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A Modified Hooke–Jeeves Algorithm in Two-Player Nonlinear Static Game: With an Application in Forest Management

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Abstract: Strategy considerations for optimal outcomes play the key roles in every decision-making problems. Game theory in itself preserves the sustainability to draw some decision-making conclusions. Different forms of game exist in literature. This article is concerned with two-player nonlinear static game. We consider here the solution of the game problems through the algorithmic structures, with some modification, of Hooke–Jeeves (HJ) algorithm which is a tool to encounter unconstrained optimization. Basically, we lay emphasis on the modification of the considered function which is to be optimized. Hence, the name of the proposed algorithm is *modified HJ algorithm.* Numerous articles present different solutions of various real-life problems using HJ algorithm. Forest management problem is considered here under *modified HJ algorithm* to get solution. This article concludes with some comparative analysis with respect to some different methods, for example, analytical method and graphical method. Our proposed methodology gives better results than the results obtained in analytical method and graphical method with respect to optimal strategy sets and optimal payoff values.

Keywords: nonlinear static game, Hooke–Jeeves algorithm, modified Hooke–Jeeves algorithm, forest management problem

1. Introduction

Game theory (von Neumann & Morgenstern, 1944) has been utilized widely in different circumstances where the selections of players collaborate to make some influences in the results. In focusing on the strategic aspects of decision making, or aspects managed by the players rather than by pure chance, the related game theory affects a huge area, from political alliances to business combinations, from marketing management to environmental issues, and from socioeconomic studies to cloud computing. This article provides an inspection of the overall problem and the state of the literature on forest management in static games, with an emphasis on Hooke–Jeeves (HJ) algorithm. The model considered here can be summarized as two-player nonlinear static game in continuous format. Basically, the characteristics of the game are a set of players, their action space with payoff features. These are the primary additives of the model. Here, we consider the game through the algorithm steps of HJ algorithm and consider a slight modification in HJ algorithm. The solution concept used here is highlighted by the behavioral structures of the players. We assume forest management problem and consequently, the components of it appear as game characteristics. These components may be the communities living in forest or in surrounding areas, who may be skilled in forest management or not. Components include skilled non-government as well as government organizations. Thus, a sustainable forest management tends to be fruitful. In this study, we consider static game model with Nash equilibrium strategies. Depending upon the degree to which the fundamental game has been determined, inference may prompt to counterfactual investigations, or it can empower the analyst to verify whether the information observed is steady with a particular social model as Nash equilibrium. Specifically, we study and compare our proposed work with other existing methods.

The focused objectives of this work are:

- to define nonlinear static game through HJ algorithm with some modifications,
- to compare our proposed work with some existing methods, and
- to make a sense of our proposed research into real-life problems considering computational studies with advantages and disadvantages.

This article is structured as follows: Section 2 is devoted for previous studies against game, HJ algorithm, forest management, etc. Section 3 describes the preliminaries of static game, unconstrained optimization, and HJ algorithm. Modified HJ algorithm is described in Section 4. Static game solution through modified HJ

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algorithm is prescribed in Section 5. Section 6 elaborately describes real-life forest management problem with results and discussion in Section 7. Section 8 concludes the whole study with future research scopes of the proposed study.

2. Previous Research Works

Decision-making problems mainly encounter the optimization criteria. Unconstrained optimization simulates an extensive part in decision making. HJ algorithm is treated as one of the fundamental tools to tackle unconstrained optimization in decision-making problems. Several articles have been written in HJ algorithm (Hooke & Jeeves, 1961) with different applications. Price et al. (2009) mathematically explained hybrid HJ method for non-smooth optimization. Alkhamis and Ahmed (2006) introduced HJ algorithm with modification into probability ratio performance extrapolation for simulation optimization. In mechanical design engineering, HJ direct search solution method was reviewed and analyzed by Kirgat & Surde (2014). Tabassum et al. (2021) considered HJ algorithm in power engineering system. Gao et al. (2018) used HJ algorithm through multi-objective optimization in case of reheating furnace operations. Costa et al. (2014) applied multistart HJ filter method for optimization in mixed variable case. Robertson et al. (2011) combined HJ algorithm and the random search CARTopt algorithm in non-smooth optimization problems. HJ algorithm was further analyzed by Moser (2009). Numerous articles (cf., Rios-Coelho et al., 2010; Jiang et al., 2010; Li & Rahman, 1990; Long & Wu, 2014; Torczon, 1997; Torres et al., 2015; Yang et al., 2017) published in different journals have considered real-life problems in different fields, such as motor designing, hydro-geological water issues, management problems, pattern search, and global optimization, using HJ method or using some modifications of HJ method.

Researchers have considered many problems (cf., Bonanno, 2013; Chen, 2021; Falcone, 2006; Gromova et al., 2016; Li, 2014; Lopez, 2020; Mazalov, 2014; Mazalov et al., 2018; Sedakov et al., 2013) simultaneously in crisp and fuzzy environments corresponding to game theory. Numerous articles have been published involving different game theory strategies to cover various real-life problems, such as online-shopping marketing management problem (Bhaumik et al., 2017), water resources management problem (Roy & Bhaumik, 2018), tourism management problem (Bhaumik et al., 2021), and pollution control (Gromova et al., 2016). Among several environmental issues, forestry, deforestation, forest management, etc. are the emerging issues of today's world. Forest management involves the interconnected relationships between nature and human lives. Coordinating different bodies, such as local communities, government and non-government organizations, and students and women, in the forest management are the now-a-days research trends. Several articles (cf., Ellis et al., 2019; Ghazanfari et al., 2004; Kahsay & Bulte, 2019; Riggs et al., 2018; Zandebasiri & Pourhashemi, 2016; Zandebasiri & Hoseini, 2019) have been written in forest management using decision-making criteria and analyze covering issues in both hemispheres. Forest management issues have been solved using different game-theoretic approaches. Shahi & Kant (2007) have analyzed joint forest management using an evolutionary-game-theoretic approach. The relationships among several groups, under state and private forest management regime, for use of forest resources, are shown as n-person asymmetric games. The concepts of evolutionary strong techniques and asymptotically stable states are accustomed to recognize the variations within the results of joint forest management program. Mohammadi Limaei (2010) considered forest industry scenario related to two paper mills where the theory of dynamic duopsony games was studied. Zandebasiri et al. (2020) proposed an incomplete information static game evaluating community-based forest management and applied on analyzing of land-use planning in Zagros forests, Iran. This tool, consisted of two modeling scenarios with low and high acceptance, aimed at providing a complete sustainable forest planning. Andrés-Domenech et al. (2015) devoted their studies for cooperation of sustainable forest management in differential game theory approaches where they retrieved the cooperative solution and showed the cases in which cooperation enables a partial reduction in the negative externality. They analyzed the case where both the abatement of emissions and reduction of net deforestation have reached to their optimality. But studies on forest management through game theory and HJ algorithm are rare. In our proposed research work, we consider static game analysis using some modifications in HJ algorithm and consider the game problem in forest management issues.

3. Preliminaries

In this section, we recapitulate definitions, properties and examples of static game, unconstrained optimization, and HJ algorithm.

3.1. Static game

In an optimization situation, we have got a single selection maker, his viable decision alternative set, and an objective function depending on the selected opportunity. In game theoretical fashions, we have numerous decision makers, known as players; every of them have a feasible opportunity set. This set is referred to as the player's strategy set and every player has a goal function, called payoff function. The payoff of every player relies upon on the strategy selections of all players, so the outcome counts upon on his own selection as well as on the opposite players.

Definition 3.1. *Let Q denotes the number of players assuming that all feasible strategy sets of each players have cardinality 2. If Γ_τ be the strategy set of player τ, then its payoff function Φ_τ is defined on the set of all simultaneous strategies, expressed as: $\Gamma = \Gamma_1 \times \Gamma_2 \times \ldots \times \Gamma_Q$; $\Phi_\tau(\gamma) \epsilon \mathrm{R}(\gamma \epsilon \Gamma)$. The normal form of the game is given by $G = \{Q;\ \Gamma_1,\ \Gamma_{2,} \ldots,\ \Gamma_Q;\ \Phi_1,\ \Phi_2,\ \ldots,\ \Phi_Q\}$.*

Definition 3.2. *The best response function of any player τ is defined as the following: $T_\tau(\gamma_{-\tau}) = \{\gamma_\tau^* | \gamma_\tau^* \epsilon \Gamma_r,\ \Phi_r(\gamma_\tau^*,\ \gamma_{-\tau}) = max\ \Phi(\gamma_r,\ \gamma_{-\tau})\}$, which is the set of all strategies γ_τ^* of player τ such that his payoff is maximal given the strategy selections $\gamma_{-\tau} = (\gamma_1,\ \gamma_2,\ \ldots,\ \gamma_{\tau-1},\ \gamma_{\tau+1},\ \ldots,\ \gamma_Q)$ of the other players.*

Definition 3.3. *The Nash equilibrium is a simultaneous strategy vector $\gamma^* = (\gamma_1^*,\ \gamma_2^*,\ \ldots,\ \gamma_Q^*)$ such that the equilibrium strategy γ_τ^* of any player τ is his best response given the strategies γ_j^* of all other players j. This property can be reformulated as for all players τ and $\gamma_\tau \epsilon \Gamma_\tau$, with the condition $\Phi_\tau(\gamma_1^*,\ \gamma_2^*,\ \ldots,\ \gamma_\tau^*,\ \ldots,\ \gamma_Q^*) \geq \Phi_\tau(\gamma_1^*,\ \gamma_2^*,\ \ldots,\ \gamma_{\tau-1}^*,\ \gamma_\tau,\ \gamma_{\tau+1}^*,\ \ldots,\ \gamma_Q^*)$ which means that no player can increase his payoff from the equilibrium by unilaterally changing strategy.*

Example 3.1. Sharing a pie, Cournot duopoly, spying game, etc. are the examples of static continuous game.

3.2. Unconstrained optimization

In many situations, the assumption of linearity in objective function or in the set of constraints might be questionable. Basically, in real-world situation, nothing is seemed to be linear. So, nonlinear optimization processes can be treated as the most effective way to solve the problems considered.

Definition 3.4. *Let n, m, and p be positive integers; X be a subset of R_n; f, g_i, and h_j be real-valued functions on X for each i ϵ {1, . . . , m} and each j ϵ {1, . . . ,p}, with at least one of f, g_i, and h_j being nonlinear. A nonlinear minimization problem is defined as:*

$$\begin{aligned} \textit{minimize} \quad & f(x); \\ \textit{subject to} \quad & g_i(x) \leq 0 \textit{ for each } i \in \{1, \ldots, m\}; \\ & h_j(x) = 0 \textit{ for each } j \in \{1, \ldots, p\}; \\ & x \in X. \end{aligned}$$

A nonlinear maximization problem is detailed in a similar manner.

If the objective function is continuous over the domain of definition, stationary points can be located using differential calculus providing the derivatives exist and the stationary point lies in the interior or at a boundary if the partial derivatives of an unconstrained function vanish at a particular solution vector.

Definition 3.5. *If the nonlinear programming problem consists of only an objective function, say f* (x)*, and if the objective function is convex (concave), then a unique optimum solution will be found at a point,*

a) interior to the feasible region where all derivatives vanish, or
b) on the boundary.

3.3. HJ algorithm

HJ algorithm (Hooke & Jeeves, 1961) is one of the interesting ways to solve unconstrained optimization problems. The HJ algorithm consists of two parts. One is *exploratory search* part and other is *pattern search* part. The initial guess point is the starting point of the algorithm, and the next points are generated by addition or reduction of variables separately using step lengths. Then, the values of the function at these points are evaluated and, based upon these values, the best next approximation is pointed out. A vector is formed connecting the original point and the best next point. A new point along this vector is fabricated and applied as a basis of a new search if it is of smaller function value.

Let the algorithm starts with the temporary point Z_{te} which is equal to Z_i (i.e., the current point used as a base point for the exploratory search part). Now, an addition is performed to the variable by a step $\pm\delta_j$ along the j-th variable with the objective of connecting a new point having smaller value in objective function. The step vector δ shows a set of accretion, acceptable for the concerned problem. Consider $Z_{te} = Z(i)$ $(= Z_i)$. Now, we calculate the value $f(Z_i)$. First we assume $j = 1$. While $j \leq n$, we perform the calculations, $Z_{te}]_j = Z_{te}]_j + \delta_j$. If, $f(Z_{te}) < f(Z_i)$, consider $j = j + 1$ and starts from the very first step. Otherwise, considering $Z_{te}]_j = Z_{te}]_j$- δ_j we verify for $f(Z_{te}) < f(Z_i)$. If this inequality remains true, we go for $j = j + 1$ and the process starts from first step and we get, $Z(i + 1)$ as new Z_{te}.

HJ algorithm is depicted through the steps in Algorithm 1.

3.3.1. Examples

In literature, one can find a huge number of examples verifying HJ algorithm. We have considered here only three.

1. Beale's function (More et al., 1981): $\min f = (1.5 - z_1(1 - z_2))^2 + (2.25 - z_1(1 - z_2^2))^2 + (2.625 - z_1(1 - z_2^3))^2$
   Initial approximation: (1,1); min = (3, 0.5); termination accuracy: 10^{-5}.

2. Wood's function (Rao, 1996): $\min f = (10(z_2 - z_1^2))^2 + (1 - z_1)^2 + 90(z_4 - z_3^2)^2 + (1 - z_3)^2 + 10\,(z_2 + z_4 - 2)^2 + 0.1\,(z_2 - z_4)$
   Initial approximation: (−3, −1, −3, −1); min = (1, 1, 1, 1); termination accuracy: 10^{-5}.

3. Himmelblau's function (Himmelblau, 1972):
   $\min f = (z_1^2 + z_2 - 11)^2 + (z_1 + z_2^2 - 7)^2$
   Initial approximation: (0, 0); min = (3, 2); termination accuracy: 10^{-3}.

Algorithm 1: Pseudo-code of Hooke–Jeeves algorithm

Step 1: Input : Initial base point, say Z; Set of step lengths, say δ;
Objective function, say f; Termination accuracy, say ϵ.
Step 2: Move the base point along every dimensional axes at a time and figure out the result;
Affirm each distinct point if it boosts on the previous point ;
if any of the moves was advantageous ***then***
go to **Step 3** ;
else
go to **Step 4** ;
end
Step 3: Rerun the advantageous moves in a combined pattern move;
If the new point has a finer fitness, assume it as the new base point;
Return to **Step 2** whatever the outcome ;
Step 4: Accustom step length to next smaller step ;
If there is a smaller step ***then***
continue from **Step 2** ;
else
terminate ;
end
Step 5: Output: Optimal solution

4. Modified HJ Algorithm

In this section, we consider modified HJ algorithm through the steps. Here, we consider initial base point and determine the set of step lengths. Then, we flow the base point alongside every one of the *d* dimensional axes at a time and examine the end result of the objective function. Now, we undertake each new point if improvement on the previous one. This takes as a minimum *d*, at most 2*d* opinions. If any of the movements became a success, then we repeat the successful moves in a combined pattern move (PM). If the new point has a better fitness, count on it as the new base point and perform the same method for better final results. Contrarily, we adjust step length to subsequent smaller step. If there is a smaller step, retain from the beginning till the process terminates.

Every steps of the original HJ algorithm are presumed having modification in the objective function. The objective function, considered here, is the weighted sum of the set of objectives functions. Hence, the name of this method is modified HJ algorithm.

Algorithm 2: Pseudo-code of modified Hooke–Jeeves algorithm

Step 1: Input : Initial base point, say Z; Set of step lengths, say δ;
Objective functions, say f s; Termination accuracy, say ϵ.
Consider objective functions in weighted form with a set of weights, say λs;
Step 2: Move the base point along every one of the s dimensional axes at a time and check out the consequence;
Endorse each new point if it enhances on the previous point;
If any of the moves was successful ***then***
go to **Step 3**;
else
go to **Step 4**;
9 **end**
Step 3: Replay the successful moves in a combined pattern move;
If the new point has a superior fitness, assume it as the new base point;
Return to **Step 2** whatever the outcome;
Step 4: Accommodate step length to next smaller step;
if there is a smaller step ***then***
continue from **Step 2**;
else
terminate;
end
Change the values of λ and perform from **Step 2**;
Step 5: Output: Optimal solution

5. Static Game Solution Through Modified HJ Algorithm

We consider in this section the static game solution through modified HJ algorithm. We consider here the strategy variables z_1 and z_2 corresponding to two players and the two payoff functions, say, for Player 1: $f^1(z_1, z_2)$ and for Player 2: $f^2(z_1, z_2)$ and consider their weighted sum. Assuming their weighted sum as $\xi(z_1, z_2)$, we consider this $\xi(z_1, z_2)$ as our desired function to be optimized. Since, sum of two convex functions is convex, negative sum of two concave functions is convex, and sum of convex and concave function possess minimum point due to quasi-differential calculus (Polyakova, 1986) and then the combination of functions can be done for performing the HJ algorithm.

Definition 5.1. *A static game in two-players with continuous payoff functions can be solved using HJ algorithm considering weighted some of payoff functions and then the game can be defined as HJ algorithm oriented continuous static game with two players.*

Theorem 5.1. *Assume that the strategy sets Γ_τ (for number of players $\tau = 2$) are nonempty, convex, closed, bounded subsets of finite-dimensional Euclidean spaces, and the payoff functions are one–one and T_γ is continuous, then there is at least one equilibrium of the game.*

Proof: Since $\Gamma_\tau (\tau = 1, 2)$ are non empty, convex, closed, and bounded, then $\Gamma = \Gamma_1 \times \Gamma_2$ is also nonempty, convex, closed, and bounded. Let $z_1 = (z_1^1, z_1^2)$ and $z_2 = (z_2^1, z_2^2)$ be two points in Γ. Then for α with $0 \le \alpha \le 1$,

$$z_1 + \alpha z_2 = (z_1^1 + \alpha z_2^1, \ z_1^2 + \alpha z_2^2) \tag{5.1}$$

and the convexity of Γ_τ implies that for $\tau = 1, 2$, we have $z_1 + \alpha z_2 \epsilon \Gamma_2$. Then, the Brouwer fixed point theorem implies the existence of at least one fixed point, which is an equilibrium.

Theorem 5.2. *Assume that the strategy sets Γ_τ(for $\tau = 1, 2$) are nonempty and closed in finite-dimensional Euclidean spaces; furthermore, function T_γ is one–one and contraction on Γ. Then, there is a unique equilibrium which can be obtained as the limit of the iteration sequence $\gamma^{(\tau+1)} = T(\gamma^\tau)$ with $\tau = 1, 2$, starting with any arbitrary initial approximation $\gamma^{(0)} \epsilon \Gamma$.*

Proof: This theorem is easy to prove due to the Banach fixed point theorem since $\Gamma_1 \times \Gamma_2$ is closed.

6. Forest Management: A Real-life Example

In developing countries, territories, and in highly populated areas, people living near about and in the forest depend upon forest resources. On the other hand, state authorities manage the forest and forest resources by their own strategies. Mainly, different socioeconomic characteristics, such as attitude to the forest environment and local community-based environment, ownership of lands, income from various points, education, and gender equality, are controlled by these two bodies—skilled body (state) and non-skilled body (local communities).

Relationships among local communities with the skilled state professionals/officials/managers constitute a game phenomenon and the game can be treated as two-player game. The game can be considered as continuous static game since the decisions, costs, or profits are related to continuous rather than discrete variables. Plantation and protection of the forest, sharing in the benefits earned from forests goods, involvement, and role of the decision makers are considered as major items while classifying participatory forest management.

Here, we consider two bodies as:

- Player I, that is, non-skilled bodies mainly communities living in forest area and involved in forest management;
- Player II, that is, skilled professionals involved in forest management.

These two bodies are competing for minimization of the deforestation.

Let z_1 and z_2 denote their efforts to get minimum loss of trees and forest resources. These bodies are the game players having their strategies $z_1, z_2 \ge 0$. So the strategy sets are $\Gamma_1 = \Gamma_2 = [0, \infty]$. For illustration of our proposed modified HJ algorithm in static game, we consider the payoff functions in combined form $\xi(Z)$ with $\lambda = 0.4$. We consider $\delta = 0.5$. We have to minimize the value of the objective function $\xi(Z)$.

Here,

$$\xi(Z) = \xi(z_1, z_2) = (z_1 - 1)^2 + (1.4)(z_1 - z_2)^2 + (0.4)(z_2 - 3)^2 \tag{6.2}$$

$$\text{Set 1: } Z^{(0)} = (1.00, 1.00); \ \xi(Z^{(0)}) = 1.60. \tag{6.3}$$

Now, $\xi(1.50, 1.00) = 0.25 + 0.35 + 1.60 = 2.20 \not< 1.60$;
$\xi(1.00, 1.50) = 0.00 + 0.35 + 0.90 = 1.25 < 1.60$; (success)
$\xi(1.50, 1.50) = 0.25 + 0.00 + 0.90 = 1.15 < 1.60$; (success)
Now, $1.15 < 1.25 \Rightarrow (1.50, 1.50)$ is better than $(1.00, 1.50)$.

$$\text{Set 1: } Z^{(0)} = (1.00, 1.00);$$

$$\text{Base 1: } Z^{(1)} = (1.50, 1.50); \ \xi(Z^{(1)}) = 1.15. \tag{6.4}$$

Now, we perform the PM (Kramer et al., 2011) as:
PM: $Z^{(2)} = 2Z^{(1)} - Z^{(0)} = 2(1.50, 1.50) - (1.00, 1.00) = (2.00, 2.00)$

So, we execute the exploratory move (EM) of searching from the point (2.00, 2.00).

Now, the ξ-values are obtained as:

EM:

$\xi(2.50, 2.00) = 2.25 + 0.35 + 0.40 = 3.00 \nless 1.15$;
$\xi(2.00, 2.50) = 1.00 + 0.35 + 0.10 = 1.45 \nless 1.15$;
$\xi(1.50, 2.00) = 0.25 + 0.35 + 0.40 = 1.00 < 1.15$; (success)
$\xi(2.00, 1.50) = 1.00 + 0.35 + 0.90 = 2.25 \nless 1.15$;
$\xi(2.50, 2.50) = 2.25 + 0.00 + 0.10 = 2.35 \nless 1.15$;

Base 2: $Z^{(2)} = (1.50, 2.00)$; $\xi(Z^{(2)}) = 1.00$; (6.5)

PM: $Z^{(3)} = 2Z^{(2)} - Z^{(1)} = 2(1.50, 2.00) - (1.50, 1.50) = (1.50, 2.50)$. (6.6)

Now, the next EMs are:

EM:

$\xi(2.00, 2.50) = 1.45 \nless 1.00$;
$\xi(1.50, 3.00) = 3.40 \nless 1.00$;
$\xi(1.00, 2.50) = 3.25 \nless 1.00$;
$\xi(2.00, 3.00) = 2.40 \nless 1.00$;

So, process ends with $\delta = 0.50$ having $Z^* = (1.50, 2.00)$; $\xi(Z^*) = 1.00$. Next, we proceed with reduced step vectors. The step size vector is reduced using the formula $\delta_1 = (\Delta z_1/2, \Delta z_2/2, \Delta z_3/2, \ldots, \Delta z_n/2)$, that is, we get $\delta_1 = 0.25$.

Now we consider the next calculations as follows:

Set 1: $Z^{(0)} = (1.50, 2.00)$; $\xi(Z^{(0)}) = 1.00$. (6.7)

EM:

$\xi(1.75, 2.00) = 1.05 \nless 1.00$;
$\xi(1.50, 2.25) = 1.2625 \nless 1.00$;
$\xi(1.25, 2.00) = 1.25 \nless 1.00$;
$\xi(1.50, 1.75) = 0.9625 < 1.00$; (success)
$\xi(1.75, 2.25) = 1.1375 \nless 1.00$;

Base 1: $Z^{(1)} = (1.25, 2.00)$; $\xi(Z^{(1)}) = 1.25$; (6.8)

PM: $Z^{(2)} = 2Z^{(1)} - Z^{(0)} = 2(1.25, 2.00) - (1.50, 2.00) = (1.00, 2.00)$. (6.9)

EM:

$\xi(1.00, 2.25) = 2.4125 \nless 1.25$;
$\xi(0.75, 2.00) = 2.65 \nless 1.25$;
$\xi(1.00, 1.75) = 1.4125 \nless 1.25$;
$\xi(1.25, 2.25) = 1.6875 \nless 1.25$;

So, process ends with $\delta_1 = 0.25$ and $0.125 < 0.25 \Rightarrow$ no next calculations.

So, we achieved $Z^* = (1.25, 2.00)$; $\xi(Z^*) = 1.25$ with $\delta = 0.25$.

7. Results and Discussion

Considering forest management problem through static game structure and solving the problems using HJ algorithm give different results. When we consider $\lambda = 0.6$, we get $Z^* = (1.50, 2.00)$; $\xi(Z^*) = 1.25$ with $\delta = 0.50$, or when we consider $\lambda = 0.9$, we get $Z^* = (1.75, 2.25)$; $\xi(Z^*) = 1.54375$ with $\delta = 0.25$. Thus, other calculations are performed with different values of λ. The whole calculations are tabulated in Table 1. We assume here $\delta = 0.50$, initially. Different δs can be taken also to get a set of tables.

Table 1
Strategies and payoffs values due to different λ and δ

λ	δ	X*		f(X*)
0.00	0.50	(1.00,	1.00)	0.0000
	0.25	—		—
	0.125	—		—
0.10	0.50	—		—
	0.25	(1.25,	1.50)	0.3563
	0.125	—		—
0.20	0.50	(1.50,	1.50)	.7000
	0.25	(1.50,	1.75)	0.6375
	0.125	—		—
0.30	0.50	(1.50,	2.00)	0.8750
	0.25	(1.50,	1.75)	0.8000
	0.125	—		—
0.40	0.50	(1.50,	2.00)	1.0000
	0.25	(1.25,	2.00)	1.2500
	0.125	—		—
0.50	0.50	(1.50,	2.00)	1.1250
	0.25	(1.50,	1.75)	1.1250
	0.125	—		—
0.60	0.50	(1.50,	2.00)	1.2500
	0.25	—		—
	0.125	—		—
0.70	0.50	(1.50,	2.00)	1.3750
	0.25	(1.75,	2.00)	1.3688
	0.125	—		—
0.80	0.50	(1.50,	2.00)	1.5000
	0.25	(1.75,	2.25)	1.4625
	0.125	—		—
0.90	0.50	(1.50,	2.00)	1.6250
	0.25	(1.75,	2.25)	1.5438
	0.125	—		—
1.00	0.50	(1.50	2.00)	1.7500
		(2.00	2.50)	1.7500
	0.25	(1.75, 2.25)		1.6250
	0.125	—		—

7.1. Comparative analysis

In this section, we consider our proposed approach in comparison with analytic method and graphical method, respectively.

7.1.1. Comparative study with analytic method

Nash equilibrium is obtained by assuming the first partial derivative of each player's payoff function with reference to corresponding strategy variable equal to zero. Assuming the static game having two players, say Player 1 and Player 2, whose payoff functions are given by:

For Player 1: $f^1(z_1, z_2) = (z_1 - 1)^2 + (z_1 - z_2)^2$; (7.10)

For Player 2: $f^2(z_1, z_2) = (z_2 - 3)^2 + (z_1 - z_2)^2$; (7.11)

The Nash point of this game, using analytical method, is derived from the equations:

$$\frac{\partial f1\,(z1, z2)}{\partial z1} = 0 \Rightarrow z_2 = 2z_1 - 1 \Rightarrow z_1 = 5/3 \quad (7.12)$$

$$\frac{\partial f1\,(z1,\,z2)}{\partial z1} = 0 \Rightarrow 2z_2 = z_1 + 3 \Rightarrow z_2 = 7/3 \quad (7.13)$$

Thus, the Nash equilibrium point is obtained from Eqs. (7.12−7.13) as: $(z_1^*, z_2^*) = (1.6667, 2.3333)$ with corresponding payoff values (0.8888445, 0.8888445). If we consider the combined form of the objective function as we assume in our proposed study, we get the combined optimal values as 1.777689 for $\lambda = 1.00$.

7.1.2. Comparative study with graphical method

We consider the graphical representation of the problem (Figure 1). The payoff functions related to the two players are considered here altogether with weight value $\lambda = 1$. From the pictorial representation, the presence of the minimum point at (1.6, 2.3) is justified. Our proposed model to encounter static game problem through modified HJ algorithm is simple to use. This is the main advantage of our proposed method. But the calculation of the objective function value, EMs, and PMs are time-consuming factors. These are the main disadvantages of our proposed study. The comparison studies of our proposed approach with analytic and graphical methods are presented in Table 2.

In HJ algorithm, we consider single-objective function for optimization. Here, we consider weighted sum objective function to be optimized in preference single-objective function due to real-life problems. In real-life situations, several constraints arise and we have to sum of those assigning with different weights and have to optimize those. In this point of view, our proposed modified HJ algorithm is better than HJ algorithm. Also, we get the optimal outcome 1.750000 (>1.700000) which is better than graphical method and almost same result (difference 1.5575%) when we use analytical method to solve our problem. We also get optimal strategy set (1.5000, 2.0000) rather than (1.6667, 2.3333) and (1.6000, 2.3000) of analytical and graphical methods, respectively. The results conclude in modified HJ algorithm's favor in the considered experiment above two other methodologies.

Table 2
Comparison of results using different methodologies

Methods	Strategies	Optimal payoffs
Analytical methods	(1.6667, 2.3333)	1.777689
Graphical methods	(1.6000, 2.3000)	1.700000
Our proposed method	(1.5000, 2.0000)	1.750000
	(2.0000, 2.5000)	1.750000

8. Conclusion

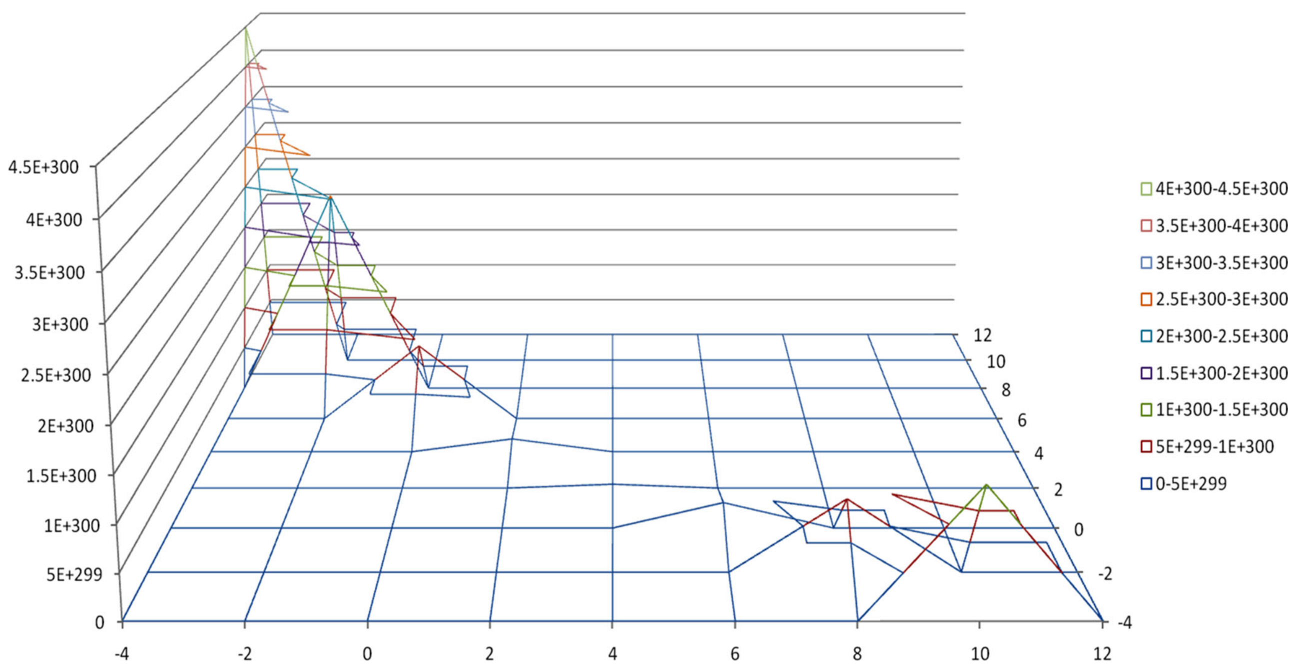
Many decision-making problems are encountered using HJ algorithm. Several modification of HJ algorithm has been verified in decision-making problems, also. HJ algorithm-based solutions in real-life problems give sustainable results with stability. The proposed method in our study contributes almost the same results with respect to other solution procedures of static game phenomena. The main advantages of this study can be considered as:

- It is simple and easy for calculations to get solutions of problems through our proposed modified HJ algorithm.
- Our proposed modified HJ algorithm can be implemented into other resources management systems, that is, in water management, soil management, air management, flora and fauna management, etc. to get sustainable solutions.

But the calculation of the objective function value, EMs, and PMs are time-consuming factors. These are the main disadvantages of our proposed study.

The present scenario of per capita trees (442 in 2015) is alarming (Crowther et al., 2015) and obviously, a proper forest

Figure 1
Graphical representation of payoffs of the players



management is very essential. Forest management problems are involved with some constraints, such as forest rights transfer market (Zhu et al., 2018) and investment in forest land (Xie et al., 2014), with other environmental issues. We must consider these facts in future research studies, also. Both the non-cooperative game theory and the cooperative game theory can be considered in the formulation of the concept of forest management problems and its natural constraints. Scientists can afford an explanation for such management issues using this type of game problem as a future study of this article. There are many open issues for future research works, as:

i) Computer programs can be developed using the proposed research methodology.
ii) Fuzzy or other uncertain environments can be implemented in this study.
iii) Researches can be done on more complicated continuous static games with HJ algorithmic steps.
iv) Constrained optimization problems can be considered using our proposed modified HJ algorithm.

Conflicts of Interest

The authors declare that they have no conflicts of interest to this work.

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