some cases that affect the validity of these results. Therefore, the concept of the cognitive impact of a SFS is introduced. Herein, the similarity is measured by the effect of a SFS on decision-making. The cognitive impact is the logical implications for what human perception ought to ensue. A novel SM is proposed based on this concept. Therefore, increase the robustness and accuracy of decision-making. A comparative analysis between the proposed SM and some of the extant SMs is performed. Finally, the CBSM is applied to solve a GSS problem. Then, the results are compared with the results of some SMs, namely the JSM, the ESM based on the Hamming distance and the Euclidean distance, and the SQCSSM. Two distance-based MCDM techniques are also employed in comparison, namely TOPSIS and VIKOR. The results of the CBSM were almost identical to the results of these methods.

In practice, the proposed SM can eliminate the drawbacks that affect the validity of the results in some cases. It is expected that the developed method can be successfully applied in various applications that utilize MCDM leading to robust decisions.

One of the difficulties in the mathematical interpretation of human cognition expressed by SF-information arises from having two independent degrees giving a negative influence. While the NMD has a decisive impact on rejection, the HD affects the whole judgment, both the MD and the NMD. For this reason, the HD is more critical than the NMD, and hesitance phenomena deserve further study. Up to now, HD is a given value in the corresponding SFSs for linguistic measures. Hesitance is a human behavior that differs from one expert to another. Each expert can express his doubts about the definition of MD and NMD. Hence, it should not be a constant value for all experts giving the same linguistic assessment. Therefore, future research will focus on the estimation of hesitation in experts' evaluation in a SF-environment.

Conflicts of Interest

The author declares that he has no conflicts of interest to this work.

References

Ashraf, S., Abdullah, S., Mahmood, T., Ghani, F., & Mahmood, T. (2019). Spherical fuzzy sets and their applications in multi-attribute decision making problems. *Journal of Intelligent and Fuzzy Systems*, *36*(3), 2829–2844. https://doi.org/10.3233/JIFS-172009

Atanassov, K. T. (1986). Intuitionistic fuzzy sets. *Fuzzy Sets and Systems*, *20*(1), 87–96. https://doi.org/10.1016/S0165-0114(86)80034-3

Banaeian, N., Mobli, H., Fahimnia, B., Nielsen, I. E., & Omid, M. (2018). Green supplier selection using fuzzy group decision making methods: A case study from the agri-food industry. *Computers and Operations Research*, *89*, 337–347. https://doi.org/10.1016/j.cor.2016.02.015

Cuong, B. C., & Kreiovich V. (2013). Picture fuzzy sets-a new concept for computational intelligence problems. In *2013 Third World Congress on Information and Communication Technologies*, 1–6. https://doi.org/10.1109/WICT.2013.7113099

Donyatalab, Y., Gündoğdu, F. K., Farid, F., Seyfi-Shishavan, S. A., Farrokhizadeh, E., & Kahraman, C. (2022). Novel spherical fuzzy distance and similarity measures and their applications to medical diagnosis. *Expert Systems with Applications*, *191*, 116330. https://doi.org/10.1016/J.ESWA.2021.116330

Jin, L. S. (2019). Eliciting and measuring hesitance in decision-making. *International Journal of Intelligent Systems*, *34*(6), 1206–1222. https://doi.org/10.1002/int.22092

Kaur, J., Sidhu, R., Awasthi, A., & Srivastava, S. K. (2019). A Pareto investigation on critical barriers in green supply chain management. *International Journal of Management Science and Engineering Management*, *14*(2), 113–123. https://doi.org/10.1080/17509653.2018.1504237

Khan, M. J., Kumam, P., Deebani, W., Kumam, W., & Shah, Z. (2020). Distance and similarity measures for spherical fuzzy sets and their applications in selecting mega projects. *Mathematics*, *8*(4), 519. https://doi.org/10.3390/math8040519

Kutlu Gündoğdu, F., & Kahraman, C. (2019). Spherical fuzzy sets and spherical fuzzy TOPSIS method. *Journal of Intelligent and Fuzzy Systems*, *36*(1), 337–352. https://doi.org/10.3233/JIFS-181401

Mahmood, T., Ullah, K., Khan, Q., & Jan, N. (2019). An approach toward decision-making and medical diagnosis problems using the concept of spherical fuzzy sets. *Neural Computing and Applications*, *31*, 7041–7053. https://doi.org/10.1007/s00521-018-3521-2

Mishra, A. R., Rani, P., Saha, A., Senapati, T., Hezam, I. M., & Yager, R. R. (2022). Fermatean fuzzy copula aggregation operators and similarity measures-based complex proportional assessment approach for renewable energy source selection. *Complex & Intelligent Systems*, *8*(6), 5223–5248. https://doi.org/10.1007/s40747-022-00743-4

Nourmohamadi Shalke, P., Paydar, M. M., & Hajiaghaei-Keshteli, M. (2018). Sustainable supplier selection and order allocation through quantity discounts. *International Journal of Management Science and Engineering Management*, *13*(1), 20–32. https://doi.org/10.1080/17509653.2016.1269246

Rafiq, M., Ashraf, S., Abdullah, S., Mahmood, T., & Muhammad, S. (2019). The cosine similarity measures of spherical fuzzy sets and their applications in decision making. *Journal of Intelligent and Fuzzy Systems*, *36*(6), 6059–6073. https://doi.org/10.3233/JIFS-181922

Samarandache, F. (1998). *Neutrosophy: Neutrosophic probability, set, and logic: Analytic synthesis & synthetic analysis*. USA: American Research Press.

Shishavan, S. A. S., Kutlu Gündoğdu, F. K., Farrokhizadeh, E., Donyatalab, Y., & Kahraman, C. (2020). Novel similarity measures in spherical fuzzy environment and their applications. *Engineering Applications of Artificial Intelligence*, *94*, 103837. https://doi.org/10.1016/J.ENGAPPAI.2020.103837

Ullah, K., Mahmood, T., & Jan, N. (2018). Similarity measures for T-spherical fuzzy sets with applications in pattern recognition. *Symmetry*, *10*(6), 193. https://doi.org/10.3390/sym10060193

Ünver, M., Olgun, M., & Türkarslan, E. (2022). Cosine and cotangent similarity measures based on Choquet integral for spherical fuzzy sets and applications to pattern recognition. *Journal of Computational and Cognitive Engineering*, 1(1), 21–31. https://doi.org/10.47852/bonviewJCCE2022010105

Wang, P., Wang, J., & Wei, G. (2021). The generalized dice similarity measures for spherical fuzzy sets and their applications. In C. Kahraman & F. Kutlu Gündoğdu (Eds.) *Decision making with spherical fuzzy sets: Theory and application* (pp. 85–110). Springer. https://doi.org/10.1007/978-3-030-45461-6_4

Wei, G. W. (2018). Some similarity measures for picture fuzzy sets and their applications. *Iranian Journal of Fuzzy Systems*, *15*(1), 77–89. https://doi.org/10.22111/IJFS.2018.3579

Wei, G., Wang, J., Lu, M., Wu, J., & Wei, C. (2019). Similarity measures of spherical fuzzy sets based on cosine function and their applications. *IEEE Access*, *7*, 159069–159080. https://doi.org/10.1109/ACCESS.2019.2949296

Xu, Z., & Cai, X. (2012). *Intuitionistic fuzzy information aggregation: Theory and applications*. Germany: Springer. https://doi.org/10.1007/978-3-642-29584-3_3

Yager, R. R. (2014). Pythagorean membership grades in multicriteria decision making. *IEEE Transactions on Fuzzy Systems*, *22*(4), 958–965. https://doi.org/10.1109/TFUZZ.2013.2278989

Yager, R. R., & Abbasov, A. M. (2013). Pythagorean membership grades, complex numbers, and decision making. *International Journal of Intelligent Systems*, *28*(5), 436–452. https://doi.org/10.1002/int.21584

Ye, J. (2011). Cosine similarity measures for intuitionistic fuzzy sets and their applications. *Mathematical and Computer Modelling*, *53*(1–2), 91–97. https://doi.org/10.1016/J.MCM.2010.07.022

Zadeh, L. A. (1965). Fuzzy sets. *Information and Control, 8,* 338–353. https://doi.org/10.1016/S0019-9958(65)90241-X

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