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2-Tuple Linguistic Fermatean Fuzzy Decision-Making Method Based on COCOSO with CRITIC for Drip Irrigation System Analysis

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Abstract: Given the increasing scarcity of water resources, especially climate change, the adoption of water-efficient irrigation systems (ISs) is becoming increasingly important. Drip irrigation systems (DISs) are the most successful method of saving water and increasing agricultural yields in water-efficient IS. DIS reduces not only the cost of water supply but also the cost of activities such as labor costs and other planting costs. DIS is the most reliable, profitable, and cost-effective agricultural irrigation technique for the vast majority of crops, and it could be a potential solution to the growing water crisis caused by climate change. The Hamacher operation is an extension of the algebraic and Einstein operations. The combination of 2-tuple linguistic Fermatean fuzzy (2TLFF) numbers and the Hamacher operation is more valuable and agile. The method based on the Combined Compromise Solution (COCOSO) with Criteria Importance Through Inter-criteria Correlation (CRITIC) is introduced to manage multiple attribute group decision-making (MAGDM) issues in a 2TLFF environment. Finally, a practical example is shown, followed by a comparison study that supports the unique approach's efficacy and generalizability. The suggested method distinguishes itself by having no paradoxical instances and a powerful ability for recognizing the optimal choice.

Keywords: 2-tuple linguistic model, Fermatean fuzzy sets, COCOSO, Hamacher aggregation operator

1. Introduction

The two most important resources for any agricultural operation are land and water. Water is a finite natural resource, and its demand is increasing at an alarming rate. The majority of agricultural fields are irrigated with underground water for guaranteed irrigation. Rainfall provides water for rainfed agriculture. Water resources are critical for development and economic planning in poor countries. Despite large investments and phenomenal growth in the irrigation sector, crop yield, farm income, and cost recovery from irrigated systems are disappointing. Aside from that, there are additional issues such as rising soil salinity, water logging, and social inequity.

There is a significant gap between the creation of irrigation potential and its utilization. In 1940, mostly people of United Kingdom used to irrigate land using small apertures in pipes. In 1974, Davis used subsurface clay pipes with irrigation and drainage systems in an experiment in the early days of drip irrigation system (DIS) development. In 1964, it is discovered that a tree near a leaking faucet grew faster than other trees in the area. He worked on it and eventually patented the current form of DIS technology. Drip irrigation (DI) gradually expanded throughout the world, particularly in countries where water was scarce. Although DISs are widely regarded as the most effective water-saving technologies in irrigated agriculture, their adoption remains low. Currently, about 2.9% of the total world irrigated area is equipped with Shock (2006). The majority of DI is mostly used in Europe and North America. Asia has the most irrigated land, but it accounts for 2.3% of the total irrigated area in Shock (2006). DI is not suitable for every agricultural crop. Also it is not suitable for every site. DI is most suitable where oils are sandy or rocky, steep slopes, high-value crops are grown, water and labor are expensive or scarce, and water is of poor quality.

Commercial field crops and horticultural crops are the main crops irrigated with DIS. This method of irrigation is still used in protected agriculture for the production of vegetables. DIS is also used in landscapes, parks, and commercial developments. DIS has the following additional benefits:

1. DIS allows for more efficient application of agricultural chemicals because only the crop root zone is watered and fertilizer provided may be utilized more effectively. Less product may be needed while using pesticides. Verify that the pesticide is labeled for DIS and abide by the directions there.
2. Wheel traffic rows may be built and controlled in DIS such that they are always dry enough to be used for tractor operations. Fungicides can be used if necessary.

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3. DISs can be used in areas with unusual shapes, uneven topography, or different soil textures. DIS can also be effective in areas where other ISs are ineffective due to excessive infiltration, water puddling, or runoff.
4. Proven production and quality responses are available with proper irrigation scheduling made feasible by DIS. Increases in productivity and quality have been shown for a number of crops, including cotton, wheat, cauliflower, mustard, cabbage, watermelon, and tomatoes.
5. If water is scarce or expensive, DIS can be beneficial. It is not essential to overwater portions of a field in order to sufficiently irrigate the more challenging areas because of the reduction in evaporation, runoff, and deep percolation, as well as the improvement in irrigation uniformity.
6. It is possible to reduce fertilizer costs and nitrate losses.

DIS has various limitations, including:

1. Manage drip tape or tubing to stop it from leaking or clogging. Silt or other particles in the irrigation water that are not filtered out can quickly block drip emitters. Emitter blockage may also be brought on by chemical buildup at the emitter or algae development in the tape.
2. The expense of post-harvest cleaning is increased by drip tape, except in permanent installations. It is difficult to plan for the reuse, recycling, or disposal of drip tape.
3. Installing DIS normally costs between $270,000$ PKR and $380,000$ PKR per acre. The equipment needed to install or remove drip tape from non-permanent installations is not included in this pricing range. Some of the significant variances in drip tubing expenses per acre can be attributed to the spacing between plant rows.
4. Your weed control plan might need to be changed. If herbicides need to be activated by rain or sprinkler irrigation, DI might not be enough. By maintaining a large portion of the soil surface dry, DI, on contrary, can minimize weed populations or difficulties in arid regions.

1.1. Literature review

Zadeh 1965 proposed the concept of fuzzy sets in 1965. An intuitionistic fuzzy set (IFS) was proposed by Atanassov 1986 in 1986. Szmidt and Kacprzyk 2001 introduced the medical applications of IFSs. Further, Yager 2014 proposed the concept of the Pythagorean fuzzy set (PFS). Although IFSs and PFSs can solve a lot of real-life applications, there are some limitations to these sets. Some applications may contain the Decision maker (DM)'s opinion as (0.8, 0.9). In those cases, PFSs and IFSs failed to apply. Senapati and Yager 2019 introduced Fermatean fuzzy sets (FFSs). The FFSs are those sets in which the cube sum of MD and ND is less than 1. Senapati and Yager 2019; 2020 introduced the solution to some MADM problems based on FFSs. Multiple attribute group decision-making (MAGDM) approaches Akram et al. (2022); Akram, Ramzan and Feng (2022); Akram, Niaz and Feng (2022) have been proposed using FFSs.

The 2-tuple linguistic (2TL) term is a useful technique that helps to prevent information loss and produce more accurate evaluation results. One of the most important methods for addressing linguistic decision-making (DeM) concerns is the 2TL representation model, which was initially put out by Herrera and Martínez 2000. Based on earlier research, Zhang et al. 2021 improved the TODIM approach and the cumulative prospect theory under the 2-tuple linguistic Pythagorean fuzzy (2TLPF) sets. Several 2TL data-based DeM techniques have been introduced. For an understandable 2TL collection, Faizi et al. 2021 developed worst-case approaches and Hamacher aggregation processes. Using linguistic data to make decisions, the method of linguistic decision analysis was developed by Herrera and Herrera-Viedma 2000. A low-cost DIS was proposed for small farmers in poor nations by Polak et al. 1997 in 1997. Studying a review of subsurface DIS was done by Camp 1998. In comparison to traditional surface ISs, Maisiri et al. 2005 examined the impact of low-cost DIS on water and agricultural productivity on farms. Smart DIS for sustainable agriculture was proposed by Kavianand et al. 2016. A smart sensor for automated DIS for rice cultivation was introduced by Barkunan et al. 2019. The algebraic, Einstein, Hamacher *t*-conorms (HTCN), and Hamacher *t*-norms (HTN) of Archimedes are widely recognized. Algebraic and Einstein are extensions of the Hamacher operation. Liu et al. 2014; 2014 examined the Hamacher operating guidelines due to the Hamacher operator's more broad nature. The Criteria Importance Through Inter-criteria Correlation (CRITIC) method was introduced in 1995 by Diakoulaki et al. 1995. A multi-attribute performance study of Initial Public Offering enterprisers utilizing CRITIC and Vlekriterijumsko KOmpromisno Rangiranje (VIKOR) methodologies was first published by Yalcin and Unlu 2018 in 2018. In 2018, Rostamzadeh et al. 2018 proposed an integrated fuzzy TOPSIS-CRITIC approach to assess sustainable supply chain risk management. The Combined Compromise Solution (COCOSO) method, a brand-new and successful MAGDM technique, was introduced by Yazdani et al. 2018. In their article Peng and Huang (2020), Peng and Huang introduced a fuzzy DeM strategy based on COCOSO with CRITIC for assessing financial risk. For the 2020 assessment of the 5G sector, Peng et al. 2020 proposed the Pythagorean fuzzy MADM technique based on COCOSO and CRITIC. Using a combined Best Worst Method and COCOSO model, Zolfani et al. 2019 established a framework for sustainable supplier selection in 2019. Ulutas et al. 2020 studied the location of logistics centers using fuzzy SWARA and COCOSO methods. In 2019, Yazdani et al. 2019 proposed a grey COCOSO method for supplier selection in construction management. In 2021, Deveci et al. 2021 proposed COCOSO method based on fuzzy exponentiation Heronian function for the prioritization of advantages of autonomous vehicles in real-time traffic management. The following is the motivation for this research article.

1. The algebraic and Einstein *t*-norm and *t*-conorm are extended in a more thorough, full, and dynamic way by the HTCN and HTN.
2. The existing averaging operators (AOs), that is, Deng et al. (2018); Senapati and Yager (2019) used to resolve MADM difficulties contain situations that defy logic and have a poor discernibility degree when differentiating the optimal option. The COCOSO technique is an adaptive algorithm that disposes of the information in a rational and practical manner. The second motivation is to address the MAGDM issues by introducing a unique algorithm that does not have the two aforementioned flaws.
3. The only weights that the known 2TLPF weight determination algorithms consider are either subjective weights (SWs) He et al. (2020) or objective weights (OWs) Das and Chakraborty (2022). DMs provide the SWs while excluding the weight data provided by the assessment matrix. The combined weights (CWs) model is hence the third incentive.

The following is a listing of innovations in the proposed approach.

1. It is demonstrated that the suggested technique can choose the optimum valve for DIS.
2. The explicative numerical example is provided to show how the suggested technique may be applied in actual DeM scenarios. Through comparison study, the supremacy and legitimacy of the suggested strategy are confirmed. The benefits of the suggested method are fully described.
3. The innovative 2TL Fermatean fuzzy (2TLFF) DeM approach, which is based on COCOSO, has been created and can provide the best alternative out of a scenario that defies logic and has a strong ability to distinguish the option that is most wanted.
4. The linear weighted integrated method (LWIM), which considers both SWs and OWs information into account, and CRITIC are the foundations of the CWs technique.

The remainder of this article is listed as follows: Section 2 elaborates some basic concepts regarding the 2TL representation model and several Hamacher operators for 2TLFFNs along with their significant properties. Section 3 elaborates the thorough procedure of the COCOSO method to tackle MAGDM issues with 2TLFFNs and real-life applications of the 2TLFF-COCOSO approach are given using the 2TLFF Hamacher weighted averaging operator (2TLFFHWAO) in Section 4. In Section 5, parametric analysis using different values of parameter γ is given. In Section 6, comparative analysis with existing operators, that is, 2TLPF weighted averaging operator (2TLPFWAO), 2TLPF weighted geometric operator (2TLPFWGO), 2TLPF weighted Hamy mean operator (2TLPFWHMO), and 2TLPF weighted dual Hamy mean operator (2TLPFWDHMO), is given. Section 7 presents the conclusion.

2. Preliminaries

Variables whose values are expressed in linguistic terms (LTs) are called linguistic variables. In other words, it is a variable whose value is a word or phrase in the language, whether natural or artificial. The concept of linguistic variables is useful when dealing with situations that are too complex or poorly defined to be well expressed in conventional quantitative terms Zadeh (1965). Linguistic phrases often have properties such as finite sets, odd cardinality, semantic symmetry, ordinal levels, and compensation operations that help identify the diversity of each evaluation item and simplify the computation Tai and Chen (2009).

Definition 2.1. Herrera and Martínez (2000) Let there exists a LT set $H=\{v_i|i=0,1,\ldots,t\}$, where v_i indicates a possible LT for a linguistic variable (LeV). A LT set H with three terms, for instance, can be explained as follows:

$$H=\{v_0=none, v_1=low, v_2=high\}.$$

If $v_i, v_k \in H$, therefore the LT set satisfies the following prerequisites:

(i) $v_i > v_k$, if and only if $i>k$.
(ii) $\max(v_i,v_k)=v_i$, if and only if $i\geq k$.
(iii) $\min(v_i,v_k)=v_i$, if and only if $i\leq k$.
(iv) $\mathrm{Neg}(v_i)=v_k$ such that $k=t-i$.

Definition 2.2. Herrera and Martínez (2000) Let $\grave{\beta}$ be the outcome of a symbolic aggregation operation, $i\in[0,t]$, where t is the cardinality of H. Let us consider two values $i=round(\grave{\beta})$ and $\alpha=\grave{\beta}-i$, such that, $i\in[0,t]$ and $\alpha\in[-\frac{1}{2},\frac{1}{2})$, then α is known as symbolic translation.

The granularity of LT sets is related to the range of $\grave{\beta}$ in traditional 2TL methods, between 0 and t. Here, $\grave{\beta}$ is the result of combining the indices of a set of labels evaluated using a linguistic word set. Herrera and Martinez 2000 proposed a generalized 2TL model to overcome the limitation.

Definition 2.3. Herrera and Martinez (2000) Let $H=\{v_i|i=0,\ldots,t\}$ be a LT set and $i\in[0,t]$ be a number value encoding the combined result of linguistic symbol. The function Δ, which is utilized to acquire the 2TL data comparable to $\grave{\beta}$, is then defined as follows:

$$\Delta:[0,t]\rightarrow H\times[-\frac{1}{2},\frac{1}{2}),$$

$$\Delta(\grave{\beta})=\begin{cases} v_i, & i=round(\grave{\beta}),\\ \alpha=\grave{\beta}-i, \alpha\in[-\frac{1}{2},\frac{1}{2}). \end{cases} \tag{1}$$

Definition 2.4. Herrera and Martínez (2000) Let $H=\{v_i|i=0,\ldots,t\}$ be a LT set and (v_i,α_i) be a 2T, there exists a function Δ^{-1} that returns the 2T to its numerical equivalents $\grave{\beta}\in[0,t]\subset R$, where

$$\Delta^{-1}:H\times[-\frac{1}{2},\frac{1}{2})\rightarrow[0,t],$$

$$\Delta^{-1}(v_i,\alpha)=i+\alpha=\grave{\beta}. \tag{2}$$

During the operation of the 2-tuple linguistic representation, the functions Δ and Δ^{-1} are both used to ensure the operation of 2TL variables can be 2-tuples without losing any information.

Definition 2.5. Senapati and Yager (2019) Let C be a fixed set. A FFS is an object with the form

$$F=\{(c,(\mu_F(c),\nu_F(c)))|c\in C\}, \tag{3}$$

where the function μ_F is from C to [0,1] specifying the MD, and ν_F is from C to [0,1] specifying the ND of an element $c\in C$ to F, respectively. For every $c\in C$, it satisfies $(\mu_F(c))^3+(\nu_F(c))^3\leq 1$. The concepts of FFSs are extended to 2TLFFSs by Akram et al. 2022.

Definition 2.6. Akram et al. (2022) Let $H=\{v_0,v_1,v_2,\ldots,v_t\}$ be a LT set, where t is an even number. If $V=\{(v_\zeta,\zeta),(v_\sigma,\sigma)\}$ is defined for $v_\zeta,v_\sigma\in H$ and $\sigma,\zeta\in[-0.5,0.5)$, where $(v_\zeta,\zeta),(v_\sigma,\sigma)$ express the membership degree (MD) and nonmembership degree (ND) by 2-tuple LT sets. Then 2TLFFS can be defined as follows:

$$D=[<x,\{(v_{\zeta_j},\zeta_j),(v_{\sigma_j},\sigma_j)\}>|x\in X],$$

where $(v_{\zeta_j},\zeta_j),(v_{\sigma_j},\sigma_j)$ are 2TLT such that $0\leq\Delta^{-1}(v_{\zeta_j},\zeta_j)\leq t$, $0\leq\Delta^{-1}(v_{\sigma_j},\sigma_j)\leq t$ and $0\leq(\Delta^{-1}(v_{\zeta_j},\zeta_j))^3+(\Delta^{-1}(v_{\sigma_j},\sigma_j))^3\leq t^3$. In order to easy computation, $V_j=\{(v_{\zeta_j},\zeta_j),(v_{\sigma_j},\sigma_j)\}$, denote 2TLFFN.

Definition 2.7. Akram et al. (2022) Let $V_1=\{(v_{\zeta_1},\zeta_1),(v_{\sigma_1},\sigma_1)\}$ be a 2TLFFN in P. Then score function is defined as:

$$\dot{S}(V_1)=\Delta\left\{\frac{t}{2}\left(1+\left(\frac{\Delta^{-1}(v_{\zeta_1},\zeta_1)}{t}\right)^3-\left(\frac{\Delta^{-1}(v_{\sigma_1},\sigma_1)}{t}\right)^3\right)\right\}, \tag{4}$$

The scoring function can be used to obtain the final ranking of alternatives.

Definition 2.8. Akram et al. (2022) Let $V_1 = \{(v_{\zeta_1}, \zeta_1), (v_{\sigma_1}, \sigma_1)\}$ and $V_2 = \{(v_{\zeta_2}, \zeta_2), (v_{\sigma_2}, \sigma_2)\}$ be two 2TLFFNs, $\lambda > 0$ be real numbers, where $v_{\zeta_1}, v_{\sigma_1}, v_{\zeta_2}, v_{\sigma_2} \in H = \{v_\alpha | v_0 \leq v_\alpha \leq v_t, \alpha \in [0, t]\}$.

Following are some basic operations on 2TLFFNs:

1. $$V_1 \oplus V_2 = \begin{pmatrix} \Delta\left(t\sqrt[3]{\left(\frac{\Delta^{-1}(v_{\zeta_1},\zeta_1)}{t}\right)^3 + \left(\frac{\Delta^{-1}(v_{\zeta_2},\zeta_2)}{t}\right)^3 - \left(\frac{\Delta^{-1}(v_{\zeta_1},\zeta_1)}{t}\right)^3\left(\frac{\Delta^{-1}(v_{\zeta_2},\zeta_2)}{t}\right)^3}\right), \\ \Delta\left(t\left(\frac{\Delta^{-1}(v_{\sigma_1},\sigma_1)}{t}\right)^3\left(\frac{\Delta^{-1}(v_{\sigma_2},\sigma_2)}{t}\right)^3\right) \end{pmatrix},$$

2. $$V_1 \otimes V_2 = \begin{pmatrix} \Delta\left(t\left(\frac{\Delta^{-1}(v_{\zeta_1},\zeta_1)}{t}\right)^3\left(\frac{\Delta^{-1}(v_{\zeta_2},\zeta_2)}{t}\right)^3\right), \\ \Delta\left(t\sqrt[3]{\left(\frac{\Delta^{-1}(v_{\sigma_1},\sigma_1)}{t}\right)^3 + \left(\frac{\Delta^{-1}(v_{\sigma_2},\sigma_2)}{t}\right)^3 - \left(\frac{\Delta^{-1}(v_{\sigma_1},\sigma_1)}{t}\right)^3\left(\frac{\Delta^{-1}(v_{\sigma_2},\sigma_2)}{t}\right)^3}\right) \end{pmatrix},$$

3. $$\lambda V_1 = \begin{pmatrix} \Delta\left(t\sqrt[3]{\left(1-\left(1-\left(\frac{\Delta^{-1}(v_{\zeta_1},\zeta_1)}{t}\right)^3\right)^\lambda\right)}\right), \\ \Delta\left(t\left(\frac{\Delta^{-1}(v_{\sigma_1},\sigma_1)}{t}\right)^{3\lambda}\right) \end{pmatrix},$$

4. $$V_1^\lambda = \begin{pmatrix} \Delta\left(t\left(\frac{\Delta^{-1}(v_{\zeta_1},\zeta_1)}{t}\right)^{3\lambda}\right), \\ \Delta\left(t\sqrt[3]{\left(1-\left(1-\left(\frac{\Delta^{-1}(v_{\sigma_1},\sigma_1)}{t}\right)^3\right)^\lambda\right)}\right) \end{pmatrix}.$$

Define operations on 2TLFFN can be used to aggregate LTs.

Definition 2.9. Akram, Niaz and Feng (2022) Let $V_1 = \{(v_{\zeta_1}, \zeta_1), (v_{\sigma_1}, \sigma_1)\}$ and $V_2 = \{(v_{\zeta_2}, \zeta_2), (v_{\sigma_2}, \sigma_2)\}$ be two 2TLFFNs, where $v_{\zeta_1}, v_{\sigma_1}, v_{\zeta_2}, v_{\sigma_2} \in H$. Then some Hamacher operational laws on 2TLFFNs are as follows:

1. $$V_1 \oplus V_2 = \begin{pmatrix} \Delta\left(t\sqrt[3]{\frac{\left(\frac{\Delta^{-1}(v_{\zeta_1},\zeta_1)}{t}\right)^3 + \left(\frac{\Delta^{-1}(v_{\zeta_2},\zeta_2)}{t}\right)^3 - \left(\frac{\Delta^{-1}(v_{\zeta_1},\zeta_1)}{t}\right)^3\left(\frac{\Delta^{-1}(v_{\zeta_2},\zeta_2)}{t}\right)^3 - (1-\kappa)\left(\frac{\Delta^{-1}(v_{\zeta_1},\zeta_1)}{t}\right)^3\left(\frac{\Delta^{-1}(v_{\zeta_2},\zeta_2)}{t}\right)^3}{1-(1-\kappa)\left(\frac{\Delta^{-1}(v_{\zeta_1},\zeta_1)}{t}\right)^3\left(\frac{\Delta^{-1}(v_{\zeta_2},\zeta_2)}{t}\right)^3}}\right), \\ \Delta\left(t\frac{\left(\frac{\Delta^{-1}(v_{\sigma_1},\sigma_1)}{t}\right)^3\left(\frac{\Delta^{-1}(v_{\sigma_2},\sigma_2)}{t}\right)^3}{\sqrt[3]{\kappa + (1-\kappa)\left(\frac{\Delta^{-1}(v_{\sigma_1},\sigma_1)}{t}\right)^3 + \left(\frac{\Delta^{-1}(v_{\sigma_2},\sigma_2)}{t}\right)^3 - \left(\frac{\Delta^{-1}(v_{\sigma_1},\sigma_1)}{t}\right)^3\left(\frac{\Delta^{-1}(v_{\sigma_2},\sigma_2)}{t}\right)^3}}\right) \end{pmatrix},$$

2. $$V_1 \otimes V_2 = \begin{pmatrix} \Delta\left(t\frac{\left(\frac{\Delta^{-1}(v_{\zeta_1},\zeta_1)}{t}\right)^3\left(\frac{\Delta^{-1}(v_{\zeta_2},\zeta_2)}{t}\right)^3}{\sqrt[3]{\kappa + (1-\kappa)\left(\frac{\Delta^{-1}(v_{\zeta_1},\zeta_1)}{t}\right)^3 + \left(\frac{\Delta^{-1}(v_{\zeta_2},\zeta_2)}{t}\right)^3 - \left(\frac{\Delta^{-1}(v_{\zeta_1},\zeta_1)}{t}\right)^3\left(\frac{\Delta^{-1}(v_{\zeta_2},\zeta_2)}{t}\right)^3}}\right), \\ \Delta\left(t\sqrt[3]{\frac{\left(\frac{\Delta^{-1}(v_{\sigma_1},\sigma_1)}{t}\right)^3 + \left(\frac{\Delta^{-1}(v_{\sigma_2},\sigma_2)}{t}\right)^3 - \left(\frac{\Delta^{-1}(v_{\sigma_1},\sigma_1)}{t}\right)^3\left(\frac{\Delta^{-1}(v_{\sigma_2},\sigma_2)}{t}\right)^3 - (1-\kappa)\left(\frac{\Delta^{-1}(v_{\sigma_1},\sigma_1)}{t}\right)^3\left(\frac{\Delta^{-1}(v_{\sigma_2},\sigma_2)}{t}\right)^3}{1-(1-\kappa)\left(\frac{\Delta^{-1}(v_{\sigma_1},\sigma_1)}{t}\right)^3\left(\frac{\Delta^{-1}(v_{\sigma_2},\sigma_2)}{t}\right)^3}}\right) \end{pmatrix},$$

3. $$\lambda V_1 = \begin{pmatrix} \Delta\left(t\sqrt[3]{\frac{\left(1+(\kappa-1)\left(\frac{\Delta^{-1}(v_{\zeta_1},\zeta_1)}{t}\right)^3\right)^\lambda - \left(1-\left(\frac{\Delta^{-1}(v_{\zeta_1},\zeta_1)}{t}\right)^3\right)^\lambda}{\left(1+(\kappa-1)\left(\frac{\Delta^{-1}(v_{\zeta_1},\zeta_1)}{t}\right)^3\right)^\lambda + (\kappa-1)\left(1-\left(\frac{\Delta^{-1}(v_{\zeta_1},\zeta_1)}{t}\right)^3\right)\lambda}}\right), \\ \Delta\left(t\frac{\sqrt{\kappa}\left(\frac{\Delta^{-1}(v_{\sigma_1},\sigma_1)}{t}\right)^{3\lambda}}{\sqrt[3]{\left(1+(\kappa-1)\left(1-\left(\frac{\Delta^{-1}(v_{\sigma_1},\sigma_1)}{t}\right)^3\right)\right)^\lambda + (\kappa-1)\left(\frac{\Delta^{-1}(v_{\sigma_1},\sigma_1)}{t}\right)^3\lambda}}\right) \end{pmatrix},$$

where κ and λ are two positive real numbers.

Definition 2.10. Akram, Niaz and Feng (2022) Let $V_j = \{(v_{\zeta_j}, \zeta_j), (v_{\sigma_j}, \sigma_j)\}, (1 \leq j \leq n)$ be a group of 2TLFFNs. Its weight vector (WV) is $\omega = (\omega_1, \omega_2, \ldots, \omega_n)$. Then 2TLFFHWAO and 2TLFFHWGO are given by:

$$2TLFFHWA_u(V_1, V_2, \ldots, V_n) = \oplus_{j=1}^n(\omega_j V_j)$$

$$= \begin{pmatrix} \Delta\left(t\sqrt[3]{\frac{\prod_{j=1}^n\left(1+(\kappa-1)\left(\frac{\Delta^{-1}(v_{\zeta_j},\zeta_j)}{t}\right)^3\right)^{\omega_j} - \prod_{j=1}^n\left(1-\left(\frac{\Delta^{-1}(v_{\zeta_j},\zeta_j)}{t}\right)^3\right)^{\omega_j}}{\prod_{j=1}^n\left(1+(\kappa-1)\left(\frac{\Delta^{-1}(v_{\zeta_j},\zeta_j)}{t}\right)^3\right)^{\omega_j} + (\kappa-1)\prod_{j=1}^n\left(1-\left(\frac{\Delta^{-1}(v_{\zeta_j},\zeta_j)}{t}\right)^3\right)^{\omega_j}}}\right), \\ \Delta\left(t\frac{\sqrt{\kappa}\prod_{j=1}^n\left(\frac{\Delta^{-1}(v_{\sigma_j},\sigma_j)}{t}\right)^{\omega_j}}{\sqrt[3]{\prod_{j=1}^n\left(1+(\kappa-1)\left(1-\left(\frac{\Delta^{-1}(v_{\sigma_j},\sigma_j)}{t}\right)^3\right)^{\omega_j}\right) + (\kappa-1)\prod_{j=1}^n\left(\frac{\Delta^{-1}(v_{\sigma_j},\sigma_j)}{t}\right)^{3\omega_j}}}\right) \end{pmatrix}.$$

$$2TLFFHWG_u(V_1, V_2, \ldots, V_n) = \otimes_{j=1}^n(V_j)^{\omega_j}$$

$$= \begin{pmatrix} \Delta\left(t\frac{\sqrt{\kappa}\prod_{j=1}^n\left(\frac{\Delta^{-1}(v_{\zeta_j},\zeta_j)}{t}\right)^{\omega_j}}{\sqrt[3]{\prod_{j=1}^n\left(1+(\kappa-1)\left(1-\left(\frac{\Delta^{-1}(v_{\zeta_j},\zeta_j)}{t}\right)^3\right)\right)^{\omega_j} + (\kappa-1)\prod_{j=1}^n\left(\left(\frac{\Delta^{-1}(v_{\zeta_j},\zeta_j)}{t}\right)^3\right)^{\omega_j}}}\right), \\ \Delta\left(t\sqrt[3]{\frac{\prod_{j=1}^n\left(1+(\kappa-1)\left(\frac{\Delta^{-1}(v_{\sigma_j},\sigma_j)}{t}\right)^3\right)^{\omega_j} - \prod_{j=1}^n\left(1-\left(\frac{\Delta^{-1}(v_{\sigma_j},\sigma_j)}{t}\right)^3\right)^{\omega_j}}{\prod_{j=1}^n\left(1+(\kappa-1)\left(\frac{\Delta^{-1}(v_{\sigma_j},\sigma_j)}{t}\right)^3\right)^{\omega_j} + (\kappa-1)\prod_{j=1}^n\left(1-\left(\frac{\Delta^{-1}(v_{\sigma_j},\sigma_j)}{t}\right)^3\right)^{\omega_j}}}\right) \end{pmatrix}.$$

where ω_j lies between 0 and 1, also $\omega_1 + \omega_2 + \ldots + \omega_n = 1$. 2TLFFHWAO and 2TLFFHWGO can be used to obtain the weighted comparable sequence (CS) and power weight (PW) of CS, respectively.

3. Extended COCOSO Method with 2TLFFNs

Let ¥ $= \{¥_1, ¥_2, \ldots, ¥_m\}$ be a series of alternatives, and $G = \{g_1, g_2, \ldots, g_n\}$ be a discrete set of attributes. Let us consider there are k DMs, n alternatives, and j attributes, and their values given by kth DM, described by linguistic expressions $l_{ij}^k(1 \leq i \leq m, 1 \leq j \leq n, 1 \leq k \leq q)$, in Table 1.

Table 1
LeV and corresponding 2TLFFNs

LeV	2TLFFNs
Extra extra small (*xexes*)	$\{(v_0, 0), (v_6, 0)\}$
Extra small (*xes*)	$\{(v_1, 0), (v_5, 0)\}$
Medium small (*mes*)	$\{(v_2, 0), (v_4, 0)\}$
Small (*s*)	$\{(v_3, 0), (v_3, 0)\}$
Large (*la*)	$\{(v_4, 0), (v_2, 0)\}$
Extra large (*xela*)	$\{(v_5, 0), (v_1, 0)\}$
Extra extra large (*xexela*)	$\{(v_6, 0), (v_0, 0)\}$

3.1. CRITIC method to determine OWs

The CRITIC Diakoulaki, Mavrotas and Papyannakis (1995) is an approach for calculating the OWs of the given attribute in MAGDM issues. The OWs derived from the preceding approach take into account both the intensity contrast of each attribute and the conflict between attributes. The correlation coefficient is used to obtain the conflict between attributes based on their intensity contrast. We extend CRITIC approach to the 2TLFF environment.

1. Compute the score function $T = (t_{ij})_{m\times n}$ of each 2TLFFN $V_i = \{(v_{\zeta_i}, \zeta_i), (v_{\sigma_i}, \sigma_i)\}$ by Equation (5).

$$S(V_1) = \Delta\left\{\frac{t}{2}\left(1 + \left(\frac{\Delta^{-1}(v_{\zeta_1}, \zeta_1)}{t}\right)^3 - \left(\frac{\Delta^{-1}(v_{\sigma_1}, \sigma_1)}{t}\right)^3\right)\right\}, \tag{5}$$

2. Convert the score matrix T into a 2TLFF matrix $T' = (t'_{ij})_{m\times n}$ by using expression given below.

$$t'_{ij} = \begin{cases} \frac{t_{ij}-t_j^-}{t_j^+-t_j^-}, & \text{if} \quad j \in \text{beneficial attribute}, \\ \frac{t_j^+-t_{ij}}{t_j^+-t_j^-}, & \text{if} \quad j \in \text{cost attribute}, \end{cases}$$

where $t_j^- = \min t_{ij}$ and $t_j^+ = \max t_{ij}$.

3. We calculate standard deviations by using Equation (6).

$$\rho_j = \sqrt{\frac{\sum_{i=1}^m (t'_{ij} - t_j)^2}{m}}. \tag{6}$$

4. Using Equation (7) calculate correlation between attribute pairs.

$$\varsigma_{jk} = \frac{\sum_{i=1}^m (t'_{ij} - t_j)(t'_{ik} - t_k)}{\sqrt{\sum_{i=1}^m (t'_{ij} - t_j)^2 \sum_{i=1}^m (t'_{ik} - t_k)^2}}. \tag{7}$$

5. Using Equation (8) calculate the quantity of information of each attribute.

$$\varpi_j = \rho_j \sum_{i=1}^m (1 - \varsigma_{jk}). \tag{8}$$

6. Obtain the OWs by using the following formula:

$$\xi_j = \frac{\varpi_j}{\sum_{j=1}^n \varpi_j}. \tag{9}$$

3.2. CWs by LWIM

SWs are denoted by $\xi' = \{\xi'_1, \xi'_2, \ldots, \xi'_n\}$. These SWs are given by DMs, where $0 \le \xi'_n \le 1$ and $\xi'_1 + \xi'_2 + \ldots + \xi'_n = 1$. Hence, we can find CWs $\omega = \{\omega_1, \omega_2, \ldots, \omega_n\}$ by combining OWs and SWs using Equation (10).

$$\omega_j = \frac{\xi'_j \xi_j}{\sum_{j=1}^n \xi'_j \xi_j}, \tag{10}$$

where $0 \le \omega_n \le 1$ and $\omega_1 + \omega_2 + \ldots + \omega_n = 1$.

3.3. The 2TLFF-COCOSO method

COCOSO is an efficient MAGDM method. To address the MAGDM problem, we propose a 2TLFF-COCOSO method. In general, the 2TLFF-COCOSO method consists of the following steps.

1. Obtain the 2TLFF decision matrix.
2. Score function of each 2TLFFN is calculated by using Equation (5).
3. Standard 2TLFF matrix $T' = (t'_{ij})_{m\times n}$ is obtained by score matrix.
4. Calculate CWs by using Equation (10).
5. Compute total of the weighted CS denoted by SE_i by using 2TLFFHWAO:

$$SE_i = \oplus_{j=1}^n (t'_j \omega_j).$$

6. Compute total of the PWs of CS denoted by PE_i by using 2TLFFHWGO:

$$PE_i = \otimes_{j=1}^n (t'_j)^{\omega_j}$$

7. Compute relative weights of alternatives by using equations given below:

$$ZE_{i\alpha} = \frac{SE_i \oplus PE_i}{\sum_{i=1}^m SE_i \oplus PE_i}, \tag{11}$$

$$ZE_{i\beta} = \frac{SE_i}{\min_i SE_i} \oplus \frac{PE_i}{\min_i PE_i}, \tag{12}$$

$$ZE_{i\gamma} = \frac{\lambda SE_i \oplus (1-\lambda) PE_i}{\lambda \max_i SE_i \oplus (1-\lambda) \max_i PE_i}, 0 \le \lambda \le 1. \tag{13}$$

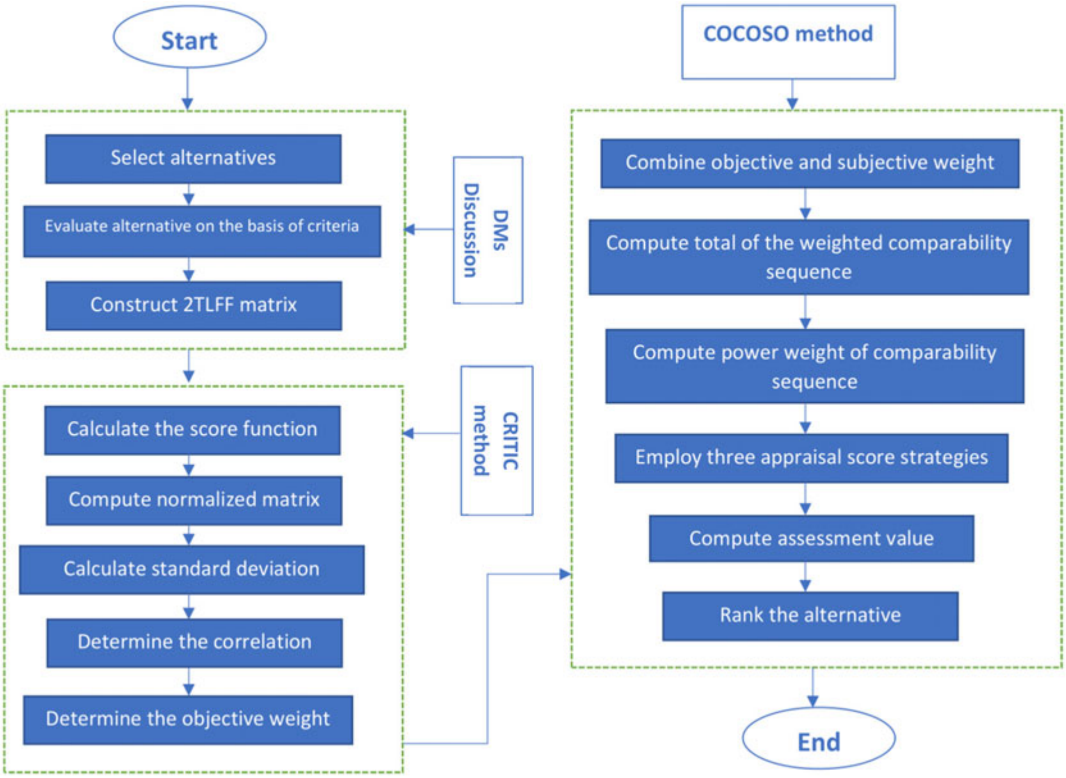
8. Compute the assessment value ZE_i by Equation (14).

$$ZE_i = \sqrt[3]{ZE_{i\alpha} \otimes ZE_{i\beta} \otimes ZE_{i\gamma}} \oplus \frac{ZE_{i\alpha} \oplus ZE_{i\beta} \oplus ZE_{i\gamma}}{3}. \tag{14}$$

9. Rank the alternatives according to the values of ZE_i.

Figure 1 displays a flowchart of our proposed extended COCOSO method with 2TLFFNs.

Figure 1
Flowchart of proposed extended COCOSO method with 2TLFFNs



4. Application

In this section, we propose the extended 2TLFF-COCOSO method.

Example 4.1 (Selection of the best valve in DIS). The farming industry uses a large amount of fresh water for irrigation purposes. Water consumption can be reduced by using DIS. This can be accomplished with the assistance of water kept at a constant level. Water will be transferred through a proper pipeline, operated by a pump, and drawn from a water tank in a DIS. DI uses less water and is very cost-effective for high-value crops in developed countries. DIS is most suitable in the area where water resources are limited. It is a very efficient method of irrigation because of its economic returns. It is valuable for the irrigation of more land with less resources. Despite the higher initial investment, DIS saves a very large amount of water and leads to excellent results.

The selection and positioning of valves in an IS are crucial because the rate of flow and pressure must be carefully regulated throughout the system to guarantee effective and timely water delivery. Valves have important functions including distribution, flow, and regulating the pressure. It helps in making management easier and maximizing pressure when needed. It also needs less maintenance.

Figure 2 displays the flowchart of selection of best valve (SBV) for DIS.

This research article aims to introduce the MAGDM methodology to select the best valve in DIS. Table 2 shows a brief description of each alt.

A pictorial representation of alternatives is shown in Figure 3.

The attributes listed below should be taken into consideration while choosing the ideal valve:

1. Pressure sustaining capacity ($\dagger_1$),
2. Price ($\dagger_2$),
3. Reliable ($\dagger_3$),
4. Easy to implant ($\dagger_4$),
5. Long-term performance ($\dagger_5$).

Figure 2
Flowchart of SBV in DIS

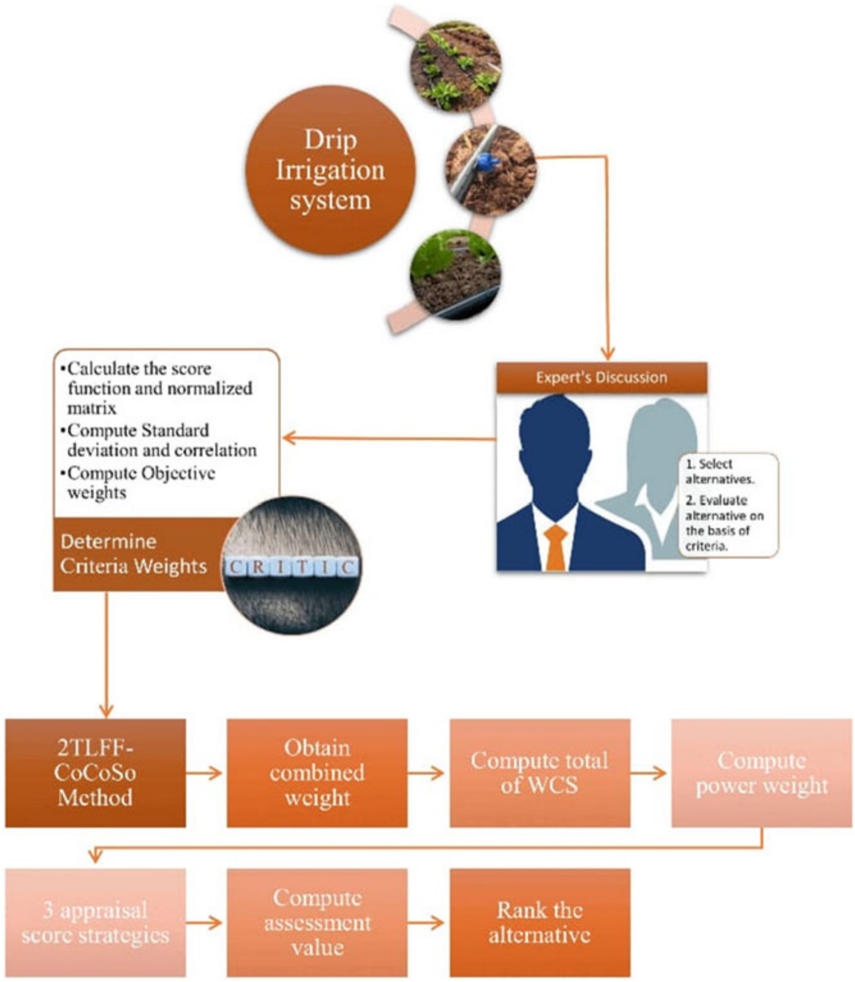


Table 2
A brief description of each alt

Alternatives	Brief description
Ball valves ($¥_1$)	This valve is also known as quarter-turn valve that contains a port through the middle of its closing mechanism. Its position is shown by a lever connected to it. The ball turns when we rotate the lever and flow occurs when the port is parallel with the pipe and blocks when the port and pipe are perpendicular to each other
Butterfly valves ($¥_2$)	It is very much similar to the ball valve except for the closing mechanism. Butterfly valves close in response to the movement of disk placed in the center. The disk is always present within the flow. It is designed to be fully opened or closed
Globe valves ($¥_3$)	It is a type of manual valve. This valve is used for regulating the flow. It also causes minimum power loss due to friction. It is operated by screw action
Gate valves ($¥_4$)	This type of valve opens when we lift a wedge from the path of fluid flow. It causes very low friction loss because it contains no obstacle in the path of its flow
Check valves ($¥_5$)	Its basic purpose includes priming of the inlet pipe. This valve prevents infiltration of the water source caused by fertilizers. It also prevents water from flowing back
Hydraulic control valve ($¥_6$)	It is a type of valve that turns on and off when we apply a pressure command on it either locally or remotely
Pressure relief valve ($¥_7$)	It opens immediately, fully, and accurately when we increase pressure of the system
Booster pump control valve ($¥_8$)	It is a double-chambered valve that responds to electrical inputs by opening and closing
Surge anticipating valve ($¥_9$)	Surge anticipating valve is an off-line valve that relives excessive system pressure
Combination air release valve ($¥_{10}$)	Combination air release valve efficiently enables air pockets in pressured pipelines to be released
Kinetic air valve ($¥_{11}$)	This valve releases a large volume of air during network draining

Figure 3
Selected alternatives

There are three DMs denoted by G_1, G_2, and G_3. The WV of DMs is (0.2, 0.3, 0.5).

1. Each DM judges the alternatives on each attribute. Table 3 displays the outcomes.

Table 3
Judgment of valves by DMs

DMs	Valves	$†_1$	$†_2$	$†_3$	$†_4$	$†_5$
G_1	$¥_1$	*mes*	*xela*	*xes*	*xes*	*xela*
	$¥_2$	*s*	*mes*	*xela*	*xes*	*mes*
	$¥_3$	*xes*	*xes*	*xela*	*s*	*xes*
	$¥_4$	*xes*	*xela*	*s*	*xes*	*xes*
	$¥_5$	*xes*	*xela*	*xes*	*xes*	*xela*
	$¥_6$	*s*	*mes*	*xes*	*xes*	*mes*
	$¥_7$	*xes*	*xela*	*xela*	*s*	*xes*
	$¥_8$	*xes*	*xela*	*s*	*xela*	*xes*
	$¥_9$	*xes*	*xela*	*xes*	*xes*	*xela*
	$¥_{10}$	*s*	*mes*	*xes*	*xes*	*mes*
	$¥_{11}$	*xes*	*xela*	*xela*	*s*	*xes*
G_2	$¥_1$	*s*	*xes*	*xes*	*s*	*s*
	$¥_2$	*s*	*s*	*xela*	*xela*	*xela*
	$¥_3$	*xes*	*xes*	*xela*	*s*	*xes*
	$¥_4$	*xes*	*xes*	*s*	*xela*	*xes*
	$¥_5$	*xela*	*xes*	*xes*	*s*	*s*
	$¥_6$	*s*	*s*	*xela*	*xela*	*xela*
	$¥_7$	*xes*	*xes*	*xela*	*s*	*xes*
	$¥_8$	*xes*	*xes*	*s*	*xela*	*xes*
	$¥_9$	*xela*	*xes*	*xes*	*s*	*xela*
	$¥_{10}$	*s*	*s*	*xela*	*xela*	*mes*
	$¥_{11}$	*xexes*	*mes*	*xela*	*s*	*xes*
G_3	$¥_1$	*xela*	*xes*	*xela*	*xela*	*xela*
	$¥_2$	*s*	*xes*	*xes*	*xes*	*mes*
	$¥_3$	*xela*	*xela*	*xes*	*s*	*xela*
	$¥_4$	*xela*	*xela*	*s*	*xela*	*xela*
	$¥_5$	*xes*	*xes*	*xela*	*xela*	*xela*
	$¥_6$	*s*	*xes*	*xes*	*xes*	*mes*
	$¥_7$	*xela*	*xes*	*xela*	*s*	*xela*
	$¥_8$	*xela*	*xela*	*s*	*xela*	*xela*
	$¥_9$	*xes*	*xes*	*xela*	*xela*	*xela*
	$¥_{10}$	*s*	*xes*	*xes*	*xes*	*mes*
	$¥_{11}$	*xela*	*xes*	*xes*	*s*	*xela*

2. Convert the linguistic assessing matrix into assessing matrix. Table 4 displays the results.

Table 4
Assessing matrix by DMs

DMs	Valves	$\dagger_1$	$\dagger_2$	$\dagger_3$	$\dagger_4$	$\dagger_5$
G_1	$¥_1$	$\{(v_2,0),(v_4,0)\}$	$\{(v_5,0),(v_1,0)\}$	$\{(v_1,0),(v_5,0)\}$	$\{(v_1,0),(v_5,0)\}$	$\{(v_4,0),(v_2,0)\}$
	$¥_2$	$\{(v_3,0),(v_3,0)\}$	$\{(v_2,0),(v_4,0)\}$	$\{(v_4,0),(v_2,0)\}$	$\{(v_1,0),(v_5,0)\}$	$\{(v_2,0),(v_4,0)\}$
	$¥_3$	$\{(v_1,0),(v_5,0)\}$	$\{(v_1,0),(v_5,0)\}$	$\{(v_5,0),(v_5,0)\}$	$\{(v_3,0),(v_3,0)\}$	$\{(v_1,0),(v_5,0)\}$
	$¥_4$	$\{(v_1,0),(v_5,0)\}$	$\{(v_4,0),(v_2,0)\}$	$\{(v_3,0),(v_3,0)\}$	$\{(v_1,0),(v_5,0)\}$	$\{(v_1,0),(v_5,0)\}$
	$¥_5$	$\{(v_1,0),(v_5,0)\}$	$\{(v_5,0),(v_1,0)\}$	$\{(v_1,0),(v_1,0)\}$	$\{(v_1,0),(v_5,0)\}$	$\{(v_4,0),(v_2,0)\}$
	$¥_6$	$\{(v_3,0),(v_3,0)\}$	$\{(v_2,0),(v_4,0)\}$	$\{(v_1,0),(v_4,0)\}$	$\{(v_1,0),(v_5,0)\}$	$\{(v_2,0),(v_4,0)\}$
	$¥_7$	$\{(v_1,0),(v_5,0)\}$	$\{(v_5,0),(v_1,0)\}$	$\{(v_5,0),(v_1,0)\}$	$\{(v_3,0),(v_3,0)\}$	$\{(v_1,0),(v_5,0)\}$
	$¥_8$	$\{(v_1,0),(v_5,0)\}$	$\{(v_4,0),(v_2,0)\}$	$\{(v_3,0),(v_3,0)\}$	$\{(v_5,0),(v_1,0)\}$	$\{(v_1,0),(v_5,0)\}$
	$¥_9$	$\{(v_1,0),(v_5,0)\}$	$\{(v_5,0),(v_1,0)\}$	$\{(v_1,0),(v_1,0)\}$	$\{(v_1,0),(v_5,0)\}$	$\{(v_4,0),(v_2,0)\}$
	$¥_{10}$	$\{(v_3,0),(v_3,0)\}$	$\{(v_2,0),(v_4,0)\}$	$\{(v_1,0),(v_4,0)\}$	$\{(v_1,0),(v_5,0)\}$	$\{(v_2,0),(v_4,0)\}$
	$¥_{11}$	$\{(v_1,0),(v_5,0)\}$	$\{(v_5,0),(v_1,0)\}$	$\{(v_5,0),(v_5,0)\}$	$\{(v_3,0),(v_3,0)\}$	$\{(v_1,0),(v_5,0)\}$
G_2	$¥_1$	$\{(v_3,0),(v_3,0)\}$	$\{(v_1,0),(v_5,0)\}$	$\{(v_1,0),(v_5,0)\}$	$\{(v_3,0),(v_3,0)\}$	$\{(v_3,0),(v_3,0)\}$
	$¥_2$	$\{(v_3,0),(v_3,0)\}$	$\{(v_3,0),(v_1,0)\}$	$\{(v_4,0),(v_2,0)\}$	$\{(v_5,0),(v_1,0)\}$	$\{(v_5,0),(v_1,0)\}$
	$¥_3$	$\{(v_1,0),(v_5,0)\}$	$\{(v_1,0),(v_5,0)\}$	$\{(v_4,0),(v_2,0)\}$	$\{(v_3,0),(v_3,0)\}$	$\{(v_1,0),(v_5,0)\}$
	$¥_4$	$\{(v_1,0),(v_5,0)\}$	$\{(v_1,0),(v_1,0)\}$	$\{(v_3,0),(v_3,0)\}$	$\{(v_4,0),(v_2,0)\}$	$\{(v_1,0),(v_5,0)\}$
	$¥_5$	$\{(v_5,0),(v_1,0)\}$	$\{(v_1,0),(v_1,0)\}$	$\{(v_1,0),(v_5,0)\}$	$\{(v_3,0),(v_3,0)\}$	$\{(v_3,0),(v_3,0)\}$
	$¥_6$	$\{(v_3,0),(v_3,0)\}$	$\{(v_3,0),(v_3,0)\}$	$\{(v_4,0),(v_2,0)\}$	$\{(v_5,0),(v_1,0)\}$	$\{(v_5,0),(v_1,0)\}$
	$¥_7$	$\{(v_1,0),(v_5,0)\}$	$\{(v_1,0),(v_5,0)\}$	$\{(v_4,0),(v_2,0)\}$	$\{(v_3,0),(v_3,0)\}$	$\{(v_1,0),(v_5,0)\}$
	$¥_8$	$\{(v_1,0),(v_5,0)\}$	$\{(v_1,0),(v_5,0)\}$	$\{(v_3,0),(v_3,0)\}$	$\{(v_4,0),(v_2,0)\}$	$\{(v_1,0),(v_5,0)\}$
	$¥_9$	$\{(v_5,0),(v_1,0)\}$	$\{(v_1,0),(v_5,0)\}$	$\{(v_1,0),(v_5,0)\}$	$\{(v_3,0),(v_3,0)\}$	$\{(v_4,0),(v_2,0)\}$
	$¥_{10}$	$\{(v_3,0),(v_3,0)\}$	$\{(v_3,0),(v_3,0)\}$	$\{(v_4,0),(v_2,0)\}$	$\{(v_5,0),(v_2,0)\}$	$\{(v_2,0),(v_4,0)\}$
	$¥_{11}$	$\{(v_0,0),(v_6,0)\}$	$\{(v_2,0),(v_4,0)\}$	$\{(v_4,0),(v_2,0)\}$	$\{(v_3,0),(v_3,0)\}$	$\{(v_1,0),(v_5,0)\}$
G_3	$¥_1$	$\{(v_4,0),(v_2,0)\}$	$\{(v_1,0),(v_5,0)\}$	$\{(v_5,0),(v_1,0)\}$	$\{(v_4,0),(v_2,0)\}$	$\{(v_4,0),(v_2,0)\}$
	$¥_2$	$\{(v_3,0),(v_3,0)\}$	$\{(v_1,0),(v_5,0)\}$	$\{(v_1,0),(v_5,0)\}$	$\{(v_1,0),(v_5,0)\}$	$\{(v_2,0),(v_4,0)\}$
	$¥_3$	$\{(v_5,0),(v_1,0)\}$	$\{(v_5,0),(v_1,0)\}$	$\{(v_1,0),(v_5,0)\}$	$\{(v_3,0),(v_3,0)\}$	$\{(v_4,0),(v_2,0)\}$
	$¥_4$	$\{(v_5,0),(v_1,0)\}$	$\{(v_5,0),(v_1,0)\}$	$\{(v_3,0),(v_3,0)\}$	$\{(v_5,0),(v_1,0)\}$	$\{(v_5,0),(v_1,0)\}$
	$¥_5$	$\{(v_1,0),(v_5,0)\}$	$\{(v_1,0),(v_5,0)\}$	$\{(v_5,0),(v_1,0)\}$	$\{(v_4,0),(v_2,0)\}$	$\{(v_4,0),(v_2,0)\}$
	$¥_6$	$\{(v_3,0),(v_3,0)\}$	$\{(v_1,0),(v_5,0)\}$	$\{(v_1,0),(v_5,0)\}$	$\{(v_1,0),(v_5,0)\}$	$\{(v_2,0),(v_4,0)\}$
	$¥_7$	$\{(v_5,0),(v_1,0)\}$	$\{(v_1,0),(v_5,0)\}$	$\{(v_5,0),(v_1,0)\}$	$\{(v_3,0),(v_3,0)\}$	$\{(v_4,0),(v_2,0)\}$
	$¥_8$	$\{(v_5,0),(v_1,0)\}$	$\{(v_5,0),(v_1,0)\}$	$\{(v_3,0),(v_3,0)\}$	$\{(v_1,0),(v_5,0)\}$	$\{(v_5,0),(v_1,0)\}$
	$¥_9$	$\{(v_1,0),(v_5,0)\}$	$\{(v_1,0),(v_5,0)\}$	$\{(v_5,0),(v_1,0)\}$	$\{(v_4,0),(v_2,0)\}$	$\{(v_4,0),(v_2,0)\}$
	$¥_{10}$	$\{(v_3,0),(v_3,0)\}$	$\{(v_1,0),(v_5,0)\}$	$\{(v_1,0),(v_5,0)\}$	$\{(v_1,0),(v_5,0)\}$	$\{(v_2,0),(v_4,0)\}$
	$¥_{11}$	$\{(v_5,0),(v_1,0)\}$	$\{(v_1,0),(v_5,0)\}$	$\{(v_1,0),(v_5,0)\}$	$\{(v_3,0),(v_3,0)\}$	$\{(v_4,0),(v_2,0)\}$

3. Calculate score values and the outcomes are given in Table 5.

Table 5
Score values

Valves	$\dagger_1$	$\dagger_2$	$\dagger_3$	$\dagger_4$	$\dagger_5$
$¥_1$	4.370884766	4.763851667	4.763851667	4.277263317	5.119855967
$¥_2$	4.119140603	3.49990616	4.790447339	4.763851667	4.955951836
$¥_3$	4.763851667	4.763851667	5.636507392	4.119140603	3.91664189
$¥_4$	4.763851667	5.636507392	4.119140603	5.636507392	4.763851667
$¥_5$	4.763851667	4.763851667	4.763851667	4.277263317	5.119855967
$¥_6$	4.119140603	3.49990616	4.790447339	4.763851667	4.955951836
$¥_7$	4.763851667	4.763851667	5.636507392	4.119140603	3.91664189
$¥_8$	4.763851667	5.636507392	4.119140603	5.636507392	4.763851667
$¥_9$	4.763851667	4.763851667	4.763851667	4.277263317	5.119855967
$¥_{10}$	4.119140603	3.49990616	4.790447339	4.763851667	4.955951836
$¥_{11}$	4.749999942	4.860813423	5.636507392	4.11914060	3.91664189

4. Switch the matrix into a standard 2TLFF matrix and we obtain normalized matrix as shown in Table 6.

Table 6
Normalized matrix

Valves	$\dagger_1$	$\dagger_2$	$\dagger_3$	$\dagger_4$	$\dagger_5$
¥$_1$	0.390475947	0.408431724	0.424888082	0.10420863	1
¥$_2$	0	1	0.442415598	0.424888082	0.863778081
¥$_3$	1	0.408431724	1	0	0
¥$_4$	1	0	0	1	0.704122228
¥$_5$	1	0.408431724	0.424888082	0.209680017	1
¥$_6$	0.28982218	1	0.442415598	0.492602344	0.863778081
¥$_7$	0.591568276	0.408431724	1	0.117741017	0
¥$_8$	0.591568276	0	0	1	0.704122228
¥$_9$	0.591568276	0.408431724	0.424888082	0.2997151	1
¥$_{10}$	0.289822187	1	0.442415598	0.704122228	1
¥$_{11}$	0.585085211	0.511210589	1	0.168298158	1

5. Calculate the standard deviation, correlation, and quantity of information using Equations (6), (7), and (8). We obtain the OWs of each attribute listed in Table 7.

Table 7
Objective weights

ξ_1	ξ_2	ξ_3	ξ_4	ξ_5
.171055978	0.169314013	0.160159666	0.183869801	0.315600541

6. The SWs of each attribute given by DMs is shown in Table 8.

Table 8
Subjective weights

ξ_1'	ξ_2'	ξ_3'	ξ_4'	ξ_5'
.2344	0.0705	0.3209	0.2345	0.1397

7. Calculate CWs using Equation (10). Table 9 displays the results.

Table 9
Combined weights

ω_1	ω_2	ω_3	ω_4	ω_5
.210326944	0.062615387	0.269601261	0.226179011	0.231277398

8. Calculate the aggregated decision matrix with 2TLFFN and the results display in Table 10.

Table 10
Aggregated decision matrix

Valves	$\dagger_1$	$\dagger_2$	$\dagger_3$	$\dagger_4$	$\dagger_5$
¥$_1$	$\{(v_5,-0.38),(v_1,-0.002)\}$	$\{(v_5,0.02),(v_0,0.009)\}$	$\{(v_5,0.02),(v_0,0.009)\}$	$\{(v_5,-0.49),(v_0,0.01)\}$	$\{(v_5,0.34),(v_0,0.001)\}$
¥$_2$	$\{(v_4,0.31),(v_, 0.01)\}$	$\{(v_3,0.3),(v_0,0.12)\}$	$\{(v_5,0.05),(v_0,0.004)\}$	$\{(v_6,-0.36),(v_0,0)\}$	$\{(v_5,0.02),(v_0,0.009)\}$
¥$_3$	$\{(v_5,0.02),(v_0,0.009)\}$	$\{(v_5,0.02),(v_0,0.009)\}$	$\{(v_5,0.02),(v_0,0.009)\}$	$\{(v_5,0.02),(v_0,0.009)\}$	$\{(v_4,0.04),(v_0,0.001)\}$
¥$_4$	$\{(v_5,0.02),(v_0,0.009)\}$	$\{(v_6,-0.36),(v_0,0)\}$	$\{(v_6,-0.36),(v_0,0)\}$	$\{(v_4,0.31),(v_0,0.011)\}$	$\{(v_5,0.02),(v_0,0.009)\}$
¥$_5$	$\{(v_5,0.02),(v_0,0.009)\}$	$\{(v_5,0.02),(v_0,0.009)\}$	$\{(v_4,0.31),(v_0,0.011)\}$	$\{(v_5,-0.49),(v_0,0.01)\}$	$\{(v_5,0.34),(v_0,0.001)\}$
¥$_6$	$\{(v_4,0.31),(v_0,0.01)\}$	$\{(v_3,0.3),(v_0,0.12)\}$	$\{(v_5,0.02),(v_0,0.009)\}$	$\{(v_5,0.02),(v_0,0.009)\}$	$\{(v_5,0.20),(v_0,0.002)\}$
¥$_7$	$\{(v_5,0.02),(v_0,0.009)\}$	$\{(v_5,0.02),(v_0,0.009)\}$	$\{(v_6,-0.36),(v_0,0)\}$	$\{(v_4,0.31),(v_0,0.011)\}$	$\{(v_4,0.04),(v_0,0.001)\}$
¥$_8$	$\{(v_5,0.02),(v_0,0)\}$	$\{(v_6,-0.36),(v_0,0)\}$	$\{(v_4,0.31),(v_0,0.011)\}$	$\{(v_6,-0.26),(v_0,0.001)\}$	$\{(v_5,0.02),(v_0,0.009)\}$
¥$_9$	$\{(v_5,0.02),(v_0,0)\}$	$\{(v_5,0.02),(v_0,0.009)\}$	$\{(v_5,0.02),(v_0,0.009)\}$	$\{(v_5,-0.49),(v_0,0.01)\}$	$\{(v_5,0.34),(v_0,0.001)\}$
¥$_{10}$	$\{(v_4,0.31),(v_0,0.01)\}$	$\{(v_3,0.3),(v_0,0.12)\}$	$\{(v_5,0.05),(v_0,0.004)\}$	$\{(v_5,0.02),(v_0,0.009)\}$	$\{(v_5,0.02),(v_0,0.009)\}$
¥$_{11}$	$\{(v_5,0.01),(v_0,0.01)\}$	$\{(v_5,0.11),(v_0,0.004)\}$	$\{(v_6,-0.36),(v_0,0)\}$	$\{(v_4,0.31),(v_0,0.011)\}$	$\{(v_4,0.04),(v_0,0.001)\}$

9. Compute weighted CS using 2TLFFHWAO. Table 11 displays the results.

Table 11
Weighted CS using 2TLFFHWAO

SE_1	$\{(v_5, 0.-05), (v_0, 0.006)\}$
SE_2	$\{(v_5, 0.-09), (v_0, 0.04)\}$
SE_3	$\{(v_5, 0.15), (v_0, 0.02)\}$
SE_4	$\{(v_5, 0.26), (v_0, 0.001)\}$
SE_5	$\{(v_5, 0.03), (v_0, 0.04)\}$
SE_6	$\{(v_5, -0.09), (v_0, 0.007)\}$
SE_7	$\{(v_5, 0.15), (v_0, 0.002)\}$
SE_8	$\{(v_5, 0.26), (v_0, 0.0044)\}$
SE_9	$\{(v_5, 0.03), (v_0, 0.006)\}$
SE_{10}	$\{(v_5, -0.09), (v_0, 0.007)\}$
SE_{11}	$\{(v_5, 0.15), (v_0, 0.002)\}$

10. Calculate PW of CS using 2TLFFHWGO. Table 12 displays the outcomes.

Table 12
PW of CS

PE_1	$\{(v_5, -0.22), (v_0, 0.01)\}$
PE_2	$\{(v_5, -0.21), (v_0, 0.07)\}$
PE_3	$\{(v_5, 0.24), (v_0, 0.04)\}$
PE_4	$\{(v_5, 0.01), (v_0, 0.009)\}$
PE_5	$\{(v_5, 0.11), (v_0, 0.01\}$
PE_6	$\{(v_5, -0.21), (v_0, 0.05)\}$
PE_7	$\{(v_5, -0.21), (v_0, 0.045)\}$
PE_8	$\{(v_5, 0.01), (v_0, 0.009)\}$
PE_9	$\{(v_5, -0.03), (v_0, 0.01)\}$
PE_{10}	$\{(v_5, -0.21), (v_0, 0.045)\}$
PE_{11}	$\{(v_5, -0.20), (v_0, 0.045)\}$

11. Calculate three appraisal score strategies and Tables 13, 14, and 15 display the results.

Table 13
Relative weights of alternatives by first appraisal score strategy

$ZE_{1\alpha}$	$\{(v_0, 0.09), (v_0, 0.001)\}$
$ZE_{2\alpha}$	$\{(v_0, 0.08), (v_1, -0.05)\}$
$ZE_{3\alpha}$	$\{(v_0, 0.09), (v_0, 0.03)\}$
$ZE_{4\alpha}$	$\{(v_0, 0.09), (v_0, 0.008)\}$
$ZE_{5\alpha}$	$\{(v_0, 0.09), (v_0, 0.004)\}$
$ZE_{6\alpha}$	$\{(v_0, 0.08), (v_0, 0.001)\}$
$ZE_{7\alpha}$	$\{(v_0, 0.09), (v_0, -0.002)\}$
$ZE_{8\alpha}$	$\{(v_0, 0.09), (v_0, -0.006)\}$
$ZE_{9\alpha}$	$\{(v_0, 0.09), (v_0, -0)\}$
$ZE_{10\alpha}$	$\{(v_0, 0.09), (v_0, -0.005)\}$
$ZE_{11\alpha}$	$\{(v_0, 0.09), (v_0, 0.001)\}$

Table 14
Relative weights of alternatives by second appraisal score strategy

$ZE_{1\beta}$	$\{(v_1, 0.27), (v_0, 0.01)\}$
$ZE_{2\beta}$	$\{(v_1, 0.25), (v_1, 0.33)\}$
$ZE_{3\beta}$	$\{(v_1, 0.34), (v_4, 0.03)\}$
$ZE_{4\beta}$	$\{(v_1, 0.33), (v_0, 0.001)\}$
$ZE_{5\beta}$	$\{(v_1, -0.31), (v_6, -0.12)\}$
$ZE_{6\beta}$	$\{(v_1, 0.25), (v_2, 0.24)\}$
$ZE_{7\beta}$	$\{(v_1, 0.28), (v_0, 0.04)\}$
$ZE_{8\beta}$	$\{(v_1, 0.33), (v_0, -0.0034)\}$
$ZE_{9\beta}$	$\{(v_1, 0.29), (v_0, 0.01)\}$
$ZE_{10\beta}$	$\{(v_1, 0.25), (v_2, 0.25)\}$
$ZE_{11\beta}$	$\{(v_1, 0.29), (v_2, 0.49)\}$

Table 15
Relative weights of alternatives by third appraisal score strategy

$ZE_{1\gamma}$	$\{(v_0, 0.5), (v_0, 0.0001)\}$
$ZE_{2\gamma}$	$\{(v_0, 0.57), (v_0, 0.0033)\}$
$ZE_{3\gamma}$	$\{(v_0, 0.61), (v_0, 0.0002)\}$
$ZE_{4\gamma}$	$\{(v_0, 0.60), (v_0, 0)\}$
$ZE_{5\gamma}$	$\{(v_0, 0.601), (v_0, 0)\}$
$ZE_{6\gamma}$	$\{(v_1, -0.43), (v_0, 0)\}$
$ZE_{7\gamma}$	$\{(v_1, -0.44), (v_0, 0.0008)\}$
$ZE_{8\gamma}$	$\{(v_1, 0.40), (v_0, 0.0003)\}$
$ZE_{9\gamma}$	$\{(v_1, -0.41), (v_0, 0.0003)\}$
$ZE_{10\gamma}$	$\{(v_1, -0.43), (v_0, 0.0005)\}$
$ZE_{11\gamma}$	$\{(v_1, -0.46), (v_0, 0.0004)\}$

12. Compute the assessment values using Equation (14). Table 16 displays the outcomes.

Table 16
Assessment values

ZE_1	$\{(v_2, -0.36), (v_0, 0)\}$
ZE_2	$\{(v_1, -0.40), (v_0, 0)\}$
ZE_3	$\{(v_2, -0.25), (v_0, 0)\}$
ZE_4	$\{(v_2, -0.31), (v_0, 0)\}$
ZE_5	$\{(v_1, -0.28), (v_0, 0)\}$
ZE_6	$\{(v_2, -0.40), (v_0, 0)\}$
ZE_7	$\{(v_2, -0.40), (v_0, 0)\}$
ZE_8	$\{(v_2, -0.31), (v_0, 0)\}$
ZE_9	$\{(v_2, -0.33), (v_0, 0)\}$
ZE_{10}	$\{(v_2, -0.40), (v_0, 0)\}$
ZE_{11}	$\{(v_2, -0.461), (v_0, 0)\}$

13. Calculate score values and on the basis of these values we rank the alternatives. Table 17 shows the score values.

Table 17
Score values

Alternatives	Score values
$¥_1$	3.061269965
$¥_2$	3.05753899547
$¥_3$	3.075655132
$¥_4$	3.066235979
$¥_5$	3.069950367
$¥_6$	3.0575389950654
$¥_7$	3.057538995876
$¥_8$	3.0645989174
$¥_9$	3.0645989172
$¥_{10}$	3.057667133
$¥_{11}$	3.057764754

Ranking of alternatives is given below:

$$¥_3 \geq ¥_5 \geq ¥_4 \geq ¥_8 \geq ¥_9 \geq ¥_1 \geq ¥_{11} \geq ¥_{10} \geq ¥_7 \geq ¥_2 \geq ¥_6.$$

5. Parametric Analysis

In our study, we use the parameter κ to describe the interdependence between several measurable features. Different values for this parameter also help to highlight the range of possible DeM scenarios. Table 18 shows the result when we change the value of the κ parameter from 1 to 6. According to Table 19, $¥_3$ is still the best choice, while $¥_6$ is the worst in any case. This validates our recommended technique.

Figure 4 shows a graph of the SBV ranking results for DIS by 2TLFFHWAO using different values of parameter κ.

6. Comparative Analysis

In this section, we compare our 2TLFF-COCOSO method with various existing operators, including 2TLPFWAO Deng et al. (2018), 2TLPFWGO Deng et al. (2018), 2TLPFWHMO Deng et al. (2018), and 2TLPFWDHMO Deng et al. (2018). We provide a comparative study to evaluate the importance and efficacy of the proposed 2TLFF-COCOSO technique. We compare the created method with four existing operators to prove that it is legal. Table 20 compares the proposed method with 2TLPFWHMO Deng et al. (2018).

Table 18
Ranking results by 2TLFFHWAO using $\kappa = 1, 2, 3, 4, 5, 6$.

Alternatives	$\kappa = 1$	$\kappa = 2$	$\kappa = 3$	$\kappa = 4$	$\kappa = 5$	$\kappa = 6$
$¥_1$	$(v_5, -0.26)$	$(v_5, -0.28)$	$(v_5, -0.28))$	$(v_5, -0.29)$	$(v_5, -0.29)$	$(v_5, -0.30)$
$¥_2$	$(v_5, -0.39)$	$(v_5, -0.42)$	$(v_5, -0.45)$	$(v_5, -0.46)$	$(v_5, -0.48)$	$(v_5, -0.49)$
$¥_3$	$(v_5, 0.003)$	$(v_5, -0.04)$	$(v_5, -0.06)$	$(v_5, -0.07)$	$(v_5, -0.08)$	$(v_5, -0.09)$
$¥_4$	$(v_5, -0.13)$	$(v_5, -0.19)$	$(v_5, -0.21)$	$(v_5, -0.23)$	$(v_5, -0.24)$	$(v_5, -0.25)$
$¥_5$	$(v_5, 0.002)$	$(v_5, -0.05)$	$(v_5, -0.07)$	$(v_5, -0.08)$	$(v_5, -0.09)$	$(v_5, -0.10)$
$¥_6$	$(v_5, -0.40)$	$(v_5, -0.44)$	$(v_5, -0.46)$	$(v_5, -0.47)$	$(v_5, -0.47)$	$(v_5, -0.50)$
$¥_7$	$(v_5, -0.38)$	$(v_5, -0.39)$	$(v_5, -0.40)$	$(v_5, -0.42)$	$(v_5, -0.42)$	$(v_5, -0.45)$
$¥_8$	$(v_5, -0.14)$	$(v_5, -0.20)$	$(v_5, -0.22)$	$(v_5, -0.23)$	$(v_5, -0.25)$	$(v_5, -0.26)$
$¥_9$	$(v_5, -0.13)$	$(v_5, -0.19)$	$(v_5, -0.21)$	$(v_5, -0.24)$	$(v_5, -0.26)$	$(v_5, -0.29)$
$¥_{10}$	$(v_5, -0.32)$	$(v_5, -0.33)$	$(v_5, -0.34)$	$(v_5, -0.34)$	$(v_5, -0.37)$	$(v_5, -0.38)$
$¥_{11}$	$(v_5, -0.27)$	$(v_5, -0.29)$	$(v_5, -0.29))$	$(v_5, -0.30)$	$(v_5, -0.30)$	$(v_5, -0.31)$

Table 19
Score values by 2TLFFHWAO using $\kappa = 1, 2, 3, 4, 5, 6$.

Alternatives	$\kappa = 1$	$\kappa = 2$	$\kappa = 3$	$\kappa = 4$	$\kappa = 5$	$\kappa = 6$
$¥_1$	4.613429278	4.265756106	4.4648882229	4.643103326	4.3638836542	4.3635480329
$¥_2$	4.0628871941	4.0608669397	4.159658227	4.588250622	4.20582028336	4.1577132871
$¥_3$	5.003849654	4.807976248	4.778708919	4.760857345	4.748617324	4.739590599
$¥_4$	4.868110177	4.662380033	4.719543779	4.7030452821	4.6922115966	4.6915916427
$¥_5$	4.948141185	4.73514085	4.727772933	4.722744923	4.718965022	4.715950478
$¥_6$	4.05628871941	4.00608669397	4.1059658227	4.578250622	4.11582028336	4.0577132871
$¥_7$	4.1868110177	4.11807976248	4.1778708919	4.610857345	4.22748617324	4.33739590599
$¥_8$	4.7003849654	4.4962380033	4.116942543779	4.6930452821	4.5922115966	4.5915916427
$¥_9$	4.648141185	4.3514085	4.5727772933	4.6722744923	4.4718965022	4.4715950478
$¥_{10}$	4.2628871941	4.1608669397	4.259658227	4.628250622	4.2582028336	4.577132871
$¥_{11}$	4.57100533	4.21812379982	4.3782702882	4.63635	4.2752260272	4.34314129

Figure 4
Ranking results of SBV for DIS by 2TLFFHWAO using $\kappa = 1, 2, 3, 4, 5, 6$.

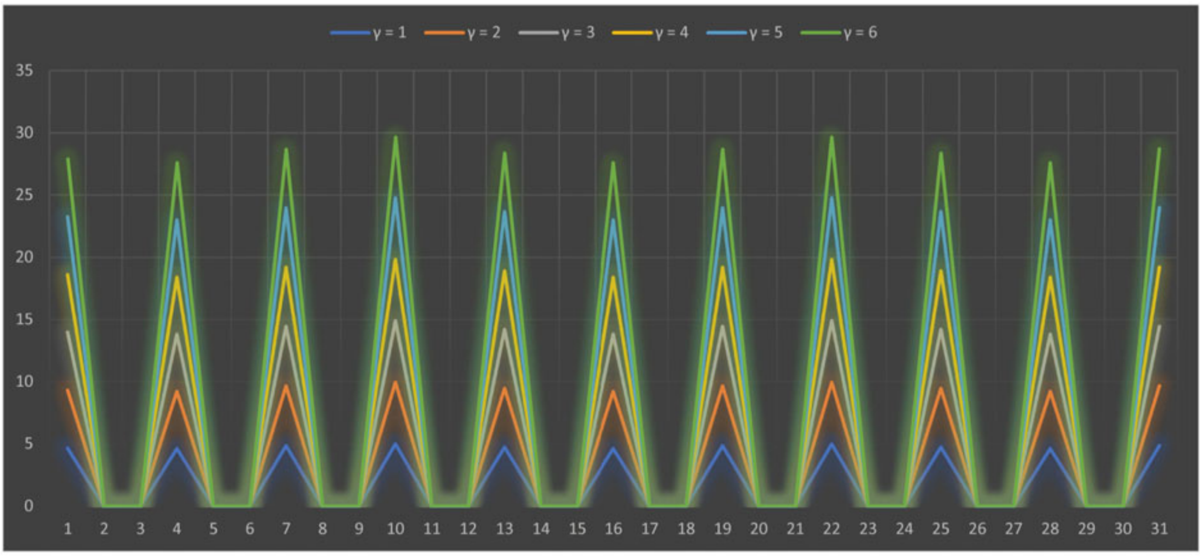


Table 20
Score functions and ranking results by 2TLPFWHMO Deng et al. (2018)

Alternatives	2TLPFWHMO Deng et al. (2018)	Score	Ranking
¥$_1$	$\{(v_6, -0.0000332), (v_0, 0)\}$	5.999953286	5
¥$_2$	$\{(v_6, -0.0001), (v_0, 0)\}$	5.99852456	10
¥$_3$	$\{(v_6, -0.00000), (v_0, 0)\}$	6.000000000	1
¥$_4$	$\{(v_6, 0.000023), (v_0, 0)\}$	5.999997900	3
¥$_5$	$\{(v_6, -0.00001), (v_0, 0)\}$	5.999997922	2
¥$_6$	$\{(v_6, -0.000081), (v_0, 0)\}$	5.998521643	11
¥$_7$	$\{(v_6, -0.000065), (v_0, 0)\}$	5.998532345	9
¥$_8$	$\{(v_6, -0.000087), (v_0, 0)\}$	5.9999978765	4
¥$_9$	$\{(v_6, -0.000076), (v_0, 0)\}$	5.999953124	6
¥$_{10}$	$\{(v_6, -0.000053), (v_0, 0)\}$	5.9985397654	8
¥$_{11}$	$\{(v_6, -0.000035), (v_0, 0)\}$	5.998539875	7

Comparative analysis of proposed method with 2TLPFDWHMO Deng et al. (2018) is shown in Table 21.

Table 21
Score functions and ranking results by 2TLPFDWHMO Deng et al. (2018)

Alternatives	2TLPFDWHMO Deng et al. (2018)	Score	Ranking
¥$_1$	$\{(v_6, -0.000017659), (v_0, 0)\}$	5.9999537892	5
¥$_2$	$\{(v_6, -0.000154), (v_0, 0)\}$	5.9766543	10
¥$_3$	$\{(v_6, 0.0000), (v_0, 0)\}$	6.00000000	1
¥$_4$	$\{(v_6, 0.00002367), (v_0, 0)\}$	5.9999978976	3
¥$_5$	$\{(v_6, -0.0000123), (v_0, 0)\}$	5.999997943	2
¥$_6$	$\{(v_6, -0.000081), (v_0, 0)\}$	5.952764369	11
¥$_7$	$\{(v_6, -0.0000356), (v_0, 0)\}$	5.98176542	9
¥$_8$	$\{(v_6, -0.00007113), (v_0, 0)\}$	5.99999775843	4
¥$_9$	$\{(v_6, -0.0000432), (v_0, 0)\}$	5.98654323	7
¥$_{10}$	$\{(v_6, -0.0000543567), (v_0, 0)\}$	5.985365423	8
¥$_{11}$	$\{(v_6, -0.00001657), (v_0, 0)\}$	5.99987536	6

Comparative analysis of proposed method with 2TLPFWAO Deng et al. (2018) is shown in Table 22.

Table 22
Score functions and ranking results by 2TLPFWAO Deng et al. (2018)

Alternatives	2TLPFWAO Deng et al. (2018)	Score	Ranking
$¥_1$	$\{(v_3, 0.0210614782), (v_0, 0)\}$	3.13456789	6
$¥_2$	$\{(v_2, 0.610588113), (v_0, 0)\}$	3.0012345678	10
$¥_3$	$\{(v_3, 0.37311553), (v_0, 0)\}$	3.93456789	1
$¥_4$	$\{(v_3, 0.18765678), (v_0, 0)\}$	3.5678909876	3
$¥_5$	$\{(v_3, 0.25679221), (v_0, 0)\}$	3.87654327	2
$¥_6$	$\{(v_2, 0.086543678), (v_0, 0)\}$	3.0007112345	11
$¥_7$	$\{(v_2, 0.98765432), (v_0, 0)\}$	3.00567890	9
$¥_8$	$\{(v_3, 0.09432113), (v_0, 0)\}$	3.34567899	4
$¥_9$	$\{(v_3, 0.05432432), (v_0, 0)\}$	3.243526789	5
$¥_{10}$	$\{(v_3, 0.0000543567), (v_0, 0)\}$	3.00876431	8
$¥_{11}$	$\{(v_3, 0.004321657), (v_0, 0)\}$	3.03456789	7

Comparative analysis of proposed method with 2TLPFWGO Deng et al. (2018) is shown in Table 23.

Table 23
Score functions and ranking results by 2TLPFWGO Deng et al. (2018)

Alternatives	2TLPFWGO Deng et al. (2018)	Score	Ranking
$¥_1$	$\{(v_0, 0.60633815), (v_5, 0.026531505)\}$	1.0042697733	6
$¥_2$	$\{(v_0, 0.253545121), (v_3, 0.301823936)\}$	0.0380953183	11
$¥_3$	$\{(v_0, 0.534015831), (v_5, 0.027063561)\}$	2.500273309	1
$¥_4$	$\{(v_1, 0.080036715), (v_5, -0.26280685)\}$	2.000273309	3
$¥_5$	$\{(v_0, 0.25679221), (v_5, 0.02653337)\}$	2.40027330	2
$¥_6$	$\{(v_0, 0.780146212), (v_3, 0.301823936)\}$	0.3809531835	10
$¥_7$	$\{(v_0, 0.253545121), (v_5, 0.027063526)\}$	1.0012376599	8
$¥_8$	$\{(v_1, 0.534015831), (v_5, -0.26280685)\}$	1.242697733	4
$¥_9$	$\{(v_0, 0.080036715), (v_5, 0.026533375)\}$	1.042697733	5
$¥_{10}$	$\{(v_0, 0.253545121), (3_0, 0.301823936)\}$	1.000140852	9
$¥_{11}$	$\{(v_0, 0.549065256), (5_0, 0.117536968)\}$	1.002379945	7

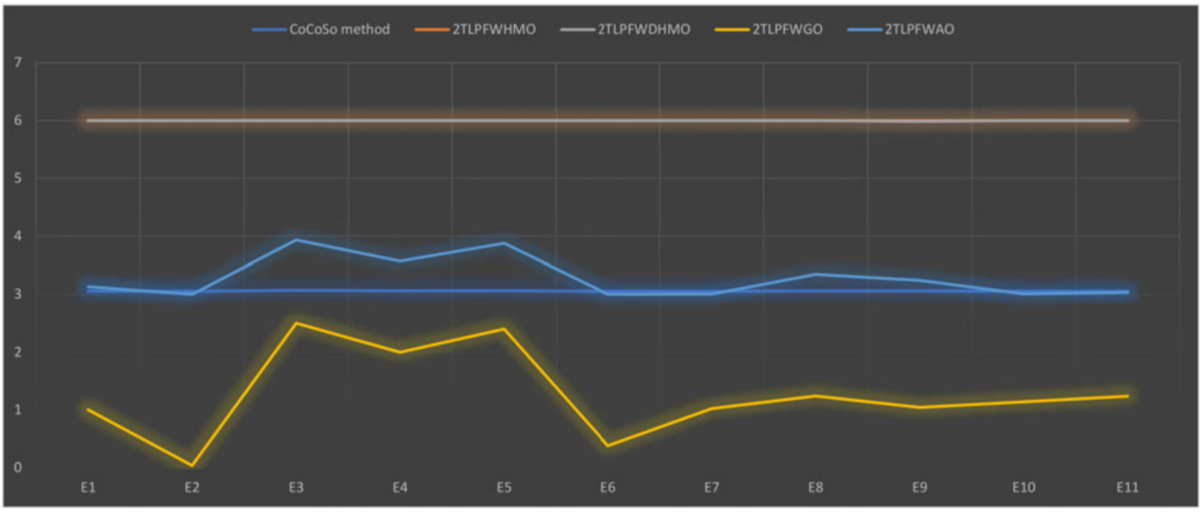
Table 24 shows the final ranking of our proposed method and its comparative analysis with four operators, namely 2TLPFWHMO, 2TLPFWDHMO, 2TLPWAO, and 2TLPFWGO.

Table 24
Final ranking of valves

Alternatives	2TLFF-COCOSO	2TLPFWHMO Deng et al. (2018)	2TLPFWDHMO Deng et al. (2018)	2TLPWAO Deng et al. (2018)	2TLPFWGO Deng et al. (2018)
$¥_1$	6	5	5	6	6
$¥_2$	10	10	10	10	11
$¥_3$	1	1	1	1	1
$¥_4$	3	3	3	3	3
$¥_5$	2	2	2	2	2
$¥_6$	11	11	11	11	10
$¥_7$	9	9	9	9	8
$¥_8$	4	4	4	4	4
$¥_9$	5	6	7	5	5
$¥_{10}$	8	8	8	8	9
$¥_{11}$	7	7	6	7	7

A graph of the final ranking results of SBV for DIS is shown in Figure 5.

Figure 5
Comparative analysis



7. Conclusion

The DIS is designed to reduce irrigation water usage. When DIS is used, crop yields increase due to adequate water supply. It also has some limitations, some of which are listed below:

1. The DIS is advantageous over conventional and modernized surface ISs in terms of water saving and crop yield; however, when economics are taken into account, drip is preferred over flood irrigation but not over modernized surface irrigation methods in a few cases.
2. There is a scarcity of research on DI effectiveness for grain crops such as wheat and rice in India. The majority of irrigation research has focused on cash crops such as cotton and sugarcane, as well as vegetables and fruit crops.
3. More technical knowledge and intensive training are required for successful DIS fabrication and operation on farms. It has a high installation cost, but the government assists farmers by providing the drip system at subsidized rates.
4. Farmers' lack of knowledge about the DIS's water-saving and cost-effectiveness has resulted in its limited use in comparison to other conventional ISs.

Despite these limitations, the DIS is a popular irrigation method on several continents due to its low cost and ability to save water. In this research article, we have chosen the best valve for DIS, that is, the globe valve is the best valve for DIS. We have solved the problem with the COCOSO method using the 2TLFFHWAO. To demonstrate the application of its integrity, we performed a comparison research with current operators, including 2TLPFWHMO Deng et al. (2018), 2TLPFWDHMO Deng et al. (2018), 2TLPFWAO Deng et al. (2018), and 2TLPFWGO Deng et al. (2018). The proposed 2TLFF-COCOSO method can be used for different MAGDM applications in industries, that is, health care, agriculture, construction companies, etc. New methods such as DNMA, GLDS, ORESE, and MARCOS in the fuzzy context of FFS and the 2TLFF can be used for future development. These methods can be used for the selection of filters, pumps, pipes, commercial fertilizers, and pesticides. Although this study produced adequate and fruitful results, it did have some limitations. Only 10 valves were included in this study. Several more complex valves are now available. Future research may increase the number of valves and use more complex valves. In addition, only five characteristics were selected, although many other factors, such as robustness, strength, and quality, are considered when selecting a valve. Future research should consider these qualities.

Data availability

No data were used to support this study.

Conflicts of Interest

Muhammad Akram is an editorial board member for *Journal of Computational and Cognitive Engineering*, and was not involved in the editorial review or the decision to publish this article. The authors declare that they have no conflicts of interest to this work.

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