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RESEARCH ARTICLE

Multiple Attribute Decision-Making Based on Bonferroni Mean Operators under Square Root Fuzzy Set Environment

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Abstract: The intuitionistic fuzzy set (IFS), which has a membership and non-membership degree, is a controlling and effective device for dealing with fuzziness and uncertainty. Recently, the square root fuzzy set which is one of the efficient generalizations of an IFS for dealing with uncertainty and haziness in information has been introduced. In this study, a novel method for multiple attribute decision-making (MADM) based on SR-fuzzy information is investigated. Since aggregation operators are significant in the decision-making (DM) process, to achieve this goal, the current paper suggests a variety of novel Bonferroni mean and weighted Bonferroni mean operators to aggregate the SR-fuzzy values for the various decision-maker preferences. To achieve this goal, the current paper suggests a variety of novel Bonferroni mean and weighted Bonferroni mean operators to aggregate the SR-fuzzy values for the various decision-maker preferences. SR-fuzzy Bonferroni mean operator and weighted SR-fuzzy Bonferroni mean operator are established and their properties are described. Then, we constructed a MADM approach using the proposed operators for the SR-fuzzy information and proved the approach with a mathematical example. In the end, a comparative study of the developed and existing approaches has been discussed to evaluate the pertinency and practicality of the proposed DM technique.

Keywords: Bonferroni mean operators, aggregation operators, SR-fuzzy sets, multiple attribute decision-making

1. Introduction

Decision-making (DM) is a very important process in the world, to obtain the most feasible alternative through the given alternatives with their attributes is very difficult. To evaluate the true alternative with their multiple attributes for a good decision, researchers introduced the method known as multiple attribute decision-making (MADM). In this method, firstly the information is aggregated and then the results are ranked. Merigo and Gil-Lafuente (2010) introduced a new DM technique and also provide its application for financial products. Falessi et al. (2011) worked on DM techniques for software designing. Mardani et al. (2015a) and Mardani et al. (2015b) reviewed the literature from 1994 to 2014 on DM techniques and its application. Pamucar et al. (2020) presented DM approach using hybrid neutrosophic fuzzy environment. Many scholars worked on this method and applied it to many fields such as computer, economics, industry, and business (Abdel-malak et al., 2017; Chai et al., 2013; Ho et al., 2006). MADM is a very helpful and time- and data-saving procedure.

For DM method, aggregation operators played a significant role. Applying aggregation operators, information is aggregated from multiple input to a single value output. During an aggregation process, there is always a circumstance in which the association between multiple criteria, such as prioritizing, assistance, and consequence on each other, plays a prominent role. For managing this relationship and to integrate it into the DM study, Yager (2008) presented prioritized aggregation operators. Xu and Yager (2010) introduced power and geometric operators and applied them for DM. Some researchers worked on Bonferroni operators and weighted Bonferroni operators for aggregation (Bonferroni, 1950; Kaur & Garg, 2018; Shi & He, 2013). For probabilistic language terms, Liu and Teng (2018) suggested several Muirhead mean operators and applied to MADM challenges. Senapati et al. (2022) presented the idea of Aczel–Alsina operators and produced their purposes. Zhang et al. (2022) established Dombi power Heronian mean aggregation operators for MAGDM. Similarly, many researchers focused on this area and developed aggregation operators (Ashraf et al., 2022; Mahmood et al., 2021; Naz et al., 2022; Tian et al., 2022; Yanhong et al., 2022). Pamucar (2020) and Pamučar et al. (2018) developed normalized weighted geometric Dombi Bonferroni mean (BM) operators. Yahya et al. (2022) analyzed medical diagnosis based on fuzzy credibility Dombi BM operator. Ganguly et al. (2022) discussed a model for plant identification using BM operator. Similar to this, several scholars (Alfaro-Garcia et al., 2022; Asan &

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Soyer, 2022; Deli, 2021; Saha et al., 2021; Tanrıverdi et al., 2022) investigated and grouped their findings using BM aggregation operators, considering the crucial role and significance of these operators.

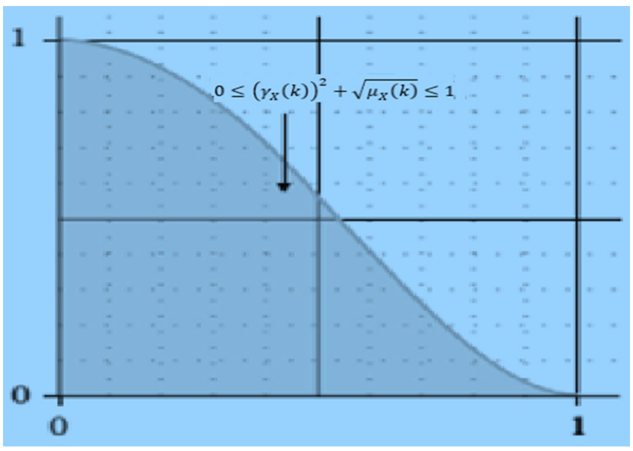
A decision-maker finds it challenging to deal with data ambiguities, and conventional techniques to DM frequently miss the optimum option. Because of this, the researchers have described the data in terms of FS and RS. The idea of FS was presented by Zadeh (1965) to manage the imprecise data, and FSs have proved fruitful for DM. Many researchers produced its application in MADM (Bansal et al., 2022; Rudnik & Franczok, 2022; Ranjbar & Effati, 2022; Zhu et al., 2022). After that, the concept of RS theory was proposed by Pawlak (1982), and FS and RS were applied to different fields (Alsager, 2022; Ayub et al., 2022; He et al., 2022; Shaheen et al., 2020).

Fuzzy sets play a designated role due to membership grade in the course of the DM procedure. But in some cases, the grade of membership is not enough to deal with the situation and overcome the problem for decision. To cope with these kinds of data, Atanassov (1986) worked very hard in this field and developed the theory of intuitionistic fuzzy set (IFS). It is a very significant and interesting expanded form of the FS with its best applicability. In different areas, IFSs are applied such as DM, medical diagnosis, and optimization (Atanassov, 1999; Kozae et al., 2020). In IFSs, it is necessary that the sum of membership grade and non-membership grade should be 1. Consequently, Yager (2013) put forward the idea of Pythagorean fuzzy set (PyFS) which is a simplification of IFS. PyFS is a more effective tool to solve unclear and vagueness difficulties and applied in DM challenges (Akram et al., 2022; Garg et al., 2022; Hussain et al., 2022; Zhang et al., 2017).

It is a trend to expand the ideas and to introduce their generalizations; Ibrahim et al. (2021) characterized a novel expansion of PyFS called (3,2)-fuzzy set. This generalized form can describe more imprecise cases than PyFS, which is very helpful during DM issues. Recently, Al-shami et al. (2022) further generalized IFSs and defined square root fuzzy sets (SR-FSs) and produced its application in MCDM. It can be described from Figure 1 that SR-fuzzy sets are stronger than the IFSs.

Based on the aforementioned literature, we discovered that all of the theories now in use have a broad focus on the extensions of FS and their usage in DM employing aggregation operators. The writing of this research piece was sparked by the following reasons:

Figure 1
Grades space of SR-Fuzzy set



(1) to use Bonferroni operators for DM under the SR-fuzzy information
(2) to propose novel SR-fuzzy Bonferroni mean (SRFBM) operators and weighted SR-fuzzy Bonferroni mean (WSRFBM) operators
(3) to establish a new MADM approach and solve a real-life problem
(4) to analyze the comparison of existing and proposed studies.

The rest of the paper is arranged as follows: Section 2 describes several fundamental definitions related to the SR-fuzzy sets. In Section 3, SR-fuzzy Bonferroni operators and weighted SR-fuzzy Bonferroni operators with their properties are discussed. In Section 4, to address the MADM concerns, a DM approach based on prospective operators has been devised. In Section 5, a mathematical example is also solved to evaluate the proposed approach and to prove its usefulness and efficiency and a comparative analysis with existing studies. Section 6 includes some remarks and conclusions.

2. Preliminaries

This section recalls SR-fuzzy sets introduced by Al-shami et al. (2022). Additionally, some related notions have also been reviewed here.

Definition 1. (Al-shami et al., 2022) Taking U as a ground set, an SR-fuzzy set (SRFS) $\mathcal{T}$ over U is formulated as below.

$$\mathcal{T} = (k, \gamma(k), \mu(k) : k \epsilon \mathrm{U}) \quad (1)$$

where $\gamma : U \to [0,1]$ and $\mu : U \to [0,1]$ are, respectively, the mappings dedicated to assign the grades of membership and non-membership such that

$$0 \le (\gamma_X(k))^2 + \sqrt{\mu_X(k)} \le 1 \text{ for all } k \epsilon \mathrm{U}$$

For ease of calculations, we call $\mathcal{T} = (\gamma, \mu)$ an SR-fuzzy number (SRFN) satisfying $\gamma^2 + \sqrt{\mu} \le 1$, where γ and μ are chosen from the unit closed interval $[0,1]$.

To integrate SRFNs, Shaheen et al. (2020) proposed the following operators.

Definition 2. Let $\mathcal{T}_1 = (\gamma_1, \mu_1)$ and $\mathcal{T}_2 = (\gamma_2, \mu_2)$ be two SRFNs. Then

(i) $\mathcal{T}_1 \cup \mathcal{T}_2 = (sup\{\gamma_1, \gamma_2\}, \inf\{\mu_1, \mu_2\})$
(ii) $\mathcal{T}_1 \cap \mathcal{T}_2 = (inf\{\gamma_1, \gamma_2\}, sup\{\mu_1, \mu_2\})$
(iii) $\mathcal{T}_1^c = (\sqrt[]{[4]}\gamma_1, (\mu_1)^4)$

Definition 3. For SRFNs $\mathcal{T} = (\gamma, \mu), \mathcal{T}_1 = (\gamma_1, \mu_1)$ and $\mathcal{T}_2 = (\gamma_2, \mu_2)$, following operators have been defined.

(i) $\mathcal{T}_1 \oplus \mathcal{T}_2 = \left(\sqrt{1 - (1-\gamma_1^2)(1-\gamma_2^2)}, \mu_1\mu_2\right)$,
(ii) $\mathcal{T}_1 \otimes \mathcal{T}_2 = \left(\gamma_1\gamma_2, \left(1 - \left(1 - \sqrt{\mu_1}\right)\left(1 - \sqrt{\mu_2}\right)\right)^2\right)$,
(iii) $\lambda\mathcal{T} = \left(\sqrt{1 - (1-\gamma)^\lambda}, \mu^\lambda\right)$
(iv) $\mathcal{T}^\lambda = \left(\gamma^\lambda, \left(1 - \left(1 - \sqrt{\mu}\right)^\lambda\right)^2\right)$,

Following the convention, Shaheen et al. (2020) designed the score function and accuracy function for the new model as below.

Definition 4. The score function to rank the SRFNs $\mathcal{T}_i = (\gamma_i, \mu_i)$ is defined as

$$V(\mathcal{T}_i) = \left(\gamma_i^2 - \sqrt{\mu_i}\right) \tag{1}$$

Also, an accuracy function is defined as

$$W(\mathcal{T}_i) = \left(\gamma_i^2 + \sqrt{\mu_i}\right) \tag{2}$$

It is evident that $-1 \leq V(\mathcal{T}_i) \leq 1$ and $0 \leq W(\mathcal{T}_i) \leq 1$.

The following comparison is proposed by Al-shami et al. (2022) to compare two SRFNs, particularly in the field of MADM problems.

Definition 5. For any two SRFNs $\mathcal{T}_1$ and $\mathcal{T}_2$, the following rule of comparison has been established.

(i) If $V(\mathcal{T}_1) > V(\mathcal{T}_2)$, then $\mathcal{T}_1$ is superior to $\mathcal{T}_2$ and is represented by $\mathcal{T}_1 \succ \mathcal{T}_2$.
(ii) If $V(\mathcal{T}_1) = V(\mathcal{T}_2)$, then
  (a) if $W(\mathcal{T}_1) > W(\mathcal{T}_2)$, then $\mathcal{T}_1 \succ \mathcal{T}_2$
  (b) if $W(\mathcal{T}_1) = W(\mathcal{T}_2)$, then $\mathcal{T}_1 \sim \mathcal{T}_2$, where $\sim$ represent "equivalent to."

3. SR-Fuzzy Sets and Bonferroni Operators

To extend and explore the utility of SRFSs, we define a few basic operational laws between the SRFNs. Additionally, a series of BM operators have been proposed in this section.

3.1 SRFBM operators

Bonferroni initiated the BM operator (Bonferroni, 1950). The important feature of the BM operator is that it can manage the connection and correlation between the aggregation elements.

Definition 6. (Xu & Yager, 2010) For $c, d \geq 0$ with $\alpha_i (i \in n)$ be the family of non-negative numbers. If

$$BM^{c,d}(\alpha_1, \alpha_2, \ldots, \alpha_n) = \left(\frac{1}{n(n-1)} \sum_{\substack{i,j=1 \\ i \neq j}}^{n} \alpha_i{}^c \alpha_j{}^d\right)^{\frac{1}{c+d}} \tag{3}$$

then $BM^{c,d}$ is known as the BM operator.

Definition 7. An SRFBM operator is a mapping SRFBM: $\mathcal{T}^n \rightarrow \mathcal{T}$ defined on a set of SRFSs $\mathcal{T}_i$ and presented by

$$\mathrm{SRF}BM^{c,d}(\mathcal{T}_1, \mathcal{T}_2, \ldots, \mathcal{T}_n) = \left(\frac{1}{n(n-1)}\left(\bigoplus_{\substack{i,j=1 \\ i \neq j}}^{n} \left(\mathcal{T}_i{}^c \otimes \mathcal{T}_j{}^d\right)\right)\right)^{\frac{1}{c+d}} \tag{4}$$

where $c, d > 0$ are the real numbers.

Theorem 1. The aggregated result applying SRFBM operator for SRFNs $\mathcal{T}_i = (\gamma_i, \mu_i)$ is again an SRFN and is given by

$$\mathrm{SRF}BM^{c,d}(\mathcal{T}_1, \mathcal{T}_2, \ldots, \mathcal{T}_n) = \left\{ \begin{array}{l} \left(1 - \prod_{\substack{i,j=1 \\ i \neq j}}^{n} \left(1 - (\mu_i)^{2c}(\mu_j)^{2d}\right)^{\frac{1}{n(n-1)}}\right)^{\frac{1}{2(c+d)}}, \\ \left(\left(1 - \left(1 - \prod_{\substack{i,j=1 \\ i \neq j}}^{n} \left(1 - \left(1 - \sqrt{\gamma_i}\right)^c \left(1 - \sqrt{\gamma_j}\right)^d\right)^{\frac{1}{n(n-1)}}\right)^{\frac{1}{c+d}}\right)\right)^2 \end{array} \right\} \tag{5}$$

Proof. From Definition 3, a collection of SRFNs $\mathcal{T}_I$, $\mathcal{T}_j$ is written as

$$\mathcal{T}_i^c = \left(\gamma_i^c, \left(1 - \left(1 - \sqrt{\mu_i}\right)^c\right)^2\right) \tag{6}$$

and

$$\mathcal{T}_j^d = \left(\gamma_j^d, \left(1 - \left(1 - \sqrt{\mu_j}\right)^d\right)^2\right) \tag{7}$$

where c, d are two positive real numbers

Therefore,

$$\mathcal{T}_i^c \otimes \mathcal{T}_j^d = \left(\gamma_i^c \gamma_j^d, \left(1 - \left(1 - \sqrt{\mu_i}\right)^c \left(1 - \sqrt{\mu_j}\right)^d\right)^2\right) \tag{8}$$

Firstly, we prove

$$\bigoplus_{\substack{i,j=1 \\ i \neq j}}^{n} \left(\mathcal{T}_i^c \otimes \mathcal{T}_j^d\right) = \left\{ \sqrt{1 - \prod_{\substack{i,j=1 \\ i \neq j}}^{n} \left(1 - (\mu_i)^{2c}(\mu_j)^{2d}\right)}, \left(\prod_{\substack{i,j=1 \\ i \neq j}}^{n} \left(1 - \left(1 - \sqrt{\gamma_i}\right)^c \left(1 - \sqrt{\gamma_j}\right)^d\right)\right)^2 \right\} \tag{9}$$

using induction method on n.

For $n = 2$, it is obtained,

$$\bigoplus_{\substack{i,j=1 \\ i \neq j}}^{2} \left(\mathcal{T}_i^c \otimes \mathcal{T}_j^d\right) = \left\{ \sqrt{1 - \prod_{\substack{i,j=1 \\ i \neq j}}^{2} \left(1 - (\mu_i)^{2c}(\mu_j)^{2d}\right)}, \left(\prod_{\substack{i,j=1 \\ i \neq j}}^{2} \left(1 - \left(1 - \sqrt{\gamma_i}\right)^c \left(1 - \sqrt{\gamma_j}\right)^d\right)\right)^2 \right\} \tag{10}$$

So, it satisfies for $n = 2$. Suppose result is true for $n = k$, i.e.,

$$\underset{\substack{i,j=1\\ i\neq j}}{\overset{k}{\oplus}} \left(\mathcal{T}_i^{c} \otimes \mathcal{T}_j^{d}\right) = \left\{ \sqrt{1 - \prod_{\substack{i,j=1\\ i\neq j}}^{k} \left(1 - (\mu_i)^{2c}(\mu_j)^{2d}\right)}, \left(\prod_{\substack{i,j=1\\ i\neq j}}^{k} \left(1 - \left(1 - \sqrt{\gamma_i}\right)^{c}\left(1 - \sqrt{\gamma_j}\right)^{d}\right) \right)^2 \right\} \tag{11}$$

Now, for $n = k + 1$, we have

$$\underset{\substack{i,j=1\\ i\neq j}}{\overset{k+1}{\oplus}} \left(\mathcal{T}_i^{c} \otimes \mathcal{T}_j^{d}\right) = \left(\underset{\substack{i,j=1\\ i\neq j}}{\overset{k}{\oplus}} \left(\mathcal{T}_i^{c} \otimes \mathcal{T}_j^{d}\right) \right) \oplus \left(\underset{i=1}{\overset{k}{\oplus}} \left(\mathcal{T}_i^{c} \otimes \mathcal{T}_{k+1}^{d}\right) \right) \oplus \left(\underset{j=1}{\overset{k}{\oplus}} \left(\mathcal{T}_{k+1}^{c} \otimes \mathcal{T}_j^{d}\right) \right) \tag{12}$$

Now, we shall prove

$$\underset{i=1}{\overset{k}{\oplus}} \left(\mathcal{T}_i^{c} \otimes \mathcal{T}_{k+1}^{d}\right) = \left\{ \begin{array}{c} \sqrt{1 - \prod_{\substack{i,j=1\\ i\neq j}}^{2} \left(1 - (\mu_i)^{2c}(\mu_{k+1})^{2d}\right)}, \\ \left(\prod_{\substack{i,j=1\\ i\neq j}}^{2} \left(1 - \left(1 - \sqrt{\gamma_i}\right)^{c}\left(1 - \sqrt{\gamma_{k+1}}\right)^{d}\right) \right)^2 \end{array} \right\} \tag{13}$$

Again for $k = 2$, using Equation (8), we have

$$\mathcal{T}_i^{c} \otimes \mathcal{T}_{2+1}^{d} = \left\{ \sqrt{1 - \left(1 - (\mu_i)^{2c}(\mu_{2+1})^{2d}\right)}, \left(\left(1 - \left(1 - \sqrt{\gamma_i}\right)^{c}\left(1 - \sqrt{\gamma_{2+1}}\right)^{d}\right)\right)^2 \right\} \tag{14}$$

And thus,

$$\underset{i=1}{\overset{2}{\oplus}} \left(\mathcal{T}_i^{c} \otimes \mathcal{T}_{2+1}^{d}\right) = \left(\mathcal{T}_i^{c} \otimes \mathcal{T}_{2+1}^{d}\right) \oplus \left(\mathcal{T}_2^{c} \otimes \mathcal{T}_{2+1}^{d}\right) = \left\{ \sqrt{1 - \prod_{\substack{i,j=1\\ i\neq j}}^{2} \left(1 - (\mu_i)^{2c}(\mu_3)^{2d}\right)}, \left(\prod_{\substack{i,j=1\\ i\neq j}}^{2} \left(1 - \left(1 - \sqrt{\gamma_i}\right)^{c}\left(1 - \sqrt{\gamma_3}\right)^{d}\right) \right)^2 \right\} \tag{15}$$

If Equation (13) holds for $k = k_0$, i.e.,

$$\underset{i=1}{\overset{k_0}{\oplus}} \left(\mathcal{T}_i^{c} \otimes \mathcal{T}_{k_0+1}^{d}\right) = \left\{ \begin{array}{c} \sqrt{1 - \prod_{\substack{i,j=1\\ i\neq j}}^{k_0} \left(1 - (\mu_i)^{2c}\left(\mu_{k_0+1}\right)^{2d}\right)}, \\ \left(\prod_{\substack{i,j=1\\ i\neq j}}^{k_0} \left(1 - \left(1 - \sqrt{\gamma_i}\right)^{c}\left(1 - \sqrt{\gamma_{k_0+1}}\right)^{d}\right) \right)^2 \end{array} \right\} \tag{16}$$

then, for $k = k_0 + 1$ using Definition (12), we have

$$\underset{i=1}{\overset{k_0+1}{\oplus}} \left(\mathcal{T}_i^{c} \otimes \mathcal{T}_{k_0+2}^{d}\right) = \underset{i=1}{\overset{k_0}{\oplus}} \left(\mathcal{T}_i^{c} \otimes \mathcal{T}_{k_0+2}^{d}\right) \oplus \left(\mathcal{T}_{k_0+1}^{c} \otimes \mathcal{T}_{k_0+2}^{d}\right) = \left\{ \begin{array}{c} \sqrt{1 - \prod_{\substack{i,j=1\\ i\neq j}}^{k_0+1} \left(1 - (\mu_i)^{2c}\left(\mu_{k_0+2}\right)^{2d}\right)}, \\ \left(\prod_{\substack{i,j=1\\ i\neq j}}^{k_0+1} \left(1 - \left(1 - \sqrt{\gamma_i}\right)^{c}\left(1 - \sqrt{\gamma_{k_0+2}}\right)^{d}\right) \right)^2 \end{array} \right\} \tag{17}$$

And therefore Equation (13) holds for $k = k_0 + 1$. Hence, it is true for each k. Likewise,

$$\overset{k}{\underset{j=1}{\oplus}}\left(\mathcal{T}_{k+1}^{c}\otimes\mathcal{T}_{j}^{d}\right)=\left\{\begin{array}{c}\sqrt{1-\prod_{\substack{i,j=1\\ i\neq j}}^{k}\left(1-(\mu_{k+1})^{2c}(\mu_j)^{2d}\right)},\\ \left(\prod_{\substack{i,j=1\\ i\neq j}}^{k}\left(1-\left(1-\sqrt{\gamma_{k+1}}\right)^{c}\left(1-\sqrt{\gamma_j}\right)^{d}\right)\right)^{2}\end{array}\right\} \tag{18}$$

Therefore, by using Equations (11), (13), and (18), Equation (12) becomes

$$\overset{k}{\underset{\substack{i,j=1\\ i\neq j}}{\oplus}}\left(\mathcal{T}_{i}^{c}\otimes\mathcal{T}_{j}^{d}\right)=\left\{\begin{array}{c}\sqrt{1-\prod_{\substack{i,j=1\\ i\neq j}}^{k}\left(1-(\mu_i)^{2c}(\mu_j)^{2d}\right)},\\ \left(\prod_{\substack{i,j=1\\ i\neq j}}^{k}\left(1-\left(1-\sqrt{\gamma_i}\right)^{c}\left(1-\sqrt{\gamma_j}\right)^{d}\right)\right)^{2}\end{array}\right\}\oplus\left\{\begin{array}{c}\sqrt{1-\prod_{\substack{i,j=1\\ i\neq j}}^{k}\left(1-(\mu_i)^{2c}(\mu_{k+1})^{2d}\right)},\\ \left(\prod_{\substack{i,j=1\\ i\neq j}}^{k}\left(1-\left(1-\sqrt{\gamma_i}\right)^{c}\left(1-\sqrt{\gamma_{k+1}}\right)^{d}\right)\right)^{2}\end{array}\right\}$$

$$\oplus\left\{\sqrt{1-\prod_{\substack{i,j=1\\ i\neq j}}^{k}\left(1-(\mu_{K+1})^{2c}(\mu_j)^{2d}\right)},\left(\prod_{\substack{i,j=1\\ i\neq j}}^{k}\left(1-\left(1-\sqrt{\gamma_{k+1}}\right)^{c}\left(1-\sqrt{\gamma_j}\right)^{d}\right)\right)^{2}\right\}$$

$$=\left\{\sqrt{1-\prod_{\substack{i,j=1\\ i\neq j}}^{k+1}\left(1-(\mu_i)^{2c}(\mu_j)^{2d}\right)},\left(\prod_{\substack{i,j=1\\ i\neq j}}^{k+1}\left(1-\left(1-\sqrt{\gamma_i}\right)^{c}\left(1-\sqrt{\gamma_j}\right)^{d}\right)\right)^{2}\right\}$$

Equation (13) valid for all positive integers n by the principle of mathematical induction since it is true for $n = k + 1$.

Now,

$$\frac{1}{n(n-1)}\overset{n}{\underset{\substack{i,j=1\\ i\neq j}}{\oplus}}\left(\mathcal{T}_{i}^{c}\otimes\mathcal{T}_{j}^{d}\right)=\left\{\begin{array}{c}\sqrt{1-\left(\prod_{\substack{i,j=1\\ i\neq j}}^{n}\left(1-(\mu_i)^{2c}(\mu_j)^{2d}\right)\right)^{\frac{1}{n(n-1)}}},\\ \left(\prod_{\substack{i,j=1\\ i\neq j}}^{n}\left(1-\left(1-\sqrt{\gamma_i}\right)^{c}\left(1-\sqrt{\gamma_j}\right)^{d}\right)\right)^{\frac{2}{n(n-1)}}\end{array}\right\} \tag{19}$$

So, by definition of SRFBM, we get

$$\mathrm{SRF}BM^{c,d}(\mathcal{T}_1,\mathcal{T}_2,\ldots,\mathcal{T}_\mathrm{n})=\left(\frac{1}{n(n-1)}\left(\bigoplus_{\substack{i,j=1\\ i\neq j}}^{n}\left(\mathcal{T}_\mathrm{i}^{c}\otimes\mathcal{T}_\mathrm{j}^{d}\right)\right)\right)^{\frac{1}{c+d}}=\left\{\begin{array}{c}\left(1-\left(\prod_{\substack{i,j=1\\ i\neq j}}^{n}\left(\left(1-(\mu_i)^{2c}\left(\mu_j\right)^{2d}\right)\right)^{\frac{1}{n(n-1)}}\right)\right)^{\frac{1}{2(c+d)}},\\ \left(1-\left(1-\prod_{\substack{i,j=1\\ i\neq j}}^{n}\left(1-(1-\sqrt{\gamma_i})^{c}\left(1-\sqrt{\gamma_j}\right)^{d}\right)^{\frac{1}{n(n-1)}}\right)^{\frac{1}{c+d}}\right)^{2}\end{array}\right\} \tag{20}$$

Hence, the result.

From the SRFBM operator, it has been noted that they satisfy specific properties for a group of SRFNs $\mathcal{T}_i$. These noteworthy properties are stated below.

Property 1. (Idempotency) If $\mathcal{T}_i=\mathcal{T}$ for all i, then SRFBM fulfills

$$SRFBM^{c,d}(\mathcal{T}_1,\mathcal{T}_2,\ldots,\mathcal{T}_\mathrm{n})=\mathcal{T}$$

Property 2. (Monotonicity) Let $\mathcal{T}_i=\left(\gamma_{\mathcal{T}_i},\mu_{\mathcal{T}_i}\right)$ and $\alpha_i=\left(\gamma_{\alpha_i},\mu_{\alpha_i}\right)$ be two SRFNs such that $\gamma_{\mathcal{T}_i}\leq\gamma_{\alpha_i}$ and $\mu_{\mathcal{T}_i}\leq\mu_{\alpha_i}$. Then

$$SRFBM^{c,}(\mathcal{T}_1,\mathcal{T}_2,\ldots,\mathcal{T}_n)\leq SRFBM^{c,d}(\alpha_1,\alpha_2,\ldots,\alpha_n)$$

Property 3. (Commutativity) Let $\left(\mathcal{T}_1^{\prime},\mathcal{T}_2^{\prime},\ldots,\mathcal{T}_n^{\prime}\right)$ be any permutation of SRFNs $(\mathcal{T}_1,\mathcal{T}_2,\ldots,\mathcal{T}_n)$. Then

$$SRFBM^{c,d}(\mathcal{T}_1,\mathcal{T}_2,\ldots,\mathcal{T}_n)=SRFBM^{c,d}\left(\mathcal{T}_1^{\prime},\mathcal{T}_2^{\prime},\ldots,\mathcal{T}_n^{\prime}\right).$$

Property 4. (Boundedness) Let $\mathcal{T}^-=(\gamma_{min},\mu_{max})$, $\mathcal{T}^+=(\gamma_{max},\mu_{min})$ be the lower and upper bounds for the family of SRFNs $\mathcal{T}_i$. Then

$$\mathcal{T}^-\leq\ SRFBM^{c,d}(\mathcal{T}_1,\mathcal{T}_2,\ldots,\mathcal{T}_n)\leq\mathcal{T}^+.$$

3.2 Weighted BM operator of SR-fuzzy sets

Definition 8. For SRFNs $\mathcal{T}_\mathrm{i}$(i=1,2,...,n) and weight vector $e=(e_1,e_2,\ldots,e_i)^T$ such that each $e_i>0$ and $\sum_{i=1}^{n}e_i=1$, a WSRFBM defined over SRFNs $\mathcal{T}$ as $WSRFBM:\mathcal{T}^n\rightarrow\mathcal{T}$ is given by

$$SRFBM^{c,d}(\mathcal{T}_1,\mathcal{T}_2,\ldots,\mathcal{T}_\mathrm{n})=\left(\frac{1}{n(n-1)}\left(\bigoplus_{\substack{i,j=1\\ i\neq j}}^{n}\left((e_i\mathcal{T}_\mathrm{i})^{c}\otimes\left(e_j\mathcal{T}_\mathrm{j}\right)^{d}\right)\right)\right)^{\frac{1}{c+d}} \tag{21}$$

where c and d are positive real numbers.

Theorem 2. The aggregated result applying WSRFBM operator for the family of SRFNs $\mathcal{T}_\mathrm{i}=(\gamma_i,\mu_i)$ is still SRFN and can be represented as

$$W\mathrm{SRF}BM_e^{c,d}(\mathcal{T}_1,\mathcal{T}_2,\ldots,\mathcal{T}_\mathrm{n})=(\gamma,\mu) \tag{22}$$

where

$$\gamma=\left(1-\prod_{\substack{i,j=1\\ i\neq j}}^{n}\left(1-(1-(1-\mu_i^2)^{e_i})^{c}\left(1-\left(1-\mu_j^2\right)^{e_j}\right)^{d}\right)^{\frac{1}{n(n-1)}}\right)^{\frac{1}{2(c+d)}}$$

$$\mu=\left(\left(1-\prod_{\substack{i,j=1\\ i\neq j}}^{n}\left(1-\left(1-\sqrt{\gamma_i^{e_i}}\right)^{c}\left(1-\sqrt{\gamma_j^{e_j}}\right)^{d}\right)^{\frac{1}{n(n-1)}}\right)^{\frac{1}{c+d}}\right)^{2}$$

and $e=(e_1,e_2,\ldots,e_n)^T$ is the associated weight vector, where $e_i>0$ and $\sum_{i=1}^{n}e_i=1$.

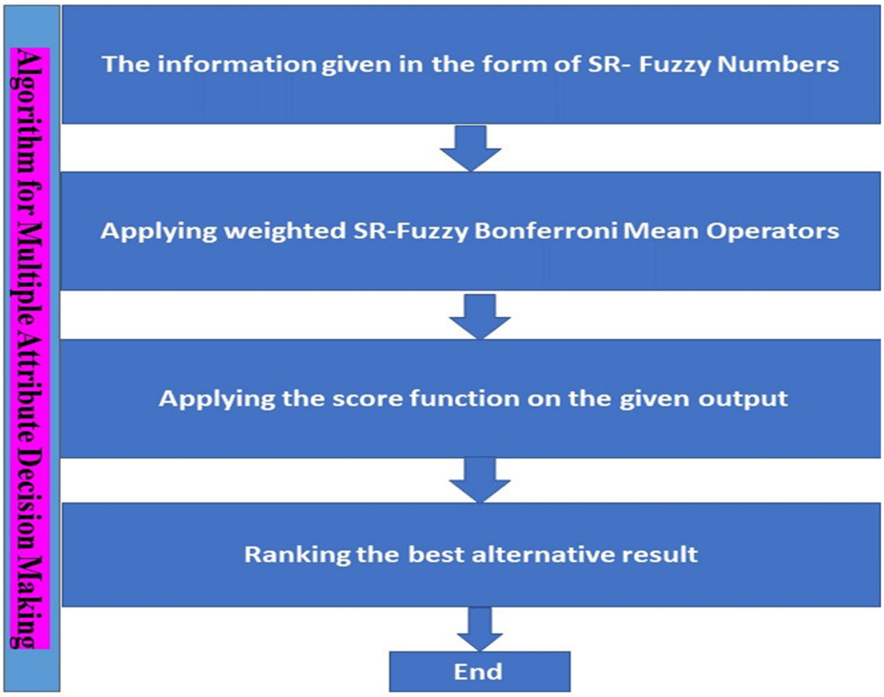
Proof. We ignore the proof because it is similar to Theorem 1.

4. Proposed DM Approach Based on SRFBM Operator

In this portion, to solve the MADM under the SR-FSs, we will use the proposed SRFBM aggregation operators. The MADM algorithm has been designed using the following notations:

Let $B=\{B_1,B_2,\ldots,B_m\}$ be the collection of *mi* distinct alternatives which must be examined under the collection of $'n'$ several criteria $\mathcal{C}=\{\mathcal{C}_1,\mathcal{C}_2,\ldots,\mathcal{C}_n\}$. Consider that these alternatives are examined using an expert who expresses his/her preferences in relation to each alternative $B_i(i\in m)$ under the SR-fuzzy data, and these statistics may be recognized as SRFSs $\mathcal{D}=\left(\mathcal{T}_{ij}\right)_{m\times n}$ such that $\mathcal{T}_{ij}=\left(\gamma_{ij},\mu_{ij}\right)$ shows the priority values of alternative $\mathcal{B}_i$ given by decision-maker. Let $e=(e_1,e_2,\ldots,e_n)^T$ be the weight vector of the criteria such that $e_i>0$ and $\sum_{i=1}^{n}e_i=1$. The suggested strategy has been divided into the following steps in order to determine the optimal alternative(s) and Figure 2 represents the algorithm step by step.

Figure 2
Representation of the algorithm



Step 1: Obtain information on alternative ratings that are related to criteria and describe it in the form of SRFS $\mathcal{T}_{ij} = (\gamma_{ij}, \mu_{ij});\ i = 1, 2, \ldots, m; j = 1, 2, \ldots, n$. These rating *i* results are stated as a decision matrix $\mathcal{D}$ as

$$\mathcal{D} = \begin{matrix} \\ \mathcal{B}_1 \\ \mathcal{B}_2 \\ \vdots \\ \mathcal{B}_m \end{matrix} \begin{matrix} \begin{matrix} \mathcal{C}_1 & \mathcal{C}_2 & \cdots & \mathcal{C}_n \end{matrix} \\ \begin{pmatrix} \mathcal{T}_{11} & \mathcal{T}_{12} & \cdots & \mathcal{T}_{12} \\ \mathcal{T}_{21} & \mathcal{T}_{22} & & \mathcal{T}_{2n} \\ \vdots & \ddots & \ddots & \vdots \\ \mathcal{T}_{m1} & \mathcal{T}_{m2} & \cdots & \mathcal{T}_{mn} \end{pmatrix} \end{matrix}$$

Step 2: Aggregate the various preference results $\breve{\Upsilon}_{ij}, j = 1, 2, \ldots, n$ of the alternatives $\mathcal{B}_i$ into the collective one $\breve{\Upsilon}_i$, using WSRFBM operators as

$$\breve{\Upsilon}_i = (\gamma_{ij}, \mu_{ij}) = WCSRFBM^{c,d}\left(\breve{\Upsilon}_1, \breve{\Upsilon}_2, \ldots, \breve{\Upsilon}_n\right)$$

$$\gamma_{ij} = \left(1 - \prod_{\substack{i,j=1 \\ i \neq j}}^{n} \left(1 - (1 - (1 - \mu_i^2)^{e_i})^c \left(1 - \left(1 - \mu_j^2\right)^{e_j}\right)^d\right)^{\frac{1}{n(n-1)}}\right)^{\frac{1}{2(c+d)}}$$

$$\mu_{ij} = \left(\left(1 - \prod_{\substack{i,j=1 \\ i \neq j}}^{n} \left(1 - \left(1 - \sqrt{\gamma_i^{e_i}}\right)^c \left(1 - \sqrt{\gamma_j^{e_j}}\right)^d\right)^{\frac{1}{n(n-1)}}\right)^{\frac{1}{c+d}}\right)^2$$

Step 3: Aggregate the score *i* value of the aggregated SRFNs $\breve{\Upsilon}_i$ applying Equation (2) as

$$V(\mathcal{T}_i) = \left(\gamma_i{}^2 - \sqrt{\mu_i}\right) \tag{23}$$

5. Illustrative Example

To illustrate the practical use of the suggested methodology, a numerical example has been given below.

The number of populations is increasing daily, and due to this factor, we face many challenges. One of the significant resulting issues is the traffic problem, which is also Pakistan's central issue. Therefore, to overcome this challenge in the town, the development authority decided to enlarge the roads and construct some new service roads, which will be helpful during rush hours. For this project, four international companies (B_i), namely "Khan Construction Service (B_1)," "WAUZ Engineers and Constructors (B_2)," "Aldean International Ltd. (B_3)," and "Faizan West Construction limited (B_4)," took interest. The three criteria—"Project Cost," "Completion Time," and "Business Status"—were configured and their respective weights were assigned by authority in order to determine which firm would be the best choice. The weight vector for it has been created based on the preferences of the DMs. The assignment's main goal is to select the top project business. The proposed technique's steps are carried out exactly as stated.

Step 1: The specialist has graded all alternatives for the distinct criteria based on the SRFNs as given in Table 1.

Table 1
SR-fuzzy information set

Alternatives	$\mathcal{C}_1$	$\mathcal{C}_2$	$\mathcal{C}_3$
B_1	(0.53, 0.49)	(0.52, 0.51)	(0.26, 0.76)
B_2	(0.51, 0.53)	(0.5, 0.54)	(0.3, 0.6)
B_3	(0.7, 0.2)	(0.8, 0.1)	(0.4, 0.7)
B_4	(1.0, 0.0)	(0.6, 0.4)	(0.5, 0.5)

Step 2: The aggregated values of the SRFNs by using WSRFBM operators are presented in Tables 2, 3, 4, and 5.

Table 2
Aggregated values using weighted SR-fuzzy Bonferroni mean operators

c	d	$\mathcal{T}_1$	$\mathcal{T}_2$	$\mathcal{T}_3$	$\mathcal{T}_4$
$c=1$	$d=1$	(0.1909, 0.0259)	(0.1961, 0.0285)	(0.3713, 0.0523)	(0.0, 0.0505)
	$d=2$	(0.2152, 0.0165)	(0.2069, 0.0197)	(0.4001, 0.0530)	(0.0, 0.0367)
	$d=3$	(0.2315, 0.0135)	(0.2129, 0.0172)	(0.4203, 0.0562)	(0.0, 0.0320)
	$d=4$	(0.2431, 0.0121)	(0.2169, 0.0162)	(0.4352, 0.0596)	(0.0, 0.0297)

Table 3
Aggregated values using weighted SR-fuzzy Bonferroni mean operators

c	d	$\mathcal{T}_1$	$\mathcal{T}_2$	$\mathcal{T}_3$	$\mathcal{T}_4$
$c=2$	$d=1$	(0.2095, 0.0916)	(0.2237, 0.0989)	(0.4181, 0.1718)	(0.0, 0.1366)
	$d=2$	(0.2240, 0.0476)	(0.2254, 0.0549)	(0.4299, 0.1278)	(0.0, 0.0838)
	$d=3$	(0.2356, 0.0328)	(0.2268, 0.0402)	(0.4410, 0.1122)	(0.0, 0.0637)
	$d=4$	(0.2446, 0.0259)	(0.2279, 0.0332)	(0.4503, 0.1049)	(0.0, 0.0534)

Table 4
Aggregated values using weighted SR-fuzzy Bonferroni mean operators

c	d	$\mathcal{T}_1$	$\mathcal{T}_2$	$\mathcal{T}_3$	$\mathcal{T}_4$
$c=3$	$d=1$	(0.2196, 0.1669)	(0.2389, 0.1770)	(0.4444, 0.2787)	(0.0, 0.2247)
	$d=2$	(0.2294, 0.0877)	(0.2374, 0.0984)	(0.4490, 0.1993)	(0.0, 0.1376)
	$d=3$	(0.2383, 0.0581)	(0.2366, 0.0688)	(0.4553, 0.1659)	(0.0, 0.1008)
	$d=4$	(0.2458, 0.0437)	(0.2362, 0.0541)	(0.4615, 0.1481)	(0.0, 0.0811)

Table 5
Aggregated values using weighted SR-fuzzy Bonferroni mean operators

c	d	$\mathcal{T}_1$	$\mathcal{T}_2$	$\mathcal{T}_3$	$\mathcal{T}_4$
$c=4$	$d=1$	(0.2258, 0.2388)	(0.2486, 0.2504)	(0.4611, 0.3638)	(0.0, 0.3028)
	$d=2$	(0.2331, 0.1316)	(0.2457, 0.1449)	(0.4623, 0.2630)	(0.0, 0.1915)
	$d=3$	(0.2403, 0.0873)	(0.2438, 0.1009)	(0.4659, 0.2160)	(0.0, 0.1399)
	$d=4$	(0.2466, 0.0646)	(0.2426, 0.0779)	(0.4701, 0.1892)	(0.0, 0.1111)

Step 3: Variation of the parameters which has an effect on the ranking c and d using score function is described in Table 6.

Table 6
Ranking results after applying score function

c	d	$V(\mathcal{T}_1)$	$V(\mathcal{T}_2)$	$V(\mathcal{T}_3)$	$V(\mathcal{T}_4)$	Ranking
$c=1$	$d=1$	-0.1245	-0.1304	-0.0908	-0.2247	$\mathcal{T}_3 > \mathcal{T}_1 > \mathcal{T}_2 > \mathcal{T}_4$
	$d=2$	-0.0821	-0.0975	-0.0701	-0.1916	$\mathcal{T}_3 > \mathcal{T}_1 > \mathcal{T}_2 > \mathcal{T}_4$
	$d=3$	-0.0626	-0.0858	-0.0604	-0.1789	$\mathcal{T}_3 > \mathcal{T}_1 > \mathcal{T}_2 > \mathcal{T}_4$
	$d=4$	-0.0509	-0.0802	-0.0547	-0.1723	$\mathcal{T}_1 > \mathcal{T}_3 > \mathcal{T}_2 > \mathcal{T}_4$
$c=2$	$d=1$	-0.2588	-0.2644	-0.2397	-0.3696	$\mathcal{T}_3 > \mathcal{T}_1 > \mathcal{T}_2 > \mathcal{T}_4$
	$d=2$	-0.1680	-0.1835	-0.1727	-0.3696	$\mathcal{T}_1 > \mathcal{T}_3 > \mathcal{T}_2 > \mathcal{T}_{i4}$
	$d=3$	-0.1256	-0.1491	-0.1405	-0.2524	$\mathcal{T}_1 > \mathcal{T}_3 > \mathcal{T}_2 > \mathcal{T}_4$
	$d=4$	-0.1011	-0.1303	-0.1211	-0.2311	$\mathcal{T}_1 > \mathcal{T}_3 > \mathcal{T}_2 > \mathcal{T}_4$
$c=3$	$d=1$	-0.3603	-0.3636	-0.3304	-0.4740	$\mathcal{T}_3 > \mathcal{T}_1 > \mathcal{T}_2 > \mathcal{T}_4$
	$d=2$	-0.2435	-0.2575	-0.2448	-0.3709	$\mathcal{T}_1 > \mathcal{T}_i > \mathcal{T}_2 > \mathcal{T}_4$
	$d=3$	-0.1843	-0.2063	-0.2000	-0.3175	$\mathcal{T}_1 > \mathcal{T}_3 > \mathcal{T}_2 > \mathcal{T}_4$
	$d=4$	-0.1486	-0.1768	-0.1719	-0.2848	$\mathcal{T}_1 > \mathcal{T}_3 > \mathcal{T}_2 > \mathcal{T}_4$
$c=4$	$d=1$	-0.4377	-0.4386	-0.3905	-0.5503	$\mathcal{T}_3 > \mathcal{T}_1 > \mathcal{T}_2 > \mathcal{T}_4$
	$d=2$	-0.3084	-0.3203	-0.2991	-0.4376	$\mathcal{T}_3 > \mathcal{T}_1 > \mathcal{T}_2 > \mathcal{T}_4$
	$d=3$	-0.2377	-0.2582	-0.2477	-0.3740	$\mathcal{T}_1 > \mathcal{T}_3 > \mathcal{T}_2 > \mathcal{T}_4$
	$d=4$	-0.1934	-0.2203	-0.2140	-0.3333	$\mathcal{T}_1 > \mathcal{T}_3 > \mathcal{T}_2 > \mathcal{T}_4$

For aggregating the results, the parameters play very important roles on the above-mentioned results as well as figures, which can be easily observed. In terms of the parameters c and d, the suggested aggregation operators are symmetrical. However, in order to determine the impact of these characteristics on the final ranking of the alternatives, more research is required, and inquiry was conducted by modifying them all at the same time, and the results are described in Table 6. The variation of parameters values shows the impact on the alternative's rankings. The score amounts of the aggregated statistics are different when various pairs of the parameters c and d are assigned; nevertheless, the ranking orders of the alternatives stay the same.

Figures 3, 4, 5, and 6 represents the as variation of parameter c and d is happened then alternative 1 and alternative 3 ranked first. Similarly, the same situation is found among the variation of parameter. Moreover, it can be observed that the alternatives $\mathcal{T}_2$ *and* $\mathcal{T}_4$ remain worst during the variation of parameters, and the alternatives $\mathcal{T}_1$ and $\mathcal{T}_3$ move during the variation of parameters. This property of the suggested operators is increasingly important in real-world DM situations. For example, it has been seen that as the parameters are increased, the score values of the alternative grow, giving us an optimistic impression of the decision-makers. As a result, if the decision-makers are optimistic, the outcome will be positive. Moreover, to analyze the suggested approach deeply,

Figure 3
Graphical representation $c=1$ and $c=2$

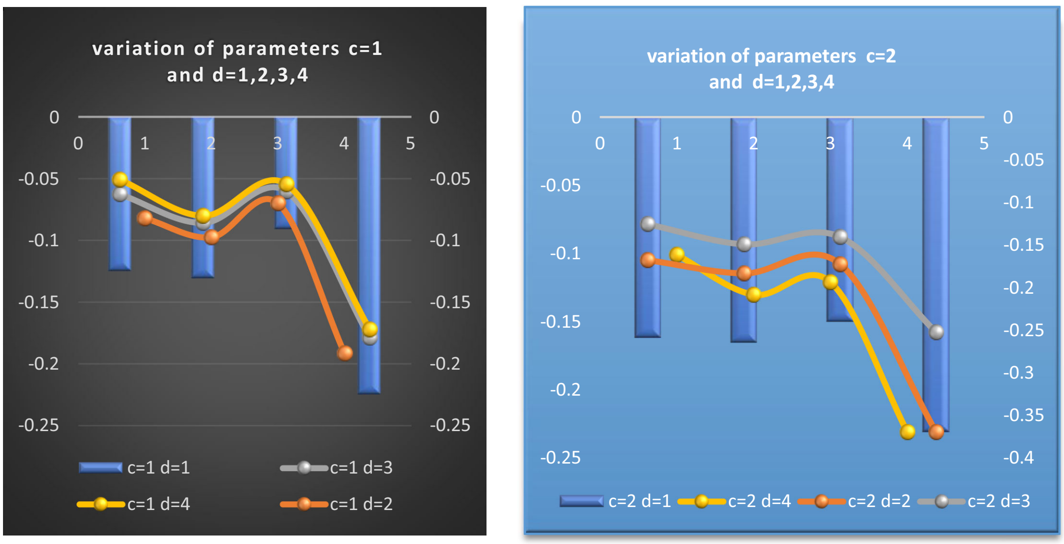


Figure 4
Graphical representation $c = 3$ and $c = 4$

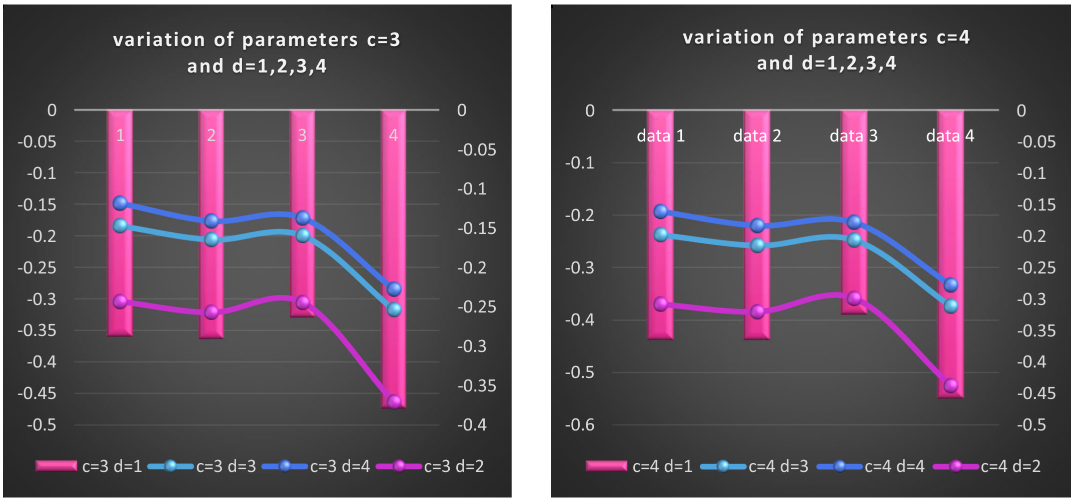
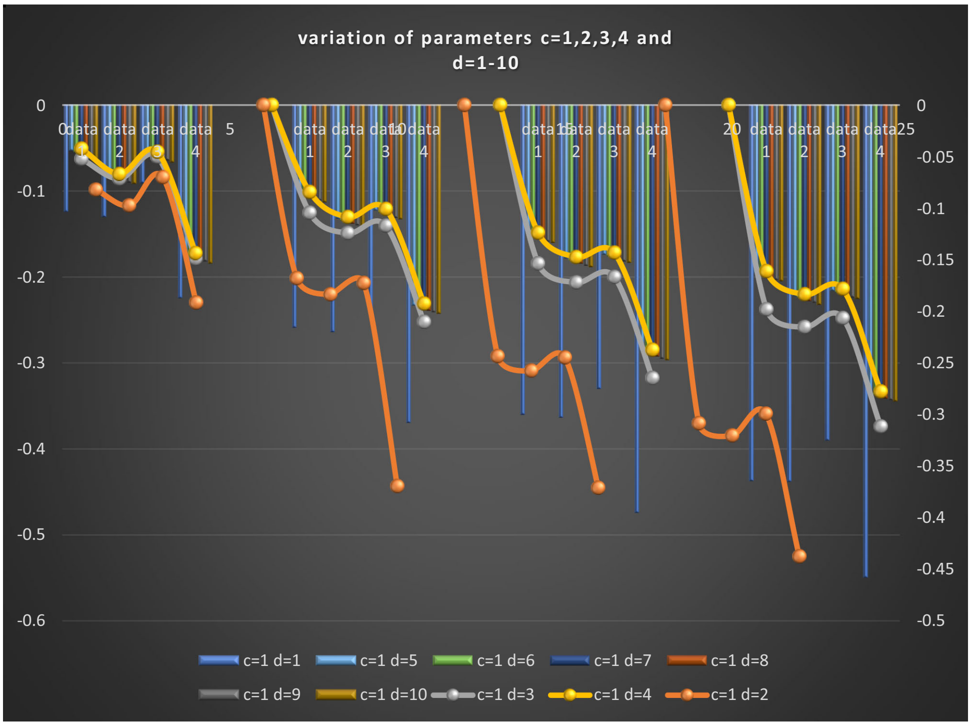


Figure 5
Graphical representation of variation parameters $c = 1$, 2, 3, 4, and $d = 1$–10



further changing of parameters c and d is applied and also the results are shown in graph in Figure 5.

5.1. Comparative analysis

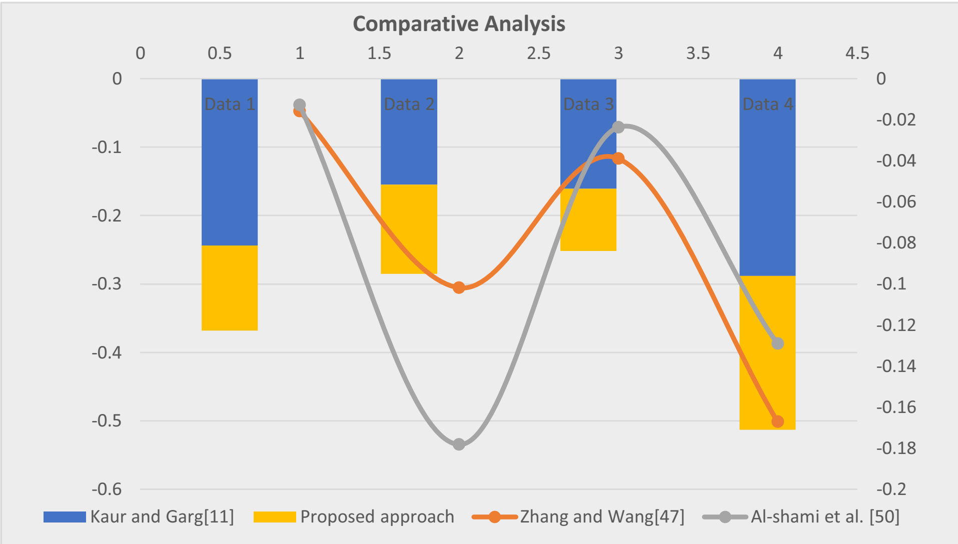
A comparative analysis has been undertaken under the IFSs (Bonferroni, 1950) by taking the IvIFS as zero and PyFSs (Ayub et al., 2022) and existing approach of Al-shami et al.(2022), during analysis the weight vector is $e = (0.2,\ 0.3,\ 0.5)^T$ to justify the superiority of our suggested mean operator over existing alternatives. The best possible score results and the alternative's ranking order are summarized in Table 7.

From this table, we observe that the study found that while the proposed approach's computational procedure differs from existing approaches in different environments, because of the constant priority degree between the pairs of arguments, a proposed approach in this research is more practical in the DM procedure. Finally, it is determined that a suggested operators consider the decision-makers parameters c as well as d, which give decision-makers additional options to choose from based on the varying

Table 7
Comparative analysis

Comparison with	Score value $\mathcal{T}_1$	$\mathcal{T}_2$	$\mathcal{T}_3$	$\mathcal{T}_4$	Ranking
Kaur and Garg (2018)	−0.2436	−0.1549	−0.1607	−0.2883	$\mathcal{T}_2 > \mathcal{T}_3 > \mathcal{T}_1 > \mathcal{T}_4$
Zhang et al. (2017)	−0.0158	−0.1018	−0.0389	−0.1670	$\mathcal{T}_1 > \mathcal{T}_3 > \mathcal{T}_2 > \mathcal{T}_4$
Al-shami et al. (2022)	−0.0128	−0.1781	−0.0237	−0.129	$\mathcal{T}_2 > \mathcal{T}_4 > \mathcal{T}_3 > \mathcal{T}_1$
Proposed approach	−0.1245	−0.1304	−0.0908	−0.2247	$\mathcal{T}_3 > \mathcal{T}_1 > \mathcal{T}_2 > \mathcal{T}_4$

Figure 6
Graphical representation of comparative studies of existing approaches with proposed approach



score results of the alternatives for various parametric variables. Moreover, the graphical representation is displayed in Figure 6.

The advantages and benefits of the proposed notion defined in this paper are the generalized form of IFS. If we reduce the square of membership and root of non-membership into 1, then SR-fuzzy set reduces to IFS. Moreover, SR-fuzzy set is reduced to fuzzy set by taking grade of non-membership zero and square of membership grade is zero. Therefore, SR-fuzzy set is capable of coping with ambiguity and vagueness of environment.

6. Conclusion

The aim of this paper is to explore the novel notion of SR-fuzzy set which is a generalized form of IFS. The properties and benefits of SR-fuzzy set are discussed, and a series of aggregation operators named BM operators and WSRFBM operators are developed. By these aggregation operators, a new MADM approach is proposed, and a problem related to road construction companies is solved. Every outcome is represented graphically and explained to assess the complexity and simplicity of the methodology. A comparison with various existing operators was conducted and information sets are also described and showed that the suggested operators and procedures give the decision-maker a more steady, realistic, and optimistic attitude during the aggregation process.

In future works, more applications of SR-fuzzy sets may be examined, and also SR-fuzzy rough sets may be investigated. Also, we will try to produce the topology from the set of SR-fuzzy sets and propose the concepts of connectedness and compactness in SR-fuzzy topology.

Conflicts of Interest

The authors declare that they have no conflicts of interest to this work.

Ethical Approval

This paper does not contain any studies with human participants or animals performed by any of the authors.

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Data Availability

Data sharing is not applicable to this paper as no datasets were generated or analyzed during the current study.

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