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New Possibility Soft Sets with Quality Assurance Application in Distance Education

Murat Kirişci[1,*]

[1]*Department of Biostatistics and Medical Informatics, Istanbul University-Cerrahpaşa, Turkey*

Abstract: The probability fuzzy soft set (FSS) is an approach that is obtained by adding a probability degree to each approximate element of the FSS and is formed by combining fuzzy set theory and soft set theory. This idea was later studied in different sets, such as intuitionistic fuzzy sets, neutrosophic fuzzy sets, Pythagorean fuzzy sets, etc. In this paper, a new possibility set is defined in the Fermatean fuzzy environment, and the essential features of new sets have been examined. The set-theoretic operations of new possibility sets, such as subset, soft equal, complement, union, intersection, AND, and OR, have been characterized with the help of elaborated examples. Their fundamental laws and properties are also discussed. A practical example concerning quality assurance in distance education is studied to demonstrate the applicability of new similarity measures (SMs) in decision-making situations. Thus, it has been shown that the newly given SM can be successfully applied to real-world decision problems. Finally, a comparison of the similarity of the proposed model is made with some existing models.

Keywords: possibility Fermatean fuzzy soft set, similarity measure, quality assurance, distance education

1. Introduction

In daily life, various uncertainties or indecisive situations are encountered many times. For decision-makers, besides qualitative values, emotions and thoughts also affect the decision to be made. In such cases, uncertainties arise. While some of the decisions that have to be made are quite simple, most of them require deep reflection and examination. Decision-making (DM) is the choice of the most appropriate one among the present options, taking into account the determined criteria, in order to achieve the determined purpose. In order to approach the DM process, which is a multistep process, analytically, it is important to predetermine the basic elements that make up the decision. The basis of the fuzzy logic set is digitizing the uncertainty between human thought and perceptions. Fuzzy logic eliminates problems and problems by offering new methods, especially in cases where basic mathematics is insufficient, cannot be solved, and is uncertain. Eliminating uncertainty is the most basic feature of fuzzy logic. The fuzzy logic theory has developed rapidly since Zadeh (1965), taking place in many application areas (Agarwal et al., 2013; Ali et al., 2009; Ali et al., 2020; Atanassov, 1986; Garg, 2016a, 2016b; Kirişci, 2022a; Mahmood & Rehman, 2021; Maji et al., 2001a, 2001b; Peng & Yang, 2015; Peng et al., 2015; Senapati & Yager, 2020; Xu, 2007; Yager, 2013, Yager & Abbasov, 2013; Zhang & Xu, 2014). Despite all of the possible solutions, these concepts have limitations. Examples of these limitations are the inadequacies in the consideration of the parametrization tool and how to specify the membership function for each individual item. Because of these restrictions, it is challenging for decision-makers to come to wise judgments during the analysis.

The Fermatean fuzzy set (FFS) was initiated by Senapati and Yager (2020). In the FFSs, the membership degree (MD) and non-membership degree (ND) perform the condition $0 \leq MD^3 + ND^3 \leq 1$, which is included in the literature as a new concept, gives better results than the intuitionistic fuzzy sets (IFSs) and Pythagorean fuzzy sets (PFSs) in defining uncertainties. For example, $0.8 + 0.7 > 1$, $0.8^2 + 0.7^2 > 1$, and $0.8^3 + 0.7^3 < 1$ (Akram et al., 2022, Kirişci, 2022b).

The current approaches have constraints due to their insufficiency, and as a result, experts cannot achieve a convenient conclusion. To overcome these drawbacks, Molodtsov (1999) worked with the soft set (SS) approach, which assigns ratings to certain factors. Maji et al. (2001a, 2001b) created the fuzzy soft set (FSS) with the intuitionistic fuzzy soft set (IFSS) by combining this theory with the current FS and IFS theoretical methods (IFSS). The IFSS has a significant benefit over the IFS in that they have evaluated information over a wide range of criteria (Deli & Cagman, 2015). The new SS, which is called Pythagorean FSS, is defined by Peng et al. (2015). Examples of other studies related to the PFSSs are Athira et al., (2019, 2020). Similar ideas were given by Fermatean fuzzy soft sets (FFSS) (Kirişci, 2022b).

Alkhazaleh et al. (2011) developed the idea of FSS possibility by giving a degree of probability to every number of FSS. Bashir et al. (2012) established the notion of the possibility IFSS (PIFSS) to handle it more effectively. The PIFSS is broader in scope than the current FSS, IFSS, and other sets. During the object assessment in PIFSS, a degree of possibility for every

*Corresponding author: Murat Kirişci, Department of Biostatistics and Medical Informatics, Istanbul University-Cerrahpaşa, Turkey. Email: mkirisci@hotmail.com

component is allocated to the IFSNs. The possibility concept for PFSS (PPFSS) is proposed by Jia-Hua et al. (2019).

The PIFSS (or PFFSS) is more generalized than the existing FSS, IFSS (or PFSS), and other sets. In PIFSS (or PPFSS), the degree of possibility of each component is appointed to the degrees of the IFSNs (or PFSNs) during the evaluation of the object. For example, consider the term "courage" and three different experts have considered evaluating the candidate. The possibility of the courage of a candidate according to the first expert can be 0.9, while for others it would differ from the first expert. To evaluate the given candidate, the rating values of it in terms of IFSS (or PFSS) are taken as (0.9, 0.5) where 0.9 represents the favorable degree of the expert toward the candidate and 0.5 against the degree. Hence, in terms of PIFSS (or PPFSS), such information is represented rather than IFSS (or PFSS) or IFS (or PFS) as (0.9, 0.5) only. Therefore, we can conclude that the evaluation of the object by using PIFSS (or PPFSS) is more reliable and robust than the other existing FSSs or IFSSs (or PFSSs).

The motivation of this study and its main contributions to the literature are as follows: this paper is aimed to define the FFS probability and examine its basic properties, inspired by the basic ideas of existing studies in the literature (such as PFSS, IPFSS, and PPFSS). In this study, the possibility FFS has been defined. By examining the properties of newly defined fuzzy sets, similarity measures (SMs) have been given based on these sets. In the application of the newly defined FS, the competencies of universities that provide distance education during the pandemic period were examined. In addition, the new method was compared with the known approaches.

The research gap, novelty, and motivation of the proposed study can be viewed from the following points:

1. The approaches such as the FSS, IFSS, neutrosophic soft set, etc., have been extensively utilized in various circumstances to resolve decision-making problems. Nevertheless, under several conditions, these structures demonstrate inadequacies in categorizing the entities according to their possibility grades. In other words, it can be interpreted that in the existing literature, the possibility degree of each element is regarded as one. However, in several realistic applications, different individuals may assign different possibility grades to each entity. To handle such concerns, Alkhazaleh et al. (2011) explored the possibility fuzzy soft set, which ensures the allocation of a possibility grade with every approximate element in the fuzzy soft set. However, such a model is not compatible with the use of the non-membership grade. In order to tackle it and address it more appropriately, Bashir et al. (2012) introduced the PIFSS. In the PIFSS, the intuitionistic fuzzy numbers are assessed through the use of the possibility grade while computing the ranking analysis, but the degree of indeterminacy is ignored. This shortcoming was addressed by developing the possibility neutrosophic soft set by Karaaslan (2016). In Jia-Hua et al. (2019), the concept of the Pythagorean fuzzy soft set has been extended by introducing a possibility of each element in the universe which is attached to the parameterization of PFSs while defining a Pythagorean fuzzy soft set.
2. FFSs can handle problems with imprecise and incomplete information more effectively than that IFSss and PFSs. FFSS, on the other hand, is a generalization of FS, IFS, NS, PFS, and PIFS. In the generalization of the FSS (or PIFSS, PNSS, PPFSS), the possibility of each element in the universe is attached to the parameterization of FSs (or IFSs, NSs, PFSs) while defining an FFS (or IFSS, NSS, PFSS). Keeping in mind this idea, PFFSSs will be described as a new approach.

Table 1
Abbreviation

Abbreviation	
DM	Decision-making
MD	Membership degree
ND	Non-membership degree
FS	Fuzzy set
IFS	Intuitionistic fuzzy set
PFS	Pythagorean fuzzy set
FFS	Fermatean fuzzy set
NS	Neutropsihic set
SS	Soft set
FFS	Fuzzy soft set
IFSS	Intuitionistic fuzzy soft set
NSS	Neutrosophic soft set
PFSS	Pythagorean fuzzy soft set
PFSS	Possibility fuzzy soft set
PIFSS	Possibility intuitionistic Fuzzy soft set
PPFSS	Possibility Pythagorean Fuzzy soft set
PFFSS	Possibility Fermatean fuzzy Soft set
SM	Similarity measure

3. The proposed model generalizes existing models to the PFFSS. It is a more flexible and generalized model to deal with uncertain data diligently. In addition, some algebraic properties have been investigated. A novel SM based on PFFSS has been given to compare two PFFSSs to deal with decision problems.
4. The validity of the proposed approach has been assessed by its implementation in a real-world problem-based scenario.
5. The advantageous aspects of the proposed approach have been judged through a structural comparison with some relevant existing approaches.

All the abbreviations used are explained in Table 1.

2. Preliminaries

Throughout the paper, U, E, and U_E will be denoted as the universe set, parameters sets, and soft universe, respectively.

Definition 2.1. (Yager, 2013) The set $A = \{(x,\ m_A(x), n_A(x)) : x\ \in U\}$ is said to be a Pythagorean FS where $m_{A,}, n_A : U \to [0,1]$, with the condition $0 \le (m_A(x))^2 + (n_A(x))^2 \le 1$.

For three PF numbers (PFN) over U_E, $M = (m_M, n_M)$, $M_1 = \left(m_{M_1}, n_{M_1}\right)$, $M_2 = \left(m_{M_2}, n_{M_2}\right)$, then the following conditions hold

$$M^c = (n_M,\ m_M),$$
$$M_1 \cup M_2 = \left(max\left(m_{M_1}, m_{M_2}\right),\quad min\left(n_{M_1}, n_{M_2}\right)\right),$$
$$M_1 \cap M_2 = \left(min\left(m_{M_1}, m_{M_2}\right),\quad max\left(n_{M_1}, n_{M_2}\right)\right),$$
$$M_1 \ge M_2 \text{ if and only if } m_{M_1} \ge m_{M_2} \text{ and } n_{M_1} \le n_{M_2},$$
$$M_1 = M_2 \text{ if and only if } m_{M_1} = m_{M_2} \text{ and } n_{M_1} = n_{M_2}.$$

Definition 2.2. (Peng et al., 2015) Take $f \subseteq E$. Let $F : f \to P_{FSS}(U)$, such that $P_{FSS}(U)$ is the family of all PF subsets of U. Therefore, The set F_f is referred to as a PFSS on U.

Definition 2.3. (Jia-Hua et al., 2019) Let $P_{FSS}(U)$ be a family of all PF subsets of U and $: E \to P_{FSS}(U)$ and $f : E \to P_{FSS}(U)$. For $x \in U$, If $F_f : E \to P_{FSS}(U) \times P_{FSS}(U)$ is a function $(F_f(e) = (F(e)(x), f(e)(x)))$, F_f is referred to as a PPFSS on U_E.

Further, $F_f(e)$ expressed as: $F_f(e) = \{\langle x, \ (m_{F(e)}(x), \ n_{F(e)}(x)), (m_{f(e)}(x), \ n_{f(e)}(x))\rangle \ : x \in U\}$.

Definition 2.4. (Senapati & Yager, 2020) The set $F = \{(x, \ m_F(x), \ n_F(x)) : x \in U\}$, is said to be an FFS in U, where m_F, n_F in the unit interval, including the condition $0 \le (m_F(x))^3 + (n_F(x))^3 \le 1$.

$h_F(x) = \sqrt[3]{1 - (m_F(x))^3 - (n_F(x))^3}$ is called the degree of indeterminacy

Definition 2.5. (Kirişci, 2022b) Let $f \subseteq E$ and $F_{FSS}(U)$ be the family of all FF subsets of U. F_f is an FFSS over U, where $F : f \to F_{FSS}(U)$.

An FFS on U is a family of parameters formed by some FF subsets on U. For any parameter, $\varepsilon \in f \quad F(\varepsilon)$ is an FSS associated with $\varepsilon \in U$. Then, $F(\varepsilon)$ is said to be a FF value set of.

3. New Possibility Set

We initiate the notion of the PFFSSs to generalize the possibility, FSS model.

Definition 3.1. Let $F_{FSS}(U)$ shows the family of all FF subsets of U. For $f \ \subseteq E$ and $F : f \to F_{FSS}(U)$, if $F_f : E \to F_{FSS}(U) \times F_{FSS}(U)$ is a function characterized as $F_f(e) = (F(e)(x), \ f(e)(x))$, then F_f is referred to as a PFFSS on U_E.

Further, $F_f(e)$ will be expressed as:

$$F_f(e) = \{\langle x, \ (m_{F(e)}(x), \ n_{F(e)}(x)), \ (m_{f(e)}(x), \ n_{f(e)}(x))\rangle \ : x \in U\}.$$

for each parameter e.

Example 1. Given that $U = \{d_1, \ d_2, \ d_3\}$ is a set of three diseases under consideration of a physician. Let the symptoms are denoted by $E = \{s_1, \ s_2, \ s_3\}$. $F_f : E \to F_{FSS}(U) \times F_{FSS}(U)$ is given as follows:

$$F_f(s_1) = \{(d_1/(0.82, \ 0.63), (0.69, 0.58)), \ (d_2/(0.64, \ 0.75), (0.55, 0.81)), (d_3/(0.88, \ 0.44), (0.76, 0.47)) \},$$

$$F_f(s_2) = \{(d_1/(0.66, \ 0.53), (0.87, 0.36)), (d_2/(0.70, \ 0.48), (0.36, 0.85)), (d_3/(0.91, \ 0.27), (0.84, 0.57)) \},$$

$$F_f(s_3) = \{(d_1/(0.58, \ 0.67), (0.72, 0.44)), \ (d/(0.94, \ 0.30), (0.86, 0.38)), (d_3/(0.35, \ 0.92), (0.42, 0.77)) \}.$$

and matrix representation of these values is

$$F_f = \begin{bmatrix} (0.82, \ 0.63), (0.69, 0.58) & (0.64, \ 0.75), (0.55, 0.81) & (0.88, \ 0.44), (0.76, 0.47) \\ (0.66, \ 0.53), (0.87, 0.36) & (0.70, \ 0.48), (0.36, 0.85) & (0.91, \ 0.27), (0.84, 0.57) \\ (0.58, \ 0.67), (0.72, 0.44) & (0.94, \ 0.30), (0.86, 0.38) & (0.35, \ 0.92), (0.42, 0.77) \end{bmatrix}$$

Definition 3.2. Suppose that F_f and G_g are two PFFSSs over U_E. Now, F_f is called a possibility FFS subset of G_g iff

$g(e)(x) \subseteq f(e)(x)$ if $m_{f(e)}(x) \ge m_{g(e)}(x), \ \ n_{q(e)}(x) \le n_{f(e)}(x)$, $G(e)(x) \subseteq F(e)(x)$ if $m_{F(e)}(x) \ge m_{G(e)}(x), \quad n_{G(e)}(x) \le n_{F(e)}(x), \forall e \ \in E$.

This relationship is denoted as $G_g \subseteq \ F_f$.

Example 2. Let's use PFFSS F_f in Example 1. Now let's define a new PFFSS G_g:

$$G_g(s_1) = \{(d_1/(0.72, \ 0.51), (0.49, 0.65)), (d/(0.55, \ 0.81), (0.3, 0.9)), (d_3/(0.80, \ 0.60), (0.67, 0.54)) \},$$

$$G_g(s_2) = \{(d_1/(0.43, \ 0.668), (0.70, 0.57)), (d_2/(0.62, \ 0.59), (0.30, 0.91)), (d_3/(0.68, \ 0.34), (0.69, 0.72)) \},$$

$$G_g(s_3) = \{(d_1/(0.51, \ 0.69), (0.62, 0.55)), (d_2/(0.81, \ 0.35), (0.77, 0.46)), (d_3/(0.24, \ 0.94), (0.35, 0.80)) \}$$

Clearly, we have $G_g \subseteq \ F_f$.

Definition 3.3. Suppose that F_f and G_g are two PFFSSs over U_E. Now, F_f and G_g are referred to as a possibility Fermatean fuzzy soft equal if and only if

(i) $F_f \subseteq \ G_g$,
(ii) $G_g \subseteq \ F_f$,

which can be denoted by $G_g = \ F_f$.

Now, some operations of PFFSSs will be defined and some of their features will be mentioned.

Definition 3.4. For the PFFSS F_f, $F_f^c = \langle F^c(e)(x), \ f^c(e)(x)\rangle$ is called the complement of F_f such that $F^c(e)(x) = \langle n_{F(e)}(x), \ m_{F(e)}(x)\rangle$ and $f^c(e)(x) = \langle n_{f(e)}(x), \ m_{f(e)}(x)\rangle$.

Proposition 3.1. $(F_f^c)^c = F$

Definition 3.5. Let F_f and G_g are two PFFSSs over U_E. The union and intersection operations on two PFFSSs F_f and G_g over U_E denoted by $F_f \ \cup \ U_E$ and $F_f \ \cap \ U_E$ are respectively defined by two mappings as follows:

$$B_b : E \to F_{FSS}(U) \times F_{FSS}(U) \quad and \quad K_k : E \to F_{FSS}(U) \times F_{FSS}(U)$$

such that for all $x \ \in U$,

$$B_b(e)(x) = (B(e)(x), \ b(e)(x))$$
$$K_k(e)(x) = (K(e)(x), \ k(e)(x)),$$

where $B(e)(x) = F(e)(x) \cup G(e)(x)$ and $b(e)(x) = f(e)(x) \cup g(e)(x)$; $K(e)(x) = F(e)(x) \cap G(e)(x)$ and $k(e)(x) = f(e)(x) \cap g(e)(x)$.

Example 3. Take F_f and G_g are two PFFSSs over U_E given in Example 1 and Example 2.

$$F_f \cup G_g = \begin{bmatrix} (0.8, \ 0.6), (0.7, 0.8) & (0.6, \ 0.7), (0.5, 0.8) & (0.9, \ 0.4), (0.8, 0.4) \\ (0.6, \ 0.5), (0.9, 0.5) & (0.7, \ 0.5), (0.4, 0.8) & (0.9, \ 0.2), (0.8, 0.6) \\ (0.6, \ 0.6), (0.7, 0.4) & (0.9, \ 0.3), (0.8, 0.4) & (0.3, \ 0.9), (0.4, 0.7) \end{bmatrix}$$

and

$$F_f \cap G_g = \begin{bmatrix} (0.7, \ 0.7), (0.5, 0.8) & (0.5, \ 0.8), (0.3, 0.9) & (0.8, \ 0.6), (0.7, 0.5) \\ (0.4, \ 0.6), (0.7, 0.7) & (0.6, \ 0.7), (0.3, 0.9) & (0.8, \ 0.3), (0.6, 0.7) \\ (0.5, \ 0.6), (0.4, 0.5) & (0.8, \ 0.5), (0.7, 0.6) & (0.2, \ 0.9), (0.3, 0.8) \end{bmatrix}$$

Definition 3.6. The function $\emptyset_\theta : E \to F_{FSS}(U) \times F_{FSS}(U)$ is called possibility null FFSS, if $\emptyset(e)(x) = (0, 1)$ and $\theta(e)(x) = (0, 1)$, $\forall\, x \in U$, where $\emptyset_\theta(e)(x) = \langle \emptyset(e)(x),\ \theta(e)(x)\rangle$.

The function $\Omega_\Lambda : E \to G(U) \times G(U)$ is called possibility absolute FFSS, if $\Omega(e)(x) = (1, 0)$ and $\Lambda(e)(x) = (1, 0)$, $\forall\, x \in U$, where $\Omega_\Lambda(e)(x) = \langle \Omega(e)(x),\ \Lambda(e)(x)\rangle$.

Theorem 3.1. Take a PFFSS F_f. The following conditions hold

(i) $F_f = F_f \cup F_f$, $F_f = F_f \cap F_f$,
(ii) $F_f \subseteq F_f \cup F_f$, $F_f \subseteq F_f \cap F_f$,
(iii) $F_f \cup \emptyset_\theta = F_f$, $F_f \cap \emptyset_\theta = \emptyset_\theta$,
(iv) $F_f \cup \Omega_\Lambda = F_f$, $F_f \cap \Omega_\Lambda = F_f$.

Proof. Straightforward.

It should be noted that, if $F_f \neq \emptyset_\theta$ or $F_f \neq \Omega_\Lambda$, then $F_f \cup F_f^c \neq \Omega_\Lambda$, $F_f \cap F_f^c \neq \emptyset_\theta$, for a PFFSS F_f over U_E.

Theorem 3.2. Let $F_f,\ G_g,\ H_h$ be three PFFSS over U_E. Then,

(i) $F_f \cup G_g = G_g \cup F_f$,
(ii) $F_f \cap G_g = G_g \cap F_f$,
(iii) $F_f \cup (G_g \cup H_h) = (F_f \cup G_g) \cup H_h$,
(iv) $F_f \cap (G_g \cap H_h) = (F_f \cap G_g) \cap H_h$.

Theorem 3.3. Let $F_f,\ G_g,\ H_h$ be three PFFSS over U_E. Then,

(i) $(F_f \cup G_g)^c = F_f^c \cap G_g^c$,
(ii) $(F_f \cap G_g)^c = F_f^c \cup G_g^c$,
(iii) $(F_f \cup G_g) \cap F_f = F_f$,
(iv) $(F_f \cap G_g) \cup F_f = F_f$,
(v) $F_f \cup (G_g \cap H_h) = (F_f \cup G_g) \cap (F_f \cup H_h)$,
(vi) $F_f \cap (G_g \cup H_h) = (F_f \cap G_g) \cup (F_f \cap H_h)$.

The proofs of these theorems can be shown immediately from Definitions.

Definition 3.7. Let $F_f,\ G_g$ be two PFFSS over U_E. Then, the **AND operation** is defined as:

$$(F_f,\ A) \vee (G_g, B) = (H_h,\ A \times B)$$

where $H_h(\zeta, \eta) = (H(\zeta,\eta)(x),\quad h(\zeta,\eta)(x))$, for all $(\zeta,\eta) \in A \times B$, such that $H(\zeta,\eta) = F(\zeta) \cap B(\eta)$, and $h(\zeta,\eta) = f(\zeta) \cap g(\eta)$.

The OR operation is defined as:

$$(F_f,\ A) \wedge (G_g, B) = (H_h,\ A \times B)$$

where $H_h(\zeta,\eta) = (H(\zeta,\eta)(x),\quad h(\zeta,\eta)(x))$, for all $(\zeta,\eta) \in A \times B$, such that $H(\zeta,\eta) = F(\zeta) \cup B(\eta)$, and $h(\zeta,\eta) = f(\zeta) \cup g(\eta)$.

It should be noted that for two PFFSS $F_f,\ G_g$ over U_E and for all $(\zeta,\eta) \in A \times B$, if $u \neq v$, then

$$(G_g, B) \wedge (F_f, A) \neq (F_f,\ A) \wedge (G_g, B),$$

$$(G_g, B) \vee (F_f, A) \neq (F_f,\ A) \vee (G_g, B).$$

Theorem 3.4. Let $F_f,\ G_g$ be two PFFSS over U_E. Then,

(i) $((F_f, A) \wedge (G_g, B))^c = (F_f,\ A)^c \vee (G_g, B)^c$,
(ii) $((F_f, A) \vee (G_g, B))^c = (F_f,\ A)^c \wedge (G_g, B)^c$.

Proof. (i) Suppose that $(F_f,) \wedge (G_g, B) = (H_h,\ A \times B)$. The definition of AND operation, for all $(\zeta,\eta) \in A \times B$, we have $H^c(\zeta,\eta) = (F(\zeta) \cap G(\eta))^c = F^c(\zeta) \cup G^c(\eta)$, $h^c(\zeta,\eta) = (f(\zeta) \cap g(\eta))^c = f^c(\zeta) \cup g^c(\eta)$, where $H_h^c(\zeta,\eta) = (H^c(\zeta,\eta)(x), h^c(\zeta,\eta)(x))$.

On the other hand, given that $(F_f,\ A)^c \vee (G_g,\ B)^c = (K_k,\ A \times B)$, where $K_k(\zeta,\eta) = (K(\zeta,\eta)(x),\ k(\zeta,\eta)(x))$, for all all $(\zeta,\eta) \in A \times B$, such that $K(\zeta,\eta) = F^c(\zeta) \cup G^c(\eta)$, $k(\zeta,\eta) = f^c(\zeta) \cup g^c(\eta)$. Hence, $H_h^c = K_k$.
Item (ii.) can be proved similarly to (i.).

4. Similarity Measure

The conditions to be used in the new definition of SM are listed below:

$$\Gamma(F(e)(x),\ G(e)(x)) = \frac{\sum_{i=1}^{n}\left(m_{F(e_i)}(x).m_{G(e_i)}(x)\right)}{\sum_{i=1}^{n}\left(1 - \sqrt[3]{\left[1 - m^3_{F(e_i)}(x)\right]\left[1 - m^3_{G(e_i)}(x)\right]}\right)} \tag{1}$$

$$\Lambda(F(e)(x),\ G(e)(x)) = \sqrt[3]{1 - \frac{\sum_{i=1}^{n}\left|n^3_{F(e_i)}(x) - n^3_{G(e_i)}(x)\right|}{\sum_{i=1}^{n}\left(1 + n^3_{F(e_i)}(x).n^3_{G(e_i)}(x)\right)}} \tag{2}$$

$$\sigma_i = \frac{m^3_{f(e_i)}(x)}{m^3_{f(e_i)}(x) + n^3_{f(e_i)}(x)} \tag{3}$$

$$\tau_i = \frac{m^3_{g(e_i)}(x)}{m^3_{g(e_i)}(x) + n^3_{g(e_i)}(x)} \tag{4}$$

$$\Phi(F, G) = \frac{\Gamma(F(e)(x),\ G(e)(x)) + \Lambda(F(e)(x),\ G(e)(x))}{2} \tag{5}$$

$$\Psi(f, g) = 1 - \frac{\sum|\sigma_i - \tau_i|}{\sum|\sigma_i + \tau_i|} \tag{6}$$

Definition 4.1. Let F_f and G_g be two PFFSSs over U_E. Using Equations (1)-(6), the SMs between F_f and G_g are as follows:

$$B(F_f,\ G_g) = \Phi(F, G).\Psi(f, g).$$

Theorem 4.1. Let F_f, G_g, and H_h are three PFFSSs over U_E. Then, we have

(i) $B(F_f,\ G_g) = B(\ G_g,\ F_f)$,
(ii) $0\ \leq B(F_f,\ G_g) \leq 1$,
(iii) $F_f =\ G_g \quad \Rightarrow \quad B(F_f,\ G_g) = 1$,
(iv) $F_f \subseteq\ G_g \subseteq H_h \quad \Rightarrow \quad B(F_f,\ G_g) \leq B(G_g,\ H_h)$,
(v) $F_f \cap\ G_g = \emptyset \quad \Rightarrow \quad B(F_f,\ G_g) = 0$.

The proof of this theorem is immediately obtained from Definition 4.1.

Example 4. Let's use the two PFFSSs F_f and G_g over U_E given in Example 3. Compute the similarity between two PFFSSs F_f and G_g. The first disease u_1 and the set of symptoms can be given as:

$$F_f = \begin{bmatrix} (0.8,\ 0.6) & (0.7,\ 0.6) \\ (0.6,\ 0.5) & (0.9,\ 0.5) \\ (0.6,\ 0.6) & (0.7,\ 0.4) \end{bmatrix} \quad and \quad G_g = \begin{bmatrix} (0.7,\ 0.7) & (0.5,\ 0.8) \\ (0.4,\ 0.6) & (0.7,\ 0.7) \\ (0.5,\ 0.6) & (0.6,\ 0.5) \end{bmatrix}$$

$\Gamma(F(e)(x),\ G(e)(x)) = 0.483$, $\Lambda(F(e)(x),\ G(e)(x)) = 0.932$ and so $\Phi(F,G) = 0.6465$. Further, $\Psi(f,g) = 0.26$. Therefore, $B(F_f,\ G_g) = \Phi(F,G).\Psi(f,g) = 0.1681$.

5. Decision-Making Application

Students from all nations' schooling were disrupted as a result of the global coronavirus outbreak. Due to school closures in response to COVID-19, more than 1,7 billion children missed class. Almost all nations have implemented nationwide shutdowns, according to UNESCO monitoring, affecting 90% of students globally (Education, 2020).

With the transition to distance education in all programs of higher education institutions during the COVID-19 pandemic, ensuring quality assurance in distance education has become a priority agenda item all over the world. In this process, there have been intense discussions on the constituents, guidelines, and quality signs of the quality assurance system in distance education for higher education institutions.

The Quality Board will oversee the distance education system of three universities. The components to be considered in the creation of a qualified distance education system and the scope of these components have been determined under certain headings at the end of these discussions: E_1 educational processes; E_2 infrastructure and access options; E_3 human resources and support services; and E_4 information security and ethics are attributes. By calculating the SM with these attributes, the closest university to the ideal system determined in the quality assurance system will be determined. The values expected to be in a university in quality measurements in distance education have been determined by the Quality Board (Table 2).

Tables 2, 3 and 4 list the PFFSS evaluations of distant learning at universities. Depending on their evaluation of the options in relation to the criteria being taken into consideration, the board members submitted the FFNs values in Tables 3, 4, and 5.

In this case, based on Definition 4.1, we should compute the SM of PFFSSs in Tables 3, 4, and 5 with the one in Table 1 in order to discover the best institution that is closest to the ideal university in remote education. The university should be used to determine the criterion of resemblance. Generally speaking, among all the available universities, the university with an SM above this threshold is the most appropriate choice.

Table 2
Ideal university criteria values in distance education

$K_k(e)(x)$	E_1	E_2	E_3	E_4
$K(e)(x)$	(1, 0)	(0.9, 0.1)	(0.8, 0.3)	(0.9, 0.1)
$k(e)(x)$	(1, 0)	(1, 0)	(1, 0)	(1, 0)

Table 3
First university criteria values in distance education

$F_f(e)(x)$	E_1	E_2	E_3	E_4
$F(e)(x)$	(0.72, 0.55)	(0.46, 0.38)	(0.81, 0.25)	(0.94, 0.22)
$f(e)(x)$	(0.81, 0.37)	(0.53, 0,66)	(0.95, 0.27)	(0.92, 0.18)

Table 4
Second university criteria values in distance education

$G_g(e)(x)$	E_1	E_2	E_3	E_4
$G(e)(x)$	(0.37, 0.83)	(0.85, 0.52)	(0.91, 0.29)	(0.76, 0.57)
$g(e)(x)$	(0.24, 0.68)	(0.75, 0,62)	(0.87, 0.35)	(0.84, 0.49)

Table 5
Third university criteria values in distance education

$H_h(e)(x)$	E_1	E_2	E_3	E_4
$H(e)(x)$	(0.35, 0.87)	(0.82, 0.56)	(0.94, 0.24)	(0.74, 0.55)
$h(e)(x)$	(0.55, 0.55)	(0.46, 0,64)	(0.48, 0.79)	(0.75, 0.61)

Calculating the SM for the universities as follows: For the first university,

Γ(K,F)=0.782, Λ(K,F)=0.913 $\Rightarrow$ Φ(K,F)=0.8475; $\sigma_{1,2,3,4}$=1, τ_1=0.913, τ_2=0.343, τ_3=0.978, τ_4=0.992, $\Rightarrow$ $\Psi(k,f)$ =0.9 and so $B(K_k,\ F_f) = \Phi(K,F).\Psi(k,f) = 0.76275$.

For the second university,

Γ(K,G)=0.992, Λ(K,G)=0.9 $\Rightarrow$ Φ(K,G)=0.946; $\sigma_{1,2,3,4}$=1, τ_1=0.5, τ_2=0.64, τ_3=0.94, τ_4=0.84, $\Rightarrow$ $\Psi(k,g)$ =0.844 and so $B(K_k,\ G_g) = \Phi(K,G).\Psi(k,g) = 0.8$.

For the third university,

Γ(K,H)=0.77, Λ(K,H)=0.231 $\Rightarrow$ Φ(K,H)=0.5005; $\sigma_{1,2,3,4}$=1, τ_1=0.5, τ_2=0.276, τ_3=0.184, τ_4=0.65, $\Rightarrow$ $\Psi(k,h)$ =0.574 and so $B(K_k,\ H_h) = \Phi(K,H).\Psi(k,h) = 0.2873$.

According to the aforementioned findings, the second university is the one that is most similar to the ideal university for distance learning, with a similarity score of 0.8 being the greatest. The second university and the third university in distance education with the values of SM as 0.76275 and 0.2873 follow the first university in distance education.

6. Comparative Analysis

In this section, we will again investigate the above-mentioned case study using the FFSS, PPFSS, and PFSS approaches. In these approaches, the effect of the possibility parameter will be considered.

For FFSS, the following results are obtained when the SMs of the three universities are calculated:

$$(K_k,\ F_f) = \Phi(K,F).\Psi(k,f) = 0.77,$$
$$B(K_k,\ G_g) = \Phi(K,G).\Psi(k,g) = 0.77,$$
$$B(K_k,\ H_h) = \Phi(K,H).\Psi(k,h) = 0.387.$$

For PFSS, the following results are obtained when the SMs of the three universities are calculated:

$$(K_k,\ F_f) = \Phi(K,F).\Psi(k,f) = 0.903,$$
$$B(K_k,\ G_g) = \Phi(K,G).\Psi(k,g) = 0.861,$$
$$B(K_k,\ H_h) = \Phi(K,H).\Psi(k,h) = 0.861.$$

For PPFSS, the following results are obtained when the SMs of the three universities are calculated:

$$\left(K_k,\ F_f\right) = \Phi(K,F).\Psi(k,f) = 0.627,$$
$$B\left(K_k,\ G_g\right) = \Phi(K,G).\Psi(k,g) = 0.8014,$$
$$B(K_k,\ H_h) = \Phi(K,H).\Psi(k,h) = 0.528.$$

The results above demonstrate that the possibility parameter significantly affects the determination of the SM of FPFSSs. It can be monitored that the possibility parameter's computations are more sensitive and produce more precise results. Similar to PFFSS, the possibility parameter's impact makes PPFSS's results much more precise.

On the other hand, without the generalization parameter, we are unable to determine which distance learning university is the best when utilizing the PFSS and FFSS techniques. Therefore, the possibility parameter has a significant impact on how similar the two colleges are for distance learning. Therefore, compared to the PFSS and FFSS techniques without the generalization parameter in the DM process, the PFFSS approach is more rational and scientific.

7. Conclusion

We can explain the contribution of this study to the literature as follows: the examined study employs the PFFSS to handle the inadequate, vague, and conflicting data by considering MDs, NDs, and the possibility of degrees toward these MDs. Thus, a PFFSS expresses the veritable circumstances of the real conditions because it represents the fuzziness and the possibility acquired in genuine issues. This paper offers new operational laws and distance measures for estimating the degree of discrimination between the two or more PFFSSs. A novel SM based on PFFSS has been given. The new approach is presented to solve the multi-attribute decision-making problems with PFFSS information. In these approaches, the expert opinion and parameters are computed with the SM and some operations of PFFSS. The essential superiority of the proposed technique as compared to existing ones is that these reflect the decision-maker's risk factor in the application fields represented by the possibility of each assessment esteem. Finally, a numerical example is presented to demonstrate the approach and compare their results with the several existing approaches.

Inspired by this study, new solutions to decision-making problems can be proposed by using hypersoft set, indetermsoft set, and tree soft set (Smarandache, 2022a, 2022b) in the future.

Conflicts of Interest

The author declares that he has no conflicts of interest to this work.

References

Agarwal, M., Biswas, K. K., & Hanmandlu, M. (2013). Generalized intuitionistic fuzzy sets with applications in decision making. *Applied Soft Computing*, *13*(8), 3552–3566. https://doi.org/10.1016/j.asoc.2013.03.015.

Akram, M., Ali, G., Alcantud, J. C. R., & Riaz, A. (2022). Group decision-making with Fermatean fuzzy soft expert knowledge. *Artificial Intelligence Review*, *55*(7), 5349–5389. https://doi.org/10.1007/s10462-021-10119-8

Ali, M. I., Feng, F., Liu, X., Min, W. K., & Shabir, M. (2009). On some new operations in soft set theory. *Computers & Mathematics with Applications*, *57*(9), 1547–1553. https://doi.org/10.1016/j.camwa.2008.11.009

Ali, Z., Mahmood, T., & Yang, M. S. (2020). TOPSIS method based on complex spherical fuzzy sets with Bonferroni mean operators. *Mathematics*, *8*(10), 1739. https://doi.org/10.3390/math8101739.

Alkhazaleh, S., Salleh, A. R., & Hassan, N. (2011). Possibility fuzzy soft set. *Advances in Decision Sciences*, *2011*. https://doi.org/10.1155/2011/479756

Athira, T. M., John, S. J., & Garg, H. (2019). Entropy and distance measures of Pythagorean fuzzy soft sets and their applications. *Journal of Intelligent & Fuzzy Systems*, *37*(3), 4071–4084. https://doi.org/10.3233/JIFS-190217

Athira, T. M., John, S. J., & Garg, H. (2020). A novel entropy measure of Pythagorean fuzzy soft sets. *AIMS Mathematics*, *5*(2), 1050–1061. https://doi.org/10.3934/math.2020073

Atanassov, K. (1986). Intuitionistic fuzzy sets. *Fuzzy Sets and Systems, 20*(1), 87–96.

Bashir, M., Salleh, A. R., & Alkhazaleh, S. (2012). Possibility intuitionistic fuzzy soft set. *Advances in Decision Sciences*, *2012*. https://doi.org/10.1155/2012/404325

Deli, I., & Cagman, N. (2015). Intuitionistic fuzzy parameterized soft set theory and its decision making. *Applied Soft Computing*, *28*, 109–113. https://doi.org/10.1016/j.asoc.2014.11.053

Education. (2020). *UNESCO: 290 million students stay home due to Coronavirus*. Retrieved from: https://learningenglish.voanews.com/a/unesco-290-million-students-stay-home-due-to-coronavirus/5317148.html

Garg, H. (2016a). A new generalized Pythagorean fuzzy information aggregation using Einstein operations and its application to decision making. *International Journal of Intelligent Systems*, *31*(9), 886–920. https://doi.org/10.1002/int.21809

Garg, H. (2016b). A novel correlation coefficients between Pythagorean fuzzy sets and its applications to decision-making processes. *International Journal of Intelligent Systems*, *31*(12), 1234–1252. https://doi.org/10.1002/int.21827

Jia-Hua, D., Zhang, H., & He, Y. (2019). Possibility Pythagorean fuzzy soft set and its application. *Journal of Intelligent & Fuzzy Systems*, *36*(1), 413–421. https://doi.org/10.3233/JIFS-181649

Karaaslan, F. (2016). Similarity measure between possibility neutrosophic soft sets and its applications. *University Politehnica of Bucharest Scientific Bulletin, Series A: Applied Mathematics and Physics*, *78*(3), 155–162.

Kirişci, M. (2022a). Group-based Pythagorean fuzzy soft sets with medical decision-making applications. *Journal of Experimental & Theoretical Artificial Intelligence*, 1–19. https://doi.org/10.1080/0952813X.2022.2079006.

Kirişci, M. (2022b). New entropy and distance measures for Fermatean fuzzy soft sets with medical decision-making and pattern recognition applications. *Research Square*. https://doi.org/10.21203/rs.3.rs-1796355/v1

Mahmood, T., & Rehman, U. (2021). A novel approach towards bipolar complex fuzzy sets and their applications in generalized similarity measures. *International Journal of Intelligent Systems*, *37*(1), 535–567. https://doi.org/10.1002/int.22639

Maji, P., Biswas, A., & Roy, A. (2001a). Fuzzy soft sets. *Journal of Fuzzy Mathematics*, *9*, 589–602.

Maji, P., Biswas, A., & Roy, A. (2001b). Intuitionistic fuzzy soft sets. *Journal of Fuzzy Mathematics*, *9*, 677–692.

Molodtsov, D. (1999). Soft set theory-first results. *Computers & Mathematics with Applications*, *37*, 19–31. https://doi.org/10.1016/S0898-1221(99)00056-5

Peng, X., Yang, Y., Song, J., & Jiang, Y. (2015). Pythagorean fuzzy soft set and its application. *Computer Engineering*, *41*(7), 224–229.

Peng, X., & Yang, Y. (2015). Some results for Pythagorean fuzzy sets. *International Journal of Intelligent Systems*, *30*(11), 1133–1160. https://doi.org/10.1002/int.21738

Senapati, T., & Yager, R. R. (2020). Fermatean fuzzy sets. *Journal of Ambient Intelligence and Humanized Computing*, *11*, 663–674. https://doi.org/10.1007/s12652-019-01377-0

Smarandache, F. (2022a). Introduction to the IndetermSoft set and IndetermHyperSoft set. *Neutrosophic Sets and Systems*, *50*, 629–650. https://doi.org/10.5281/zenodo.6774960

Smarandache, F. (2022b). Practical applications of IndetermSoft set and IndetermHyperSoft set and introduction to TreeSoft set as an extension of the MultiSoft set. *Neutrosophic Sets and Systems, 51*, 939–947.

Xu, Z. S. (2007). Intuitionistic fuzzy aggregation operator. *IEEE Transactions on Fuzzy Systems*, *15*(6), 1179–1187. https://doi.org/10.1109/TFUZZ.2006.890678

Yager, R. R. (2013). Pythagorean fuzzy subsets. In *Proceedings of Joint IFSA World Congress and NAFIPS Annual Meeting*, 57–61. https://doi.org/10.1109/IFSA-NAFIPS.2013.6608375

Yager, R. R., & Abbasov, A. M. (2013). Pythagorean membership grades, complex numbers, and decision making. *International Journal of Intelligent Systems*, *28*(5), 436–452. https://doi.org/10.1002/int.21584

Zadeh, L. A., (1965). Fuzzy sets. *Information and Control*, *8*(3), 338–353. https://doi.org/10.1016/S0019-9958(65)90241-X

Zhang, X. L., & Xu, Z. S. (2014). Extension of TOPSIS to multi-criteria decision making with Pythagorean fuzzy sets. *International Journal of Intelligent Systems*, *29*(12), 1061–1078. https://doi.org/10.1002/int.21676

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