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RESEARCH ARTICLE

An MCGDM Technique for Weighted Average Interaction Aggregation Operator Under Interval-Valued Intuitionistic Fuzzy Hypersoft Set Environment

Rana Muhammad Zulqarnain[1,*] and Imran Siddique[2]

[1]*Department of Mathematics, Zhejiang Normal University, China*

[2]*Department of Mathematics, University of Management and Technology, Pakistan*

Abstract: The intuitionistic fuzzy hypersoft set (IFHSS) is the most generalized form of the intuitionistic fuzzy soft set used to resolve ambiguous and elusive data in the decision-making (DM) process, considering the parameters' multi-sub-attributes. Aggregation operators (AOs) execute a dynamic role in assessing the two prospect sequences and eliminating anxieties from this perception. This paper prolongs the IFHSS to interval-valued IFHSS (IVIFHSS), which proficiently contracts with hesitant and unclear data. It is the most potent technique for incorporating insecure data into DM. The core impartial of this investigation is to develop the algebraic operational laws for IVIFHSS. Furthermore, using the interaction operational law, an interaction AO for IVIFHSS has been presented, such as interval-valued intuitionistic fuzzy hypersoft interactive weighted average (IVIFHSIWA) with its essential properties. Multi-criteria group decision-making (MCGDM) technique is vigorous for material selection (MS). However, conventional methods of MCGDM regularly provide inconsistent results. Based on the expected AOs, industrial enterprises propose a robust MCGDM MS method to meet this shortfall. The real-world application of the planned MCGDM method for cryogenic storing vessel MS is presented. The implication is that the designed model is more efficient and consistent in handling information based on IVIFHSS.

Keywords: IVIFHSS, IVIFHSIWA operator, MCGDM

1. Introduction

Multi-criteria group decision-making (MCGDM) is deliberated as the most proper technique for the verdict of the adequate alternative from all probable choices, following conditions or features. Maximum judgments are taken when the intentions and confines are usually unspecified or unclear in real-life surroundings. Zadeh presented the notion of the fuzzy set (FS) (Zadeh, 1965) to overcome such vagueness and doubts in decision-making (DM). Turksen (1986) gave the interval-valued FS (IVFS) with fundamental operations. If the experts consider a membership degree (MD) and a non-membership degree (NMD) in the DM procedure, the FS theories cannot grip the situation. Atanassov (1986) resolved the above-mentioned limitations and developed the intuitionistic fuzzy set (IFS). Garg (2018) developed the cosine similarity measures (SMs) for IFS considering the interaction between the couples of MD and NMD. Atanassov (2022) introduced the topological operators and discussed some essential properties. Ejegwa and Agbetayo (2022) developed several SM and distance measures under the IFS environment and used their presented measures to resolve DM complications. Khan et al. (2022) offered a multi attribute decision making (MADM) technique using complex T-spherical fuzzy power aggregation operator (AOs). Atanassov (1999) presented the interval-valued IFS (IVIFS). Garg and Rani (2022) settled the MULTIMOORA technique under IFS information using their presented AOs. Xu and Gou (2017) developed several DM methodologies under the IVIFS setting and utilized their methodologies in various real-life problems. Ze-Shui (2007) proposed the weighted arithmetic and geometric AOs for IVIFS. Zhang (2018) developed the Bonferroni mean geometric AOs under the IVIFS setting and presented the multi attribute group decision making (MAGDM) approach. Park et al. (2009) proposed the hybrid geometric AO for IVIFS and utilized it for MAGDM problems. Gupta et al. (2018) developed a corrective model for determining the weight of experts. Garg and Kumar (2019) extended the AOs with their fundamental properties under the linguistic IVIFS environment.

The above-stated FS, IVFS, IFS, IVIFS, Pythagorean fuzzy set (PFS), and Interval valued Pythagorean fuzzy set (IVPFS) cannot contract with the parametrized values of the alternatives.

*Corresponding author: Rana Muhammad Zulqarnain, Department of Mathematics, Zhejiang Normal University, China. Email: ranazulqarnain7777@gmail.com

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Molodtsov (1999) introduced soft sets (SS) theory and explained some basic operations with their features to handle confusion and uncertainties. Maji et al. (2001) offered the Fuzzy soft set (FSS) theory by merging SS and FS. Maji et al. (2001) developed basic operations for their properties for the IFSS. Gurmani et al. (2022) presented the concept of a T-spherical hesitant FS. Jiang et al. (2010) offered the interval-valued IFSS (IVIFSS) with some basic operations. Zulqarnain et al. (2021) planned the TOPSIS technique for IVIFSS using correlation measures to resolve MADM complications. Gurmani et al. (2021) established the VIKOR approach for linguistic interval-valued q-rung orthopair fuzzy settings. Smarandache (2018) projected the hypersoft set (HSS) idea. Smarandache HSS is the most appropriate model that grips the deliberated constraints' multiple sub-attributes. Jafar et al. (2021) developed the intuitionistic fuzzy hypersoft matrices with fundamental operations. Zulqarnain et al. (2016) utilized the interval-valued fuzzy soft matrix for DM. Debnath (2022) offered the interval-valued intuitionistic fuzzy hypersoft set (IVIFHSS) with several fundamental operations and their properties. IFHSS is an amalgam intelligent erection of IFSS. A boosted sorting development captivates the investigators to crash unsolved and insufficient facts. Interpreting the exploration consequences, it is concluded that the IFHSS performs an energetic part in DM by assembling several causes into a solitary value. Therefore, to inspire the current research on IVIFHSS, we will describe interaction AO built on irregular information. The core objectives of the present study are as follows:

- IVIFHSS deals competently with multidimensional concerns by looking at the multi-sub-attributes of the deliberated parameters in the DM procedure. To preserve this benefit in concentration, we extend IFHSS to IVIFHSS and set up interaction AO for IVIFHSS.
- Interaction AO for IVIFHSS is a well-known attractive estimate AO. It has been observed that the prevailing AO aspect is irresponsible for scratching the correct detection of the DM process. To overcome these specific complications, these existing AO must be appraised.
- Interval-valued intuitionistic fuzzy hypersoft interactive weighted average (IVIFHSIWA) operator has been introduced with their essential features using developed operational laws.
- A new algorithm based on planned operators has been established to crack the complications of MCGDM under the IVIFHSS scenario.
- Material selection (MS) is an essential feature of manufacturing as it understands the stable conditions for all components. MS is a complex but essential step in professional development. Lack of MS will damage the manufacturer's efficiency, productivity, and eccentricity.
- A comparative analysis of the latest MCGDM technique and prevalent methods is presented to consider the utility and superiority.

The organization of this research is estimated to be as follows: the second part of this study contains some basic impressions that support us growing the organization of the later research. Section 3 introduces some new interactional operational laws for IVIFHSN. Also, in the same section, the IVIFHSIWA operator is presented based on the basic features of our developed operators. Section 4 develops an MCGDM approach built on the anticipated AO. A numerical example for MS in the industry is discussed in the same section to confirm the practicality of the established technique. In addition, Section 5 provides a brief comparative analysis to confirm the validity of the advanced approach.

2. Preliminaries

This section contains some basic definitions that will structure the following work.

Definition 2.1. (Smarandache, 2018) Let U be a universe of discourse and $\mathcal{P}(U)$ be a power set of U and $t = \{t_1, t_2, t_3, \ldots, t_n\}$, $(n \geq 1)$, and T_i denotes the set of attributes and their conforming sub-attributes, such as $T_i \cap T_j = \varphi$, where $i \neq j$ for each $n \geq 1$ and $i, j \in \{1,2,3 \ldots n\}$. Assume that $T_1 \times T_2 \times T_3 \times \ldots \times T_n = \dddot{A} = \{d_{1h} \times d_{2k} \times \cdots \times d_{nl}\}$ is a collection of sub-attributes. Then the pair $(\mathcal{F}, T_1 \times T_2 \times T_3 \times \ldots \times T_n = (\Omega, \dddot{A})$ is known as HSS and defined as follows:

$$\Omega: T_1 \times T_2 \times T_3 \times \ldots \times T_n = \dddot{A} \to \mathcal{P}(U).$$

It is also defined as:

$$(\Omega, \dddot{A}) = \left\{\check{d}, \Omega_{\dddot{A}}(\check{d}): \check{d} \in \dddot{A}, \Omega_{\dddot{A}}(\check{d}) \in \mathcal{P}(U)\right\}.$$

where $1 \leq h \leq \alpha$, $1 \leq k \leq \beta$, and $1 \leq l \leq \gamma$, and $\alpha, \beta, \gamma \in$ N.

Definition 2.2. (Atanassov, 1999) Let U be a universe of discourse, and A be any subset of U. Then, the IVIFS A over U is defined as:

$$A = \left\{\left(x, \left(\left[\kappa_A^l(t), \kappa_A^u(t)\right], \left[\delta_A^l(t), \delta_A^u(t)\right]\right)\right) | t \in U\right\}$$

where $[\kappa_A^l(t), \kappa_A^u(t)]$ and $[\delta_A^l(t), \delta_A^u(t)]$ represent the MD and NMD intervals, respectively. Also, $\kappa_A^l(t), \kappa_A{}^u(t), \delta_A^l(t), \delta_A^u(t) \in [0,1]$ and satisfied the subsequent condition $0 \leq \kappa_A^u(t) + \delta_A^u(t) \leq 1$.

Definition 2.3. (Jiang et al., 2010) Let U be a universe of discourse and $\mathbb{N}$ be a set of attributes. Then a pair $(\Omega, \mathbb{N})$ is called an IVIFSS over U. Its mapping can be expressed as:

$$\Omega: \mathbb{N} \to IK^U$$

where IK^U represents the collection of interval-valued intuitionistic fuzzy subsets of the universe of discourse U.

$$(\Omega, \mathbb{N}) = \left\{x, \left(\left[\kappa_A^l(t), \kappa_A^u(t)\right], \left[\delta_A^l(t), \delta_A^u(t)\right]\right) | t \in A\right\}$$

where $[\kappa_A^l(t), \kappa_A^u(t)]$ and $[\delta_A^l(t), \delta_A^u(t)]$ represent the MD and NMD intervals, respectively. Also, $\kappa_A^l(t), \kappa_A^u(t), \delta_A^l(t), \delta_A^u(t) \in [0,1]$ and satisfied the subsequent condition $0 \leq \kappa_A^u(t) + \delta_A^u(t) \leq 1$ and $A \subset \mathbb{N}$.

Definition 2.4. (Debnath, 2022) Let U be a universe of discourse and $\mathcal{P}(U)$ be a power set of U and $t = \{t_1, t_2, t_3, \ldots, t_n\}$, $(n \geq 1)$, and T_i represents the set of attributes and their corresponding sub-attributes, such as $T_i \cap T_j = \varphi$, where $i \neq j$ for each $n \geq 1$ and $i, j \in \{1,2,3 \ldots n\}$. Assume that $T_1 \times T_2 \times T_3 \times \ldots \times T_n = \dddot{A} = \{d_{1h} \times d_{2k} \times \cdots \times d_{nl}\}$ is a collection of sub-attributes. Then the pair $(\mathcal{F}, T_1 \times T_2 \times T_3 \times \ldots \times T_n = (\Omega, \dddot{A})$ is known as IVIFHSS and defined as follows:

$$\Omega: T_1 \times T_2 \times T_3 \times \ldots \times T_n = \dddot{A} \to IVIFHS^U.$$

It is also defined as $(\Omega, \dddot{A}) = \left\{(\check{d}, \Omega_{\dddot{A}}(\check{d})): \check{d} \in \dddot{A}, \Omega_{\dddot{A}}(\check{d}) \in IVPFS^U \in [0, 1]\right\}$, where $\Omega_{\dddot{A}}(\check{d}) = \{\langle \zeta, \kappa_{\Omega(\check{d})}(\zeta), \delta_{\Omega(\check{d})}(\zeta)\rangle : \zeta \in U\}$, and $\kappa_{\Omega(\check{d})}(\zeta) = [\kappa_{\Omega(\check{d})}^l(\zeta), \kappa_{\Omega(\check{d})}^u(\zeta)]$, $\delta_{\Omega(\check{d})}(\zeta) = [\delta_{\Omega(\check{d})}^l(\zeta), \delta_{\Omega(\check{d})}^u(\zeta)]$, where $\kappa_{\Omega(\check{d})}(\zeta)$ and $\delta_{\Omega(\check{d})}(\zeta)$ represent the MD and NMD intervals, respectively, such as $\kappa_{\Omega(\check{d})}^l(\zeta), \kappa_{\Omega(\check{d})}^u(\zeta), \delta_{\Omega(\check{d})}^l(\zeta), \delta_{\Omega(\check{d})}^l(\zeta) \in [0,1]$, and $0 \leq \kappa_{\Omega(\check{d})}^u(\zeta)\ \delta_{\Omega(\check{d})}^u(\zeta) \leq +1$.

The IVIFHSN can be stated as $\mathcal{F} = ([\kappa_{\Omega(\check{d})}^l(\zeta), \kappa_{\Omega(\check{d})}^u(\zeta)], [\delta_{\Omega(\check{d})}^l(\zeta), \delta_{\Omega(\check{d})}^u(\zeta)])$.

To compute the alternative ranking, the score function and accuracy function for IVIFHSS can be stated as, if $\mathcal{F} = ([\kappa_{\Omega(\check{d})}^l(\zeta), \kappa_{\Omega(\check{d})}^u(\zeta)], [\delta_{\Omega(\check{d})}^l(\zeta), \delta_{\Omega(\check{d})}^u(\zeta)])$ be an IVIFHSN,

$$S(\mathcal{F}) = \frac{\kappa^l_{\Omega(\check{d})}(\zeta)+\kappa^u_{\Omega(\check{d})}(\zeta)+\delta^l_{\Omega(\check{d})}(\zeta)+\delta^u_{\Omega(\check{d})}(\zeta)}{4}$$

And

$$A(\mathcal{F}) = \frac{\left(\kappa^l_{\Omega(\check{d})}(\zeta)\right)^2+\left(\kappa^u_{\Omega(\check{d})}(\zeta)\right)^2+\left(\delta^l_{\Omega(\check{d})}(\zeta)\right)^2+\left(\delta^u_{\Omega(\check{d})}(\zeta)\right)^2}{2}$$

Definition 2.5. (Zulqarnain et al., 2022) Let $\mathcal{F}_{\check{d}_k} = ([\kappa^l_{\check{d}_k}, \kappa^u_{\check{d}_k}], [\delta^l_{\check{d}_k}, \delta_{\check{d}_k}{}^u])$, $\mathcal{F}_{\check{d}_{11}} = ([\kappa^l_{\check{d}_{11}}, \kappa^u_{\check{d}_{11}}], [\delta^l_{\check{d}_{11}}, \delta^u_{\check{d}_{11}}])$, and $\mathcal{F}_{\check{d}_{12}} = ([\kappa^l_{\check{d}_{12}}, \kappa^u_{\check{d}_{12}}], [\delta^l_{\check{d}_{12}}, \delta^u_{\check{d}_{12}}])$ be three IVIFHSNs and $\beta > 0$, and by algebraic norms, we have

1. $\mathcal{F}_{\check{d}_{11}} \oplus \mathcal{F}_{\check{d}_{12}} = ([\kappa^l_{\check{d}_{11}}+\kappa^l_{\check{d}_{12}}-\kappa^l_{\check{d}_{11}}\kappa^l_{\check{d}_{12}}, \kappa^u_{\check{d}_{11}}+\kappa^u_{\check{d}_{12}}-\kappa^u_{\check{d}_{11}}\kappa^u_{\check{d}_{12}}], [\delta^l_{\check{d}_{11}}\delta^l_{\check{d}_{12}}, \delta^u_{\check{d}_{11}}\delta^u_{\check{d}_{12}}])$
2. $\mathcal{F}_{\check{d}_{11}} \otimes \mathcal{F}_{\check{d}_{12}} = ([\kappa^l_{\check{d}_{11}}\kappa^l_{\check{d}_{12}}, \kappa^u_{\check{d}_{11}}\kappa^u_{\check{d}_{12}}], [\delta^l_{\check{d}_{11}}+\delta^l_{\check{d}_{12}}-\delta^l_{\check{d}_{11}}\delta^l_{\check{d}_{12}}, \delta^u_{\check{d}_{11}}+\delta^u_{\check{d}_{12}}-\delta^u_{\check{d}_{11}}\delta^u_{\check{d}_{12}}])$
3. $\beta\mathcal{F}_{\check{d}_k} = ([1-(1-\kappa^l_{\check{d}_k})^\beta, 1-(1-\kappa^u_{\check{d}_k})^\beta], [\delta^{l\ \beta}_{\check{d}_k}, \delta^{u\ \beta}_{\check{d}_k}]) = (1-(1-[\kappa^l_{\check{d}_k}, \kappa^u_{\check{d}_k}])^\beta, [\delta^{l\ \beta}_{\check{d}_k}, \delta^{u\ \beta}_{\check{d}_k}])$
4. $\mathcal{F}_{\check{d}_k}{}^\beta = ([\kappa^{l\ \beta}_{\check{d}_k}, \kappa^{u\ \beta}_{\check{d}_k}], [1-(1-\delta^l_{\check{d}_k}), 1-(1-\delta^u_{\check{d}_k})^\beta]) = ([\kappa^{l\ \beta}_{\check{d}_k}, \kappa^{u\ \beta}_{\check{d}_k}], 1-(1-[\delta^l_{\check{d}_k}, \delta^u_{\check{d}_k}])^\beta)$.

For the assortment of IVIFHSNs $\mathcal{F}_{\check{d}_{ij}}$, where ω_i and ν_j are weights for professionals and attributes, correspondingly, with given conditions $\omega_i > 0, \sum_{i=1}^n \omega_i = 1; \nu_j > 0, \sum_{j=1}^m \nu_j = 1$. Zulqarnain et al. (2022) proposed a weighted average AO for IVIFHSNs as follows:

$$\text{IVIFHSWA}\left(\mathcal{F}_{\check{d}_{11}}, \mathcal{F}_{\check{d}_{12}}, \ldots\ldots\ldots, \mathcal{F}_{\check{d}_{nm}}\right) = \left(1-\prod_{j=1}^m\left(\prod_{i=1}^n\left(1-\left[\kappa^l_{\check{d}_{ij}}, \kappa^u_{\check{d}_{ij}}\right]\right)^{\omega_i}\right)^{\nu_j}, \prod_{j=1}^m\left(\prod_{i=1}^n\left(\left[\delta^l_{\check{d}_{ij}}, \delta^u_{\check{d}_{ij}}\right]\right)^{\omega_i}\right)^{\nu_j}\right)$$

Example 2.1. Let $\mathcal{R} = \{\mathcal{R}_1, \mathcal{R}_2, \mathcal{R}_3\}$ be a set of experts with the given weight vector $\omega_i = (0.38, 0.45, 0.17)^T$. The group of experts describes the beauty of a house under considered attributes $\text{Å} = \{e_1 = \textit{lawn}, e_2 = \textit{security system}\}$ with their corresponding sub-attributes Lawn $= e_1 = \{e_{11} = \textit{with grass}, e_{12} = \textit{without grass}\}$ Security system $= e_2 = \{e_{21} = \textit{guards}, e_{22} = \textit{cameras}\}$. Let $\text{Å} = e_1 \times e_2$ be a set of sub-attributes

$\text{Å} = \{\check{d}_1, \check{d}_2, \check{d}_3, \check{d}_4\}$ be a set of multi-sub-attributes with weights $\nu_j = (0.2, 0.2, 0.2, 0.4)^T$. The rating values for each alternative in terms of IVIFHSN $(\mathcal{F}, \text{Å}) = ([\kappa^l_{\check{d}_{ij}}, \kappa^u_{\check{d}_{ij}}], [\delta^l_{\check{d}_{ij}}, \delta^u_{\check{d}_{ij}}])_{3\times4}$ are given as:

$$(\mathcal{F}, \text{Å}) = \begin{bmatrix} ([0.3,0.5],[0.4,0.5]) & ([0.4,0.6],[0.3,0.4]) & ([0.5,0.7],[0.1,0.3]) & ([0.4,0.5],[0.3,0.4]) \\ ([0.1,0.5],[0.2,0.3]) & ([0.3,0.4],[0.0,0.0]) & ([0.2,0.4],[0.2,0.3]) & ([0.1,0.3],[0.6,0.7]) \\ ([0.2,0.6],[0.2,0.3]) & ([0.5,0.6],[0.2,0.4])38 & ([0.2,0.4],[0.2,0.6]) & ([0.3,0.4],[0.5,0.6]) \end{bmatrix}$$

$$\text{IVIFHSWA}\left(\mathcal{F}_{\check{d}_{11}}, \mathcal{F}_{\check{d}_{12}}, \ldots\ldots\ldots, \mathcal{F}_{\check{d}_{34}}\right) = \left(1-\prod_{j=1}^4\left(\prod_{i=1}^3\left(1-\left[\kappa^l_{\check{d}_{ij}}, \kappa^u_{\check{d}_{ij}}\right]\right)^{\omega_i}\right)^{\nu_j}, \prod_{j=1}^4\left(\prod_{i=1}^3\left(\left[\delta^l_{\check{d}_{ij}}, \delta^u_{\check{d}_{ij}}\right]\right)^{\omega_i}\right)^{\nu_j}\right)$$

$$= ([0.3198, 0.4719], [0.0, 0.0])$$

From the IVIFHSWA operator, it is detected that, in certain conditions, they provide some unattractive outcomes. To overcome such scenarios, we introduce the interaction AO for IVIFHSNs.

3. Interaction Weighted Average Aggregation Operator for Interval-valued Intuitionistic Fuzzy Hypersoft Sets

We will extend the IVIFHSS with some fundamental concepts and present the operational laws for IVIFHSNs in the following section. Moreover, we prolong the IVIFHSIWA operator by utilizing the developed operational laws.

Definition 3.1. Let $\mathcal{F}_{\check{d}_k} = ([\kappa^l_{\check{d}_k}, \kappa^u_{\check{d}_k}], [\delta^l_{\check{d}_k}, \delta^u_{\check{d}_k}])$, $\mathcal{F}_{\check{d}_{11}} = ([\kappa^l_{\check{d}_{11}}, \kappa^u_{\check{d}_{11}}], [\delta^l_{\check{d}_{11}}, \delta^u_{\check{d}_{11}}])$, and $\mathcal{F}_{\check{d}_{12}} = ([\kappa^l_{\check{d}_{12}}, \kappa^u_{\check{d}_{12}}], [\delta^l_{\check{d}_{12}}, \delta^u_{\check{d}_{12}}])$ be three IVIFHSNs and $\beta > 0$, and by algebraic norms, we have

1. $\mathcal{F}_{\check{d}_{11}} \oplus \mathcal{F}_{\check{d}_{12}} =$

$$\begin{pmatrix} 1-\left(1-\left[\kappa^l_{\check{d}_{11}}, \kappa^u_{\check{d}_{11}}\right]\right)\left(1-\left[\kappa^l_{\check{d}_{12}}, \kappa^u_{\check{d}_{12}}\right]\right), \\ \left(1-\left[\kappa^l_{\check{d}_{11}}, \kappa^u_{\check{d}_{11}}\right]\right)\left(1-\left[\kappa^l_{\check{d}_{12}}, \kappa^u_{\check{d}_{12}}\right]\right) - \left(1-\left(\left[\kappa^l_{\check{d}_{11}}, \kappa^u_{\check{d}_{11}}\right], \left[\delta^l_{\check{d}_{11}}, \delta^u_{\check{d}_{11}}\right]\right)\right)\left(1-\left(\left[\kappa^l_{\check{d}_{12}}, \kappa^u_{\check{d}_{12}}\right], \left[\delta^l_{\check{d}_{12}}, \delta^u_{\check{d}_{12}}\right]\right)\right) \end{pmatrix}$$

2. $\mathcal{F}_{\check{d}_{11}} \otimes \mathcal{F}_{\check{d}_{12}} =$

$$\begin{pmatrix} \left(1-\left[\delta^l_{\check{d}_{11}}, \delta^u_{\check{d}_{11}}\right]\right)\left(1-\left[\delta^l_{\check{d}_{12}}, \delta^u_{\check{d}_{12}}\right]\right) - \left(1-\left[\kappa^l_{\check{d}_{11}}, \kappa^u_{\check{d}_{11}}\right] - \left[\delta^l_{\check{d}_{11}}, \delta^u_{\check{d}_{11}}\right]\right), \\ 1-\left(1-\left[\delta^l_{\check{d}_{11}}, \delta^u_{\check{d}_{11}}\right]\right)\left(1-\left[\delta^l_{\check{d}_{12}}, \delta^u_{\check{d}_{12}}\right]\right) \end{pmatrix}$$

3. $\beta\mathcal{F}_{\check{d}_k} =$

$$\left(1-\left(1-\left[\kappa^l_{\check{d}_k}, \kappa^u_{\check{d}_k}\right]\right)^\beta, \left(1-\left[\kappa^l_{\check{d}_k}, \kappa^u_{\check{d}_k}\right]\right)^\beta - \left(\left(1-\left[\kappa^l_{\check{d}_k}, \kappa^u_{\check{d}_k}\right], \left[\delta^l_{\check{d}_k}, \delta^u_{\check{d}_k}\right]\right)\right)^\beta\right)$$

4. $\mathcal{F}_{\check{d}_k}{}^\beta = ((1-[\kappa^l_{\check{d}_k}, \kappa^u_{\check{d}_k}])^\beta - ((1-[\kappa^l_{\check{d}_k}, \kappa^u_{\check{d}_k}], [\delta^l_{\check{d}_k}, \delta^u_{\check{d}_k}]))^\beta, 1-(1-[\delta_{\check{d}_k}{}^l, \delta_{\check{d}_k}{}^u])^\beta)$

Definition 3.2. Let $\mathcal{F}_{\check{d}_k} = ([\kappa^l_{\check{d}_k}, \kappa^u_{\check{d}_k}], [\delta^l_{\check{d}_k}, \delta^u_{\check{d}_k}])$ be a collection of IVIFHSNs, and ω_i and ν_j represent the weights of experts and multi-sub-parameters, respectively, with specified circumstances $\omega_i > 0, \sum_{i=1}^n \omega_i = 1; \nu_j > 0, \sum_{j=1}^m \nu_j = 1$. Then, the IVIFHSIWA operator is defined as IVIFHSIWA: $\Psi^n \to \Psi$:

$$\text{IVIFHSIWA}\left(\mathcal{F}_{\check{d}_{11}}, \mathcal{F}_{\check{d}_{12}}, \ldots\ldots\ldots, \mathcal{F}_{\check{d}_{nm}}\right) = \oplus_{j=1}^m \nu_j\left(\oplus_{i=1}^n \omega_i \mathcal{F}_{\check{d}_{ij}}\right)$$

Theorem 3.1. Let $\mathcal{F}_{\check{d}_{ij}} = ([\kappa^l_{\check{d}_{ij}}, \kappa^u_{\check{d}_{ij}}], [\delta^l_{\check{d}_{ij}}, \delta^u_{\check{d}_{ij}}])$ be a collection of IVIFHSNs and the aggregated value is also an IVIFHSN, such as:

$$\text{IVIFHSIWA}\left(\mathcal{F}_{\check{d}_{11}}, \mathcal{F}_{\check{d}_{12}}, \ldots\ldots\ldots, \mathcal{F}_{\check{d}_{nm}}\right)$$

$$= \begin{pmatrix} 1-\prod_{j=1}^m\left(\prod_{i=1}^n\left(1-\left[\kappa^l_{\check{d}_{ij}}, \kappa^u_{\check{d}_{ij}}\right]\right)^{\omega_i}\right)^{\nu_j}, \\ \prod_{j=1}^m\left(\prod_{i=1}^n\left(1-\left[\kappa^l_{\check{d}_{ij}}, \kappa^u_{\check{d}_{ij}}\right]\right)^{\omega_i}\right)^{\nu_j} - \prod_{j=1}^m\left(\prod_{i=1}^n\left(1-\left[\kappa^l_{\check{d}_{ij}}, \kappa^u_{\check{d}_{ij}}\right]-\left[\delta^l_{\check{d}_{ij}}, \delta^u_{\check{d}_{ij}}\right]\right)^{\omega_i}\right)^{\nu_j} \end{pmatrix}$$

ω_i and ν_j show the experts and multi-sub-attributes weights, respectively, such as $\omega_i > 0$, $\sum_{i=1}^n \omega_i = 1$, $\nu_j > 0$, $\sum_{j=1}^m \nu_j = 1$.

Proof. The proof of the above-presented IVIFHSIWA operator can be proved by mathematical induction:

For $n = 1$, we get $\omega_1 = 1$. Then, we have

$$\mathrm{IVIFHSIWA}\left(\mathcal{F}_{\check{d}_{11}}, \mathcal{F}_{\check{d}_{12}}, \ldots\ldots\ldots, \mathcal{F}_{\check{d}_{nm}}\right) = \oplus_{j=1}^{m} \nu_j \mathcal{F}_{\check{d}_{1j}}$$

$$\mathrm{IVIFHSIWA}\left(\mathcal{F}_{\check{d}_{11}}, \mathcal{F}_{\check{d}_{12}}, \ldots\ldots\ldots, \mathcal{F}_{\check{d}_{nm}}\right)$$

$$= \left(\begin{array}{c} 1 - \prod_{j=1}^{m}\left(\left(1 - \left[\kappa^{l}_{\check{d}_{ij}}, \kappa^{u}_{\check{d}_{ij}}\right]\right)\right)^{\nu_j}, \\ \prod_{j=1}^{m}\left(\left(1 - \left[\kappa^{l}_{\check{d}_{ij}}, \kappa^{u}_{\check{d}_{ij}}\right]\right)\right)^{\nu_j} - \prod_{j=1}^{m}\left(\left(1 - \left[\kappa^{l}_{\check{d}_{ij}}, \kappa^{u}_{\check{d}_{ij}}\right] - \left[\delta^{l}_{\check{d}_{ij}}, \delta^{u}_{\check{d}_{ij}}\right]\right)\right)^{\nu_j} \end{array}\right)$$

$$= \left(\begin{array}{c} 1 - \prod_{j=1}^{m}\left(\prod_{i=1}^{1}\left(1 - \left[\kappa^{l}_{\check{d}_{ij}}, \kappa^{u}_{\check{d}_{ij}}\right]\right)^{\omega_i}\right)^{\nu_j}, \\ \prod_{j=1}^{m}\left(\prod_{i=1}^{1}\left(1 - \left[\kappa^{l}_{\check{d}_{ij}}, \kappa^{u}_{\check{d}_{ij}}\right]\right)^{\omega_i}\right)^{\nu_j} - \prod_{j=1}^{m}\left(\prod_{i=1}^{1}\left(1 - \left[\kappa^{l}_{\check{d}_{ij}}, \kappa^{u}_{\check{d}_{ij}}\right] - \left[\delta^{l}_{\check{d}_{ij}}, \delta^{u}_{\check{d}_{ij}}\right]\right)^{\omega_i}\right)^{\nu_j} \end{array}\right)$$

For $m = 1$, we get $\nu_1 = 1$. Then, we have

$$\mathrm{IVIFHSIWA}\left(\mathcal{F}_{\check{d}_{11}}, \mathcal{F}_{\check{d}_{12}}, \ldots\ldots\ldots, \mathcal{F}_{\check{d}_{nm}}\right) = \oplus_{i=1}^{n} \omega_i \mathcal{F}_{\check{d}_{i1}}$$

$$= \left(\begin{array}{c} 1 - \left(\prod_{i=1}^{n}\left(1 - \left[\kappa^{l}_{\check{d}_{ij}}, \kappa^{u}_{\check{d}_{ij}}\right]\right)^{\omega_i}\right), \qquad \omega_i \\ \prod_{i=1}^{n}\left(1 - \left[\kappa^{l}_{\check{d}_{ij}}, \kappa^{u}_{\check{d}_{ij}}\right]\right)^{\omega_i} - \prod_{i=1}^{n}\left(1 - \left[\kappa^{l}_{\check{d}_{ij}}, \kappa^{u}_{\check{d}_{ij}}\right] - \left[\delta^{l}_{\check{d}_{ij}}, \delta^{u}_{\check{d}_{ij}}\right]\right) \end{array}\right)$$

$$= \left(\begin{array}{c} 1 - \prod_{j=1}^{1}\left(\prod_{i=1}^{n}\left(1 - \left[\kappa^{l}_{\check{d}_{ij}}, \kappa^{u}_{\check{d}_{ij}}\right]\right)^{\omega_i}\right)^{\nu_j}, \\ \prod_{j=1}^{1}\left(\prod_{i=1}^{n}\left(1 - \left[\kappa^{l}_{\check{d}_{ij}}, \kappa^{u}_{\check{d}_{ij}}\right]\right)^{\omega_i}\right)^{\nu_j} - \prod_{j=1}^{1}\left(\prod_{i=1}^{n}\left(1 - \left[\kappa^{l}_{\check{d}_{ij}}, \kappa^{u}_{\check{d}_{ij}}\right] - \left[\delta^{l}_{\check{d}_{ij}}, \delta^{u}_{\check{d}_{ij}}\right]\right)^{\omega_i}\right)^{\nu_j} \end{array}\right)$$

So, the above theorem is proved for $n = 1$ and $m = 1$.

Assume that for $m = \alpha_1 + 1, n = \alpha_2$ and $m = \alpha_1, n = \alpha_2 + 1$, the above theorem holds

$$\oplus_{j=1}^{\alpha_1+1} \nu_j \left(\oplus_{i=1}^{\alpha_2} \omega_i \mathcal{F}_{\check{d}_{ij}}\right) = \left(\begin{array}{c} 1 - \prod_{j=1}^{\alpha_1+1}\left(\prod_{i=1}^{\alpha_2}\left(1 - \left[\kappa^{l}_{\check{d}_{ij}}, \kappa^{u}_{\check{d}_{ij}}\right]\right)^{\omega_i}\right)^{\nu_j}, \\ \prod_{j=1}^{\alpha_1+1}\left(\prod_{i=1}^{\alpha_2}\left(1 - \left[\kappa^{l}_{\check{d}_{ij}}, \kappa^{u}_{\check{d}_{ij}}\right]\right)^{\omega_i}\right)^{\nu_j} - \prod_{j=1}^{\alpha_1+1}\left(\prod_{i=1}^{\alpha_2}\left(1 - \left[\kappa^{l}_{\check{d}_{ij}}, \kappa^{u}_{\check{d}_{ij}}\right] - \left[\delta^{l}_{\check{d}_{ij}}, \delta^{u}_{\check{d}_{ij}}\right]\right)^{\omega_i}\right)^{\nu_j} \end{array}\right)$$

$$\oplus_{j=1}^{\alpha_1} \nu_j \left(\oplus_{i=1}^{\alpha_2+1} \omega_i \mathcal{F}_{\check{d}_{ij}}\right) = \left(\begin{array}{c} 1 - \prod_{j=1}^{\alpha_1}\left(\prod_{i=1}^{\alpha_2+1}\left(1 - \left[\kappa^{l}_{\check{d}_{ij}}, \kappa^{u}_{\check{d}_{ij}}\right]\right)^{\omega_i}\right)^{\nu_j}, \\ \prod_{j=1}^{\alpha_1}\left(\prod_{i=1}^{\alpha_2+1}\left(1 - \left[\kappa^{l}_{\check{d}_{ij}}, \kappa^{u}_{\check{d}_{ij}}\right]\right)^{\omega_i}\right)^{\nu_j} - \prod_{j=1}^{\alpha_1}\left(\prod_{i=1}^{\alpha_2+1}\left(1 - \left[\kappa^{l}_{\check{d}_{ij}}, \kappa^{u}_{\check{d}_{ij}}\right] - \left[\delta^{l}_{\check{d}_{ij}}, \delta^{u}_{\check{d}_{ij}}\right]\right)^{\omega_i}\right)^{\nu_j} \end{array}\right)$$

For $m = \alpha_1 + 1$ and $n = \alpha_2 + 1$, we have

$$\oplus_{j=1}^{\alpha_1+1}\nu_j\left(\oplus_{i=1}^{\alpha_2+1}\omega_i\mathcal{F}_{\check{d}_{ij}}\right) = \oplus_{j=1}^{\alpha_1+1}\nu_j\left(\oplus_{i=1}^{\alpha_2}\omega_i\mathcal{F}_{\check{d}_{ij}} \oplus \omega_{\alpha_2+1}\mathcal{F}_{\check{d}_{(\alpha_2+1)j}}\right)$$

$$= \oplus_{j=1}^{\alpha_1+1}\oplus_{i=1}^{\alpha_2}\nu_j\omega_i\mathcal{F}_{\check{d}_{ij}}\oplus_{j=1}^{\alpha_1+1}\nu_j\omega_{\alpha_2+1}\mathcal{F}_{\check{d}_{(\alpha_2+1)j}}$$

$$= \begin{pmatrix} 1-\prod_{j=1}^{\alpha_1+1}\left(\prod_{i=1}^{\alpha_2}\left(1-\left[\kappa^l_{\check{d}_{ij}},\kappa^u_{\check{d}_{ij}}\right]\right)^{\omega_i}\right)^{\nu_j} \bigoplus 1-\prod_{j=1}^{\alpha_1+1}\left(\left(1-\left[\kappa^l_{\check{d}_{(\alpha_2+1)j}},\kappa^u_{\check{d}_{(\alpha_2+1)j}}\right]\right)^{\omega_{\alpha_2+1}}\right)^{\nu_j}, \\ \prod_{j=1}^{\alpha_1+1}\left(\prod_{i=1}^{\alpha_2}\left(1-\left[\kappa^l_{\check{d}_{ij}},\kappa^u_{\check{d}_{ij}}\right]\right)^{\omega_i}\right)^{\nu_j} - \prod_{j=1}^{\alpha_1+1}\left(\prod_{i=1}^{\alpha_2}\left(1-\left[\kappa^l_{\check{d}_{ij}},\kappa^u_{\check{d}_{ij}}\right]-\left[\delta^l_{\check{d}_{ij}},\delta^u_{\check{d}_{ij}}\right]\right)^{\omega_i}\right)^{\nu_j} \bigoplus \\ \prod_{j=1}^{\alpha_1+1}\left(\left(1-\left[\kappa^l_{\check{d}_{(\alpha_2+1)j}},\kappa^u_{\check{d}_{(\alpha_2+1)j}}\right]\right)^{\omega_i}\right)^{\nu_j} - \prod_{j=1}^{\alpha_1+1}\left(\left(1-\left[\kappa^l_{\check{d}_{(\alpha_2+1)j}},\kappa^u_{\check{d}_{(\alpha_2+1)j}}\right]-\left[\delta^l_{\check{d}_{(\alpha_2+1)j}},\delta^u_{\check{d}_{(\alpha_2+1)j}}\right]\right)^{\omega_i}\right)^{\nu_j} \end{pmatrix}$$

$$= \begin{pmatrix} 1-\prod_{j=1}^{\alpha_1+1}\left(\prod_{i=1}^{\alpha_2+1}\left(1-\left[{}^{l}_{\check{d}_{ij}},\kappa^u_{\check{d}_{ij}}\right]\right)^{\omega_i}\right)^{\nu_j}, \\ \prod_{j=1}^{\alpha_1+1}\left(\prod_{i=1}^{\alpha_2+1}\left(1-\left[\kappa^l_{\check{d}_{ij}},\kappa^u_{\check{d}_{ij}}\right]\right)^{\omega_i}\right)^{\nu_j} - \prod_{j=1}^{\alpha_1+1}\left(\prod_{i=1}^{\alpha_2+1}\left(1-\left[\kappa^l_{\check{d}_{ij}},\kappa^u_{\check{d}_{ij}}\right]-\left[\delta^l_{\check{d}_{ij}},\delta^u_{\check{d}_{ij}}\right]\right)^{\omega_i}\right)^{\nu_j} \end{pmatrix}$$

Hence, it holds for $m = \alpha_1 + 1$ and $n = \alpha_2 + 1$. So, we can say that Theorem 3.1 holds for all values of m and n.

Example 3.1. Let $\mathcal{R} = \{\mathcal{R}_1, \mathcal{R}_2, \mathcal{R}_3\}$ be a set of experts with the given weight vector$\omega_i = (0.38, 0.45, 0.17)^T$. The group of experts describes the beauty of a house under considered attributes $Å = \{e_1 = lawn, e_2 = security\ system\}$ with their corresponding sub-attributes Lawn $= e_1 = \{e_{11} = with\ grass, e_{12} = without\ grass\}$Security system $= e_2 = \{e_{21} = guards, e_{22} = cameras\}$. Let $Å = e_1 \times e_2$ be a set of sub-attributes $Å = \{\check{d}_1, \check{d}_2, \check{d}_3, \check{d}_4\}$ be a set of multi-sub-attributes with weights $\nu_j = (0.2,0.2,0.2,0.4)^T$. The rating values for each alternative in terms of IVIFHSN $(\mathcal{F}, Å) = ([\kappa_{\check{d}_{ij}}{}^l, \kappa_{\check{d}_{ij}}{}^u],[\delta_{\check{d}_{ij}}{}^l,\delta_{\check{d}_{ij}}{}^u])_{3\times 4}$ are given as:

$$(\mathcal{F}, Å) =$$

$$\begin{bmatrix} ([0.3,0.5],[0.4,0.5]) & ([0.4,0.6],[0.3,0.4]) & ([0.5,0.7],[0.1,0.3]) & ([0.4,0.5],[0.3,0.4]) \\ ([0.1,0.5],[0.2,0.3]) & ([0.3,0.4],[0.5,0.6]) & ([0.2,0.4],[0.2,0.3]) & ([0.1,0.3],[0.6,0.7]) \\ ([0.2,0.6],[0.2,0.3]) & ([0.5,0.6],[0.2,0.4]) & ([0.2,0.4],[0.2,0.6]) & ([0.3,0.4],[0.5,0.6]) \end{bmatrix}$$

$$\text{IVIFHSIWA}\left(\mathcal{F}_{\check{d}_{11}},\mathcal{F}_{\check{d}_{12}},\ldots\ldots\ldots,\mathcal{F}_{\check{d}_{nm}}\right) =$$

$$\begin{pmatrix} 1-\prod_{j=1}^{4}\left(\prod_{i=1}^{3}\left(1-\left[\kappa^l_{\check{d}_{ij}},\kappa^u_{\check{d}_{ij}}\right]\right)^{\omega_i}\right)^{\nu_j}, \\ \prod_{j=1}^{4}\left(\prod_{i=1}^{3}\left(1-\left[\kappa^l_{\check{d}_{ij}},\kappa^u_{\check{d}_{ij}}\right]\right)^{\omega_i}\right)^{\nu_j} - \prod_{j=1}^{4}\left(\prod_{i=1}^{3}\left(1-\left[\kappa^l_{\check{d}_{ij}},\kappa^u_{\check{d}_{ij}}\right]-\left[\delta^l_{\check{d}_{ij}},\delta^u_{\check{d}_{ij}}\right]\right)^{\omega_i}\right)^{\nu_j} \end{pmatrix}$$

$$= \left(\begin{array}{c} 1 - \left(\begin{array}{c} \left\{ \begin{array}{c} [0.5,0.7]^{0.38}[0.5,0.9]^{0.45} \\ [0.4,0.8]^{0.17} \end{array} \right\}^{0.2} \left\{ \begin{array}{c} [0.4,0.6]^{0.38}[0.6,0.7]^{0.45} \\ [0.4,0.5]^{0.17} \end{array} \right\}^{0.2} \\ \left\{ \begin{array}{c} [0.3,0.5]^{0.38}[0.6,0.8]^{0.45} \\ [0.6,0.8]^{0.17} \end{array} \right\}^{0.2} \left\{ \begin{array}{c} [0.5,0.6]^{0.38}[0.7,0.9]^{0.45} \\ [0.6,0.7]^{0.17} \end{array} \right\}^{0.4} \end{array} \right), \\ \left(\begin{array}{c} \left\{ \begin{array}{c} [0.5,0.7]^{0.38}[0.5,0.9]^{0.45} \\ [0.4,0.8]^{0.17} \end{array} \right\}^{0.2} \left\{ \begin{array}{c} [0.4,0.6]^{0.38}[0.6,0.7]^{0.45} \\ [0.4,0.5]^{0.17} \end{array} \right\}^{0.2} \\ \left\{ \begin{array}{c} [0.3,0.5]^{0.38}[0.6,0.8]^{0.45} \\ [0.6,0.8]^{0.17} \end{array} \right\}^{0.2} \left\{ \begin{array}{c} [0.5,0.6]^{0.38}[0.7,0.9]^{0.45} \\ [0.6,0.7]^{0.17} \end{array} \right\}^{0.4} \end{array} \right) - \\ \left(\begin{array}{c} \left\{ \begin{array}{c} [0.0,0.3]^{0.38}[0.2,0.7]^{0.45} \\ [0.1,0.6]^{0.17} \end{array} \right\}^{0.2} \left\{ \begin{array}{c} [0.0,0.3]^{0.38} \\ [0.0,0.2]^{0.45}[0.0,0.3]^{0.17} \end{array} \right\}^{0.2} \\ \left\{ \begin{array}{c} [0.0,0.4]^{0.38}[0.3,0.6]^{0.45} \\ [0.0,0.6]^{0.17} \end{array} \right\}^{0.2} \left\{ \begin{array}{c} [0.1,0.3]^{0.38}[0.1,0.3]^{0.45} \\ [0.0,0.2]^{0.17} \end{array} \right\}^{0.4} \end{array} \right) \end{array} \right)$$

$= ([0.3,0.6],[0.40.4])$.

3.1. Properties of IVIFHSIWA operator

3.1.1. (Idempotency):

If $\mathcal{F}_{\check{d}_{ij}} = \mathcal{F}_{\check{d}_k} = ([\kappa^l_{\check{d}_{ij}}, \kappa^u_{\check{d}_{ij}}], [\delta^l_{\check{d}_{ij}}, \delta^u_{\check{d}_{ij}}]) \forall i,\ j$. Then

$$\text{IVIFHSIWA}\left(\mathcal{F}_{\check{d}_{11}}, \mathcal{F}_{\check{d}_{12}}, \ldots\ldots\ldots, \mathcal{F}_{\check{d}_{nm}}\right) = \mathcal{F}_{\check{d}_k}$$

Proof: As we know that all $\mathcal{F}_{\check{d}_{ij}} = \mathcal{F}_{\check{d}_k} = ([\kappa^l_{\check{d}_{ij}}, \kappa^u_{\check{d}_{ij}}], [\delta^l_{\check{d}_{ij}}, \delta^u_{\check{d}_{ij}}])$, we have

$$\text{IVIFHSIWA}\left(\mathcal{F}_{\check{d}_{11}}, \mathcal{F}_{\check{d}_{12}}, \ldots\ldots\ldots, \mathcal{F}_{\check{d}_{nm}}\right)$$

$$= \left(\begin{array}{c} 1 - \prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(1 - \left[\kappa^l_{\check{d}_{ij}}, \kappa^u_{\check{d}_{ij}}\right]\right)^{\omega_i}\right)^{\nu_j}, \\ \prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(1 - \left[\kappa^l_{\check{d}_{ij}}, \kappa^u_{\check{d}_{ij}}\right]\right)^{\omega_i}\right)^{\nu_j} - \prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(1 - \left[\kappa^l_{\check{d}_{ij}}, \kappa^u_{\check{d}_{ij}}\right] - \left[\delta^l_{\check{d}_{ij}}, \delta^u_{\check{d}_{ij}}\right]\right)^{\omega_i}\right)^{\nu_j} \end{array} \right)$$

$$= \left(\begin{array}{c} 1 - \left(\left(1 - \left[\kappa^l_{\check{d}_{ij}}, \kappa^u_{\check{d}_{ij}}\right]\right)^{\sum_{i=1}^{n}\omega_i}\right)^{\sum_{j=1}^{m}\nu_j}, \\ \left(\left(1 - \left[\kappa^l_{\check{d}_{ij}}, \kappa^u_{\check{d}_{ij}}\right]\right)^{\sum_{i=1}^{n}\omega_i}\right)^{\sum_{j=1}^{m}\nu_j} - \left(\left(1 - \left[\kappa^l_{\check{d}_{ij}}, \kappa^u_{\check{d}_{ij}}\right] - \left[\delta^l_{\check{d}_{ij}}, \delta^u_{\check{d}_{ij}}\right]\right)^{\sum_{i=1}^{n}\omega_i}\right)^{\sum_{j=1}^{m}\nu_j} \end{array} \right)$$

As $\sum_{j=1}^{m}\nu_j = 1$and $\sum_{i=1}^{n}\omega_i = 1$, we have

$$= \left(1 - \left(1 - \left[\kappa^l_{\check{d}_{ij}}, \kappa^u_{\check{d}_{ij}}\right]\right), 1 - \left[\kappa^l_{\check{d}_{ij}}, \kappa^u_{\check{d}_{ij}}\right] - \left(1 - \left[\kappa^l_{\check{d}_{ij}}, \kappa^u_{\check{d}_{ij}}\right] - \left[\delta^l_{\check{d}_{ij}}, \delta^u_{\check{d}_{ij}}\right]\right)\right)$$

$$= \left(\left[\kappa^l_{\check{d}_{ij}}, \kappa^u_{\check{d}_{ij}}\right], \left[\delta^l_{\check{d}_{ij}}, \delta^u_{\check{d}_{ij}}\right]\right)$$

$= \mathcal{F}_{\check{k}}$.

3.1.2. Boundedness:

Let $\mathcal{F}_{\check{d}_{ij}} = ([\kappa^l_{\check{d}_{ij}}, \kappa^u_{\check{d}_{ij}}], [\delta^l_{\check{d}_{ij}}, \delta^u_{\check{d}_{ij}}])$ be a collection of IVIFHSNs where $\mathcal{F}^-_{\check{d}_{ij}} = \left(\min_j \min_i \left\{\left[\kappa^l_{\check{d}_{ij}}, \kappa^u_{\check{d}_{ij}}\right]\right\}, \max_j \max_i \left\{\left[\delta^l_{\check{d}_{ij}}, \delta^u_{\check{d}_{ij}}\right]\right\}\right)$ and $\mathcal{F}^+_{\check{d}_{ij}} = \left(\max_j \max_i \left\{\left[\kappa^l_{\check{d}_{ij}}, \kappa^u_{\check{d}_{ij}}\right]\right\}, \min_j \min_i \left\{\left[\delta^l_{\check{d}_{ij}}, \delta^u_{\check{d}_{ij}}\right]\right\}\right)$, then

$$\mathcal{F}^-_{\check{d}_{ij}} \le \text{IVIFHSIWA}\left(\mathcal{F}_{\check{d}_{11}}, \mathcal{F}_{\check{d}_{12}}, \ldots\ldots\ldots, \mathcal{F}_{\check{d}_{nm}}\right) \le \mathcal{F}^+_{\check{d}_{ij}}$$

Proof. As we know that $\mathcal{F}_{\check{d}_{ij}} = ([\kappa^l_{\check{d}_{ij}}, \kappa^u_{\check{d}_{ij}}], [\delta^l_{\check{d}_{ij}}, \delta^u_{\check{d}_{ij}}])$ be an IVIFHSN, then

$$\min_j \min_i \left\{\left[\kappa^l_{\check{d}_{ij}}, \kappa^u_{\check{d}_{ij}}\right]\right\} \le \left[\kappa^l_{\check{d}_{ij}}, \kappa^u_{\check{d}_{ij}}\right] \le \max_j \max_i \left\{\left[\kappa^l_{\check{d}_{ij}}, \kappa^u_{\check{d}_{ij}}\right]\right\}$$

$$\Rightarrow 1 - \max_j \max_i \left\{\left[\kappa^l_{\check{d}_{ij}}, \kappa^u_{\check{d}_{ij}}\right]\right\} \le 1 - \left[\kappa^l_{\check{d}_{ij}}, \kappa^u_{\check{d}_{ij}}\right] \le 1 - \min_j \min_i \left\{\left[\kappa^l_{\check{d}_{ij}}, \kappa^u_{\check{d}_{ij}}\right]\right\}$$

$$\Leftrightarrow \left(1 - \max_j \max_i \left\{\left[\kappa^l_{\check{d}_{ij}}, \kappa^u_{\check{d}_{ij}}\right]\right\}\right)^{\omega_i} \le \left(1 - \left[\kappa^l_{\check{d}_{ij}}, \kappa^u_{\check{d}_{ij}}\right]\right)^{\omega_i} \le \left(1 - \min_j \min_i \left\{\left[\kappa^l_{\check{d}_{ij}}, \kappa^u_{\check{d}_{ij}}\right]\right\}\right)^{\omega_i}$$

$$\Leftrightarrow \left(1 - \max_j \max_i \left\{\left[\kappa^l_{\check{d}_{ij}}, \kappa^u_{\check{d}_{ij}}\right]\right\}\right)^{\sum_{i=1}^{n}\omega_i} \le \prod_{i=1}^{n}\left(1 - \left[\kappa^l_{\check{d}_{ij}}, \kappa^u_{\check{d}_{ij}}\right]\right)^{\omega_i} \le \left(1 - \min_j \min_i \left\{\left[\kappa^l_{\check{d}_{ij}}, \kappa^u_{\check{d}_{ij}}\right]\right\}\right)^{\sum_{i=1}^{n}\omega_i}$$

$$\Leftrightarrow \left(1 - \max_j \max_i \left\{\left[\kappa^l_{\check{d}_{ij}}, \kappa^u_{\check{d}_{ij}}\right]\right\}\right)^{\sum_{j=1}^{n}\nu_j} \le \prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(1 - \left[\kappa^l_{\check{d}_{ij}}, \kappa^u_{\check{d}_{ij}}\right]\right)^{\omega_i}\right)^{\nu_j} \le \left(1 - \min_j \min_i \left\{\left[\kappa^l_{\check{d}_{ij}}, \kappa^u_{\check{d}_{ij}}\right]\right\}\right)^{\sum_{j=1}^{n}\nu_j}$$

$$\Leftrightarrow 1 - \max_j \max_i \left\{\left[\kappa^l_{\check{d}_{ij}}, \kappa^u_{\check{d}_{ij}}\right]\right\} \le \prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(1 - \left[\kappa^l_{\check{d}_{ij}}, \kappa^u_{\check{d}_{ij}}\right]\right)^{\omega_i}\right)^{\nu_j} \le 1 - \min_j \min_i \left\{\left[\kappa^l_{\check{d}_{ij}}, \kappa^u_{\check{d}_{ij}}\right]\right\}$$

$$\Leftrightarrow \min_j \min_i \left\{\left[\kappa^l_{\check{j}}, \kappa^u_{\check{d}_{ij}}\right]\right\} \le 1 - \prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(1 - \left[\kappa^l_{\check{d}_{ij}}, \kappa^u_{\check{d}_{ij}}\right]\right)^{\omega_i}\right)^{\nu_j} \le \max_j \max_i \left\{\left[\kappa^l_{\check{d}_{ij}}, \kappa^u_{\check{d}_{ij}}\right]\right\} \quad (a)$$

Similarly,

$$\min_j \min_i \left\{\left[\delta^l_{\check{d}_{ij}}, \delta^u_{\check{d}_{ij}}\right]\right\} \le \prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(1 - \left[\kappa^l_{\check{d}_{ij}}, \kappa^u_{\check{d}_{ij}}\right]\right)^{\omega_i}\right)^{\nu_j} - \prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(1 - \left[\kappa^l_{\check{d}_{ij}}, \kappa^u_{\check{d}_{ij}}\right] - \left[\delta^l_{\check{d}_{ij}}, \delta^u_{\check{d}_{ij}}\right]\right)^{\omega_i}\right)^{\nu_j} \le \max_j \max_i \left\{\left[\delta^l_{\check{d}_{ij}}, \delta^u_{\check{d}_{ij}}\right]\right\} \quad (b)$$

Let IVIFHSIWA $(\mathcal{F}_{\check{d}_{11}}, \mathcal{F}_{\check{d}_{12}}, \ldots\ldots\ldots, \mathcal{F}_{\check{d}_{nm}}) = ([\kappa_{\check{d}_{ij}}, \kappa^u_{\check{d}_k}], [\delta^l_{\check{d}_k}, \delta^u_{\check{d}_k}]) = \mathcal{F}_{\check{d}_{ij}}$. So, (a) and (b) can be transferred into the form:

$\min_j \min_i \left\{\left[\kappa^l_{\check{d}_{ij}}, \kappa^u_{\check{d}_{ij}}\right]\right\} \le \mathcal{F}_{\check{d}_k} \le \max_j \max_i \left\{\left[\kappa^l_{\check{d}_{ij}}, \kappa^u_{\check{d}_{ij}}\right]\right\}$ and $\min_j \min_i \left\{\left[\delta^l_{\check{d}_{ij}}, \delta^u_{\check{d}_{ij}}\right]\right\} \le \mathcal{F}_{\check{d}_k} \le \max_j \max_i \left\{\left[\delta^l_{\check{d}_{ij}}, \delta^u_{\check{d}_{ij}}\right]\right\}$ respectively.

Using the score function, we have

$$S\left(\mathcal{F}_{\check{d}_k}\right) = \frac{\kappa^l_{\check{d}_k} + \kappa^u_{\check{d}_k} + \delta^l_{\check{d}_k} + \delta^u_{\check{d}_k}}{4} \leq \underset{j}{max}\ \underset{i}{max}\left\{\left[\kappa^l_{\check{d}_{ij}}, \kappa^u_{\check{d}_{ij}}\right]\right\} - \underset{j}{min}\ \underset{i}{min}\left\{\left[\delta^l_{\check{d}_{ij}}, \delta^u_{\check{d}_{ij}}\right]\right\}$$

$$= S\left(\mathcal{F}^-_{\check{d}_k}\right)$$

$$S\left(\mathcal{F}_{\check{d}_k}\right) = \frac{\kappa^l_{\check{d}_k} + \kappa^u_{\check{d}_k} + \delta^l_{\check{d}_k} + \delta^u_{\check{d}_k}}{4} \geq \underset{j}{min}\ \underset{i}{min}\left\{\left[\kappa^l_{\check{d}_{ij}}, \kappa^u_{\check{d}_{ij}}\right]\right\} - \underset{j}{max}\ \underset{i}{max}\left\{\left[\delta^l_{\check{d}_{ij}}, \delta^u_{\check{d}_{ij}}\right]\right\} = S(\mathcal{F}_{\check{d}_k}{}^+)$$

Using order relation among two IVIFHSNs, we have

$\mathcal{F}^-_{\check{d}_k} \leq$ IVIFHSIWA $(\mathcal{F}_{\check{d}_{11}}, \mathcal{F}_{\check{d}_{12}}, \ldots\ldots\ldots, \mathcal{F}_{\check{d}_{nm}}) \leq \mathcal{F}_{\check{d}_k}{}^+$.

3.1.3. Shift invariance:

Let $\mathcal{F}_{\check{d}_k} = ([\kappa^l_{\check{d}_k}, \kappa^u_{\check{d}_k}], [\delta^l_{\check{d}_k}, \delta^u_{\check{d}_k}])$ be an IVIFHSN. Then

IVIFHSIWA $(\mathcal{F}_{\check{d}_{11}} \oplus \mathcal{F}_{\check{d}_k}, \mathcal{F}_{\check{d}_{12}} \oplus \mathcal{F}_{\check{d}_k}, \ldots\ldots\ldots, \mathcal{F}_{\check{d}_{nm}} \oplus \mathcal{F}_{\check{d}_k}) =$ *IVIFHSIWA* $(\mathcal{F}_{\check{d}_{11}}, \mathcal{F}_{\check{d}_{12}}, \ldots\ldots\ldots, \mathcal{F}_{\check{d}_{nm}}) \oplus \mathcal{F}_{\check{d}_k}$

Proof: Let $\mathcal{F}_{\check{d}_k} = ([\kappa^l_{\check{d}_k}, \kappa^u_{\check{d}_k}], [\delta^l_{\check{d}_k}, \delta^u_{\check{d}_k}])$ and $\mathcal{F}_{\check{d}_{ij}} = ([\kappa^l_{\check{d}_{ij}}, \kappa^u_{\check{d}_{ij}}], [\delta^l_{\check{d}_{ij}}, \delta^u_{\check{d}_{ij}}])$ be two IVIFHSNs. Then, using Definition 3.1 (1), we have

$$\mathcal{F}_{\check{d}_k} \oplus \mathcal{F}_{\check{d}_{ij}} = \begin{pmatrix} 1 - \left(1 - \left[\kappa^l_{\check{d}_k}, \kappa^u_{\check{d}_k}\right]\right)\left(1 - \left[\kappa^l_{\check{}}, \kappa^u_{\check{d}_{ij}}\right]\right), \\ \left(1 - \left[\kappa^l_{\check{d}_k}, \kappa^u_{\check{d}_k}\right]\right)\left(1 - \left[\kappa^l_{\check{d}_{ij}}, \kappa^u_{\check{d}_{ij}}\right]\right) - \left(1 - \left(\left[\kappa^l_{\check{d}_k}, \kappa^u_{\check{d}_k}\right], \left[\delta^l_{\check{d}_k}, \delta^u_{\check{d}_k}\right]\right)\right)\left(1 - \left(\left[\kappa^l_{\check{d}_{ij}}, \kappa^u_{\check{d}_{ij}}\right], \left[\delta^l_{\check{d}_{ij}}, \delta^u_{\check{d}_{ij}}\right]\right)\right) \end{pmatrix}$$

So,

$$IVIFHSIWA\left(\mathcal{F}_{\check{d}_{11}} \oplus \mathcal{F}_{\check{d}_k}, \mathcal{F}_{\check{d}_{12}} \oplus \mathcal{F}_{\check{d}_k}, \ldots\ldots\ldots, \mathcal{F}_{\check{d}_{nm}} \oplus \mathcal{F}_{\check{d}_k}\right) = \oplus_{j=1}^m \nu_j\left(\oplus_{i=1}^n \omega_i\left(\mathcal{F}_{\check{d}_{ij}} \oplus \mathcal{F}_{\check{d}_k}\right)\right)$$

$$= \begin{pmatrix} 1 - \prod_{j=1}^m\left(\prod_{i=1}^n\left(1 - \left[\kappa^l_{\check{d}_{ij}}, \kappa^u_{\check{d}_{ij}}\right]\right)^{\omega_i}\left(1 - \left[\kappa^l_{\check{d}_k}, \kappa^u_{\check{d}_k}\right]\right)^{\omega_i}\right)^{\nu_j}, \\ \prod_{j=1}^m\left(\prod_{i=1}^n\left(1 - \left[\kappa^l_{\check{d}_{ij}}, \kappa^u_{\check{d}_{ij}}\right]\right)^{\omega_i}\left(1 - \left[\kappa^l_{\check{d}_k}, \kappa^u_{\check{d}_k}\right]\right)^{\omega_i}\right)^{\nu_j} - \prod_{j=1}^m\left(\prod_{i=1}^n\left(1 - \left[\kappa^l_{\check{d}_{ij}}, \kappa^u_{\check{d}_{ij}}\right] - \left[\delta^l_{\check{d}_{ij}}, \delta^u_{\check{d}_{ij}}\right]\right)^{\omega_i}\left(1 - \left[\kappa^l_{\check{d}_k}, \kappa^u_{\check{d}_k}\right] - \left[\delta^l_{\check{d}_k}, \delta^u_{\check{d}_k}\right]\right)^{\omega_i}\right)^{\nu_j} \end{pmatrix}$$

$$= \begin{pmatrix} 1 - \left(1 - \left[\kappa^l_{\check{d}_k}, \kappa^u_{\check{d}_k}\right]\right)\prod_{j=1}^m\left(\prod_{i=1}^n\left(1 - \left[\kappa^l_{\check{d}_{ij}}, \kappa^u_{\check{d}_{ij}}\right]\right)^{\omega_i}\right)^{\nu_j}, \\ \left(1 - \left[\kappa^l_{\check{d}_k}, \kappa^u_{\check{d}_k}\right]\right)\prod_{j=1}^m\left(\prod_{i=1}^n\left(1 - \left[\kappa^l_{\check{d}_{ij}}, \kappa^u_{\check{d}_{ij}}\right]\right)^{\omega_i}\right)^{\nu_j} - \left(1 - \left[\kappa^l_{\check{d}_k}, \kappa^u_{\check{d}_k}\right] - \left[\delta^l_{\check{d}_k}, \delta^u_{\check{d}_k}\right]\right)\prod_{j=1}^m\left(\prod_{i=1}^n\left(1 - \left[\kappa^l_{\check{d}_{ij}}, \kappa^u_{\check{d}_{ij}}\right] - \left[\delta^l_{\check{d}_{ij}}, \delta^u_{\check{d}_{ij}}\right]\right)^{\omega_i}\right)^{\nu_j} \end{pmatrix}$$

$$= \begin{pmatrix} 1 - \prod_{j=1}^m\left(\prod_{i=1}^n\left(1 - \left[\kappa^l_{\check{d}_{ij}}, \kappa^u_{\check{d}_{ij}}\right]\right)^{\omega_i}\right)^{\nu_j}, \\ \prod_{j=1}^m\left(\prod_{i=1}^n\left(1 - \left[\kappa^l_{\check{d}_{ij}}, \kappa^u_{\check{d}_{ij}}\right]\right)^{\omega_i}\right)^{\nu_j} - \prod_{j=1}^m\left(\prod_{i=1}^n\left(1 - \left[\kappa^l_{\check{d}_{ij}}, \kappa^u_{\check{d}_{ij}}\right] - \left[\delta^l_{\check{d}_{ij}}, \delta^u_{\check{d}_{ij}}\right]\right)^{\omega_i}\right)^{\nu_j} \end{pmatrix}$$

$$\oplus\left(\left[\kappa^l_{\check{d}_k}, \kappa^u_{\check{d}_k}\right], \left[\delta^l_{\check{d}_k}, \delta^u_{\check{d}_k}\right]\right)$$

$$= IVIFHSIWA\left(\mathcal{F}_{\check{d}_{11}}, \mathcal{F}_{\check{d}_{12}}, \ldots\ldots\ldots, \mathcal{F}_{\check{d}_{nm}}\right) \oplus \mathcal{F}_{\check{d}_k}$$

3.1.4. Homogeneity:

Prove that IVIFHSIWA $(\beta\mathcal{F}_{\check{d}_{11}}, \beta\mathcal{F}_{\check{d}_{12}}, \ldots\ldots\ldots, \beta\mathcal{F}_{\check{d}_{nm}}) = \beta$ IVIFHSIWA $(\mathcal{F}_{\check{d}_{11}}, \mathcal{F}_{\check{d}_{12}}, \ldots\ldots\ldots, \mathcal{F}_{\check{d}_{nm}})$ $\beta > 0$.

Proof: Let $\mathcal{F}_{\check{d}_{ij}} = ([\kappa^l_{\check{d}_{ij}}, \kappa^u_{\check{d}_{ij}}], [\delta^l_{\check{d}_{ij}}, \delta^u_{\check{d}_{ij}}])$ be an IVIFHSN and $\beta > 0$. Then using Definition 3.1, we have

$$\beta\mathcal{F}_{\check{d}_k} = \left(\left[1-\left(1-\kappa^l_{\check{d}_k}\right)^{\beta}, 1-\left(1-\kappa^u_{\check{d}_k}\right)^{\beta}\right], \left[\delta^l_{\check{d}_k}\beta, \delta^u_{\check{d}_k}\beta\right]\right) = \left(1-\left(1-\left[\kappa^l_{\check{d}_k}, \kappa^u_{\check{d}_k}\right]\right)^{\beta}, \left[\delta^l_{\check{d}_k}\beta, \delta^u_{\check{d}_k}\beta\right]\right)$$

So,

$$\left(\beta\mathcal{F}_{\check{d}_{11}}, \beta\mathcal{F}_{\check{d}_{12}}, \ldots\ldots\ldots, \beta\mathcal{F}_{\check{d}_{nm}}\right)$$

$$= \begin{pmatrix} 1-\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(1-\left[\kappa^l_{\check{d}_{ij}}, \kappa^u_{\check{d}_{ij}}\right]\right)^{\beta\omega_i}\right)^{\nu_j}, \\ \prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(1-\left[\kappa^l_{\check{d}_{ij}}, \kappa^u_{\check{d}_{ij}}\right]\right)^{\beta\omega_i}\right)^{\nu_j} - \prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(1-\left[\kappa^l_{\check{d}_{ij}}, \kappa^u_{\check{d}_{ij}}\right]-\left[\delta^l_{\check{d}_{ij}}, \delta^u_{\check{d}_{ij}}\right]\right)^{\beta\omega_i}\right)^{\nu_j} \end{pmatrix}$$

$$= \begin{pmatrix} 1-\left(\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(1-\left[\kappa^l_{\check{d}_{ij}}, \kappa^u_{\check{d}_{ij}}\right]\right)^{\omega_i}\right)^{\nu_j}\right)^{\beta}, \\ \left(\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(1-\left[\kappa^l_{\check{d}_{ij}}, \kappa^u_{\check{d}_{ij}}\right]\right)^{\omega_i}\right)^{\nu_j}\right)^{\beta} - \left(\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(1-\left[\kappa^l_{\check{d}_{ij}}, \kappa^u_{\check{d}_{ij}}\right]-\left[\delta^l_{\check{d}_{ij}}, \delta^u_{\check{d}_{ij}}\right]\right)^{\omega_i}\right)^{\nu_j}\right)^{\beta} \end{pmatrix}$$

$= \beta$IVIFHSIWA $(\mathcal{F}_{\check{d}_{11}}, \mathcal{F}_{\check{d}_{12}}, \ldots\ldots\ldots, \mathcal{F}_{\check{d}_{nm}})$.

4. Multi-Criteria Group Decision-making Approach Based on Proposed Operators

To validate the implications of planned AOs, a DM approach is developed to remove MCGDM obstacles. In addition, numerical illustration is provided to endorse the convenience of the proposed method.

4.1. Proposed MCGDM approach

Consider $\mathfrak{I} = \{\mathfrak{I}^1, \mathfrak{I}^2, \mathfrak{I}^3, \ldots, \mathfrak{I}^s\}$ and $\mathcal{U} = \{\mathcal{U}_1, \mathcal{U}_2, \mathcal{U}_3, \ldots, \mathcal{U}_r\}$ be the set of substitutes and specialists, respectively. The weights of specialists are prearranged as $\omega_i = (\omega_1, \omega_2, \omega_3, \ldots, \omega_n)^T$ such that $\omega_i > 0$, $\sum_{i=1}^{n}\omega_i = 1$. Let $\mathcal{L} = \{e_1, e_2, e_3, \ldots, e_m\}$ be the set of attributes with their conforming multi-sub-attributes such as $\mathcal{L}' = \{(e_{1\rho} \times e_{2\rho} \times \ldots \times e_{m\rho})\ \forall\ \rho \epsilon \{1, 2, \ldots, t\}\}$ and weights $\nu = (\nu_1, \nu_2, \nu_3, \ldots, \nu_n)^T$ such as $\nu_i > 0$, $\sum_{i=1}^{n}\nu_i = 1$, and be detailed as $\mathcal{L}' = \{\check{d}_\partial : \partial \epsilon \{1, 2, \ldots, m\}\}$. The team of specialists $\{\kappa^i: i = 1, 2, \ldots, n\}$ judge the substitutes $\{\mathfrak{L}^{(z)}: z = 1, 2, \ldots, s\}$ under the preferred sub-attributes $\{\check{d}_\partial: \partial = 1, 2, \ldots, k\}$ in terms of IVIFHSNs such as $(\mathfrak{I}_{\check{d}_{ik}}{}^{(z)})_{n\times m} = ([\kappa^l_{\check{d}_{ik}}, \kappa^u_{\check{d}_{ik}}], [\delta^l_{\check{d}_{ik}}, \delta^u_{\check{d}_{ik}}])_{n*m}$, where $0 \le \kappa^l_{\check{d}_{ik}}, \kappa^u_{\check{d}_{ik}}, \delta^l_{\check{d}_{ik}}, \delta^u_{\check{d}_{ik}} \le 1$ and $0 \le (\kappa^u_{\check{d}_{ik}})^2 (\delta^u_{\check{d}_{ik}})^2 \le +1$ for all i, k. The group of experts gives their opinion on each alternative in IVIFHSNs. The algorithmic rule based on developed operators is given as follows:

Step-1: Expert's estimation for each substitute in the form of IVIFHSNs:

$$\left(\mathfrak{I}^{(z)}_{\check{d}_{ik}}\right)_{n\times m} = \left(\left[\kappa^l_{\check{d}_{ik}}, \kappa^u_{\check{d}_{ik}}\right], \left[\delta^l_{\check{d}_{ik}}, \delta^u_{\check{d}_{ik}}\right]\right)_{n*m}$$

$$= \begin{bmatrix} \left(\left[\kappa^l_{d_{11}}, \kappa^u_{d_{11}}\right], \left[\delta^l_{d_{11}}, \delta^u_{d_{11}}\right]\right) & \left(\left[\kappa^l_{d_{12}}, \kappa^u_{d_{12}}\right], \left[\delta^l_{d_{12}}, \delta^u_{d_{12}}\right]\right) & \cdots & \left(\left[\kappa^l_{d_{1m}}, \kappa^u_{d_{1m}}\right], \left[\delta^l_{d_{1m}}, \delta^u_{d_{1m}}\right]\right) \\ \left(\left[\kappa^l_{d_{21}}, \kappa^u_{d_{21}}\right], \left[\delta^l_{d_{21}}, \delta^u_{d_{21}}\right]\right) & \left(\left[\kappa^l_{d_{22}}, \kappa^u_{d_{22}}\right], \left[\delta^l_{d_{22}}, \delta^u_{d_{22}}\right]\right) & & \left(\left[\kappa^l_{d_{2n}}, \kappa^u_{d_{2n}}\right], \left[\delta^l_{d_{2m}}, \delta^u_{d_{2m}}\right]\right) \\ \vdots & & \ddots & \vdots \\ \left(\left[\kappa^l_{d_{n1}}, \kappa^u_{d_{n1}}\right], \left[\delta^l_{d_{n1}}, \delta^u_{d_{n1}}\right]\right) & \left(\left[\kappa^l_{d_{n2}}, \kappa^u_{d_{n2}}\right], \left[\delta^l_{d_{n2}}, \delta^u_{d_{n2}}\right]\right) & \cdots & \left(\left[\kappa^l_{d_{nm}}, \kappa^u_{d_{nm}}\right], \left[\delta^l_{d_{nm}}, \delta^u_{d_{nm}}\right]\right) \end{bmatrix}$$

Step-2: Grow the normalized matrices for separate substitutes employing the normalization rule:

$$\mathcal{F}_{\check{d}_{ik}} = \begin{cases} \mathcal{F}^c_{\check{d}_{ij}} = \left(\left[\delta^l_{\check{d}_{ik}}, \delta^u_{\check{d}_{ik}}\right], \left[\kappa^l_{\check{d}_{ik}}, {}^u_{\check{d}_{ik}}\right]\right)_{n\times m} & \text{cost type parameter} \\ \mathcal{F}_{\check{d}_{ij}} = \left(\left[\kappa^l_{\check{d}_{ik}}, \kappa^u_{\check{d}_{ik}}\right], \left[\delta^l_{\check{d}_{ik}}, \delta^u_{\check{d}_{ik}}\right]\right)_{n\times m} & \text{benefit type parameter} \end{cases}$$

Step-3: Compute the aggregated values employing the IVIFHSIWA operator for each substitute.

Step-4: Analyze the score values for each substitute employing the score function.

Step-5: Determine the most suitable alternative.

Step-6: Alternatives ranking.

4.2. Numerical example

The aspect of material assortment is specified as follows: $\mathcal{L} = \{d_1$ = Specific gravity = attaining data around the meditation of resolutions of numerous materials, d_2 = Toughness index, d_3 = Yield stress, d_4 = Easily accessible$\}$. The corresponding sub-attributes of the considered parameters are as follows: Specific gravity = attaining data around the meditation of resolutions of numerous materials = d_1 =$\{d_{11}$ = assess corporal variations, d_{12} = govern the degree of regularity among tasters$\}$, Toughness index = d_2 = $\{d_{21}$ = CharpyV−Notch Impact Energy, d_{22} = Plane Strain Fracture Toughness$\}$, Yield stress = d_3 = $\{d_{31}$ = Yield stress$\}$, and Easily accessible = d_4 = $\{d_{41}$ = Easily accessible$\}$. Let $\mathcal{L}' = d_1 \times d_2 \times d_3 \times d_4$ be a set of sub-attributes.

$\mathcal{L}' = \left\{ \begin{array}{l} (d_{11}, d_{21}, d_{31} d_{41}), (d_{11}, d_{22}, d_{31}, d_{41}), \\ (_{12}, d_{21}, d_{31}, d_{41}), (d_{12}, d_{22}, d_{31}, d_{41}) \end{array} \right\}, \mathcal{L}' = \{\check{d}_1, \check{d}_2, \check{d}_3, \check{d}_4\}$ be a set of sub-attributes with weights $(0.3, 0.1, 0.2, 0.4)^T$. Let $\{\mathcal{U}_1, \mathcal{U}_2, \mathcal{U}_3, \mathcal{U}_4\}$ be a group of specialists with weights $(0.1, 0.2, 0.4, 0.3)^T$. To judge the optimal substitute, professionals deliver their predilections in IVIFHSNs from Zulqarnain et al. (2022).

4.2.1. By IVIFHSIWA operator

Step-1: The specialist's estimation in the IVIFHSNs form for each alternative is shown in Tables 1, 2, 3, and 4.

Table 1
Decision matrix for $\Im^1$ in the form of IVIFHSN

	$\check{d}_1$	$\check{d}_2$	$\check{d}_3$	$\check{d}_4$
$\mathcal{U}_1$	([0.4,0.5],[0.2,0.5])	([0.2,0.4],[0.5,0.6])	([0.1,0.3],[0.2,0.5])	([0.2,0.4],[0.2,0.6])
$\mathcal{U}_2$	([0.2,0.4],[0.2,0.6])	([0.1,0.3],[0.4,0.5])	([0.2,0.3],[0.3,0.7])	([0.2,0.4],[0.2,0.5])
$\mathcal{U}_3$	([0.3,0.5],[0.1,0.4])	([0.4,0.5],[0.2,0.4])	([0.4,0.5],[0.3,0.4])	([0.2,0.6],[0.2,0.4])
$\mathcal{U}_4$	([0.4,0.6],[0.3,0.4])	([0.1,0.3],[0.3,0.6])	([0.3,0.4],[0.3,0.5])	([0.3,0.4],[0.3,0.5])

Table 2
Decision matrix for $\Im^2$ in the form of IVIFHSN

	$\check{d}_1$	$\check{d}_2$	$\check{d}_3$	$\check{d}_4$
$\mathcal{U}_1$	([0.3,0.4],[0.5,0.5])	([0.2,0.4],[0.4,0.5])	([0.2,0.4],[0.4,0.5])	([0.4,0.5],[0.3,0.5])
$\mathcal{U}_2$	([0.3,0.5],[0.3,0.4])	([0.1,0.4],[0.4,0.5])	([0.1,0.5],[0.3,0.4])	([0.4,0.5],[0.3,0.4])
$\mathcal{U}_3$	([0.2,0.6],[0.1,0.4])	([0.1,0.2],[0.2,0.8])	([0.4,0.5],[0.3,0.5])	([0.3,0.6],[0.2,0.4])
$\mathcal{U}_4$	([0.2,0.3],[0.3,0.6])	([0.3,0.5],[0.1,0.4])	([0.3,0.4],[0.2,0.6])	([0.1,0.3],[0.3,0.6])

Table 3
Decision matrix for $\Im^3$ in the form of IVIFHSN

	$\check{d}_1$	$\check{d}_2$	$\check{d}_3$	$\check{d}_4$
$\mathcal{U}_1$	([0.3,0.4],[0.2,0.5])	([0.3,0.4],[0.4,0.6])	([0.3,0.4],[0.4,0.5])	([0.3,0.4],[0.3,0.6])
$\mathcal{U}_2$	([0.4,0.6],[0.3,0.4])	([0.2,0.5],[0.2,0.3])	([0.3,0.5],[0.3,0.5])	([0.2,0.6],[0.2,0.4])
$\mathcal{U}_3$	([0.2,0.4],[0.3,0.5])	([0.3,0.4],[0.3,0.6])	([0.3,0.5],[0.3,0.4])	([0.1,0.3],[0.4,0.5])
$\mathcal{U}_4$	([0.3,0.6],[0.3,0.4])	([0.3,0.5],[0.2,0.4])	([0.2,0.5],[0.3,0.4])	([0.3,0.4],[0.3,0.6])

Table 4
Decision matrix for $\Im^4$ in the form of IVIFHSN

	$\check{d}_1$	$\check{d}_2$	$\check{d}_3$	$\check{d}_4$
$\mathcal{U}_1$	([0.3,0.5],[0.2,0.4])	([0.2,0.6],[0.1,0.4])	([0.2,0.5],[0.3,0.4])	([0.3,0.4],[0.4,0.5])
$\mathcal{U}_2$	([0.2,0.7],[0.1,0.3])	([0.1,0.5],[0.4,0.5])	([0.3,0.5],[0.4,0.5])	([0.2,0.5],[0.3,0.4])
$\mathcal{U}_3$	([0.2,0.5],[0.1,0.4])	([0.2,0.5],[0.1,0.5])	([0.2,0.4],[0.2,0.6])	([0.3,0.5],[0.1,0.5])
$\mathcal{U}_4$	([0.2,0.4],[0.5,0.5])	([0.2,0.5],[0.2,0.4])	([0.2,0.4],[0.3,0.6])	([0.2,0.5],[0.4,0.5])

Step-2: No need to normalize.

Step-3: Calculate the aggregated values for each alternate using the IVIFHSIWA operator.

$$\Theta_1=\begin{pmatrix} 1-\prod_{j=1}^{4}\left(\prod_{i=1}^{4}\left(1-\left[\kappa^l_{\check{d}_{ij}},\kappa^u_{\check{d}_{ij}}\right]\right)^{\omega_i}\right)^{\nu_j}, \\ \prod_{j=1}^{4}\left(\prod_{i=1}^{4}\left(1-\left[\kappa^l_{\check{d}_{ij}},\kappa^u_{\check{d}_{ij}}\right]\right)^{\omega_i}\right)^{\nu_j}-\prod_{j=1}^{4}\left(\prod_{i=1}^{4}\left(1-\left[\kappa^l_{\check{d}_{ij}},\kappa^u_{\check{d}_{ij}}\right]-\left[\delta^l_{\check{d}_{ij}},\delta^u_{\check{d}_{ij}}\right]\right)^{\omega_i}\right)^{\nu_j} \end{pmatrix}$$

$=([0.4156,0.4971],[0.2415,0.4173])$

$$\Theta_2=\begin{pmatrix} 1-\prod_{j=1}^{4}\left(\prod_{i=1}^{4}\left(1-\left[\kappa^l_{\check{d}_{ij}},\kappa^u_{\check{d}_{ij}}\right]\right)^{\omega_i}\right)^{\nu_j}, \\ \prod_{j=1}^{4}\left(\prod_{i=1}^{4}\left(1-\left[\kappa^l_{\check{d}_{ij}},\kappa^u_{\check{d}_{ij}}\right]\right)^{\omega_i}\right)^{\nu_j}-\prod_{j=1}^{4}\left(\prod_{i=1}^{4}\left(1-\left[\kappa^l_{\check{d}_{ij}},\kappa^u_{\check{d}_{ij}}\right]-\left[\delta^l_{\check{d}_{ij}},\delta^u_{\check{d}_{ij}}\right]\right)^{\omega_i}\right)^{\nu_j} \end{pmatrix}$$

$=([0.2959,0.5116],[0.4446,0.4753])$

$$\Theta_3=\begin{pmatrix} 1-\prod_{j=1}^{4}\left(\prod_{i=1}^{4}\left(1-\left[\kappa^l_{\check{d}_{ij}},\kappa^u_{\check{d}_{ij}}\right]\right)^{\omega_i}\right)^{\nu}, \\ \prod_{=1}^{4}\left(\prod_{i=1}^{4}\left(1-\left[\kappa^l_{\check{d}_{ij}},\kappa^u_{\check{d}_{ij}}\right]\right)^{\omega_i}\right)^{\nu_j}-\prod_{j=1}^{4}\left(\prod_{i=1}^{4}\left(1-\left[\kappa^l_{\check{d}_{ij}},\kappa^u_{\check{d}_{ij}}\right]-\left[\delta^l_{\check{d}_{ij}},\delta^u_{\check{d}_{ij}}\right]\right)^{\omega_i}\right)^{\nu_j} \end{pmatrix}$$

$=([0.5347,0.5756],[0.4719,0.4928])$.

$$\Theta_4=\begin{pmatrix} 1-\prod_{j=1}^{4}\left(\prod_{i=1}^{4}\left(1-\left[\kappa^l_{\check{d}_{ij}},\kappa^u_{\check{d}_{ij}}\right]\right)^{\omega_i}\right)^{\nu_j}, \\ \prod_{j=1}^{4}\left(\prod_{i=1}^{4}\left(1-\left[\kappa^l_{\check{d}_{ij}},\kappa^u_{\check{d}_{ij}}\right]\right)^{\omega_i}\right)^{\nu_j}-\prod_{j=1}^{4}\left(\prod_{i=1}^{4}\left(1-\left[\kappa^l_{\check{d}_{ij}},\kappa^u_{\check{d}_{ij}}\right]-\left[\delta^l_{\check{d}_{ij}},\delta^u_{\check{d}_{ij}}\right]\right)^{\omega_i}\right)^{\nu_j} \end{pmatrix}$$

$=([0.2956,0.6754],[0.3729,0.6935])$.

Step-4: Applying the score function $\frac{S=\kappa^l_{\check{d}_k}+\kappa^u_{\check{d}_k}+\delta^l_{\check{d}_k}+\delta^u_{\check{d}_k}}{4}$ to determine the score values for all alternatives. $S(\Theta_1)=0.3929$, $S(\Theta_2)=0.4319$, $S(\Theta_3)=0.5188$, and $S(\Theta_4)=0.5094$.

Step-5: From the above calculation, we get $S(\Theta_3)>S(\Theta_4)>S(\Theta_2)>S(\Theta_1)$, which shows that $\Im^3$ is the best alternative.

Step-6: So, $\Im^3>\Im^4>\Im^2>\Im^1$ is the obtained ranking of alternatives.

5. Comparative Studies

To authenticate the practicality of the proposed procedure, an assessment between the proposed model and the prevailing methods is planned in the next section.

5.1. Supremacy of the proposed technique

The proposed method competently delivers realistic decisions in the DM procedure. We introduced the MCGDM approach using our developed IVIFHSIWA operator. Our plan MCGDM technique provides the most subtle and precise information on DM complications. The proposed model is multipurpose and communicative, adapting to changing instability, commitment, and productivity. Dissimilar replicas have particular classification processes, so there is a straight change in the classification of expected methods according to their expectations. This systematic study and evaluation determined that the consequences attained from the conventional method are erroneously equal to the amalgam organization. In addition, due to some favorable conditions, several composite configurations of FS such as IVFS, IVIFS, and IVIFSS concentrate in IVIFHSS. It is easy to syndicate insufficient and obscure data in the DM method. Data about the matter can be described more accurately and rationally. Therefore, our projected technique is extra proficient, meaningful, superior, and enhanced than multiple mixed FS structures. Table 5 provides an analysis of the technique presented and the features of some existing models.

5.2. Comparative analysis

To prove the utility of the planned process, we equate the attained consequences with some prevailing approaches under IVPFS, IVIFSS, and IVPFSS. A summary of all values is specified in Table 6. Xu and Gou (2017) developed the IVIFWA operator that cannot compute the parametrized values of the alternatives. Furthermore, if any expert considers the MD and NMD whose sum exceeds 1, the aforementioned AOs fail to accommodate the scenario. Zulqarnain et al. (2021) established AOs for IVIFSS that cannot accommodate the decision-makers selection when the sum of upper MD and NMD parameters surpasses 1. It is detected that, in certain conditions, the existing AOs provide some unattractive outcomes. So, to resolve

Table 5
Feature analysis of different models with a proposed model

	Membership information	Non-membership information	Aggregated attributes information	Aggregated information in intervals form	Aggregated sub-attributes information of any attribute	Interactional aggregation information
IVFS (Turksen, 1986)	✓	×	×	✓	×	×
IVIFWA (Xu and Gou, 2017)	✓	✓	×	✓	×	×
IVIFSWA (Zulqarnain et al., 2021)	✓	✓	✓	✓	×	×
IVIFHSWA(Zulqarnain et al., 2022)	✓	✓	✓	✓	✓	×
Proposed IVIFHSIWA	✓	✓	✓	✓	✓	✓

Table 6
Comparison of planned operators with some prevailing operators

AO	$\mathfrak{J}^1$	$\mathfrak{J}^2$	$\mathfrak{J}^3$	$\mathfrak{J}^4$	Alternatives ranking	Optimal choice
IVIFWA (Xu and Gou, 2017)	0.3681	0.2116	0.3509	0.4573	$\mathfrak{J}^3 > \mathfrak{J}^1 > \mathfrak{J}^3 > \mathfrak{J}^2$	$\mathfrak{J}^3$
IVIFWIA (Ze-Shui, 2007)	0.3104	0.2753	0.2914	0.3952	$\mathfrak{J}^3 > \mathfrak{J}^1 > \mathfrak{J}^3 > \mathfrak{J}^2$	$\mathfrak{J}^3$
IVIFSWA (Zulqarnain et al., 2021)	0.0235	0.0253	0.0584	0.0723	$\mathfrak{J}^3 > \mathfrak{J}^3 > \mathfrak{J}^2 > \mathfrak{J}^1$	$\mathfrak{J}^3$
IVIFHSWA (Zulqarnain et al., 2022)	0.2365	0.3734	0.5840	0.7134	$\mathfrak{J}^3 > \mathfrak{J}^3 > \mathfrak{J}^2 > \mathfrak{J}^1$	$\mathfrak{J}^3$
IVIFHSIWA	0.3929	0.4319	0.5188	0.5094	$\mathfrak{J}^3 > \mathfrak{J}^4 > \mathfrak{J}^2 > \mathfrak{J}^1$	$\mathfrak{J}^3$

such complications, we developed the AOs for IVIFHSS, which capably deal with the multi-sub-attributes compared to existing AOs. Thus, IVIFHSS is the most generalized form of IVIFSS. Hence, based on the above-mentioned details, the anticipated operators in this paper are more influential, consistent, and prosperous. A comparison of the projected model with prevailing replicas is given in the subsequent Table 6.

6. Conclusion

DM is a process for arranging and choosing rational preferences from numerous substitutes. The most operational methodology in DM is paying adjacent consideration and focusing on your objectives. In manufacturing, the better stability of manipulation is neutral; authoritative material and fabricated surround extensive content. In a real DM, assessing alternative facts as told by a professional is permanently incorrect, irregular, and impressive. Therefore, IVIFHSNs can be used to match this uncertain data. The main determination of this work is to extend the AO for IVIFHSS. First, we introduced the interactional operational laws for the IVIFHSS environment. By seeing the developed operational laws, we introduced the IVIFHSIWA operator with its fundamental properties. Also, a DM method is planned to deal with the complications of MCGDM based on established operators. To demonstrate the strength of the established method, we present a comprehensive mathematical illustration for MS in manufacturing engineering. Lastly, based on the results obtained, it is resolute that the method proposed in this study is the most concrete and operative one to resolve MCGDM obstacles compared to existing techniques. Future studies highlight growing DM approaches, such as Einstein's hybrid AOs in the IVIFHSS setting. Many other hybrid structures are topological structures, Bonferroni mean AOs, hammy mean AOs, etc. We are confident that these extensive growths and conjectures will support considered professional consideration extents convoluted in the world's environment.

Conflicts of Interest

The authors declare that they have no conflicts of interest to this work.

References

Atanassov, K. (1986). Intuitionistic fuzzy sets. *Fuzzy Sets and Systems*, *20*(1), 87–96. https://doi.org/10.1016/S0165-0114(86)80034-3.

Atanassov, K. T. (1999). Interval valued intuitionistic fuzzy sets. In K. T. Atanassov (Eds.), *Intuitionistic fuzzy sets*: Theory and applications (pp. 139–177). Springer. https://doi.org/10.1007/978-3-7908-1870-3

Atanassov, K. T. (2022). New topological operator over intuitionistic fuzzy sets. *Journal of Computational and Cognitive Engineering*, *1*(3), 94–102. https://doi.org/10.47852/bonviewJCCE2202197

Debnath, S. (2022). Interval-valued intuitionistic hypersoft sets and their algorithmic approach in multi-criteria decision making. *Neutrosophic Sets and Systems*, *48*, 226–250.

Ejegwa, P. A., & Agbetayo, J. M. (2023). Similarity-distance decision-making technique and its applications via intuitionistic fuzzy pairs. *Journal of Computational and Cognitive Engineering*, *2*(1), 68–74 https://doi.org/10.47852/bonviewJCCE512522514

Garg, H. (2018). An improved cosine similarity measure for intuitionistic fuzzy sets and their applications to decision-making process. *Hacettepe Journal of Mathematics and Statistics*, *47*(6), 1578–1594. http://dx.doi.org/10.15672/HJMS.2017.510.

Garg, H., & Kumar, K. (2019). Linguistic interval-valued atanassov intuitionistic fuzzy sets and their applications to group decision making problems. *IEEE Transactions on Fuzzy Systems*, *27*(12), 2302–2311. https://doi.org/10.1109/TFUZZ.2019.2897961.

Garg, H., & Rani, D. (2022). An efficient intuitionistic fuzzy MULTIMOORA approach based on novel aggregation operators for the assessment of solid waste management techniques. *Applied Intelligence*, *52*(4), 4330–4363. https://doi.org/10.1007/s10489-021-02541-w.

Gupta, P., Mehlawat, M. K., Grover, N., & Pedrycz, W. (2018). Multi-attribute group decision making based on extended TOPSIS method under interval-valued intuitionistic fuzzy environment. *Applied Soft Computing*, *69*(4), 554–567. https://doi.org/10.1016/j.asoc.2018.04.032.

Gurmani, S. H., Chen, H., & Bai, Y. (2021). The operational properties of linguistic interval valued q-Rung orthopair fuzzy information and its VIKOR model for multi-attribute group decision making. *Journal of Intelligent & Fuzzy Systems*, *41*(6), 7063–7079. https://doi.org/10.3233/JIFS-210940.

Gurmani, S. H., Chen, H., & Bai, Y. (2022). An extended MABAC method for multiple-attribute group decision making under probabilistic T-spherical hesitant fuzzy environment. Kybernetes. https://doi.org/10.1108/K-01-2022-0137.

Jafar, M. N., Saeed, M., Haseeb, M., & Habib, A. (2021). Matrix theory for intuitionistic fuzzy hypersoft sets and its application in multi-attributive decision-making problems. Theory and Application of Hypersoft Set, *65*.

Jiang, Y., Tang, Y., Chen, Q., Liu, H., & Tang, J. (2010). Interval-valued intuitionistic fuzzy soft sets and their properties. *Computers & Mathematics with Applications*, *60*(3), 906–918. https://doi.org/10.1016/j.camwa.2010.05.036.

Khan, R., Ullah, K., Pamucar, D., & Bari, M. (2022). Performance measure using a multi-attribute decision making approach based on complex T-spherical fuzzy power aggregation operators. *Journal of Computational and Cognitive Engineering*, *1*(3), 138–146. https://doi.org/10.47852/bonviewJCCE696205514.

Maji, P. K., Biswas, R., & Roy, A. R. (2001). Fuzzy soft sets. *Journal of Fuzzy Mathematics*, 2009. https://doi.org/10.1155/2009/586507.

Maji, P. K., Biswas, R., & Roy, A. R. (2001). Intuitionistic fuzzy soft sets. *Journal of Fuzzy Mathematics*, *9*, 677–692.

Molodtsov, D. (1999). Soft set theory—first results. *Computers & Mathematics with Applications*, *37*(4-5), 19–31. https://doi.org/10.1016/S0898-1221(99)00056-5.

Park, D. G., Kwun, Y. C., Park, J. H., & Park, I. Y. (2009). Correlation coefficient of interval-valued intuitionistic fuzzy sets and its application to multiple attribute group decision making problems. *Mathematical and Computer Modelling*, *50*(9-10), 1279–1293. https://doi.org/10.1016/j.mcm.2009.06.010.

Smarandache, F. (2018). Extension of the soft set to hypersoft set, and then to plithogenic hypersoft set. *Neutrosophic Sets and Systems*, *22*(1), 168–170.

Turksen, I. B. (1986). Interval valued fuzzy sets based on normal forms. *Fuzzy Sets and Systems*, *20*(2), 191–210. https://doi.org/10.1016/0165-0114(86)90077-1.

Xu, Z., & Gou, X. (2017). An overview of interval-valued intuitionistic fuzzy information aggregations and applications. *Granular Computing*, *2*(1), 13–39.

Zadeh, L. A. (1965). Fuzzy sets. *Information and Control*, *8*(3), 338–353. https://doi.org/10.1016/S0019-9958(65)90241-X.

Xu, Z. (2007). Methods for aggregating interval-valued intuitionistic fuzzy information and their application to decision making. *Control and decision*, *22*(2), 215–219.

Zhang, Z. (2018). Geometric Bonferroni means of interval-valued intuitionistic fuzzy numbers and their application to multiple attribute group decision making. *Neural Computing and Applications*, *29*(11), 1139–1154.

Zulqarnain, M., Saeed, M., & Tabasum, M. F. (2016). Comparison between fuzzy soft matrix (FSM) and interval valued fuzzy soft matrix (IVFSM) in decision making. *Science International*, *28*, 4277–4283.

Zulqarnain, R. M., Siddique, I., Jarad, F., & Iampan, A. (2022). Aggregation operators for interval-valued intuitionistic fuzzy hypersoft set with their application in material selection. *Mathematical Problems in Engineering*, 2022.

Zulqarnain, R. M., Xin, X. L., Saqlain, M., & Khan, W. A. (2021). TOPSIS method based on the correlation coefficient of interval-valued intuitionistic fuzzy soft sets and aggregation operators with their application in decision-making. *Journal of Mathematics*, 2021, 1–16. https://doi.org/10.1155/2021/6656858. 2021.

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