Received: 26 April 2024 | Revised: 15 July 2024 | Accepted: 22 July 2024 | Published online: 6 August 2024

RESEARCH ARTICLE

Journal of Computational and Cognitive Engineering
2024, Vol. 00(00) 1–16
DOI: 10.47852/bonviewJCCE42023253

BON VIEW PUBLISHING

A New Solution Technique for Fuzzy Transportation Problem Using Novel Ranking Functions on Heptagonal Fuzzy Numbers: A Case Study of Regional Shipment

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Abstract: This paper presented a fuzzy transportation problem to represent the uncertainty of transportation in parameters to produce an appropriate solution. In addition to making reliable solutions for real-world decision-making problems, the objective of transportation problem cost, demand, and supply parameters are demonstrated using heptagonal fuzzy numbers (HFN). The multiplication and division operations are presented with an example to make the inner performance of any two HFNs. A new ranking technique has been derived to defuzzify the linear HFN using an alpha-cut-based ranking technique, and it has been compared with the existing ranking technique. Furthermore, a generalized ranking heptagonal fuzzy method (GRHFM) algorithm is framed by incorporating defined operations and ranking techniques to solve the heptagonal fuzzy regional shipment problem. The effectiveness of our proposed method is demonstrated through a numerical example of an adopted case study. It determines the feasible shipment cost for vegetable transportation from the regional markets utilizing HFNs, which further compare solutions obtained through various algorithms. The conclusion section discusses the suggested GRHFM and alpha-cut ranking efficiency, outcomes, decision management, and future directions based on the existing transportation study.

Keywords: heptagonal fuzzy numbers, centroid of centroid, alpha-cut, ranking function, transportation problem

1. Introduction

The regional transportation industry is one of the many that the COVID-19 pandemic has severely impacted. Regional vegetable transports have experienced significant setbacks due to the virus spreading in urban areas. The resolution of transportation issues of regional shipment needs to be developed and modified since the local shipment reflects on the region's economy. In the literature, various transportation problems (TPs) [1, 2], such as urban and linear continuous transportation, were proposed and solved. To derive an optimal fuzzy allocation for ambiguous TPs, Pandian and Natarajan [3] developed an algorithm. Korukoglu and Ballı [4] enhanced the Vogel approximation method for the TP by incorporating accurate parameters. In real-world applications of TPs, all the parameters may not be treated as exact values due to unpredictable circumstances. In the context of the computational complexity of linear programming, finding an initial basic feasible solution (IBFS) [5] is crucial for solving optimization problems. It serves as a starting point for algorithms such as the simplex method or interior point methods to iteratively improve and find the optimal solution. As per the literature, scientists are still working to create an adaptive heuristic method to reach an optimal solution space for TP, notably [6, 7]. The fully rough integer interval TP [8] has been solved by applying the rough slice sum method.

The idea of fuzziness is introduced into transportation costs to address real-world decision-making difficulties, using a heptagonal fuzzy number (HFN). Researchers later developed fuzzy numbers for broader applications [9, 10]. The optimal solution to TPs [11] with a fuzzy cost coefficient was presented. A new fuzzy transportation algorithm was introduced to find optimal fuzzy solutions [12]. An algorithm for solving a specific type of fuzzy transportation problem (FTP) was proposed by Luo et al. [13]. A straightforward heuristic approach for the triangular fuzzy unbalanced TP [14]. A simplified approach using a generalized trapezoidal fuzzy number was created [15], for solving FTPs in pentagonal and triangular fuzzy environments [16]. A goal programming approach for the fuzzy octagonal TP under budgetary constraints [17]. A TP using fuzzy octagonal numbers [18]. The fuzzy programming approach to multi-objective solid TPs. Further study on fuzziness and its application to transportation problems is needed [19–21], along with solution approaches for the logarithmic transportation problem using column generation techniques [22]. The collaborative TP based on energy consumption with overlapping coalitions was solved [23].

A few studies have been devoted to concurrently addressing the local shipment based on uncertainty, focusing on distribution functions at the scheduling and planning levels. Furthermore, to our knowledge, limited research has been done based on the case study

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of local shipment in an uncertain environment. Section 2 provides the literature review to incorporate this study's research gap and justification. Section 3 discusses the recent well-known fuzzy numbers and the linear heptagonal fuzzy numbers (LHFN) and their operations. Section 4 deals with formulating a fuzzy transportation model and incorporating fuzziness concepts. In Section 5, two types of ranking functions of LHFN are discussed. Section 6 elaborates the solution methodology procedure to solve the heptagonal fuzzy transportation problem (HFTP). Section 7 explains the case study and numerical examples of the HFTP. Finally, a conclusion is presented in Section 8.

2. Literature Review

The transportation system naturally has uncertainties, which is why fuzzy numbers were introduced to represent such uncertainties. The variables in TP are not always well understood and consistent. Lack of precise information, uncertainty about transportation costs, varying demand-supply incidents, and other factors may contribute to imprecision. In 1965, Zadeh [24] introduced the notion of fuzziness that Bellman and Zadeh reinforced. This concept has since been extended into a theory that applies fuzzy linear programming to solve problems with diverse objective functions.

Many ranking functions are available in the literature for fuzzified numbers to defuzzify for further use. The ranking functions are in a major role in solving optimization problems like TPs, decision-making problems, traveling salesman problems [25], etc. Song and Leland [26] and Garg and Rizk-Allah [27] also contributed to the ranking function and to the adaptive learning technique for solving these behavioral management problems. Srivastava et al. [28] present the merits and demerits of various defuzzification strategies that are used in fuzzy set theory, etc. The trapezoidal type-2 fuzzy number is an extension of the trapezoidal fuzzy number. Das et al. [29] proposed a trapezoidal type-2 fuzzy variable defuzzification process centered on a critical value-based reduction method and nearest interval approximation. Kaur and Kumar [30] used the ranking function to solve a four-dimensional trapezoidal FTP. Many applications can be used with FTPs to solve the realistic issues of transportation, time, cost, reliability, etc. [31, 32]. Ghatee and Hashemi [33] derived ranking function-based solutions of a fully fuzzified minimal cost flow problem.

This article emphasizes ranking-based solutions for transportation network problems involving FTPs, aiming to optimize all uncertain parameters of the transportation network using the ordering technique introduced by Nishad and Abhishekh [34]. The advantage of the proposed method of Rani et al. [35] over the existing procedure is that the fuzzy optimum result is obtained without any dummy destination. A fuzzy transshipment network model for a well-balanced diet plan and ranking function-based transportation problems are solved in different fuzzy environments [36, 37]. Silmi Juman et al. [38] contributed an approach to creating an efficient result for the fuzzy TP by using the ranking function. A zero-position maximum allocation technique for solving an intuitionistic environment by using the ranking function and adaptive technique to solve an unknown demand fuzzy TPs with dual analysis. Aliakbari et al. [39] presented a relief logistics planning model to optimize the total cost. Bisht and Srivastava [40] applied a fuzzy trapezoidal technique to optimize the transport model based on information with interval values. Singh and Singh [41] extended the particle swarm optimization algorithm to solve the transport system in an uncertain situation. Fuzzy transportation systems for sustainable and damaged items have been constructed and solved in disaster response operations in fuzzy environments by researchers [42–46].

Various researchers have developed many fuzzy numbers, but triangle fuzzy numbers only provide a three-dimensional representation of uncertainty (low, medium, and high-range values). However, capturing uncertainty often requires a more flexible approach, leading to the concept of HFN. It is also known as seven-sided fuzzy numbers, and it offers a wider range of options for representing uncertainty compared to triangle fuzzy numbers. They allow for a more nuanced representation of uncertainty, enabling decision-makers to make more informed and robust decisions. Additionally, HFNs offer more accurate and precise representation by including intermediate values between the extreme ranges.

This study considered a fundamental problem of everyday food and vegetables bought from a nearby store. Vegetables come from places where they are readily available, and the transportation network resembles a vast spider web. Every year, the price of vegetables fluctuates primarily due to transportation costs. Many researchers are studying the transportation problem because it is an important part of human life, involving not just moving from one place to another but also shifting commodities as a kind of journey.

2.1. Research gap and contributions

According to the literature review, most of the ranking functions are defined for other existing fuzzy numbers, but the newly introduced LHFNs have limited ranking functions only. As well, this fuzzy number is not used in fully fuzzified regional transshipment problems. To utilize fuzzy numbers the constrained operations present in the existing literature, additional investigation is required. In addition, solutions for heptagonal FTP have not yet been investigated. Therefore, more research is needed to understand the capabilities of LHFNs in this context.

A few ranking functions are not widely applicable due to their complex mathematical formulations. Additionally, current ranking functions fail to differentiate between highly uncertain LHFNs, which significantly respect fuzzy membership grades and often arise in contexts with limited information about a system. The creation of a new defuzzification method surpasses current techniques. This innovative approach notably decreases computational effort while improving the clarity of fuzzy systems. Additionally, it exhibits enhanced accuracy and robustness when managing uncertain data. The method also yields promising outcomes in practical applications. Furthermore, this research addresses identified gaps and makes the following key contributions:

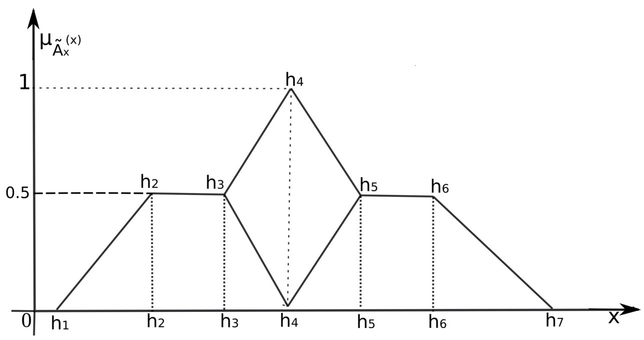
1) An advanced alpha-cut (α-cut) based ranking function is proposed to determine the defuzzification of fuzzified LHFNs.
2) The concept of multiplication and division operations for LHFNs is introduced as an extension to optimization problems, enhancing computational techniques.
3) A Heptagonal fuzzy transportation network model has been developed to examine the proposed algorithm, focusing on optimizing cost and other factors effectively.
4) The developed ranking function has been implemented in practical situations and algorithms, demonstrated through a specific case study.
5) The presented weighted operator algorithm shows superior performance in resource allocation and efficiency compared to the non-weighted operator algorithm, with sensitivity analysis conducted through systematic parameter variations in membership functions.

3. Definition and Operations of Heptagonal Fuzzy Number

Definition 3.1. A fuzzy number $\tilde{A}_x = (h_1, h_2, h_3, h_4, h_5, h_6, h_7)$ in R is an HFN, and then its membership function $\mu_{\tilde{A}} : R \longrightarrow [0, 1]$ has the following characteristics [44], as presented in Figure 1. The membership function is,

$$\mu_{A_{\tilde{x}}}(x) = \begin{cases} 0 & \text{for } X < h_1, \\ \frac{X-h_1}{2(h_2-h_1)} & \text{for } h_1 \le X \le h_2, \\ \frac{1}{2} & \text{for } h_2 \le X \le h_3, \\ \frac{1}{2} + \frac{X-h_3}{2(h_4-h_3)} & \text{for } h_3 \le X \le h_4, \\ 1 & \text{for } X = h_4, \\ \frac{1}{2} + \frac{h_5-X}{2(h_5-h_4)} & \text{for } h_4 \le X \le h_5, \\ \frac{1}{2} & \text{for } h_5 \le X \le h_6, \\ \frac{1}{2}\frac{h_7-X}{h_7-h_6} & \text{for } h_6 \le X \le h_7, \\ 0 & \text{for } X \ge h_7. \end{cases}$$

Figure 1
Graphical representation of linear heptagonal fuzzy number



Definition 3.2. *(Linear heptagonal fuzzy number asymmetry)* A LHFN with asymmetry can be written as $\tilde{A}_{Lasx} = \{(h_1, h_2, h_3, h_4, h_5, h_6, h_7 : r, s) : \mu_{\tilde{A}_{Lasx}}(x)\}$, where membership function can be described as

$$\mu_{\tilde{A}_{Lasx}}(x) = \begin{cases} 0 & \text{for } X < h_1 \\ r\left(\frac{X-h_1}{h_2-h_1}\right) & \text{for } h_1 \le X \le h_2 \\ r & \text{for } h_2 \le X \le h_3 \\ r + (1-r)\left(\frac{X-h_3}{h_4-h_3}\right) & \text{for } h_3 \le X \le h_4 \\ w & \text{for } X = h_4 \\ s + (1-s)\left(\frac{h_5-X}{h_5-h_4}\right) & \text{for } h_4 \le X \le h_5 \\ s & \text{for } h_5 \le X \le h_6 \\ s\left(\frac{h_7-X}{h_7-h_6}\right) & \text{for } h_6 \le X \le h_7 \\ 0 & \text{for } X \ge h_7 \end{cases}$$

The heights of h_2 and h_3 are "r", and the height of h_5 and h_6 is "s", and if both heights of the heptagonal asymmetry fuzzy are the same ($r = s$), then it's called a heptagonal symmetry fuzzy number as shown in Figure 2.

3.1. Arithmetic operations of LHFNs

Let $\tilde{k}_x = (k_1, k_2, k_3, k_4, k_5, k_6, k_7)$, $\tilde{L}_x = (l_1, l_2, l_3, l_4, l_5, l_6, l_7)$ be two LHFNs, and then addition and subtraction are already available in the literature. Novel operations for LHFNs involving multiplication and division are presented and described below:

Definition 3.3. *(Addition)* The + symbol represents the process of combining two or more LHFNs into a single sum. The addition formula is derived as follows:

$$\tilde{K}_x + \tilde{L}_x = (k_1 + l_1, k_2 + l_2, k_3 + l_3, k_4 + l_4, k_5 + l_5, k_6 + l_6, k_7 + l_7)$$

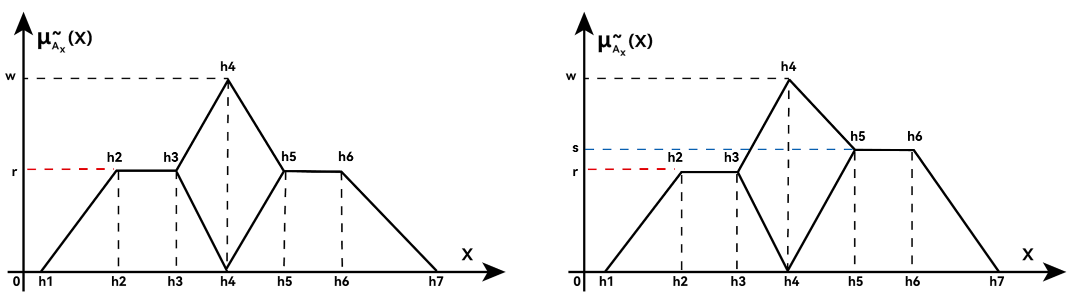
Definition 3.4. *(Subtraction)* The subtraction formula is derived as follows: $\tilde{K}_x - \tilde{L}_x = (k_1 - l_7, k_2 - l_6, k_3 - l_5, k_4 - l_4, k_5 - l_3, k_6 - l_2, k_7 - l_1)$

Definition 3.5. *(Multiplication)* The following is the formula for the multiplication of LHFN, where $R(L)$ denotes the rank of LHFN $\tilde{L}_x$

$$\tilde{K}_x \times \tilde{L}_x = (k_1 \times R(l), k_2 \times R(l), k_3 \times R(l), k_4 \times R(l), k_5 \times R(l), k_6 \times R(l), k_7 \times R(l))$$

Definition 3.6. *(Division)* The division formula for any two LHFNs is as follows: $\tilde{K}_x / \tilde{L}_x = (k_1/R(l), k_2/R(l), k_3/R(l), k_4/R(l), k_5/R(l), k_6/R(l), k_7/R(l))$

Figure 2
Graphical representation of symmetry and asymmetry LHFN



Example: Let us consider two HFNs: one is $\tilde{K} = \{12, 15, 19, 20, 21, 25, 28\}$, and another one is $\tilde{L} = \{14, 18, 21, 24, 27, 30, 34\}$.

The multiplication of these two fuzzy numbers is $\tilde{K} \times \tilde{L} = \{288, 360, 456, 480, 504, 600, 672\}$, and the division of these two fuzzy numbers is $\tilde{K}/\tilde{L} = \{0.50, 0.62, 0.79, 0.84, 0.87, 1.042, 1.17\}$.

4. Problem Formulation for Fuzzy Transportation

The HFTP is developed using LHFN as an extension of an FTP. In the TP, the coefficient of the objective function and constraints, along with all other parameters, are not assumed to be real numbers. However, in the practical situation of a transportation system, considering a transportation parameter's fixed value is unrealistic. Transportation costs for vector, supply, and demand are not always accurate. Thus, the FTP is more realistic in literature and transportation management practice. Several scholars have investigated FTP in fuzzy environments. Fuzziness on cost vector, supply, and demand is used to construct the study as follows:

$$\text{Minimize} Z = \sum_{s=1}^{m} \sum_{t=1}^{n} \tilde{C}_{st}\tilde{X}_{st}$$

subject to constraints

$$\begin{aligned} &\sum_{t=1}^{n} \tilde{X}_{st} = \tilde{\beta}_s, \qquad s = 1, 2, \ldots, m \\ &\sum_{s=1}^{m} \tilde{X}_{st} = \tilde{\delta}_t, \qquad t = 1, 2, \ldots, n \\ &\sum_{s=1}^{m} \tilde{\beta}_s = \sum_{t=1}^{m} \tilde{\delta}_t, \ \tilde{X}_{st} \geq 0, \text{ for all } s \text{ and } t \end{aligned} \tag{1}$$

The following parameters are used in this paper to develop the proposed FTP:

- $\tilde{\beta}_s$: Fuzzy accessibility at s^{th} source,
- $\tilde{\delta}_t$: Fuzzy demand at t^{th} destination,
- $\tilde{C}_{st}$: Fuzzy transportation cost from source (s) to destination (t),
- $\tilde{X}_{st}$: Number of fuzzy units carries over from source to destination.

The FTP problem can be represented as shown in Table 1. Now using the instructions given below to assign the supply and demand units in source and destination, respectively: For combination $\{\leq \tilde{\beta}, \ \approx \tilde{\delta}\}$ and $\{\geq \tilde{\beta}, \ \geq \tilde{\delta}\}$ the most desirable allocation is $\begin{Bmatrix} \tilde{\delta} \text{ if } \tilde{\beta} \geq \tilde{\delta} \\ \tilde{\beta} \text{ if } \tilde{\beta} \prec \tilde{\delta} \end{Bmatrix}$. For combination $\{\approx \tilde{\beta}, \ \approx \tilde{\delta}\}$ and $\{\leq \tilde{\beta}, \ \leq \tilde{\delta}\}$, the most desirable allocation is minimum of $\{\tilde{\beta}, \ \tilde{\delta}\}$. For combination $\{\geq \tilde{\beta}, \ \approx \tilde{\delta}\}$, the most desirable allocation is $\tilde{\delta}$. For combination $\{\geq \tilde{\beta}, \ \geq \tilde{\delta}\}$, the most desirable allocation is maximum of $\{\tilde{\beta}, \ \tilde{\delta}\}$.

New fuzzy extension parameters are provided under uncertainty for numerous real-world applications. Many researchers have used fuzzy numbers to evaluate real-life transportation studies, introducing triangular, trapezoidal, and pentagonal fuzzy numbers and extending pentagonal fuzzy numbers to hexagonal and HFNs [44]. This paper presents an α-cut-based new ranking function as a theoretical extension of an HFN and its application in FTP.

4.1. Fuzzy basic feasible solution

Standardized form of fuzzy transportation network model conversion involves max $\tilde{z} \approx \tilde{C}\tilde{\mathbf{x}}$, subject to $\delta\tilde{\mathbf{x}} \approx \tilde{\beta}$, and $\tilde{x} \geq \tilde{0}$. Now, δ represents an $(s \times t)$ nonnegative matrix, while $\tilde{\beta}, \tilde{C}, \tilde{\mathbf{x}}$ are nonnegative fuzzy matrices of dimensions $(s \times 1), (1\times\ t, (t \times 1)$, respectively, comprising LHFNs.

Theorem 4.1. Let's consider a fuzzy basic feasible solution $\tilde{\mathbf{x}}_\Gamma = \Gamma^{-1}\tilde{\beta}$ for (1). Suppose there exists a column δ_t in δ that is not part of Γ. If $(\tilde{z}_t - \tilde{c}_t) \leq \tilde{0}$ is satisfied, then $y_{st} > 0$ for some $s, s \in \{1, 2, 3, \cdots, m\}$, and then achieved fuzzy feasible basic solution can be constructed through reforming the Γ with δ_t values.

Proof. Suppose $\tilde{\mathbf{x}}_\Gamma = (\tilde{x}_{\Gamma 1}, \tilde{x}_{\Gamma 2}, \tilde{x}_{\Gamma 3}, \cdots, \tilde{x}_{\Gamma m})$ is a fuzzy basic feasible solution with **k** nonnegative components such that

$\Gamma\tilde{x}_\Gamma \approx \tilde{\beta}$ or $\tilde{x}_\Gamma = \Gamma^{-1}\tilde{\beta}$, where $\tilde{x}_{\Gamma s} = [\rho_s, \sigma_s, \kappa_s, \kappa_s], \rho_s \leq \sigma_s, \kappa_s \geq 0$ for $s = 1, 2, 3, \ldots, m$, and $\frac{\rho_s+\sigma_s}{2} > 0$ for $s = 1, 2, 3, \ldots, \mathbf{k}$, $\frac{\rho_s+\sigma_s}{2} = 0$ to the case of $s = \mathbf{k} + \mathrm{i}, \Rightarrow \mathbf{i} = \mathbf{1}, \mathbf{2}, \ldots, m$.

That is, $\tilde{x}_{\Gamma s} > \tilde{0}$ case of $s = 1, 2, 3, \ldots, \mathbf{k}$, $\tilde{x}_{\Gamma s} = [-\sigma_s, \sigma_s, -\kappa_s, \kappa_s]$ for $s = \mathbf{k} + 1, \mathbf{k} + 2, \ldots, m$.

Now the equation $\Gamma\tilde{x}_\Gamma \approx \tilde{\beta}$ becomes:

$$\begin{aligned} \sum_{s=1}^{m} \tilde{x}_{\Gamma s}\beta_s &+ [-\sigma_{\mathbf{k}+1}, \sigma_{\mathbf{k}+1}, \kappa_{\mathbf{k}+1}, \kappa_{\mathbf{k}+1}]\,\beta_{\sigma k+1} \\ &+ [-\sigma_{\mathbf{k}+2}, \sigma_{\mathbf{k}+2}, \kappa_{\mathbf{k}+2}, \kappa_{\mathbf{k}+2}]\,\beta_{\mathbf{k}+2} \\ &+ \cdots + [-\sigma_m, \sigma_m, \kappa_m, \kappa_m]\,\beta_m \approx \tilde{\beta} \end{aligned} \tag{2}$$

That is, $\sum_{s=1}^{k} \tilde{x}_{\Gamma i}\beta_s + \sum_{s=\mathbf{k}+1}^{m} [-\sigma_s, \sigma_s, \kappa_s, \kappa_s]\,\beta_s \approx \tilde{\beta}$.

Then for any column δ_t of Δ which is not in Γ, we write $\delta_t = \sum_{s=1}^{m} y_{st}\beta_s = y_{1t}\beta_1 + y_{2t}\beta_2 + \cdots + y_{rt}\beta_r + \cdots + y_{mt}\beta_m = \mathbf{y}_t\Gamma$.

Table 1
Tabular form of fuzzy transportation problem

		Objectives				
		1	2	3	…n	Accessibility $\tilde{S}$
	1	$\tilde{C}_{11}$	$\tilde{C}_{12}$	$\tilde{C}_{13}$	… $\tilde{C}_{1n}$	$\leq / \approx / \geq \tilde{\beta}_1$
	2	$\tilde{C}_{21}$	$\tilde{C}_{22}$	$\tilde{C}_{23}$	… $\tilde{C}_{2n}$	$\leq / \approx / \geq \tilde{\beta}_2$
Constraints	3	$\tilde{C}_{31}$	$\tilde{C}_{32}$	$\tilde{C}_{33}$	… $\tilde{C}_{3n}$	$\leq / \approx / \geq \tilde{\beta}_3$
	⋮		⋮	⋮	… ⋮	⋮
	m	$\tilde{C}_{m1}$	$\tilde{C}_{m2}$	$\tilde{C}_{m3}$	… $\tilde{C}_{mn}$	$\leq / \approx / \geq \tilde{\beta}_m$
	Requirement $\tilde{D}$	$\leq / \approx / \geq \tilde{\delta}_1$	$\leq / \approx / \geq \tilde{\delta}_2$	$\leq / \approx / \geq \tilde{\delta}_3$	… $\leq / \approx / \geq \tilde{\delta}_n$	

If the basis vector $\beta_{\mathbf{r}}$ for which $y_{\mathbf{r}} \neq 0$ is replaced by δ_t of Δ, subsequently, the updated vectors $(\beta_1, \beta_2, \cdots, \beta_{\mathbf{r}-1}, \delta_t, \beta_{\mathbf{r}+1}, \cdots, \beta_m)$ continuously frame a basis.

For $y_{\mathbf{r}t} \neq 0$ and $t \leq \mathbf{k}$, can be rewritten $\beta_{\mathbf{r}} = \frac{\delta_t}{y_{\mathbf{r}t}} - \sum_{\substack{s=1 \\ a \neq \mathbf{r}}}^{m} \frac{y_{st}}{y_{\mathbf{r}t}} \beta_s = \frac{\delta_y y_{\mathbf{r}y}}{-} \sum_{\substack{s=1 \\ s \neq \mathbf{r}}}^{\mathbf{k}} \frac{y_{st}}{y_{\mathbf{r}t}} \beta_s - \sum_{s=\mathbf{k}+1}^{m} \frac{y_{st}}{y_{\mathbf{r}t}} \beta_i$. Equation (2) transforms

$$\begin{aligned} &\sum_{\substack{s=1 \\ s\neq\mathbf{r}}}^{\mathbf{k}} \tilde{x}_{\Gamma s}\beta_s + \tilde{x}_{\Gamma \mathbf{r}}\beta_r + \sum_{i=k+1}^{m} [-\sigma_s, \sigma_s, \kappa_s, \kappa_s]\,\sigma_s \approx \tilde{\beta} \\ &\Rightarrow \sum_{\substack{s=1 \\ s\neq\mathbf{r}}}^{\mathbf{k}} \left(\tilde{x}_{\Gamma s} - \frac{\tilde{x}_{\Gamma \mathbf{r}}}{y_{\mathbf{r}t}} y_{st}\right)\sigma_s + \frac{\tilde{x}_{\Gamma \mathbf{r}}}{y_{\mathbf{r}t}}\delta_t \\ &+ \sum_{s=\mathbf{k}+1}^{m} \left([-\sigma_s, \sigma_s, \kappa_s, \kappa_s] - \frac{\tilde{x}_{\Gamma \mathbf{r}}}{y_{\mathbf{r}t}} y_{st}\right)\sigma_s \approx \tilde{\beta} \end{aligned} \tag{3}$$

Since $\tilde{x}_{\Gamma s} = [-\sigma_s, \sigma_s, \kappa_s, \kappa_s]$, for $s = \mathbf{k}+1, \mathbf{k}+2, \cdots, m$, it follows that,

$$\sum_{\substack{s=1 \\ s\neq\mathbf{r}}}^{\mathbf{k}} \left(\tilde{x}_{\Gamma s} - \frac{\tilde{x}_{\Gamma \mathbf{r}}}{y_{\mathbf{r}t}} y_{st}\right)\sigma_s + \frac{\tilde{x}_{\Gamma \mathbf{r}}}{y_{\mathbf{r}t}}\delta_t + \sum_{s=\mathbf{k}+1}^{m} \left(\tilde{x}_{\Gamma s} - \frac{\tilde{x}_{\gamma \mathbf{r}}}{y_{\mathbf{r}t}} y_{st}\right)\sigma_s \approx \tilde{\beta}$$

where $\hat{\tilde{x}}_{\Gamma s} = \left(\tilde{x}_{\Gamma s} - \frac{\tilde{x}_{\Gamma \mathbf{r}}}{y_{\mathbf{r}t}} y_{st}\right), s \neq \mathbf{r}$ and $\hat{\tilde{x}}_{\Gamma \mathbf{r}} = \frac{\tilde{x}_{\Gamma \mathbf{r}}}{y_{\mathbf{r}t}}$, it will renew an updated fuzzy solution to $\Delta\tilde{\mathbf{x}} \approx \tilde{\beta}$.

It shows that this updated fuzzy solution is also reliable and feasible. It needs to

$$\left(\tilde{x}_{\Gamma s} - \frac{\tilde{x}_{\Gamma \mathbf{r}}}{y_{\mathbf{r}t}} y_{st}\right) \geq \tilde{0}, s \neq r\# \tag{4}$$

and $\frac{\tilde{x}_{\Gamma \mathbf{r}}}{y_{\mathbf{r}t}} \geq \tilde{0}$.

Select $y_{\mathbf{r}t} > 0$ such that $\frac{\tilde{x}_{\Gamma \mathbf{r}}}{y_{\mathbf{r}t}} \approx \min_s \left\{\frac{\tilde{x}_{\Gamma s}}{y_{st}} : y_{st} > 0\right\}$. Then $\frac{\tilde{x}_{\Gamma \mathbf{r}}}{y_{\mathbf{r}t}} \leq \frac{\tilde{x}_{\Gamma s}}{y_{st}}$

$$\Rightarrow \left[\frac{\rho_{\mathbf{r}}}{y_{\mathbf{r}t}}, \frac{\sigma_{\mathbf{r}}}{y_{\mathbf{r}t}}, \frac{\kappa_{\mathbf{r}}}{y_{\mathbf{r}t}}, \frac{\kappa_{\mathbf{r}}}{y_{\mathbf{r}t}}\right] \leq \left[\frac{\rho_s}{y_{st}}, \frac{\rho_s}{y_{st}}, \frac{\kappa_s}{y_{st}}, \frac{\kappa_s}{y_{st}}\right] \Rightarrow \left[\frac{\alpha_s}{y_{st}} - \frac{\rho_{\mathbf{r}}}{y_{\mathbf{r}t}}, \frac{\rho_s}{y_{st}} - \frac{\rho_{\mathbf{r}}}{y_{\mathbf{r}t}}, \frac{\kappa_{\mathbf{r}}}{y_{\mathbf{r}t}} + \frac{\kappa_s}{y_{st}}, \frac{\kappa_{\mathbf{r}}}{y_{\mathbf{r}t}} + \frac{\kappa_s}{y_{st}}\right] \geq \tilde{0}.$$

$$\begin{aligned} &\Rightarrow \left\{\frac{\left(\frac{\rho_s}{y_{st}} - \frac{\sigma_{\mathbf{r}}}{y_{\mathbf{r}t}}\right) + \left(\frac{\sigma_s}{y_{st}} - \frac{\rho_{\mathbf{r}}}{y_{\mathbf{r}t}}\right)}{2}\right\} \geq 0 \Rightarrow \left(\frac{\rho_s + \sigma_s}{y_{st}}\right) - \left(\frac{\rho_{\mathbf{r}} + \sigma_{\mathbf{r}}}{y_{\mathbf{r}t}}\right) \geq 0 \\ &\Rightarrow \left(\frac{\tilde{x}_{\Gamma s}}{y_{st}} - \frac{\tilde{x}_{\Gamma \mathbf{r}}}{y_{\mathbf{r}t}}\right) \geq \tilde{0}. \end{aligned}$$

Here, the updated results are fuzzy, feasible results basically.

Once the basic solution vectors are updated, then the resulting vector $\tilde{\Gamma} = (\tilde{\sigma}_1, \tilde{\sigma}_2, \ldots, \tilde{\sigma}_m)$, where $\tilde{\sigma}_s = \sigma_s$ for $s \neq \mathbf{r}$ and $\tilde{\sigma}_{\mathbf{r}} = \delta_s$. The updated basic fuzzy feasible solution $\hat{\mathbf{x}}_\Gamma$, where $\hat{\tilde{x}}_{\gamma s} = \left(\tilde{x}_{\Gamma s} - \frac{\tilde{x}_{\Gamma \mathbf{r}}}{y_{\mathbf{r}t}} y_{st}\right), s \neq \mathbf{r}$ and $\hat{\tilde{x}}_{\Gamma \mathbf{r}} = \frac{\tilde{x}_{\Gamma \mathbf{r}}}{y_{\mathbf{r}t}}$ are the basic variables.

5. Development of Ranking in Heptagonal Fuzzy Number

The existing and new ranking functions and their defuzzification are explained in this section. Many fuzzy decision issues use fuzzy numbers to rate options. A technique is needed to make a crisp value from imprecise parameters or fuzzy numbers to convey alternate preferences. Since many ranking methods are available in the literature, this study focuses on the α-cut method and the centroid of centroid method. Defuzzification is the process of turning an imprecise quantity into a precise one, whereas fuzzification converts precise to fuzzy. The logical union of two or more fuzzy membership functions defined on the output variable's universe of discourse can be the output of a fuzzy process. The defuzzification and max membership principle, the centroid method, the weighted average method, the first (or last) of maxima, mean max, center of largest area, and center of sum techniques are also available here, only focusing on methods such as those mentioned earlier.

5.1. Exiting ranking function

The existing ranking function serves as a crucial tool for ordering elements within a fuzzy environment based on their degrees of membership. It facilitates more effective decision-making in respective environments characterized by uncertainty and imprecision parameters. Available ranking functions are the following:

5.1.1. Average ranking method

Khalifa et al. [47] presented an Average Ranking Method (ARM) method to identify the critical path for project networks with normalized HFNs. The ranking functions as

$$R\left(\tilde{A}_x\right) = \frac{h_1 + h_2 + h_3 + 2h_4 + h_5 + h_6 + h_7}{8} \tag{5}$$

5.1.2. Centroid of centroid method

For the centroid of LHFNs, as shown in Figure 3, we make two trapezoidal and one rhombus divisions of the heptagon, namely, AHID, DKLG, and IJKD, respectively, and find the centroid of these separated regions as Z_1, Z_2, and Z_3. The Centroid of Centroid Method (CCM) [44] is used as a point of reference to establish the ordering of generalized LHFNs. Compared to the centroid point of the heptagon, the centroid of centroids Z_1, Z_2, and Z_3 would be a superior position. Let the generalized LHFN $\tilde{A}_x = (h_1, h_2, h_3, h_4, h_5, h_6, h_7 : W)$, then the centroids of these triangles are

$$Z_1 = \left(\frac{2h_1 + 7h_2 + 7h_3 + 2h_4}{18}, \frac{7(w)}{36}\right),$$

$$Z_2 = \left(\frac{2h_4 + 7h_5 + 7h_6 + 2h_7}{18}, \frac{7(w)}{36}\right),$$

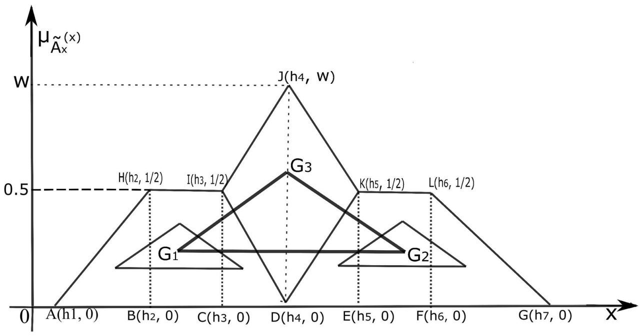
$Z_3 = \left(h_4, \frac{w}{2}\right)$. As Z_1, Z_2, and Z_3 form a triangle because they are non-collinear. Therefore, the centroid $Z_{\tilde{A}_H} = (\tilde{x}, \tilde{y})$ of a triangle with the generalized LHFNs Z_1, Z_2, and Z_3, is

$$Z_{\tilde{A}_H}(\tilde{x}, \tilde{y}) = \left(\frac{2h_1 + 7h_2 + 7h_3 + 22h_4 + 7h_5 + 7h_6 + 2h_7}{54}, \frac{11(w)}{54}\right)$$

The following definition describes the generalized LHFNs' ranking function:

$$R_{\tilde{A}_H}(\tilde{x}, \tilde{y}) = \left(\frac{2h_1 + 7h_2 + 7h_3 + 22h_4 + 7h_5 + 7h_6 + 2h_7}{54}, \frac{11(w)}{54}\right) \tag{6}$$

Figure 3
Graphical representation centroid of centroid ranking of LHFN



The generalized LHFN's ranking is given as follows:

$$R_{\tilde{A}_H}(\tilde{x}, \tilde{y}) = \sqrt{\tilde{x}^2 + \tilde{y}^2}$$

5.2. Proposed ranking function

To evaluate the ranking of LHFNs, we developed a ranking technique based on α-cut. The α-cut of $\tilde{A} = (h_1, h_2, h_3, h_4, h_5, h_6, h_7), 0 \le \alpha \le 1$ is $\tilde{A} = [\tilde{A}_L(\alpha), \tilde{A}_R(\alpha)]$ as follows:

The left α-cut from h_1 to h_2 is denoted and defined as

$$\frac{(X-h_1)}{(h_2-h_1)} = \alpha \to X = h_1 + \alpha(h_2 - h_1) \text{ then } \tilde{A}_{L_1}(\alpha) = L_1^{-1} = h_1 + \alpha(h_2 - h_1)$$

α - cut from h_2 to h_3 is denoted and defined as

$$\frac{(X-h_2)}{(h_3-h_2)} = \alpha \to X = h_2 + \alpha(h_3 - h_2) \text{ then } \tilde{A}_{L_2}(\alpha) = L_2^{-1} = h_2 + \alpha(h_3 - h_2)$$

α - cut from h_3 to h_4 is denoted and defined as

$$\frac{(X-h_3)}{(h_4-h_3)} = \alpha \to X = h_3 + \alpha(h_4 - h_3) \text{ then } \tilde{A}_{L_3}(\alpha) = L_3^{-1} = h_3 + \alpha(h_4 - h_3)$$

and the right $\alpha-$ cut from h_4 to h_5 is as follows

$$\frac{(h_3-X)}{(h_5-h_4)} = \alpha \to X = h_5 + \alpha(h_5 - h_4) \text{ then } \tilde{A}_{R_1}(\alpha) = R_1^{-1} = h_5 - \alpha(h_5 - h_4)$$

α - cut from h_5 to h_6 is given by

$$\frac{(h_6-X)}{(h_6-h_5)} = \alpha \to X = h_6 + \alpha(h_6 - h_5) \text{ then } \tilde{A}_{R_2}(\alpha) = R_2^{-1} = h_6 - \alpha(h_6 - h_5)$$

α - cut from h_6 to h_7 is follows

$$\frac{(h_7-X)}{(h_7-h_6)} = \alpha \to X = h_7 + \alpha(h_7 - h_6) \text{ then } \tilde{A}_{R_3}(\alpha) = R_3^{-1} = h_7 - \alpha(h_7 - h_6)$$

Now the left α-cut is $L^{-1}(\alpha) = \frac{L_1^{-1}(\alpha)+L_2^{-1}(\alpha)+L_3^{-1}(\alpha)}{3}$, where

$$L^{-1}(\alpha) = \frac{h_1+\alpha(h_2-h_1)+h_2+\alpha(h_3-h_2)+h_3+\alpha(h_4-h_3)}{3} = \frac{h_5+h_6+h_7+\alpha(h_4-h_7)}{3} \quad (7)$$

Again, the right α - cut is $R^{-1}(\alpha) = \frac{R_1^{-1}(\alpha)+R_2^{-1}(\alpha)+R_3^{-1}(\alpha)}{3}$, where

$$R^{-1}(\alpha) = \frac{h_5-\alpha(h_5-h_4)+h_6-\alpha(h_6-h_5)+h_7-\alpha(h_7-h_6)}{3} = \frac{h_5+h_6+h_7+\alpha(h_4-h_7)}{3} \quad (8)$$

The α-cut-based new ranking function:

$$R(\tilde{A_H}) = \frac{1}{2}\frac{\int_0^w \alpha^7\left[\frac{h_1+h_2+h_3+\alpha(h_4-h_1)}{3} + \frac{h_5+h_6+h_7+\alpha(h_4-h_7)}{3}\right]d\alpha}{\int_0^w \alpha^7 d\alpha} \quad (9)$$

Note. The above formula applies to any fuzzy number heights, even normalized ones. For x = 1, the formula is derived as follows:

$$R(\tilde{A_H}) = \frac{h_1+9h_2+9h_3+16h_4+9h_5+9h_6+h_7}{54} \quad (10)$$

5.3. Comparison of ranking function

Let's consider the LHFN divided into two types: LHFN symmetry and asymmetry. The general form of the numbers is $(h_1, h_2, h_3, h_4, h_4, h_5, h_6, h_7; w(\alpha))$. The examples of the abovementioned fuzzy numbers are tabulated in Table 2, and the derived ranking methods are compared for HFNs. We compared ranking approaches using different examples with varying membership values. Table 2

The ranking function results of linear heptagonal fuzzy asymmetric, symmetric numbers are defuzzified through the introduced ranking functions in Equation (9). The centroid of centroid and α-cut defuzzification methods are used to defuzzify the fuzzy numbers displayed in Table 2. It clearly explains that the proposed ranking method gives an exact rank of the taken numbers. If the membership value of the fuzzy number (1, 2, 3, 4, 5, 6, 7) is 0.1, and if we defuzzify using existing and proposed ranking methods, the resulting numbers are 4.0000, 4.0001, and 3.8347, respectively. The center value of the number is 4 by the membership value 0.1, which should be reflected, but the existing methods provide slightly different outcomes from this value. Furthermore, for the membership value 0.7, the defuzzification value of the proposed method

Table 2
Ranking methods comparison by varying membership values

Value of $w\&\alpha$	Fuzzy number	ARM	CCM	Proposed
0.1	(1, 2, 3, 4, 5, 6, 7)	4.0000	4.0001	3.8347
0.3	(5, 9, 10, 12, 14, 15, 19)	12.1250	12.1298	11.8550
0.5	(6, 9, 12, 14, 16, 19, 21)	13.8750	13.9633	13.5680
0.7	(4, 10, 12, 14, 16, 19, 21)	13.7500	14.0192	13.6477
0.9	(3, 5, 9, 11, 13, 15, 19)	10.7500	10.7423	10.2904
1.0	(2, 4, 7, 11, 13, 15, 17)	10.0000	10.4511	9.6863

(13.6477) for the asymmetric LHFN (4, 10, 12, 14, 16, 19, 21) provides better results compared to the existing method (13.7500) because the center of the LHFN is intermediate of two values 13 and 14 of asymmetric LHFN.

To better understand the proposed α-method efficiency, we considered LHFN {3, 7, 8, 12, 15 , 17, 20: w (α) with changing values of $w(\alpha)$ between $[0, 1]$, and then the results of the ranking methods for this example are graphically represented in Figure 4. However, the existing ranking function does not affect the ranking function or expose the ranking method results. The fuzzy set/number concept emphasizes the membership function. If membership values increase, the suggested ranking function enhances the results of defuzzification through the ranking method.

Based on the obtained defuzzification values by using the above-described procedure on LHFNs; the alpha-cut defuzzified value for the HFNs yields a more accurate value than the centroid of centroids technique. Furthermore, when employing LHFN, the α-cut method (Equation (9)) for defuzzification consistently outperforms the centroid of centroid method (Equation (6)).

6. Solution Technique for Fuzzy Heptagonal Transportation

A detailed description of the generalized ranking heptagonal fuzzy method (GRHFM) is given to solve the FTP. Two novel fuzzy matrices X (wfcd) and Y (wfcs) are created using the obtained ratios ($\tilde{p}_{ij} = \frac{d_j}{s_i}$, i = 1, 2, 3, . . . , m, and $\tilde{p}_{ji} = \frac{s_i}{d_j}$, j = 1, 2, 3, . . . , n) incorporating a cost multiplier to assign allocations. The suggested technique executes the allocations using the resultant value of the fuzzy requirement/availability, beginning with the lowest values in the new weighted fuzzy transportation cost matrix. The technique can generate effective IBFS to both balanced and unbalanced problems, and the algorithm for this purpose is presented as follows in Algorithm 1:

Algorithm 1 Generalized Ranking Heptagonal Fuzzy Method

Input:	Sum of heptagonal fuzzy demands and supplies. Analyze the equality: If it is equal, then go to the compute step; otherwise, add dummy rows or columns to equalize the demand and supply, and then go to the compute step.
Compute:	Proportional fuzzy demand $(\tilde{p}_{ij})$, supply, $(\tilde{p}_{ji})$ matrix (PFDM, PFSM): $\tilde{p}_{ij} = \frac{\tilde{d}_j}{s_i}, \tilde{p}_{ji} = \frac{\tilde{s}_i}{\tilde{d}_j}, i = 1, 2, 3, \ldots, m, j = 1, 2, 3, \ldots, n$
Update:	Compute the weighted-demand (X), supply (Y) fuzzy transportation cost matrix (DWFCM, SWFCM) by multiplying $\tilde{p}_{ij}, \tilde{p}_{ji}$ and cost values.
Allocation:	Starting with the matrices DWFCM and SWFCM, make assignments with the lowest weighted costs, taking into account fuzzy supply and demand limitations.
Check feasibility:	Finish the algorithm if all fuzzy requirements are fulfilled. If not, return back to allocation.
Output:	Compare the fuzzy matrix allocation values; set the smaller value. For the defuzzification of the obtained IBFS, use the ranking function.

Figure 4
Representation of existing and proposed ranking function defuzzification results

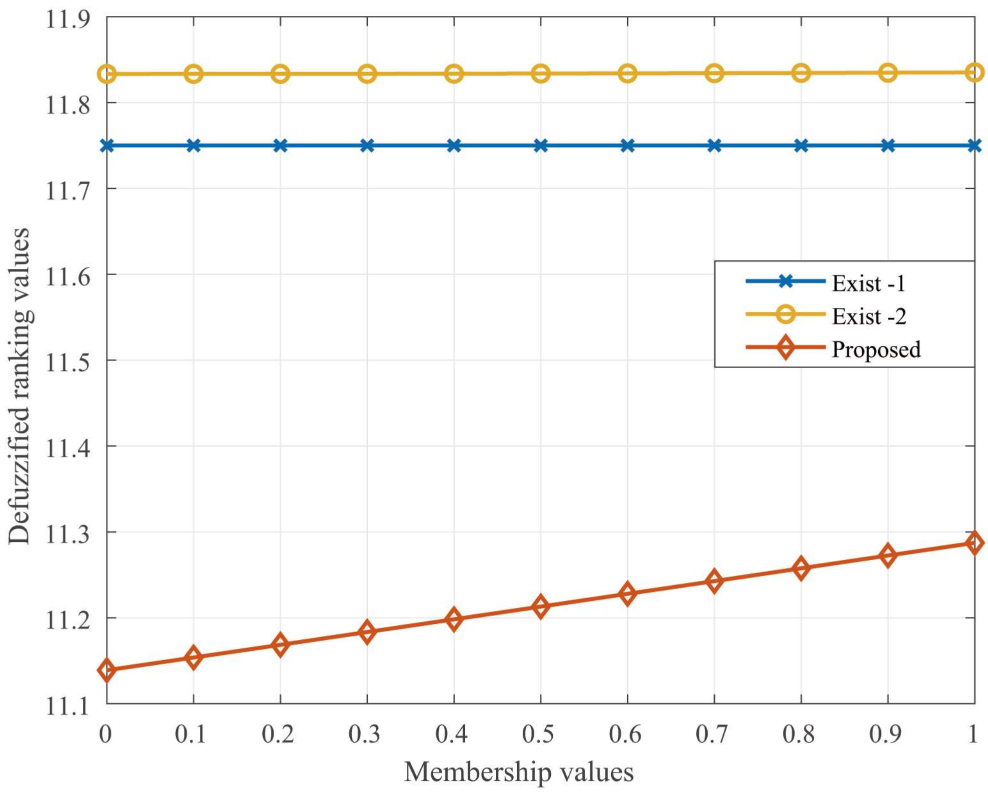
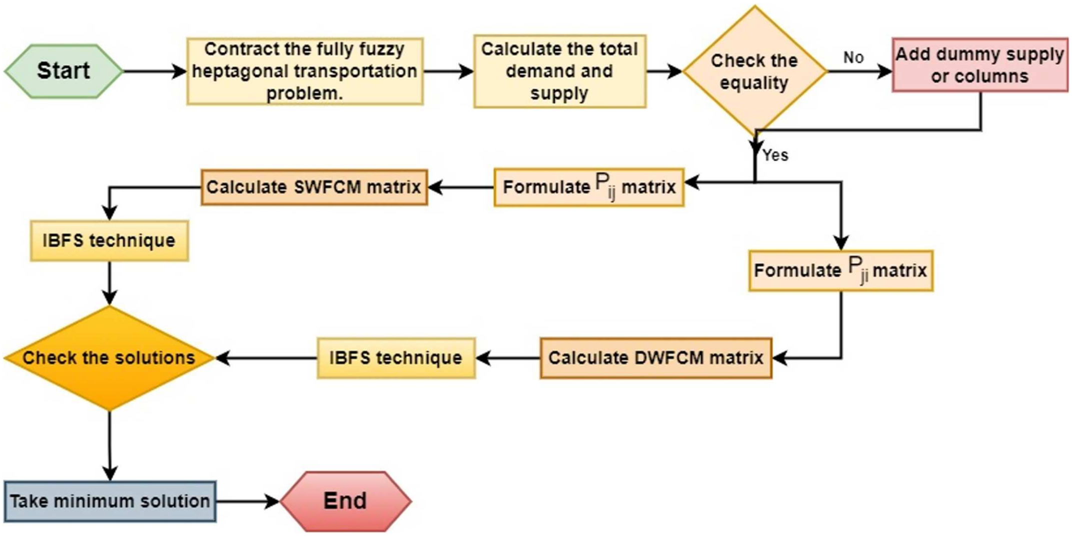


Figure 5
Flowchart for the representation of GRHFM steps for HFTP



A numerical illustration of the GRHFM through the case study by applying the presented procedure (Section 7) consisting of five demands and five supplies. The proposed fuzzy heptagonal transportation problem is described through the algorithm Figure 5.

7. Numerical Exposure

The perishable nature of vegetables makes them unfit for long-term storage, necessitating their transportation. The transporter has many problems with many types of vehicles, and vehicle rentals fluctuate according to fuel cost, capacity, and route. Furthermore, in some instances, the cost to transport those vegetables from the farm to the market is more than the cost to produce a kilogram of vegetables at the farmer's end. In that case, the cost of transportation is nearly equal to the cost of growing vegetables, so the vegetable price is generally higher when they reach the people. Therefore, a clear choice can be made to incorporate transportation costs with the LHFN for any transportation problem that persists seven days a week.

7.1. Case study

This case study considers the transport network of famous markets in Tamil Nadu state, namely, Chennai, Madurai, Trichy, Coimbatore, and Kumbakonam. Vegetables are used in our daily lives almost seven (heptagonal) days a week to see how much the prices go up and down in the months mentioned. The consideration of seven-face fuzzy numbers has been used to represent the uncertainties of the cost data in the adapted case study problem. LHFN has explored the opportunity to represent the cost data variations collected in October from seven local transporters, as tabulated in Table 3.

The LHFN was used to estimate the price of seven transporter data from five suppliers to receivers for five markets examined in the aforementioned months. The ideal cost to transport five vegetable varieties will be determined by this investigation. The difficulty of this study is that the city will not have the natural environment to group all the vegetables, so they are shifting the vegetables from one market to another according to need. LHFNs are used to calculate vegetable transportation costs for five markets. Demand and supply are also implemented in LHFN environments since product costs vary in availability and demand. This study and strategy will reduce transportation costs (purchasing).

The HFTP is shown in Table 3, whose sum of all demands $\sum \tilde{a}_i$ and sum of all supply $\sum \tilde{b}_j$ are the same so that the given HFTP is balanced $\sum \tilde{a}_i = \sum \tilde{b}_j$ = (43,51,65,72,79,93,101). The above fully fuzzy transportation problem is solved by using the generalized ranking heptagonal fuzzy method and obtaining the minimum fuzzy total transportation cost as sources.

Table 3
Imprecise cost input of fuzzy transportation problem through LHFN

		Destinations					
		$\tilde{\beta}_1$	$\tilde{\beta}_2$	$\tilde{\beta}_3$	$\tilde{\beta}_4$	$\tilde{\beta}_5$	$\tilde{\beta}$
Sources	$\tilde{\delta}_1$	(62, 65, 68, 73, 75, 79, 83)	(25, 29, 36, 40, 47, 53, 60)	(3, 4, 8, 9, 10, 13, 14)	(70, 73, 75, 79, 83, 85, 88)	(12, 16, 17, 20, 23, 24, 28)	(10, 12, 15, 16, 17, 20, 22)
	$\tilde{\delta}_2$	(55, 59, 60, 62, 63, 64, 68)	(86, 89, 91, 93, 95, 96, 99)	(87, 89, 91, 96, 98, 102, 107)	(2, 4, 7, 8, 9, 10, 12)	(5, 7, 8, 13, 15, 17, 19)	(12, 13, 14, 15, 16, 17, 18)
	$\tilde{\delta}_3$	(90, 93, 94, 96, 97, 100, 104)	(56, 58, 62, 65, 68, 72, 75)	(72, 73, 79, 80, 84, 87, 89)	(44, 45, 47, 50, 53, 55, 56)	(59, 62, 64, 65, 66, 68, 71)	(11, 13, 15, 17, 19, 21, 23)
	$\tilde{\delta}_4$	(48, 50, 53, 57, 59, 63, 65)	(50, 53, 56, 58, 60, 64, 67)	(23, 25, 26, 29, 31, 34, 36)	(6, 7, 9, 12, 13, 15, 17)	(81, 83, 86, 87, 88, 91, 93)	(4, 6, 10, 11, 12, 16, 18)
	$\tilde{\delta}_5$	(48, 52, 53, 56, 59, 60, 64)	(15, 17, 20, 23, 24, 28, 30)	(81, 84, 86, 87, 88, 90, 93)	(11, 15, 16, 18, 19, 21, 23)	(4, 6, 9, 12, 15, 17, 19)	(6, 7, 11, 13, 15, 19, 20)
	$\tilde{\delta}$	(8, 10, 13, 14, 15, 18, 20)	(11, 13, 14, 16, 18, 19, 21)	(11, 12, 16, 18, 20, 24, 25)	(6, 7, 11, 12, 13, 17, 18)	(7, 9, 11, 12, 13, 15, 17)	**(43, 51, 65, 72, 79, 93, 101)**

Table 4
Transportation problem using formula $\tilde{p}_{ij} = \frac{d_j}{s_i}$ to find PFDM

		Destinations					
		$\tilde{\beta}_1$	$\tilde{\beta}_2$	$\tilde{\beta}_3$	$\tilde{\beta}_4$	$\tilde{\beta}_5$	$\tilde{\beta}$
Sources	$\tilde{\delta}_1$	(0.50, 0.63, 0.81, 0.88, 0.94, 1.13, 1.25)	(0.69, 0.81, 0.88, 1.00, 1.13, 1.19, 1.31)	(0.69, 0.75, 1.00, 1.13, 1.25, 1.50, 1.56)	(0.38, 0.44, 0.69, 0.75, 0.81, 1.06, 1.13)	(0.44, 0.56, 0.69, 0.75, 0.81, 0.94, 1.06)	(10, 12, 15, 16, 17, 20, 22)
	$\tilde{\delta}_2$	(0.53, 0.67, 0.87, 0.93, 1.00, 1.20, 1.33)	(0.73, 0.87, 0.93, 1.07, 1.20, 1.27, 1.40)	(0.73, 0.80, 1.07, 1.20, 1.33, 1.60, 1.56)	(0.40, 0.47, 0.73, 0.80, 0.87, 1.13, 1.20)	(0.47, 0.60, 0.73, 0.80, 0.87, 1.00, 1.13)	(12, 13, 14, 15, 16, 17, 18)
	$\tilde{\delta}_3$	(0.47, 0.59, 0.76, 0.82, 0.88, 1.06, 1.18)	(0.65, 0.76, 0.82, 0.94, 1.06, 1.12, 1.24)	(0.65, 0.71, 0.94, 1.06, 1.18, 1.41, 1.47)	(0.35, 0.41, 0.65, 0.71, 0.76, 1.00, 1.06)	(0.41, 0.53, 0.65, 0.71, 0.76, 0.88, 1.00)	(11, 13, 15, 17, 19, 21, 23)
	$\tilde{\delta}_4$	(0.73, 0.91, 1.18, 1.27, 1.45, 1.64, 1.82)	(1.00, 1.18, 1.27, 1.45, 1.64, 1.73, 1.91)	(1.00, 1.09, 1.45, 1.64, 1.82, 2.18, 2.27)	(0.55, 0.64, 1.00, 1.09, 1.18, 1.55, 1.64)	(0.64, 0.82, 1.00, 1.09, 1.18, 1.36, 1.55)	(4, 6, 10, 11, 12, 16, 18)
	$\tilde{\delta}_5$	(0.62, 0.77, 1.00, 1.08, 1.15, 1.38, 1.54)	(0.85, 1.00, 1.08, 1.23, 1.38, 1.43, 1.62)	(0.85, 0.92, 1.23, 1.38, 1.54, 1.85, 1.92)	(0.46, 0.54, 0.85, 0.92, 1.00, 1.31, 1.38)	(0.54, 0.69, 0.85, 0.92, 1.00, 1.15, 1.31)	(6, 7, 11, 13, 15, 19, 20)
	$\tilde{\delta}$	(8, 10, 13, 14, 15, 18, 20)	(11, 13, 14, 16, 18, 19, 21)	(11, 12, 16, 18, 20, 24, 25)	(6, 7, 11, 12, 13, 17, 18)	(7, 9, 11, 12, 13, 15, 17)	

Table 5
Constructing PFSM of the transportation problem using the formula $\tilde{p}_{ji} = \frac{s_i}{d_j}$

		Destinations					
		$\tilde{\beta}_1$	$\tilde{\beta}_2$	$\tilde{\beta}_3$	$\tilde{\beta}_4$	$\tilde{\beta}_5$	$\tilde{\beta}$
Sources	$\tilde{\delta}_1$	(51.83, 62.78, 78.11, 83.22, 88.33, 104.39, 114.61)	(25.2, 30.0, 37.6, 40.0, 42.4, 50.0, 55.2)	(5.04, 6.03, 7.47, 8.01, 8.46, 9.99, 10.98)	(65.57, 102.7, 98.75, 105.07, 112.18, 131.93, 144.57)	(16.6, 26.0, 25.0, 26.6, 28.4, 33.4, 36.6)	(10, 12, 15, 16, 17, 20, 22)
	$\tilde{\delta}_2$	(53.32, 57.66, 62,66.34, 70.68, 75.02, 79.98)	(69.75, 75.33, 81.84, 180.42, 93.0, 98.58, 105.09)	(64.32, 69.12, 74.88, 79.68, 85.44, 90.24, 96.00)	(8.00, 8.64, 9.36, 10.00, 10.64, 11.36, 12.00)	(13.00, 14.04, 15.21, 16.25, 17.29, 18.46, 19.50)	(12, 13, 14, 15, 16, 17, 18)
	$\tilde{\delta}_3$	(75.84, 89.28, 102.72, 116.16, 130.56, 144.00, 157.44)	(44.85, 48.75, 61.1, 68.9, 77.35, 85.15, 93.60)	(48.8, 57.6, 66.4, 75.2, 84.8, 93.6, 102.4)	(46.0, 54.0, 62.5, 71.0, 79.0, 87.5, 96.0)	(59.80, 70.20, 81.25, 92.30, 102.70, 113.75, 124.80)	(11, 13, 15, 17, 19, 21, 23)
	$\tilde{\delta}_4$	(16.53, 24.51, 40.47, 45.03, 49.02, 64.98, 73.53)	(14.5, 22.04, 36.54, 40.02, 43.5, 58.0, 65.54)	(6.38, 9.57, 16.24, 17.69, 19.43, 25.81, 29.00)	(3.96, 6.00, 11.04, 12.00, 15.96, 15.96, 18.00)	(28.71, 43.50, 80.04, 87.00, 115.71, 115.71, 130.50)	(4, 6, 10, 11, 12, 16, 18)
	$\tilde{\delta}_5$	(24.08, 28.00, 44.24, 52.08, 59.92, 76.16, 80.08)	(8.74, 10.12, 15.87, 18.63, 21.62, 27.37, 28.75)	(28.71, 33.93, 53.07, 62.64, 72.21, 92.22, 96.57)	(9.00, 10.44, 16.56, 19.44, 22.50, 28.44, 30.06)	(6.00, 6.96, 11.04, 12.96, 15.00, 18.96, 20.04)	(6, 7, 11, 13, 15, 19, 20)
	$\tilde{\delta}$	(8, 10, 13, 14, 15, 18, 20)	(11, 13, 14, 16, 18, 19, 21)	(11, 12, 16, 18, 20, 24, 25)	(6, 7, 11, 12, 13, 17, 18)	(7, 9, 11, 12, 13, 15, 17)	

The derived Table 4 is the second step of the proposed algorithm. In the first column of this table, demand $\tilde{D}_1$ is divided by one by one $\tilde{S}_1, \tilde{S}_2, \tilde{S}_3, \tilde{S}_4$, and $\tilde{S}_5$. The same way proceeds for all the columns. It means the heptagonal fuzzy demand is divided by the heptagonal fuzzy supply by using the proposed division formula.

Table 5 is framed based on the compute comment of the proposed algorithm. In the first row of this table, supply $\tilde{S}_1$ is divided by $\tilde{D}_1, \tilde{D}_2, \tilde{D}_3, \tilde{D}_4$, and $\tilde{D}_5$. The same way calculates for all supply rows. The proposed division formula for LHFNs has been implemented to divide the heptagonal fuzzy supply by the heptagonal fuzzy demands.

Using the proportional fuzzy demand matrix $(\tilde{P}_{ij})$ in Table 4, we can proceed to derive the next step (Table 6) by an operation on each cell that could be multiplied by the cost matrix table, which was

Table 6
Demand-weighted fuzzy transportation cost matrix

		Destinations					
		$\tilde{\beta}_1$	$\tilde{\beta}_2$	$\tilde{\beta}_3$	$\tilde{\beta}_4$	$\tilde{\beta}_5$	$\tilde{\beta}$
	$\tilde{\delta}_1$	(36.5, 45.99, 59.13, 64.24, 68.62, 82.49, 91.25)	(27.6, 32.4, 35.2, 40.0, 45.2, 47.6, 52.4)	(6.21, 6.75, 9, 10.17, 11.25, 13.5, 14.04)	(30.02, 34.76, 54.51, 59.25, 63.99, 83.74, 89.27)	(8.8, 11.2, 13.8, 15, 16.2, 18.8, 21.2)	(10, 12, 15, 16, 17, 20, 22)
	$\tilde{\delta}_2$	(32.86, 41.54, 53.94, 57.66, 62, 74.4, 82.46)	(67.89, 80.91, 86.49, 99.51, 111.6, 118.11, 130.2)	(70.08, 76.80, 102.72, 115.20127.68, 153.60, 149.76)	(3.20, 3.76, 5.84, 6.40, 6.96, 9.04, 9.60)	(6.11, 7.80, 9.49, 10.40, 11.31, 13.0, 14.69)	(12, 13, 14, 15, 16, 17, 18)
Sources	$\tilde{\delta}_3$	(45.12, 56.64, 72.96, 78.72, 84.48, 101.76, 113.28).60)	(42.25, 49.40, 53.30, 61.10, 68.90, 78.65, 80.60)	(52.0, 56.80, 75.20, 84.80, 94.40, 112.80, 117.60)	(17.5, 20.5, 32.5, 35.5, 38.0, 50.0, 53.0)	(26.65, 34.45, 42.25, 46.15, 49.4, 57.2, 65.0)	(11, 13, 15, 17, 19, 21, 23)
	$\tilde{\delta}_4$	(41.61, 51.87, 67.26, 72.39, 82.65, 93.48, 103.74)	(58.0, 68.44, 73.66, 84.1, 95.12, 100.34, 110.78)	(29.0, 31.61, 42.05, 47.56, 52.78, 63.22, 65.83)	(6.60, 7.68, 12.0, 13.08, 14.16, 18.6, 19.68)	(55.68, 71.34, 87.0, 94.83, 102.66, 118.32, 134.85)	(4, 6, 10, 11, 12, 16, 18)
	$\tilde{\delta}_5$	(34.72, 43.12, 56.060.48, 64.4, 77.28, 86.24)	(19.55, 23.0, 24.84, 28.29, 31.74, 32.89, 37.26)	(73.95, 80.04, 107.01, 120.06, 133.98, 160.95, 167.04)	(8.28, 9.72, 15.3, 16.56, 18.0, 23.58, 24.84)	(6.48, 8.28, 10.2, 11.04, 12.0, 13.8, 15.72)	(6, 7, 11, 13, 15, 19, 20)
	$\tilde{\delta}$	(8, 10, 13, 14, 15, 18, 20)	(11, 13, 14, 16, 18, 19, 21)	(11, 12, 16, 18, 20, 24, 25)	(6, 7, 11, 12, 13, 17, 18)	(7, 9, 11, 12, 13, 15, 17)	

represented in the initial FTP. For example, let's take the value of PFDM in cell $\tilde{X}_{11}$ and take the value of $\tilde{X}_{11}$ in Table 1 and multiply via the proposed multiplication formula to obtain the weighted fuzzy cost matrix by demand (DWFCM).

The proportional fuzzy supply matrix ($\tilde{P}_{ji}$) in Table 5 has been used to derive the SWFCM matrix in Table 7 by the following process: multiplying the initial corresponding FTP $\tilde{X}_{11}$ cell value of Table1 with PFSM cell value $\tilde{X}_{11}$ in Table 5 using multiplication formula to obtain the results. These are tabulated in a supply-weighted fuzzy transportation cost matrix.

Allocate the fuzzy supply and fuzzy demand depending upon the fuzzy costs of the vegetables for Table 6 based on the proposed fuzzy transport problem model Equation (1). We obtain the one set of optimal results tabulated in Table 8 using DWFCM.

Table 6 obtained fuzzy total transportation cost as, (1783, 1916, 2140, 2310, 2438, 2643, 2793). Allocate the fuzzy supply and fuzzy demand depending upon the fuzzy costs of the vegetables for Table 7, through the proposed fuzzy transport problem model (1) using SWFCM. It provided another set of optimal results for SWFCM tabulated in Table 9 using SWFCM.

Table 7 provided fuzzy total transportation costs as (2122.8, 2276.7, 2482.8, 2619.9, 2739, 2929, 3100). Finally, the proposed method for solving the HFTP produced two results based on DWFCM results (1783, 1916, 2140, 2310, 2438, 2643, 2793 as shown in Table 8, and SWFCM is (2122.8, 2276.7, 2482.8, 2619.9, 2739, 2929, 3100) as presented in Table 9. The result of DWFCM is that after taking the ranking, it provides 2608.64 and for SWFCM, 2292.02. By comparing these results, the minimum value obtained from the results (1783, 1916, 2140, 2310, 2438, 2643, 2793) after ranking the fuzzy solutions is 2292.02. Comparing the As mentioned above, a new methodology for resolving the heptagonal fuzzy environment transportation problem, upon getting solutions, is compared to select the minimum solution. The minimum solution is the optimal feasible solution for the solved FTP by this algorithm.

7.2. Comparison of solution

This study conducts a comprehensive comparison of solution methods to address the fuzzy feasible solution of the FTP. Additionally, the FTP is solved using various well-known existing methods, namely, (i) generalized fuzzy least cost method (GFLCM), (ii) generalized fuzzy north-west corner method (GFNWCM), and (iii) generalized fuzzy Vogel's approximation method (GFVAM).

Through rigorous experimentation and analysis, we evaluate the efficiency, accuracy, and computational complexity of each method. This comparative study aims to unveil the strengths and limitations of diverse approaches, providing decision-makers and practitioners with valuable insights to choose the most suitable method based on the specific characteristics and constraints of the transportation scenarios. The obtained solutions from the mentioned methods providing heptagonal fuzzy solutions are tabulated in Table 10. The solution obtained by using GFNWCM is (3558, 3708, 3955, 4154, 4342, 4527, 4726), and after ranking, the solution is 4139.56. The same way for GFLCM obtained results is (1798, 1934, 2167, 2331, 2468, 2667, 2820), and then the ranking value is 2315.52, and by using GFVAM, the obtained result is (1837, 1970, 2140, 2328, 2420, 2625, 2784), and then the ranking of the obtained solution is 2301.19. The above obtained fuzzy total transportation costs by using various methods are tabulated below:

Table 7
Supply-weighted fuzzy transportation cost matrix

		Destination					
		$\tilde{\beta}_1$	$\tilde{\beta}_2$	$\tilde{\beta}_3$	$\tilde{\beta}_4$	$\tilde{\beta}_5$	$\tilde{\beta}$
Sources	$\tilde{\delta}_1$	(51.83, 62.78, 78.11, 83.22, 88.33, 104.39, 114.61)	(25.2, 30.0, 37.6, 40.0, 42.4, 50.0, 55.2)	(5.04, 6.03, 7.47, 8.01, 8.46, 9.99, 10.98)	(65.57, 102.7, 98.75, 105.07, 112.18, 131.93, 144.57)	(16.6, 26.0, 25.0, 26.6, 28.4, 33.4, 36.6)	(10, 12, 15, 16, 17, 20, 22)
	$\tilde{\delta}_2$	(53.32, 57.66, 62, 66.34, 70.68, 75.02, 79.98)	(69.75, 75.33, 81.84, 180.42, 93.0, 98.58, 105.09)	(64.32, 69.12, 74.88, 79.68, 85.44, 90.24, 96.00)	(8.00, 8.64, 9.36, 10.00, 10.64, 11.36, 12.00)	(13.00, 14.04, 15.21, 16.25, 17.29, 18.46, 19.50)	(12, 13, 14, 15, 16, 17, 18)
	$\tilde{\delta}_3$	(75.84, 89.28, 102.72, 116.16, 130.56, 144.00, 157.44)	(44.85, 48.75, 61.1, 68.9, 77.35, 85.15, 93.60)	(48.8, 57.6, 66.4, 75.2, 84.8, 93.6, 102.4)	(46.0, 54.0, 62.5, 71.0, 79.0, 87.5, 96.0)	(59.80, 70.20, 81.25, 92.30, 102.70, 113.75, 124.80)	(11, 13, 15, 17, 19, 21, 23)
	$\tilde{\delta}_4$	(16.53, 24.51, 40.47, 45.03, 49.02, 64.98, 73.53)	(14.5, 22.04, 36.54, 40.02, 43.5, 58.0, 65.54)	(6.38, 9.57, 16.24, 17.69, 19.43, 25.81, 29.00)	(3.96, 6.00, 11.04, 12.00, 15.96, 15.96, 18.00)	(28.71, 43.50, 80.04, 87.00, 115.71, 115.71, 130.50)	(4, 6, 10, 11, 12, 16, 18)
	$\tilde{\delta}_5$	(24.08, 28.00, 44.24, 52.08, 59.92, 76.16, 80.08)	(8.74, 10.12, 15.87, 18.63, 21.62, 27.37, 28.75)	(28.71, 33.93, 53.07, 62.64, 72.21, 92.22, 96.57)	(9.00, 10.44, 16.56, 19.44, 22.50, 28.44, 30.06)	(6.00, 6.96, 11.04, 12.96, 15.00, 18.96, 20.04)	(6, 7, 11, 13, 15, 19, 20)
	$\tilde{\delta}$	(8, 10, 13, 14, 15, 18, 20)	(11, 13, 14, 16, 18, 19, 21)	(11, 12, 16, 18, 20, 24, 25)	(6, 7, 11, 12, 13, 17, 18)	(7, 9, 11, 12, 13, 15, 17)	

Table 8
Solution of proposed heptagonal fuzzy transportation problem obtained by DWFCM (Result set-1)

		Destination					
		$\tilde{\beta}_1$	$\tilde{\beta}_2$	$\tilde{\beta}_3$	$\tilde{\beta}_4$	$\tilde{\beta}_5$	$\tilde{\beta}$
Sources	$\tilde{\delta}_1$			(10, 12, 15, 16, 17, 20, 22)			(10, 12, 15, 16, 17, 20, 22)
	$\tilde{\delta}_2$			(6, 7, 11, 12, 13, 17, 18)	(–6, –4, 1, 3, 5, 10, 12)	(–12, –6,–3, 0, 3, 8, 12, 18)	(12, 13, 14, 15, 16, 17, 18)
	$\tilde{\delta}_3$	(–27, –18, –4, 5, 14, 28, 37)	(–14, –7, 5, 12, 19, 31, 38)				(11, 13, 15, 17, 19, 21, 23)
	$\tilde{\delta}_4$	(–11, –6, 5, 9, 13, 23, 29)		(–11, –8, –1, 2, 5, 12, 15)			(4, 6, 10, 11, 12, 16, 18)
	$\tilde{\delta}_5$		(–17,–12, –1, 4, 9, 20, 25)			(–5, –1, 6, 9, 12, 19, 23)	(6, 7, 11, 13, 15, 19, 20)
	$\tilde{\delta}$	(8, 10, 13, 14, 15, 18, 20)	(11, 13, 14, 16, 18, 19, 21)	(11, 12, 16, 18, 20, 24, 25)	(6, 7, 11, 12, 13, 17, 18)	(7, 9, 11, 12, 13, 15, 17)	

Here the comparison of the solution table is very effective in understanding the effectiveness of the proposed method, and all the methods of LHFNs height are 1, which means the value of α and the value of w for the centroid method. The proposed method GRHFM gives the least fuzzy total transportation cost. From the result, it can be concluded that the proposed method will be more efficient in finding the optimal transportation cost than any other method.

7.3. Sensitivity analysis

In this section, we discuss the different solutions and their ranges of α and w. The table below explains that if the value of α is 1, then the fuzzy solution after ranking is more reliable. When the value of α decreases, the solution's reliability and effectiveness also decrease. The last table (Table 10) in the problem is tabulated

Table 9
Solution of proposed heptagonal fuzzy transportation problem by applying SWFCM concept (Result set-2)

		Destination					
		$\tilde{\beta}_1$	$\tilde{\beta}_2$	$\tilde{\beta}_3$	$\tilde{\beta}_4$	$\tilde{\beta}_5$	$\tilde{\beta}$
Sources	$\tilde{\delta}_1$			(10, 12, 15, 16, 17, 20, 22)			(10, 12, 15, 16, 17, 20, 22)
	$\tilde{\delta}_2$	(–6, –4, 1, 3, 5, 10, 12)			(6, 7, 11, 12, 13, 17, 18)		(12, 13, 14, 15, 16, 17, 18)
	$\tilde{\delta}_3$	(–4, 0, 8, 11, 14, 22, 26)	(–31, –20, –3, 6, 15, 33, 43)				(11, 13, 15, 17, 19, 21, 23)
	$\tilde{\delta}_4$		(–11, –6, 5, 9, 13, 23, 29)	(–11, –8, –1, 2, 5, 12, 15)			(4, 6, 10, 11, 12, 16, 18)
	$\tilde{\delta}_5$		(–11, –8, –2, 1, 4, 10, 13)			(7, 9, 11, 12, 13, 15, 17)	(6, 7, 11, 13, 15, 19, 20)
	$\tilde{\delta}$	(8, 10, 13, 14, 15, 18, 20)	(11, 13, 14, 16, 18, 19, 21)	(11, 12, 16, 18, 20, 24, 25)	(6, 7, 11, 12, 13, 17, 18)	(7, 9, 11, 12, 13, 15, 17)	

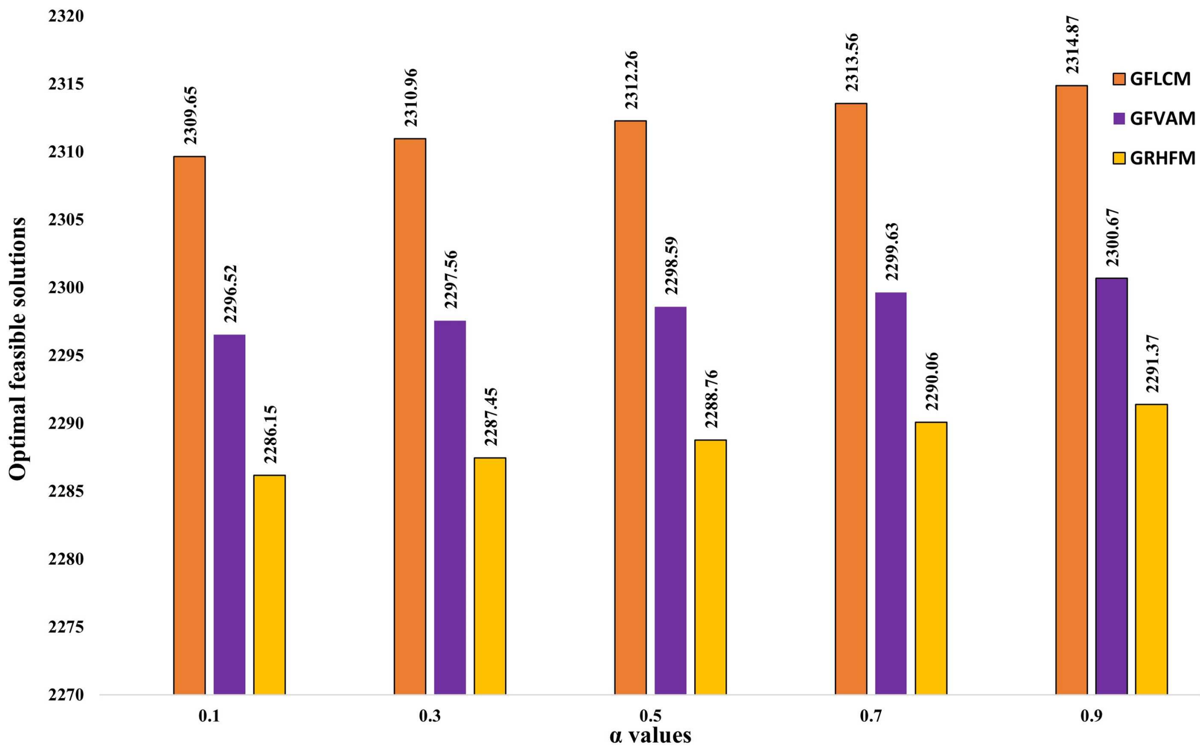
Table 10
Comparison of solutions for different methods based on ranking concepts under fuzziness

Method	Fuzzy total transportation cost	Optimal transportation cost using ranking
GFNWCM	(3558, 3708, 3955, 4154, 4342, 4527, 4726)	4139.56
GFLCM	(1798, 1934, 2167, 2331, 2468, 2667, 2820)	2315.52
GFVAM	(1837, 1970, 2140, 2328, 2420, 2625, 2784)	2301.19
GRHFM	(1783, 1916, 2140, 2310, 2438, 2643, 2793)	2292.02

Table 11
Comparison of solutions and their deviation for varying α for different techniques

α -value	Method	Fuzzy total transportation cost	α-cut total	Deviation(%)
0.1	GFNWCM	(3558, 3708, 3955, 4154, 4342, 4527, 4726)	4136.36	57.616%
	GFLCM	(1798, 1934, 2167, 2331, 2468, 2667, 2820)	2309.65	1.0227%
	GFVAM	(1837, 1970, 2140, 2328, 2420, 2625, 2784)	2296.52	0.4526%
	GRHFM	(1783, 1916, 2140, 2310, 2438, 2643, 2793)	2286.15	0.0000%
0.3	GFNWCM	(3558, 3708, 3955, 4154, 4342, 4527, 4726)	4137.07	57.579%
	GFLCM	(1798, 1934, 2167, 2331, 2468, 2667, 2820)	2310.96	1.0221%
	GFVAM	(1837, 1970, 2140, 2328, 2420, 2625, 2784)	2297.56	0.4406%
	GRHFM	(1783, 1916, 2140, 2310, 2438, 2643, 2793)	2287.46	0.0000%
0.5	GFNWCM	(3558, 3708, 3955, 4154, 4342, 4527, 4726)	4137.78	57.543%
	GFLCM	(1798, 1934, 2167, 2331, 2468, 2667, 2820)	2312.26	1.0215%
	GFVAM	(1837, 1970, 2140, 2328, 2420, 2625, 2784)	2298.59	0.4286%
	GRHFM	(1783, 1916, 2140, 2310, 2438, 2643, 2793)	2288.76	0.0000%
0.7	GFNWCM	(3558, 3708, 3955, 4154, 4342, 4527, 4726)	4137.78	57.543 %
	GFLCM	(1798, 1934, 2167, 2331, 2468, 2667, 2820)	2313.56	1.0209%
	GFVAM	(1837, 1970, 2140, 2328, 2420, 2625, 2784)	2299.63	0.4170%
	GRHFM	(1783, 1916, 2140, 2310, 2438, 2643, 2793)	2290.06	0.0000%
0.9	GFNWCM	(3558, 3708, 3955, 4154, 4342, 4527, 4726)	4139.20	57.470%
	GFLCM	(1798, 1934, 2167, 2331, 2468, 2667, 2820)	2314.87	1.0204%
	GFVAM	(1837, 1970, 2140, 2328, 2420, 2625, 2784)	2300.67	0.4050%
	GRHFM	(1783, 1916, 2140, 2310, 2438, 2643, 2793)	2291.37	0.0000%

Figure 6
Graphical visualization of feasible solutions for different values of α



for the value of $\alpha = 1$. But here we have varied the values of α like $0.1, 0.3, 0.5, 0.7, 0.9$.

The value of $\alpha = 0.1$ means the solution of α-cut ranking function has defuzzifying the fuzzified solution, which is 2286.15; for the value of α is increasing, it means that the proposed ranking function and solution methodology have provided better results compared with another ranking function centroid of the centroid based on Table 2. For the value of α 0.9, the ranking function for GFNWCM, GFLCM, GFVAM, and GRHFM method has provided results, which are 4139.20, 2314.87, 2300.67, and 2291.37. It appears that the lowest price is obtained by the proposed method for the HFTP. It gives a more effective and reliable heptagonal fuzzy cost for the study of this problem. The different levels of α-cut and height (w) are analyzed, and the solutions are tabulated in Table 11, based on the solutions. We take the result that the value of α (= w) gives the same result, but our proposed classification method gives an optimal range solution based on the value of α. As part of our proposed solution methodology and classification function, we performed a sensitivity analysis to improve comprehension.

The results of this study's sensitivity have been analyzed by varying the values of α, as represented in Figure 6. The proposed framework can effectively solve the FTP under generalized linear heptagonal fuzzy with the conditions. The framework can handle various transportation modes, scenarios, and constraints and can rank the alternatives and select the best option according to the criteria and preferences of the decision-makers. Therefore, we claim that our proposed GRHFM method for the transportation problem in a heptagonal fuzzy environment gives an efficient solution in addition to the existing methods.

From the results of optimal feasible solutions for various methods based on the experimental data, the different values of membership (α) provide different solutions. The corresponding values for GFNWCM, GFLCM, GFVAM, and GRHFM provide insights into the impact of these conditions on the measured variables. The objective of minimizing the cost of vegetable transshipment extends to the results of all methods of comparison. GRHFM provides better results. We compared the other method's results with this method's solution, and the deviations are significant as visualized in Table 11. The GFNWCM for the values of α is $0.1, 0.3, 0.5, 0.7$, and 0.9 having deviations are 57.616, 57.579, 57.543, 57.506, and 57.470. As for other methods, deviations are 1.0227 1.0221, 1.0215, 1.0209, and 1.0204 for GFLCM, and the resulting deviations in percentage for GFVAM are 0.4526, 0.4406, 0.4286, 0.4170, and 0.4050. According to the outcome of the transportation costs as per the changes of α-values, all the methods' results are decreasing; if the values of α have increased, then the optimal feasible result has decreased to reach a better decision for the solution for FTP.

8. Conclusion

In this study, we have developed a ranking technique for LHFNs using α-cut-based technique. For evaluating this α-cut-based technique from the available literature, the average and centroid of the centroid ranking technique are adopted and then analyzed by changing the values of the height of the fuzzy number. The main advantage of the proposed approach is that it provides a consistent ranking order of generalized fuzzy numbers, as demonstrated through several numerical examples and comparisons with existing methods. A novel approximate solution technique generates the crucial IBFS for FTP as a unique problem under fuzziness. The proposed method, based on a heuristic structure, differs from the earlier methods in that it considers the fuzzy cost and the fuzzy supply-demand coverage ratio (weights). Compared to an existing ranking function, the proposed ranking function defuzzy the fuzzy numbers better, indicating that the α-cut ranking approach is appropriate for defuzzifying the LHFN.

The proposed method can solve all FTPs in a similar way, without any restrictions, which is another superior feature. This technique has allocated sources according to the demand for goods, which is assigned much more quickly than GFVAM and GFNWCM. According to the results obtained, the ranking of the proposed α-cut method gives an appreciated value for finding the reliable transshipment cost compared with the solution of the ranking technique through the centroid of centroids for LHFNs. The proposed GRHFM is for solving the heptagonal FTP and gives more effective results compared with other existing methods. This method is also suitable for solving both balanced and unbalanced transportation problems.

For future research, LHFNs can be applied in various real-life applications such as cargo loading, networking, assignment, linear programming problems, etc. The observed trends and relationships for the fuzzy-based results of FTP can be developed by proposing new defuzzification techniques for LHFN to provide valuable insights into the experimental validations.

Acknowledgment

The authors sincerely thank the reviewers for their valuable suggestions for improving this article.

Ethical Statement

This study does not contain any studies with human or animal subjects performed by any of the authors.

Conflicts of Interest

The authors declare that they have no conflicts of interest to this work.

Data Availability Statement

Data available on request from the corresponding author upon reasonable request.

Author Contribution Statement

Balasundaram Baranidharan: Conceptualization, Software, Validation, Investigation, Data curation, Writing – original draft, Visualization. **Ghanshaym Singha Mahapatra**: Methodology, Software, Formal analysis, Resources, Data curation, Writing – review & editing, Visualization, Supervision.

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How to Cite: Baranidharan, B., & Mahapatra, G. S. (2024). A New Solution Technique for Fuzzy Transportation Problem Using Novel Ranking Functions on Heptagonal Fuzzy Numbers: A Case Study of Regional Shipment. *Journal of Computational and Cognitive Engineering*. https://doi.org/10.47852/bonviewJCCE42023253