Received: 12 May 2022 | Revised: 12 August 2022 | Accepted: 17 August 2022 | Published online: 22 August 2022

Journal of Computational and Cognitive Engineering
2023, Vol. 2(1) 75–87
DOI: 10.47852/bonviewJCCE2202249

BON VIEW PUBLISHING

RESEARCH ARTICLE

A Novel Approach to Model the Economic Characteristics of an Organization by Interval-Valued Complex Pythagorean Fuzzy Information

Hawraa Akram Yazbek[1], Fatima Surriya[2], Sami Ullah Khan[2], Naeem Jan[2] and Dragan Marinkovic[3,4]

[1]*Doctoral School of Mathematics and Informatics, University of Bordeaux, France*

[2]*Department of Mathematics, Gomal University, Pakistan*

[3]*Department of Structural Analysis, Technische Universität Berlin, Germany*

[4]*Department of Transport and Logistics, University of Nis, Serbia*

Abstract: The success of a nation mainly depends on its economic power. The economic growth plays an important place in enhancing the caliber of life as it produces job opportunities. In this research, the effect of uncertainty on economic growth and investments is examined with gross domestic product (GDP). Currently, the relationship between financial market and economic growth is one of the most important issues in economics. These departments are often accompanied by uncertainty. Through that regard, the idea of interval-valued complex Pythagorean fuzzy relation (IVCPFR) is introduced that is handy in such situations. Moreover, the Cartesian product between interval-valued complex Pythagorean fuzzy sets and verities of IVCPFRs was described. Furthermore, these IVCPFRs are applied to analyze GDP, unemployment, price, demand, supply, interest rate, investment, and money supply effecting business markets. The possibility of the Hasse diagram is more suitable to make sense of the relationship between various economic factors. IVCPFRs will clearly help to set up the impact of one component on the other and change the grades of supportive and distinctive outcomes concerning the time. IVCPFR is used for tracking various multidimensional issues and produces quick and better outcomes rather than the previous methodologies. Finally, the proposed method is the best method to model uncertainty in economics rather than previous methods.

Keywords: interval-valued complex Pythagorean fuzzy set, interval-valued complex Pythagorean fuzzy relation, economics, GDP, Hasse diagram

1. Introduction

A fuzzy set (FS) is a productive version with uncertainty. The manner of modifying realistic problems and actual life activities into a mathematical shape is referred to as mathematical modeling. Humans thinking and opinions are typically obscure and unclear. Also, the experimental mistakes and errors in calculations lead to ambiguous results. It is not possible to model uncertainty before 1965, when Zadeh (1965) introduced FSs and fuzzy logics. An FS is identified by a mapping ranging in the interval [0,1]. Zimmermann et al. (2011) develop the conception of FS theory and its applications. This plotting is known as membership. FS theory is used to approximate information and uncertainty. In 1986, Atanassov (1999) proposed the conception of intuitionistic FSs (IFSs). A group of objects specified by a couple of plottings ranging in [0,1] are known as IFS. These mappings define the membership and non-membership levels of an object. Operation will fail, if the total values of membership and non-membership grades increased to 1. De et al. (2001) develop the applications of IFS. Yager (2013) ends this limitation by introducing the modified version of IFS known as Pythagorean FS (PFS). PFS containing the levels of membership as well as levels of non-membership varies in [0,1]. This producer succeeds only if the sum of the square of both degrees ranges to the unit interval. Garg (2018) represents the application of PFS in different aspect of analyzing process.

Ramot et al. (2002) invent the idea of including complex numbers in FS theory, investigated as complex FS (CFS). The

*Corresponding author: Hawraa Akram Yazbek, Doctoral School of Mathematics and Informatics, University of Bordeaux, France. Email: hawraa.yazbek@u-bordeaux.fr

CFS describes the complex-valued mappings. CFS is used to solve multidimensional problems. Alkouri and Salleh (2012) present the conception of complex IFS (CIFS). CIFS defines complex mappings with membership and non-membership levels. The sum of amplitude terms and phase terms is all fuzzy numbers. Dick et al. (2015) develop the plan of complex PFSs (CPFSs). CPFSs discuss the degrees of membership and non-membership in complex-valued mappings. Relying on the prerequisite that the amount of their square likewise a complex number in a unit circle. Akram and Naz (2019) describe the decision-taking approach using CPFS. Moreover, CPFS is the generalized form of CIFS.

Interval-valued FS (IVFS) is developed by Cornelis et al. (2004). In 1975, Zadeh (1975) gave the idea of FS into IVFS. Greenfield et al. (2016) convert the level of membership of CFS in the form of interval-valued CFS (IVCFS). Greenfield et al. (2016) introduced the join and meet calculations of IVCFS. In 1999, Atanassov (1999) express the levels of membership and non-membership of an IFS in intervals and introduced a method known as interval-valued IFS (IVIFS). The universalization of IFS is known as PFS. Peng (2019) invents the idea of interval-valued PFS (IVPFS). Garg (2017) gave the idea of a new improved score function of an IVPFS. Interval-valued CIFS (IVCIFS) is developed by Garg and Rani (2019). Ali et al. (2021) develop a notion of interval-valued CPFS (IVCPFS).

Mendel (1995) develops the theory of relations for FSs, known as fuzzy relations (FRs). The enumeration of FRs specifies the grade, strength, and better relations among any pairs of FSs. This relies on its membership degree, if the value of membership degree is close to at least 1, then it represents the strong relation, but if the value is near to zero then it suggests the weaker relationship. Also, if we restrict the value of membership degree only to 0 and 1, then it is recognized as crisp relation. Complex FR (CFR) is introduced by Zhang et al. (2010). Bustince and Burillo (2000) discover the idea of intuitionistic FR (IFR). Dinakaran (2021) develops the conception of Pythagorean FR (PFR). Complex PFR (CPFR) is developed by Akram and Naz (2019). FR is an improved form of crisp relation. The IVFS spilt the single value of membership grade into the subintervals of the interval [0,1]. The notion of relation was organized as IVFSs by Hur et al. (2009), known as interval-valued FR. Goguen (1974) presents a system of axioms for a comparable simple shape of FS theory. Nasir et al. (2021) define the conception of the interval-valued CFRs applied to study the relationships among two IVCFSs. Zhang (2011) presented the origination of interval-valued IFR. Nasir et al. (2021) proposed the interval-valued complex IFR (IVCIFR). According of FSs, that defines the only membership but can also covers the non-membership in [0,1]. The single value of the level of membership and non-membership of PFSs was in the form of subintervals of interval [0,1]. Ali et al. (2021) evolved an interval-valued CPFR (IVCPFR).

This research represents the Cartesian product (CP) of two IVCPFSs modeled by Chung and Yang (2012). Additionally, the modern methodology of an IVCPFR is described by using the idea of CP of IVCPFSs. The representation of Hasse diagram for IVCPFSs and IVCPFRs is introduced. Different notions are mentioned in Hasse diagram. These notions can manage ambiguity higher than specific different concepts. The complex form of IVIFS is known as IVCIFS. IVCIFS used complex-valued mappings in a unit circle for solving different problems. Moreover, IVCPFS is an improved form of IVCIFS. The proposed application discussed the quality and grade of economic relationship. This examination produces the impact of uncertainty on economic growth and investments analyzed with gross domestic product (GDP). An integrated qualitative group decision-making method for assessing healthcare waste treatment technologies based on linguistic terms with weakened hedges is introduced by Wang and Wang (2022). This application is best used in this methodology for solving different economic problems. IVCPFS was utilized to discuss the membership and non-membership in multidimensional variables. This method has the ability to solve any type of problem beneficially and takes less time for tracking any issue. Moreover, the predefined frameworks are all limited like FS only discusses the membership, CFS expresses complex-valued variables, and IVCFS describes the intervals of complex-valued function in the membership and less ability for solving any problem rather than IVCPFS. Additionally, IFS is used to discuss the membership and non-membership for tracking issues, CIFS defines the complex-valued variables, and IVCIFS discusses the intervals of complex-valued functions, but all these frameworks have less ability and functionalities for solving various issues beneficially, correctly, and in a time-consuming way. The biggest factor for promoting economic development is the sustained economic growth. It is the major factor that proceeds financial development in the economics. Economic growth performs a vital part in the development of stander of life as it produces high-salary jobs. An application assists in the evaluation of nature of relationships between different economic standers. The economic relationship is a huge point. The interest in economic relationships arises inspiration about tracking down the responses of various complexities and creating an interest for obtaining a different outcome. Using this application, we can deal with multidimensional problems and achieve better results.

This research is represented as follows: Section 1 represents the introduction. Section 2 defines the reviews of some predefined concepts used in this study. In Section 3, the IVCPFRs and their types are explained with examples. In Section 4, we defined the Hasse diagrams for proposed concepts and its related topics. Section 5 proposed the application of IVCPFS with its verities for the explanation of direct and indirect effects of economic indicators. In Section 6, the comparison with various structures and proposed work is carried out. Section 7 summarized the research work.

2. Preliminaries

In this section, we discuss some of the basic concepts and their examples like FS, CFS, CP of two IVCFS, IFS, PFS, CPFS, CPFR, IVFS, IVCFS, IVPFS, IVCPFS, and CP of two IVBCPFRs and economic characteristics of an organization.

Economic characteristics produce high-salary jobs and a level of business while connecting in an organization. The organizational examiners considered the relation of organizations and industry characteristics for the role of organizations in the economy as a whole.

Definition 1. (Zadeh, 1965) Let σ over a universal set $\boldsymbol{\tau}$ is said to be an FS if

$$\sigma = \{\acute{n}, \varpi(\acute{n}) | \acute{n} \in \tau\}$$

where $\varpi(\acute{n})$ is the membership degree of FS such that $\varpi: \sigma \to [0,1]$.

Definition 2. (Ramot et al., 2002) Let σ over a universal set $\boldsymbol{\tau}$ is said to be a CFS if

$$\sigma = \{\acute{n}, \eta(\acute{n}) e^{\xi(\acute{n})2\pi i} | \acute{n} \in \tau\},$$

where $\eta(\acute{n})$ and $\xi(\acute{n})$ are amplitude and phase terms of membership degrees, respectively. Such that

$$\eta(\dot{n}), \xi(\dot{n}): \sigma \to [0, 1]$$

Definition 3. (Ramot et al., 2002) Let $\sigma = \{\dot{n}, \eta(\dot{n})e^{\xi(\dot{n})2\pi i} | \dot{n} \in \tau\}$ and $\theta = \{\mu, \eta(\mu)e^{\xi(\mu)2\pi i} | \mu \in \tau\}$ be two CFSs over a universal set τ with the membership degrees of amplitude term and phase term, respectively.

Then the CP of σ and θ is

$$\sigma \times \theta = \{(\dot{n}, \mu), \eta(\dot{n}, \mu)e^{\xi(\dot{n},\mu)2\pi i} | \dot{n} \in \dot{n}, \mu \in Q'\}$$

where $\eta(\dot{n} \times \mu): \sigma \times \theta \to [0,1]$ and $e^{\xi(\dot{n},\mu)2\pi i}: \sigma \times \theta \to [0, 1]$.

Such that

$$\eta(\dot{n} \times \mu)e^{\xi(\dot{n},\mu)2\pi i} \le \min\{\eta(\dot{n})e^{\xi(\dot{n})2\pi i}, \eta(\mu)e^{\xi(\mu)2\pi i}\}$$

Definition 4. (Zhang et al., 2010) Let $\sigma = \{\dot{n}, \eta(\dot{n})e^{\xi(\dot{n})2\pi i} | \dot{n} \in \sigma\}$ and $\theta = \{\mu, \eta(\mu)e^{\xi(\mu)2\pi i} | \mu \in \theta\}$ be two CFSs on τ. Then the subset of CP is known as CFR, denoted and defined as

$$\wp = \{(\dot{n}, \mu), \eta(\dot{n}, \mu)e^{\xi(\dot{n},\mu)2\pi i} | (\dot{n}, \mu) \in \sigma \times \theta\},$$

where $\eta(\dot{n}, \mu)$ and $\xi(\dot{n}, \mu)$ are the amplitude and phase terms of $\wp$, respectively.

Example 1. Consider a CFS σ on τ.

$$\sigma = \{(\dot{n}_1, 0.7e^{(0.3)2\pi i}), (\dot{n}_2, 0.4e^{(0.7)2\pi i}), (\dot{n}_3, 0.1e^{(1)2\pi i})\}$$

The CP $\sigma \times \sigma$ is

$$\sigma \times \sigma = \left\{ \begin{array}{l} ((\dot{n}_1, \dot{n}_1)0.7e^{(0.3)2\pi i}), ((\dot{n}_1, \dot{n}_2)0.4e^{(0.3)2\pi i}), ((\dot{n}_1, \dot{n}_3)0.1e^{(0.3)2\pi i}) \\ ((\dot{n}_2, \dot{n}_1)0.4e^{(0.3)2\pi i}), ((\dot{n}_2, \dot{n}_2)0.4e^{(0.7)2\pi i}), ((\dot{n}_2, \dot{n}_3)0.1e^{(0.7)2\pi i}) \\ ((\dot{n}_3, \dot{n}_1)0.1e^{(0.3)2\pi i}), ((\dot{n}_3, \dot{n}_2)0.1e^{(0.7)2\pi i}), ((\dot{n}_3, \dot{n}_3)0.1e^{(1)2\pi i}) \end{array} \right\}$$

The CFR $\wp$ is

$$\wp = \left\{ \begin{array}{l} ((\dot{n}_1, \dot{n}_1)0.7e^{(0.3)2\pi i}), ((\dot{n}_1, \dot{n}_2)0.4e^{(0.3)2\pi i}), \\ ((\dot{n}_2, \dot{n}_3)0.1e^{(0.7)2\pi i}), ((\dot{n}_3, \dot{n}_1)0.1e^{(0.3)2\pi i}) \end{array} \right\}$$

Definition 5. (Atanassov, 1999) Let σ over a universal set τ is said to be an IFS if

$$\sigma = \{\dot{n}, \varpi(\dot{n}), \omega(\dot{n}) | \dot{n} \in \tau\},$$

where $\varpi(\dot{n})$ and $\omega(\dot{n})$ are membership and non-membership degrees of the IFS σ, respectively.

Provided that $\varpi(\dot{n}) + \omega(\dot{n}) \in [0,1]$.

Definition 6. (Yager, 2013) Let σ over universal set τ is said to be a PFS if

$$\sigma = \{\dot{n}, \varpi(\dot{n}), \omega(\dot{n}) | \dot{n} \in \tau\},$$

where $\varpi(\dot{n})$ and $\omega(\dot{n})$ are membership and non-membership degrees of PFS σ, in sequence.

Provided that $(\varpi^2(\dot{n}) + \omega^2(\dot{n})) \in [0,1]$.

Definition 7. (Dick et al., 2015) Let σ over universal set τ is said to be a CPFS if

$$\sigma = \{\dot{n}, \alpha_{\varpi}(\dot{n})e^{\beta_{\varpi}(\dot{n})2\pi i}, \alpha_{\omega}(\dot{n})e^{\beta_{\omega}(\dot{n})2\pi i} | \dot{n} \in \tau\},$$

where $_{\varpi}(\dot{n}), \alpha_{\omega}(\dot{n})$ and $\beta_{\varpi}(\dot{n}), \beta_{\omega}(\dot{n})$ are the amplitude and phase terms of membership and non-membership degree of CPFS σ, in sequence.

Such that

$$\alpha_{\varpi}(\dot{n}), \alpha_{\omega}(\dot{n}), \beta_{\varpi}(\dot{n}), \beta_{\omega}(\dot{n}): \tau \to [0,1],$$

Provided that $(\alpha_{\varpi}(\dot{n})^2 + \alpha_{\omega}(\dot{n})^2) \in [0,1]$.

Definition 8. (Dick et al., 2015) Let $\sigma = \{\dot{n}, \alpha_{\varpi}(\dot{n})e^{\beta_{\varpi}(\dot{n})2\pi i}, \alpha_{\omega}(\dot{n})e^{\beta_{\omega}(\dot{n})2\pi i} | \dot{n} \in \tau\}$ and $\theta = \{\mu, \alpha_{\varpi}(\mu)e^{\beta_{\varpi}(\mu)2\pi i}, \alpha_{\omega}(\mu)e^{\beta_{\omega}(\omega)2\pi i} | \mu \in \tau\}$ be two CPFSs, then the CP is defined as

$$\sigma \times \theta = \{(\dot{n}, \mu), \alpha_{\varpi}(\dot{n}, \mu)e^{\beta_{\varpi}(\dot{n},\mu)2\pi i}, \alpha_{\omega}(\dot{n}, \mu)e^{\beta_{\omega}(\dot{n},\mu)2\pi i} | \dot{n} \in \sigma, \mu \in \theta\}.$$

where

$$\alpha_{\varpi}(\dot{n}, \mu), \alpha_{\omega}(\dot{n}, \mu), \beta_{\varpi}(\dot{n}, \mu), \beta_{\omega}(\dot{n}, \mu): \tau \to [0,1],$$

$$\alpha_{\varpi}(\dot{n}, \mu)e^{\beta_{\varpi}(\dot{n},\mu)2\pi i} = \min\{\alpha_{\varpi}(\dot{n}), \beta_{\varpi}(\dot{n})\} \text{ and}$$

$$\alpha_{\omega}(\dot{n}, \mu)e^{\beta_{\omega}(\dot{n},\mu)2\pi i} = \max\{\alpha_{\omega}(\dot{n}), \beta_{\omega}(\mu)\}$$

With conditions

$$0 \le (\alpha_{\varpi}(\dot{n}, \mu))^2 + (\alpha_{\omega}(\dot{n}, \mu))^2 \le 1,$$

$$0 \le (\beta_{\varpi}(\dot{n}, \mu))^2 + (\beta_{\omega}(\dot{n}, \mu))^2 \le 1.$$

Definition 9. (Akram & Naz, 2019) A CPFR is defined as the subset of a CP $(\sigma \times \theta)$.

Example 2. Take the CPFS

$$\sigma = \left\{ \begin{array}{l} (\dot{n}_1 0.5e^{(0.4)2\pi i}, 0.6e^{(0.5)2\pi i}), \\ (\dot{n}_2 0.6e^{(0.5)2\pi i}, 0.4e^{(0.6)2\pi i}) \end{array} \right\}$$

Then for finding $\sigma \times \sigma$, we have

$$\sigma \times \sigma = \left\{ \begin{array}{l} ((\dot{n}_1, \dot{n}_1)0.5e^{(0.4)2\pi i}, 0.6e^{(0.5)2\pi i}), ((\dot{n}_1, \dot{n}_2)(0.5)e^{(0.4)2\pi i}, 0.6e^{(0.6)2\pi i}) \\ ((\dot{n}_2, \dot{n}_1)0.5e^{(0.4)2\pi i}, 0.6e^{(0.6)2\pi i}), ((\dot{n}_2, \dot{n}_2)(0.6)e^{(0.5)2\pi i}, (0.4)e^{(0.6)2\pi i}) \end{array} \right\}$$

The relation $\wp$ is defined as

$$\wp = \{((\dot{n}_1, \dot{n}_2)(0.5)e^{(0.4)2\pi i}, 0.6e^{(0.6)2\pi i}), ((\dot{n}_2, \dot{n}_2)(0.6)e^{(0.5)2\pi i}, (0.4)e^{(0.6)2\pi i})\}$$

Definition 10. (Cornelis et al., 2004) Let σ over a universal set τ is said to be an IVFS if

$$\sigma = \{(\dot{n}, [\varpi^-(\dot{n}), \varpi^+(\dot{n})]: \dot{n} \in \tau)\}$$

where $\varpi^-: \sigma \to [0,1]$ and $\varpi^+: \sigma \to [0,1]$ are the left and right terms of membership interval, in sequence.

Example 3. $\sigma = \{\dot{n}_1, [0.32,0.68]), (\dot{n}_2,[0.16,0.26])(\dot{n}_3, [0.58,0.63])\}$ is an IVFS.

Definition 11. (Greenfield et al., 2016) Let σ over a universal set τ is said to be an IVCFS if

$$\sigma = \left\{\left(\grave{n}, [\alpha^-(\grave{n}), \alpha^+(\grave{n})]e^{[\beta^-(\grave{n}),\beta^+(\grave{n})]2\pi i}\right): \grave{n} \in \tau\right\}$$

where the mappings $\alpha^-, \alpha^+, \beta^-, \beta^+ : \sigma \to [0,1]$ are known as lower and upper amplitude term, lower and upper phase term of the degree of membership and $i = \sqrt{-1}$.

Example 4. $\sigma = \left\{\left(\grave{n}_1, [0.27, 0.31]e^{[0.43,0.61]2\pi i}\right), \left(\grave{n}_2, [0.17, 0.23]e^{[0.67,0.81]}\ 2\pi i\right), \left(\grave{n}_3, [0.45, 0.50]e^{[0.67,0.7.9]}\right)\right\}$ is an IVCFS.

Definition 12. (Peng, 2019) Let σ over a universal set τ is said to be an IVPFS if

$$\sigma = \{[\varpi^-(\grave{n}), \varpi^+(\grave{n})], [\omega^-(\grave{n}), \omega^+(\grave{n})]: \grave{n} \in \tau\}$$

where $0 \le \varpi^-(\grave{n}) \le \varpi^+(\grave{n}) \le 1, 0 \le \omega^-(\grave{n}) \le \omega^+(\grave{n}) \le 1$

Provided that $\varpi^+(\grave{n})^2 + \omega^+(\grave{n})^2 \le 1, \forall \grave{n} \in \tau$.

Definition 13. (Ali et al., 2021) Let σ over a universal set τ is said to be an IVCPFS if

$$\sigma = \left\{\left(\grave{n}, [\alpha^-{}_{\varpi}(\grave{n}), \alpha^+{}_{\varpi}(\grave{n})]e^{[\beta^-{}_{\varpi}(\grave{n}),\beta^+{}_{\varpi}(\grave{n})]2\pi i}, [\alpha^-{}_{\omega}(\grave{n}), \alpha^+{}_{\omega}(\grave{n})]e^{[\beta^-{}_{\omega}(\grave{n}),\beta^+{}_{\omega}(\grave{n})]2\pi i}: \grave{n} \in \tau\right)\right\}$$

where $\alpha_{\varpi}{}^-(\grave{n}), \alpha_{\varpi}{}^+(\grave{n}), \beta_{\varpi}{}^+(\grave{n}), \beta_{\varpi}{}^-(\grave{n}),: \sigma \to [0,1]$, $\alpha_{\omega}{}^-(\grave{n}), \alpha_{\omega}{}^+(\grave{n}), \beta_{\omega}{}^-(\grave{n}), \beta_{\omega}{}^+(\grave{n}): \sigma \to [0,1]$ are the plotting of membership and non-membership levels, in sequence. So $i = \sqrt{-1}$.

Provided that $(\alpha_{\varpi}{}^-(\grave{n})^2 + \alpha_{\omega}{}^-(\grave{n})^2) \in [0,1]$.

Definition 14. (Ali et al., 2021) An IVCPFR $\wp$ is the subset of a CP of two IVCPFSs.

Such that $\wp \subseteq \sigma \times \theta$

Example 5. The CP of two IVCPFS

$$\sigma = \left\{\begin{array}{c} \left(\grave{n}_1, [0.3, 0.5]e^{[0.4,0.6]2\pi i}, [0.2, 0.4]e^{[0.1,0.3]2\pi i}\right), \\ \left(\grave{n}_2, [0.1, 0.3]e^{[0.2,0.5]2\pi i}, [0.3, 0.6]e^{[0.0.2]2\pi i}\right), \\ \left(\grave{n}_3, [0.2, 0.7]e^{[0.3,0.5]2\pi i}, [0, 0.2]e^{[0.3,0.5]2\pi i}\right) \end{array}\right\} \text{and}$$

$$\theta = \left\{\begin{array}{c} \left(\mu_1, [0.2, 0.4]e^{[0,0.3]2\pi i}, [0.5, 0.7]e^{[0.2,0.5]2\pi i}\right), \\ \left(\mu_2, [0, 0.6]e^{[0.2,0.6]2\pi i}, [0.3, 0.5]e^{[0.1,0.5]2\pi i}\right), \\ \left(\mu_3, [0.1, 0.7]e^{[0.2,0.4]2\pi i}, [0, 0.6]e^{[0.2,0.7]2\pi i}\right) \end{array}\right\}$$

is given as

$$\sigma \times \theta = \left\{\begin{array}{c} \left((\grave{n}_1, \mu_1), [0.2, 0.4]e^{[0,0.3]2\pi i}, [0.5, 0.7]e^{[0.2,0.5]2\pi i}\right), \\ \left((\grave{n}_1, \mu_2), [0, 0.5]e^{[0.2,0.6]2\pi i}, [0.3, 0.5]e^{[0.1,0.5]2\pi i}\right), \\ \left((\grave{n}_1, \mu_3), [0.1, 0.5]e^{[0.2,0.4]2\pi i}, [0.2, 0.6]e^{[0.2,0.7]2\pi i}\right), \\ \left((\grave{n}_2, \mu_1), [0.1, 0.3]e^{[0,0.3]2\pi i}, [0.5, 0.7]e^{[0.2,0.5]2\pi i}\right), \\ \left((\grave{n}_2, \mu_2), [0, 0.3]e^{[0.2,0.5]2\pi i}, [0.3, 0.6]e^{[0.1,0.5]2\pi i}\right), \\ \left((\grave{n}_2, \mu_3), [0.1, 0.3]e^{[0.2,0.4]2\pi i}, [0.3, 0.6]e^{[0.2,0.7]2\pi i}\right), \\ \left((\grave{n}_3, \mu_1), [0.2, 0.4]e^{[0,0.3]2\pi i}, [0.5, 0.7]e^{[0.3,0.5]2\pi i}\right), \\ \left((\grave{n}_3, \mu_2), [0, 0.6]e^{[0.2,0.5]2\pi i}, [0.3, 0.5]e^{[0.3,0.5]2\pi i}\right), \\ \left((\grave{n}_3, \mu_3), [0.1, 0.7]e^{[0.2,0.4]2\pi i}, [0, 0.6]e^{[0.3,0.7]2\pi i}\right) \end{array}\right\}$$

Then the relation of $\sigma \times \theta$ is

$$\wp = \left\{\begin{array}{c} \left((\grave{n}_1, \mu_2), [0, 0.5]e^{[0.2,0.6]2\pi i}, [0.3, 0.5]e^{[0.1,0.5]2\pi i}\right), \\ \left((\grave{n}_2, \mu_1), [0.1, 0.3]e^{[0,0.3]2\pi i}, [0.5, 0.7]e^{[0.2,0.5]2\pi i}\right), \\ \left((\grave{n}_2, \mu_3), [0.1, 0.3]e^{[0.2,0.4]2\pi i}, [0.3, 0.6]e^{[0.2,0.7]2\pi i}\right), \\ \left((\grave{n}_3, \mu_2), [0, 0.6]e^{[0.2,0.5]2\pi i}, [0.3, 0.5]e^{[0.3,0.5]2\pi i}\right), \\ \left((\grave{n}_3, \mu_3), [0.1, 0.7]e^{[0.2,0.4]2\pi i}, [0, 0.6]e^{[0.3,0.7]2\pi i}\right) \end{array}\right\}$$

3. Main Results

Goals of this section describe some of the new concepts in IVCPFS, like CP of IVCPFSs and IVCPFRs. Moreover, a few exciting results and properties of these IVCPFRs exist.

Definition 15. Let σ over a universal set τ be an IVCPFS and $\wp$ be an IVCPFR on σ. Then,

a. An IVCPFR$\wp$ is called interval-valued complex Pythagorean reflexive FR (IVCP-reflexive-FR) if

$$\forall\left(\begin{array}{c} \grave{n}, \left[\varpi^-{}_{\sigma_c}(\grave{n}), \varpi^+{}_{\sigma_c}(\grave{n})\right]e^{\left[\varpi^-{}_{\sigma_c}(\grave{n}),\varpi^+{}_{\sigma_c}(\grave{n})\right]2\pi i}, \\ \left[\omega^-{}_{\sigma_c}(\grave{n}), \omega^+{}_{\sigma_c}(\grave{n})\right]e^{\left[\omega^-{}_{\sigma_c}(\grave{n}),\omega^+{}_{\sigma_c}(\grave{n})\right]2\pi i} \end{array}\right) \in \sigma$$

$$\Rightarrow \left(\begin{array}{c} (\grave{n}, \grave{n}), [\varpi^-{}_{\sigma_c}(\grave{n}, \grave{n}), \varpi^+{}_{\sigma_c}(\grave{n}, \grave{n})]e^{\left[\varpi^-{}_{\sigma_c}(\grave{n}),\varpi^+{}_{\sigma_c}(\grave{n})\right]2\pi i}, \\ [\omega^-{}_{\sigma_c}(\grave{n}, \grave{n}), \omega^+{}_{\sigma_c}(\grave{n}, \grave{n})]e^{\left[\omega^-{}_{\sigma_c}(\grave{n}),\omega^+{}_{\sigma_c}(\grave{n})\right]2\pi i} \end{array}\right) \in \wp$$

b. An IVCPFR $\wp$ is known as IVCP-irreflexive-FR if

$$\left(\begin{array}{c} (\grave{n}, \grave{n}), [\varpi^-{}_{\sigma_c}(\grave{n}, \grave{n}), \varpi^+{}_{\sigma_c}(\grave{n}, \grave{n})]e^{\left[\varpi^-{}_{\sigma_c}(\grave{n}),\varpi^+{}_{\sigma_c}(\grave{n})\right]2\pi i}, \\ [\omega^-{}_{\sigma_c}(\grave{n}, \grave{n}), \omega^+{}_{\sigma_c}(\grave{n}, \grave{n})]e^{\left[\omega^-{}_{\sigma_c}(\grave{n}),\omega^+{}_{\sigma_c}(\grave{n})\right]2\pi i} \end{array}\right) \notin \wp$$

c. An IVCPFR $\wp$ is known as IVCP-symmetric-FR if

$$\forall\left(\begin{array}{c} (\grave{n}_1, \grave{n}_2), \left[\varpi^-{}_{\sigma_c}(\grave{n}_1, \grave{n}_2), \varpi^+{}_{\sigma_c}(\grave{n}_1, \grave{n}_2)\right]e^{\left[\varpi^-{}_{\sigma_c}(\grave{n}_1,\grave{n}_2),\varpi^+{}_{\sigma_c}(\grave{n}_1,\grave{n}_2)\right]2\pi i}, \\ \left[\omega^-{}_{\sigma_c}(\grave{n}_1, \grave{n}_2), \omega^+{}_{\sigma_c}(\grave{n}_1, \grave{n}_2)\right]e^{\left[\omega^-{}_{\sigma_c}(\grave{n}_1,\grave{n}_2),\omega^+{}_{\sigma_c}(\grave{n}_1,\grave{n}_2)\right]2\pi i} \end{array}\right) \in \wp$$

$$\Rightarrow \left(\begin{array}{c} (\grave{n}_2, \grave{n}_1), \left[\varpi^-{}_{\sigma_c}(\grave{n}_2, \grave{n}_1), \varpi^+{}_{\sigma_c}(\grave{n}_2, \grave{n}_1)\right]e^{\left[\varpi^-{}_{\sigma_c}(\grave{n}_2,\grave{n}_1),\varpi^+{}_{\sigma_c}(\grave{n}_2,\grave{n}_1)\right]2\pi i}, \\ \left[\omega^-{}_{\sigma_c}(\grave{n}_2, \grave{n}_1), \omega^+{}_{\sigma_c}(\grave{n}_2, \grave{n}_1)\right]e^{\left[\omega^-{}_{\sigma_c}(\grave{n}_2,\grave{n}_1),\omega^+{}_{\sigma_c}(\grave{n}_2,\grave{n}_1)\right]2\pi i} \end{array}\right) \in \wp$$

where $(\grave{n}_1, \grave{n}_2) \in \sigma$.

d. An IVCPFR $\wp$ is known as IVCP-antisymmetric-FR $\wp$ if

$$\left(\begin{array}{c} (\grave{n}_1, \grave{n}_2), \left[\varpi^-{}_{\sigma_c}(\grave{n}_1, \grave{n}_2), \varpi^+{}_{\sigma_c}(\grave{n}_1, \grave{n}_2)\right]e^{\left[\varpi^-{}_{\sigma_c}(\grave{n}_1,\grave{n}_2),\varpi^+{}_{\sigma_c}(\grave{n}_1,\grave{n}_2)\right]2\pi i}, \\ \left[\omega^-{}_{\sigma_c}(\grave{n}_1, \grave{n}_2), \omega^+{}_{\sigma_c}(\grave{n}_1, \grave{n}_2)\right]e^{\left[\omega^-{}_{\sigma_c}(\grave{n}_1,\grave{n}_2),\omega^+{}_{\sigma_c}(\grave{n}_1,\grave{n}_2)\right]2\pi i} \end{array}\right) \in \wp,$$

$$\left(\begin{array}{c} (\grave{n}_2, \grave{n}_1), \left[\varpi^-{}_{\sigma_c}(\grave{n}_2, \grave{n}_1), \varpi^+{}_{\sigma_c}(\grave{n}_2, \grave{n}_1)\right]e^{\left[\varpi^-{}_{\sigma_c}(\grave{n}_2,\grave{n}_1),\varpi^+{}_{\sigma_c}(\grave{n}_2,\grave{n}_1)\right]2\pi i}, \\ \left[\omega^-{}_{\sigma_c}(\grave{n}_2, \grave{n}_1), \omega^+{}_{\sigma_c}(\grave{n}_2, \grave{n}_1)\right]e^{\left[\omega^-{}_{\sigma_c}(\grave{n}_2,\grave{n}_1),\omega^+{}_{\sigma_c}(\grave{n}_2,\grave{n}_1)\right]2\pi i} \end{array}\right) \in \wp,$$

$$\Rightarrow \left(\grave{n}_1, \left[\varpi^-{}_{\sigma_c}(\grave{n}_1), \varpi^+{}_{\sigma_c}(\grave{n}_1)\right]e^{\left[\varpi^-{}_{\sigma_c}(\grave{n}_1),\varpi^+{}_{\sigma_c}(\grave{n}_1)\right]2\pi i}, \left[\omega^-{}_{\sigma_c}(\grave{n}_1), \omega^+{}_{\sigma_c}(\grave{n}_1)\right]e^{\left[\omega^-{}_{\sigma_c}(\grave{n}_1),\omega^+{}_{\sigma_c}(\grave{n}_1)\right]2\pi i}\right)$$

$$= \left(\grave{n}_2, \left[\varpi^-{}_{\sigma_c}(\grave{n}_2), \varpi^+{}_{\sigma_c}(\grave{n}_2)\right]e^{\left[\varpi^-{}_{\sigma_c}(\grave{n}_2),\varpi^+{}_{\sigma_c}(\grave{n}_2)\right]2\pi i}, \left[\omega^-{}_{\sigma_c}(\grave{n}_2), \omega^+{}_{\sigma_c}(\grave{n}_2)\right]e^{\left[\omega^-{}_{\sigma_c}(\grave{n}_2),\omega^+{}_{\sigma_c}(\grave{n}_2)\right]2\pi i}\right)$$

e. An IVCPFR $\wp$ is known as IVCP-asymmetric-FR $\wp$ if

$$\left(\begin{array}{c} (\grave{n}_2, \grave{n}_1), \left[\varpi^-{}_{\sigma_c}(\grave{n}_2, \grave{n}_1), \varpi^+{}_{\sigma_c}(\grave{n}_2, \grave{n}_1)\right]e^{\left[\varpi^-{}_{\sigma_c}(\grave{n}_2,\grave{n}_1),\varpi^+{}_{\sigma_c}(\grave{n}_2,\grave{n}_1)\right]2\pi i}, \\ \left[\omega^-{}_{\sigma_c}(\grave{n}_2, \grave{n}_1), \omega^+{}_{\sigma_c}(\grave{n}_2, \grave{n}_1)\right]e^{\left[\omega^-{}_{\sigma_c}(\grave{n}_2,\grave{n}_1),\omega^+{}_{\sigma_c}(\grave{n}_2,\grave{n}_1)\right]2\pi i} \end{array}\right) \in \wp,$$

$$\left(\begin{array}{c}(\hbar_1,\hbar_2),\left[\varpi^-{}_{\sigma_c}(\hbar_1,\hbar_2),\varpi^+{}_{\sigma_c}(\hbar_1,\hbar_2)\right]e^{\left[\varpi^-{}_{\sigma_c}(\hbar_1,\hbar_2),\varpi^+{}_{\sigma_c}(\hbar_1,\hbar_2)\right]2\pi i},\\ \left[\omega^-{}_{\sigma_c}(\hbar_1,\hbar_2),\omega^+{}_{\sigma_c}(\hbar_1,\hbar_2)\right]e^{\left[\omega^-{}_{\sigma_c}(\hbar_1,\hbar_2),\omega^+{}_{\sigma_c}(\hbar_1,\hbar_2)\right]2\pi i}\end{array}\right)\notin\wp$$

f. An IVCPFR $\wp$ is known as IVCP-complete-FR if

$$\left(\begin{array}{c}(\hbar_1,\hbar_2),\left[\varpi^-{}_{\sigma_c}(\hbar_1,\hbar_2),\varpi^+{}_{\sigma_c}(\hbar_1,\hbar_2)\right]e^{\left[\varpi^-{}_{\sigma_c}(\hbar_1,\hbar_2),\varpi^+{}_{\sigma_c}(\hbar_1,\hbar_2)\right]2\pi i},\\ \left[\omega^-{}_{\sigma_c}(\hbar_1,\hbar_2),\omega^+{}_{\sigma_c}(\hbar_1,\hbar_2)\right]e^{\left[\omega^-{}_{\sigma_c}(\hbar_1,\hbar_2),\omega^+{}_{\sigma_c}(\hbar_1,\hbar_2)\right]2\pi i}\end{array}\right)\in\wp$$

or

$$\left(\begin{array}{c}(\hbar_2,\hbar_1),\left[\varpi^-{}_{\sigma_c}(\hbar_2,\hbar_1),\varpi^+{}_{\sigma_c}(\hbar_2,\hbar_1)\right]e^{\left[\varpi^-{}_{\sigma_c}(\hbar_2,\hbar_1),\varpi^+{}_{\sigma_c}(\hbar_2,\hbar_1)\right]2\pi i},\\ \left[\omega^-{}_{\sigma_c}(\hbar_2,\hbar_1),\omega^+{}_{\sigma_c}(\hbar_2,\hbar_1)\right]e^{\left[\omega^-{}_{\sigma_c}(\hbar_2,\hbar_1),\omega^+{}_{\sigma_c}(\hbar_2,\hbar_1)\right]2\pi i}\end{array}\right)\in\wp$$

g. An IVCPFR $\wp$ is known as IVCP-transitive-FR if

$$\left(\begin{array}{c}(\hbar_1,\hbar_2),\left[\varpi^-{}_{\sigma_c}(\hbar_1,\hbar_2),\varpi^+{}_{\sigma_c}(\hbar_1,\hbar_2)\right]e^{\left[\varpi^-{}_{\sigma_c}(\hbar_1,\hbar_2),\varpi^+{}_{\sigma_c}(\hbar_1,\hbar_2)\right]2\pi i},\\ \left[\omega^-{}_{\sigma_c}(\hbar_1,\hbar_2),\omega^+{}_{\sigma_c}(\hbar_1,\hbar_2)\right]e^{\left[\omega^-{}_{\sigma_c}(\hbar_1,\hbar_2),\omega^+{}_{\sigma_c}(\hbar_1,\hbar_2)\right]2\pi i}\end{array}\right)\in\wp,$$

$$\left(\begin{array}{c}(\hbar_2,\hbar_3),\left[\varpi^-{}_{\sigma_c}(\hbar_2,\hbar_3),\varpi^+{}_{\sigma_c}(\hbar_2,\hbar_3)\right]e^{\left[\varpi^-{}_{\sigma_c}(\hbar_2,\hbar_3),\varpi^+{}_{\sigma_c}(\hbar_2,\hbar_3)\right]2\pi i},\\ \left[\omega^-{}_{\sigma_c}(\hbar_2,\hbar_3),\omega^+{}_{\sigma_c}(\hbar_2,\hbar_3)\right]e^{\left[\omega^-{}_{\sigma_c}(\hbar_2,\hbar_3),\omega^+{}_{\sigma_c}(\hbar_2,\hbar_3)\right]2\pi i}\end{array}\right)\in\wp,$$

$$\Rightarrow\left(\begin{array}{c}(\hbar_1,\hbar_3),\left[\varpi^-{}_{\sigma_c}(\hbar_1,\hbar_3),\varpi^+{}_{\sigma_c}(\hbar_1,\hbar_3)\right]e^{\left[\varpi^-{}_{\sigma_c}(\hbar_1,\hbar_3),\varpi^+{}_{\sigma_c}(\hbar_1,\hbar_3)\right]2\pi i},\\ \left[\omega^-{}_{\sigma_c}(\hbar_1,\hbar_3),\omega^+{}_{\sigma_c}(\hbar_1,\hbar_3)\right]e^{\left[\omega^-{}_{\sigma_c}(\hbar_1,\hbar_3),\omega^+{}_{\sigma_c}(\hbar_1,\hbar_3)\right]2\pi i}\end{array}\right)\in\wp,$$

where $\hbar_1,\hbar_2,\hbar_3\in\sigma$

h. An IVCPFR $\wp$ is known as IVCP-equivalence-FR if $\wp$ is IVCP-reflexive-FR, IVCP-symmetric-FR, and IVCP-transitive-FR.

i. An IVCPFR $\wp$ is known as IVCP-preorder-FR if $\wp$ is IVCP-reflexive-FR and IVCP-transitive-FR.

j. An IVCPFR $\wp$ is known as IVCP-strict order-FR if $\wp$ is IVCP-irreflexive-FR and IVCP-transitive-FR.

k. An IVCPFR $\wp$ is known as IVCP-partial order-FR if $\wp$ is IVCP-preorder-FR and IVCP-antisymmetric-FR.

l. An IVCPFR $\wp$ is known as IVCP-linear order-FR if $\wp$ is IVCP-partial order-FR and interval-valued complex Pythagorean complete FR (IVCP-complete-FR).

Example 6. For an IVCPFS

$$\sigma=\left\{\begin{array}{c}\left(\hbar_1,[0.3,0.5]e^{[0.4,0.6]2\pi i},[0.2,0.4]e^{[0.1,0.3]2\pi i}\right),\\ \left(\hbar_2,[0.1,0.3]e^{[0.2,0.5]2\pi i},[0.3,0.6]e^{[0,0.2]2\pi i}\right),\\ \left(\hbar_3,[0.2,0.7]e^{[0.3,0.5]2\pi i},[0,0.2]e^{[0.3,0.5]2\pi i}\right)\end{array}\right\}$$

The CP $\sigma\times\sigma$ is

$$\sigma\times\sigma=\left\{\begin{array}{c}\left((\hbar_1,\hbar_1),[0.3,0.5]e^{[0.4,0.6]2\pi i},[0.2,0.4]e^{[0.1,0.3]2\pi i}\right),\\ \left((\hbar_1,\hbar_2),[0.1,0.3]e^{[0.2,0.5]2\pi i},[0.3,0.6]e^{[0.1,0.3]2\pi i}\right),\\ \left((\hbar_1,\hbar_3),[0.2,0.5]e^{[0.3,0.5]2\pi i},[0.2,0.4]e^{[0.3,0.5]2\pi i}\right),\\ \left((\hbar_2,\hbar_1),[0.1,0.3]e^{[0.2,0.5]2\pi i},[0.3,0.6]e^{[0.1,0.3]2\pi i}\right),\\ \left((\hbar_2,\hbar_2),[0.1,0.3]e^{[0.2,0.5]2\pi i},[0.3,0.6]e^{[0,0.2]2\pi i}\right),\\ \left((\hbar_2,\hbar_3),[0.1,0.3]e^{[0.2,0.5]2\pi i},[0.3,0.6]e^{[0.3,0.5]2\pi i}\right),\\ \left((\hbar_3,\hbar_1),[0.2,0.5]e^{[0.3,0.5]2\pi i},[0.2,0.4]e^{[0.3,0.5]2\pi i}\right),\\ \left((\hbar_3,\hbar_2),[0.1,0.3]e^{[0.2,0.5]2\pi i},[0.3,0.6]e^{[0.3,0.5]2\pi i}\right),\\ \left((\hbar_3,\hbar_3),[0.2,0.7]e^{[0.3,0.5]2\pi i},[0,0.2]e^{[0.3,0.5]2\pi i}\right)\end{array}\right\}$$

1. The IVCP-equivalence-FR $\wp_1$ on σ is given below

$$\wp_1=\left\{\begin{array}{c}\left((\hbar_1,\hbar_1),[0.3,0.5]e^{[0.4,0.6]2\pi i},[0.2,0.4]e^{[0.1,0.3]2\pi i}\right),\\ \left((\hbar_1,\hbar_3),[0.2,0.5]e^{[0.3,0.5]2\pi i},[0.2,0.4]e^{[0.3,0.5]2\pi i}\right),\\ \left((\hbar_2,\hbar_2),[0.1,0.3]e^{[0.2,0.5]2\pi i},[0.3,0.6]e^{[0,0.2]2\pi i}\right),\\ \left((\hbar_3,\hbar_1),[0.2,0.5]e^{[0.3,0.5]2\pi i},[0.2,0.4]e^{[0.3,0.5]2\pi i}\right),\\ \left((\hbar_3,\hbar_3),[0.2,0.7]e^{[0.3,0.5]2\pi i},[0,0.2]e^{[0.3,0.5]2\pi i}\right)\end{array}\right\}$$

2. The IVCP-preorder-FR $\wp_2$ on σ is given below

$$\wp_2=\left\{\begin{array}{c}\left((\hbar_1,\hbar_1),[0.3,0.5]e^{[0.4,0.6]2\pi i},[0.2,0.4]e^{[0.1,0.3]2\pi i}\right),\\ \left((\hbar_2,\hbar_3),[0.1,0.3]e^{[0.2,0.5]2\pi i},[0.3,0.6]e^{[0.3,0.5]2\pi i}\right),\\ \left((\hbar_2,\hbar_2),[0.1,0.3]e^{[0.2,0.5]2\pi i},[0.3,0.6]e^{[0,0.2]2\pi i}\right),\\ \left((\hbar_3,\hbar_3),[0.2,0.7]e^{[0.3,0.5]2\pi i},[0,0.2]e^{[0.3,0.5]2\pi i}\right)\end{array}\right\}$$

3. The IVCP-strict order-FR $\wp_3$ on σ is given below

$$\wp_3=\left\{\begin{array}{c}\left((\hbar_2,\hbar_1),[0.1,0.3]e^{[0.2,0.5]2\pi i},[0.3,0.6]e^{[0.1,0.3]2\pi i}\right),\\ \left((\hbar_3,\hbar_1),[0.2,0.5]e^{[0.3,0.5]2\pi i},[0.2,0.4]e^{[0.3,0.5]2\pi i}\right),\\ \left((\hbar_3,\hbar_2),[0.1,0.3]e^{[0.2,0.5]2\pi i},[0.3,0.6]e^{[0.3,0.5]2\pi i}\right)\end{array}\right\}$$

4. The IVCP-partial order-FR $\wp_4$ on σ is given below

$$\wp_4=\left\{\begin{array}{c}\left((\hbar_1,\hbar_1),[0.3,0.5]e^{[0.4,0.6]2\pi i},[0.2,0.4]e^{[0.1,0.3]2\pi i}\right),\\ \left((\hbar_2,\hbar_1),[0.1,0.3]e^{[0.2,0.5]2\pi i},[0.3,0.6]e^{[0.1,0.3]2\pi i}\right),\\ \left((\hbar_2,\hbar_2),[0.1,0.3]e^{[0.2,0.5]2\pi i},[0.3,0.6]e^{[0,0.2]2\pi i}\right),\\ \left((\hbar_3,\hbar_1),[0.2,0.5]e^{[0.3,0.5]2\pi i},[0.2,0.4]e^{[0.3,0.5]2\pi i}\right),\\ \left((\hbar_3,\hbar_3),[0.2,0.7]e^{[0.3,0.5]2\pi i},[0,0.2]e^{[0.3,0.5]2\pi i}\right)\end{array}\right\}$$

5. The IVCP-linear order-FR $\wp_5$ on σ is given below

$$\wp_5=\left\{\begin{array}{c}\left((\hbar_1,\hbar_1),[0.3,0.5]e^{[0.4,0.6]2\pi i},[0.2,0.4]e^{[0.1,0.3]2\pi i}\right),\\ \left((\hbar_2,\hbar_1),[0.1,0.3]e^{[0.2,0.5]2\pi i},[0.3,0.6]e^{[0.1,0.3]2\pi i}\right),\\ \left((\hbar_2,\hbar_2),[0.1,0.3]e^{[0.2,0.5]2\pi i},[0.3,0.6]e^{[0,0.2]2\pi i}\right),\\ \left((\hbar_3,\hbar_1),[0.2,0.5]e^{[0.3,0.5]2\pi i},[0.2,0.4]e^{[0.3,0.5]2\pi i}\right),\\ \left((\hbar_3,\hbar_2),[0.1,0.3]e^{[0.2,0.5]2\pi i},[0.3,0.6]e^{[0.3,0.5]2\pi i}\right),\\ \left((\hbar_3,\hbar_3),[0.2,0.7]e^{[0.3,0.5]2\pi i},[0,0.2]e^{[0.3,0.5]2\pi i}\right)\end{array}\right\}$$

Definition 16. Let $\wp$ be an IVCPFR on σ, then $\wp^c$ is said to be a converse of $\wp$ if

$$\wp^c=\{(\hbar_2,\hbar_1):(\hbar_1,\hbar_2)\in\wp\}$$

Example 7. If

$$\sigma=\left\{\begin{array}{c}\left(\hbar_1,[0.3,0.5]e^{[0.4,0.6]2\pi i},[0.2,0.4]e^{[0.1,0.3]2\pi i}\right),\\ \left(\hbar_2,[0.1,0.3]e^{[0.2,0.5]2\pi i},[0.3,0.6]e^{[0,0.2]2\pi i}\right),\\ \left(\hbar_3,[0.2,0.7]e^{[0.3,0.5]2\pi i},[0,0.2]e^{[0.3,0.5]2\pi i}\right)\end{array}\right\}\text{ and}$$

$$\theta=\left\{\begin{array}{c}\left(\mu_1,[0.2,0.4]e^{[0,0.3]2\pi i},[0.5,0.7]e^{[0.2,0.5]2\pi i}\right),\\ \left(\mu_2,[0,0.6]e^{[0.2,0.6]2\pi i},[0.3,0.5]e^{[0.1,0.5]2\pi i}\right),\\ \left(\mu_3,[0.1,0.7]e^{[0.2,0.4]2\pi i},[0,0.6]e^{[0.2,0.7]2\pi i}\right)\end{array}\right\}$$

Then the CP of $\sigma \times \theta$ is

$$\sigma \times \theta = \left\{ \begin{array}{c} \left((ŉ_1, \mu_1), [0.2, 0.4]e^{[0,0.3]2\pi i}, [0.5, 0.7]e^{[0.2,0.5]2\pi i}\right), \\ \left((ŉ_1, \mu_2), [0, 0.5]e^{[0.2,0.6]2\pi i}, [0.3, 0.5]e^{[0.1,0.5]2\pi i}\right), \\ \left((ŉ_1, \mu_3), [0.1, 0.5]e^{[0.2,0.4]2\pi i}, [0.2, 0.6]e^{[0.2,0.7]2\pi i}\right), \\ \left((ŉ_2, \mu_1), [0.1, 0.3]e^{[0,0.3]2\pi i}, [0.5, 0.7]e^{[0.2,0.5]2\pi i}\right), \\ \left((ŉ_2, \mu_2), [0, 0.3]e^{[0.2,0.5]2\pi i}, [0.3, 0.6]e^{[0.1,0.5]2\pi i}\right), \\ \left((ŉ_2, \mu_3), [0.1, 0.3]e^{[0.2,0.4]2\pi i}, [0.3, 0.6]e^{[0.2,0.7]2\pi i}\right), \\ \left((ŉ_3, \mu_1), [0.2, 0.4]e^{[0,0.3]2\pi i}, [0.5, 0.7]e^{[0.3,0.5]2\pi i}\right), \\ \left((ŉ_3, \mu_2), [0, 0.6]e^{[0.2,0.5]2\pi i}, [0.3, 0.5]e^{[0.3,0.5]2\pi i}\right), \\ \left((ŉ_3, \mu_3), [0.1, 0.7]e^{[0.2,0.4]2\pi i}, [0, 0.6]e^{[0.3,0.7]2\pi i}\right) \end{array} \right\}$$

be two IVCPFSs and a relation $\wp$ on these sets is given below

$$\wp = \left\{ \begin{array}{c} \left((ŉ_1, \mu_2), [0, 0.5]e^{[0.2,0.6]2\pi i}, [0.3, 0.5]e^{[0.1,0.5]2\pi i}\right), \\ \left((ŉ_2, \mu_1), [0.1, 0.3]e^{[0,0.3]2\pi i}, [0.5, 0.7]e^{[0.2,0.5]2\pi i}\right), \\ \left((ŉ_2, \mu_3), [0.1, 0.3]e^{[0.2,0.4]2\pi i}, [0.3, 0.6]e^{[0.2,0.7]2\pi i}\right), \\ \left((ŉ_3, \mu_2), [0, 0.6]e^{[0.2,0.5]2\pi i}, [0.3, 0.5]e^{[0.3,0.5]2\pi i}\right), \\ \left((ŉ_3, \mu_3), [0.1, 0.7]e^{[0.2,0.4]2\pi i}, [0, 0.6]e^{[0.3,0.7]2\pi i}\right) \end{array} \right\}$$

So, the converse $\wp^c$ of $\wp$ is

$$\wp^c = \left\{ \begin{array}{c} \left((\mu_2, ŉ_1), [0, 0.5]e^{[0.2,0.6]2\pi i}, [0.3, 0.5]e^{[0.1,0.5]2\pi i}\right), \\ \left((\mu_1, ŉ_2), [0.1, 0.3]e^{[0,0.3]2\pi i}, [0.5, 0.7]e^{[0.2,0.5]2\pi i}\right), \\ \left((\mu_3, ŉ_2), [0.1, 0.3]e^{[0.2,0.4]2\pi i}, [0.3, 0.6]e^{[0.2,0.7]2\pi i}\right), \\ \left((\mu_2, ŉ_3), [0, 0.6]e^{[0.2,0.5]2\pi i}, [0.3, 0.5]e^{[0.3,0.5]2\pi i}\right), \\ \left((\mu_3, ŉ_3), [0.1, 0.7]e^{[0.2,0.4]2\pi i}, [0, 0.6]e^{[0.3,0.7]2\pi i}\right) \end{array} \right\}$$

The IVCP-equivalence-FR set forwards the concept of IVCPF-equivalence classes that are defined below.

Definition 17. For an IVCP-equivalence-FR $\wp$, the IVCPF-equivalence class of $ŉ$ modulo $\wp$ is described

$$\wp[ŉ] = \{\mu | (\mu, ŉ) \in \wp\}$$

Example 8. If

$$\wp = \left\{ \begin{array}{c} \left((ŉ_1, ŉ_1), [0.3, 0.5]e^{[0.4,0.6]2\pi i}, [0.2, 0.4]e^{[0.1,0.3]2\pi i}\right), \\ \left((ŉ_1, ŉ_3), [0.2, 0.5]e^{[0.3,0.5]2\pi i}, [0.2, 0.4]e^{[0.3,0.5]2\pi i}\right), \\ \left((ŉ_2, ŉ_2), [0.1, 0.3]e^{[0.2,0.5]2\pi i}, [0.3, 0.6]e^{[0,0.2]2\pi i}\right), \\ \left((ŉ_3, ŉ_1), [0.2, 0.5]e^{[0.3,0.5]2\pi i}, [0.2, 0.4]e^{[0.3,0.5]2\pi i}\right), \\ \left((ŉ_3, ŉ_3), [0.2, 0.7]e^{[0.3,0.5]2\pi i}, [0, 0.2]e^{[0.3,0.5]2\pi i}\right) \end{array} \right\}$$

is an IVCPFR on an IVCFS

$$\sigma = \left\{ \begin{array}{c} \left(ŉ_1, [0.3, 0.5]e^{[0.4,0.6]2\pi i}, [0.2, 0.4]e^{[0.1,0.3]2\pi i}\right), \\ \left(ŉ_2, [0.1, 0.3]e^{[0.2,0.5]2\pi i}, [0.3, 0.6]e^{[0,0.2]2\pi i}\right), \\ \left(ŉ_3, [0.2, 0.7]e^{[0.3,0.5]2\pi i}, [0, 0.2]e^{[0.3,0.5]2\pi i}\right) \end{array} \right\}$$

then an IVCPF-equivalence is a class of

a. $ŉ_1$ modulo $\wp$ given as

$$\wp[ŉ_1] = \left\{ \begin{array}{c} \left(ŉ_1, [0.3, 0.5]e^{[0.4,0.6]2\pi i}, [0.2, 0.4]e^{[0.1,0.3]2\pi i}\right), \\ \left(ŉ_3, [0.2, 0.7]e^{[0.3,0.5]2\pi i}, [0, 0.2]e^{[0.3,0.5]2\pi i}\right) \end{array} \right\}$$

b. $ŉ_2$ modulo $\wp$ given as

$$\wp[ŉ_2] = \left\{ \left(ŉ_2, [0.1, 0.3]e^{[0.2,0.5]2\pi i}, [0.3, 0.6]e^{[0,0.2]2\pi i}\right) \right\}$$

c. $ŉ_3$ modulo $\wp$ given as

$$\wp[ŉ_3] = \left\{ \begin{array}{c} \left(ŉ_1, [0.3, 0.5]e^{[0.4,0.6]2\pi i}, [0.2, 0.4]e^{[0.1,0.3]2\pi i}\right), \\ \left(ŉ_3, [0.2, 0.7]e^{[0.3,0.5]2\pi i}, [0, 0.2]e^{[0.3,0.5]2\pi i}\right) \end{array} \right\}$$

Definition 18. For an IVCPFR $\wp$ on an IVCPFS σ, then the IVCP-composite-FR $\wp \circ \wp$ is defined as:

For each $(ŉ_1, ŉ_2) \in \wp$ and $(ŉ_2, ŉ_3) \in \wp \Rightarrow (ŉ_1, ŉ_3) \in \wp \circ \wp$, $\forall ŉ_1, ŉ_2, ŉ_3 \in \tau$.

Example 9. For some IVCPFRs

$$\wp_1 = \left\{ \begin{array}{c} \left((ŉ_1, ŉ_2), [0.1, 0.3]e^{[0.2,0.5]2\pi i}, [0.3, 0.6]e^{[0.1,0.3]2\pi i}\right), \\ \left((ŉ_2, ŉ_1), [0.1, 0.3]e^{[0.2,0.5]2\pi i}, [0.3, 0.6]e^{[0.1,0.3]2\pi i}\right), \\ \left((ŉ_3, ŉ_2), [0.1, 0.3]e^{[0.2,0.5]2\pi i}, [0.3, 0.6]e^{[0.3,0.5]2\pi i}\right) \end{array} \right\}$$

$$\wp_2 = \left\{ \begin{array}{c} \left((ŉ_1, ŉ_3), [0.2, 0.5]e^{[0.3,0.5]2\pi i}, [0.2, 0.4]e^{[0.3,0.5]2\pi i}\right), \\ \left((ŉ_2, ŉ_1), [0.1, 0.3]e^{[0.2,0.5]2\pi i}, [0.3, 0.6]e^{[0.1,0.3]2\pi i}\right), \\ \left((ŉ_2, ŉ_3), [0.1, 0.3]e^{[0.2,0.5]2\pi i}, [0.3, 0.6]e^{[0.3,0.5]2\pi i}\right), \\ \left((ŉ_3, ŉ_3), [0.2, 0.7]e^{[0.3,0.5]2\pi i}, [0, 0.2]e^{[0.3,0.5]2\pi i}\right) \end{array} \right\}$$

on an IVCP-composite-FR $\wp_1 \circ \wp_2$ is given

$$\wp_1 \circ \wp_2 = \left\{ \begin{array}{c} \left((ŉ_1, ŉ_1), [0.3, 0.5]e^{[0.4,0.6]2\pi i}, [0.2, 0.4]e^{[0.1,0.3]2\pi}\right), \\ \left((ŉ_2, ŉ_3), [0.1, 0.3]e^{[0.2,0.5]2\pi i}, [0.3, 0.6]e^{[0.3,0.5]2\pi i}\right), \\ \left((ŉ_3, ŉ_1), [0.2, 0.5]e^{[0.3,0.5]2\pi i}, [0.2, 0.4]e^{[0.3,0.5]2\pi i}\right), \end{array} \right\}$$

Theorem 1. An IVCPFR $\wp$ is an IVCP-symmetric-FR on an IVCPFS σ if and only if $\wp = \wp^c$.

Proof. Consider that $\wp$ is an IVCP-symmetric-FR on an IVCPFS σ, then

$$(ŉ_1, ŉ_2) \in \wp \Rightarrow (ŉ_2, ŉ_1) \in \wp$$

So, $(ŉ_2, ŉ_1) \in \wp^c \Rightarrow \wp = \wp^c$

On the contrary, assume that $\wp = \wp^c$, then

$$(ŉ_1, ŉ_2) \in \wp \Rightarrow (ŉ_2, ŉ_1) \in \wp^c \Rightarrow (ŉ_2, ŉ_1) \in \wp.$$

Theorem 2. Consider an IVCPFR $\wp$ is an IVCP-transitive-FR on an IVCPFS σ, if and only if $\wp \circ \wp \subseteq \wp$.

Proof. Consider that $\wp$ is an IVCP-transitive-FR on an IVCPFS σ.

Let $(ŉ_2, ŉ_3) \in \wp \circ \wp$,

According to IVCP-transitive-FR,

$$(ŉ_1, ŉ_2) \in \wp \text{ and } (ŉ_2, ŉ_3) \in \wp \Rightarrow (ŉ_1, ŉ_3) \in \wp \Rightarrow \wp \circ \wp \subseteq \wp$$

On the contrary, suppose that $\wp \circ \wp \subseteq \wp$, then

For $(ŉ_1, ŉ_2) \in \wp$ and $(ŉ_2, ŉ_3) \in \wp \Rightarrow (ŉ_1, ŉ_3) \in \wp \circ \wp \subseteq \wp \Rightarrow (ŉ_1, ŉ_3) \in \wp$.

Thus, $\wp$ is an IVCP-transitive-FR on σ.

Theorem 3. Consider $\wp$ is an IVCP-equivalence-FR on an IVCPFS σ, then $\wp \circ \wp = \wp$.

Proof. Suppose this $(\grave{n}_1, \grave{n}_2) \in \wp$.

According to IVCP-symmetric-FR,

$$(\grave{n}_2, \grave{n}_1) \in \wp.$$

Now, as stated by IVCP-transitive-FR,

$$(\grave{n}_1, \grave{n}_1) \in \wp.$$

Also, as mentioned by IVCP-composite-FR,

$$(\grave{n}_1, \grave{n}_1) \in \wp \circ \wp.$$

Thus,

$$\wp \subseteq \wp \circ \wp \tag{1}$$

Again, assume that $(\grave{n}_1, \grave{n}_2) \in \wp \circ \wp$, then $\exists \grave{n}_3 \in \wp \ni (\grave{n}_1, \grave{n}_3) \in \wp$ and $(\grave{n}_3, \grave{n}_2) \in \wp$

Provided that $\wp$ is an IVCP-equivalence-FR on σ, so $\wp$ as well an IVCP-transitive-FR. Consequently,

$$(\grave{n}_1, \grave{n}_2) \in \wp \Rightarrow \wp \circ \wp \subseteq \wp \tag{2}$$

Hence, by (1) and (2),

$$\wp \circ \wp = \wp$$

Hence proved.

Theorem 4. Consider $\wp$ is an IVCP-partial order-FR on an IVCPFS σ, then the converse relation $\wp^c$ of $\wp$ is also an IVCP-partial order-FR on σ.

Proof. To prove the claim, this is enough to expose that the inverse of a complex Pythagorean-partial order-FR $\wp^c$ pleases the three characteristics of a complex Pythagorean-partial order-FR.

With the characteristics of IVCP-partial order-FR $\wp$,

1. It is provided that $\wp$ is an IVCP-reflexive-FR. However, for any $\grave{n}_1 \in \tau$, $(\grave{n}_1, \grave{n}_1) \in \wp \Rightarrow (\grave{n}_1, \grave{n}_1) \in \wp^c$. Thus, $\wp^c$ is an IVCP-reflexive-FR.
2. Suppose that $(\grave{n}_1, \grave{n}_1) \in \wp^c$ and $(\grave{n}_2, \grave{n}_1) \in \wp^c$, then $(\grave{n}_1, \grave{n}_2) \in \wp$ and $(\grave{n}_2, \grave{n}_1) \in \wp$. Consequently, $\wp$ is an IVCP-antisymmetric-FR. Therefore, $(\grave{n}_1, \grave{n}_2) = (\grave{n}_2, \grave{n}_1)$. So, $\wp$ is also an IVCP-antisymmetric-FR.
3. Assume that $(\grave{n}_1, \grave{n}_2) \in \wp^c$ and $(\grave{n}_2, \grave{n}_3) \in \wp^c$, then, $(\grave{n}_3, \grave{n}_2) \in \wp$ and $(\grave{n}_2, \grave{n}_1) \in \wp$. Now, it is stated that $\wp$ is an IVCP-transitive-FR. So, $(\grave{n}_3, \grave{n}_1) \in \wp \Rightarrow (\grave{n}_1, \grave{n}_3) \in \wp^c$. Furthermore, $\wp^c$ is an IVCP-transitive-FR.

Through 1, 2, and 3, the converse relation $\wp^c$ of an IVCP-partial order-FR $\wp$ is also an IVCP-partial order-FR.

Theorem 5. Consider $\wp$ is an IVCP-equivalence-FR on an IVCPFS σ, then $(\grave{n}_1, \grave{n}_2) \in \wp$, if and only if

$$\wp[\grave{n}_1] = \wp[\grave{n}_2].$$

Proof. Let $(\grave{n}_1, \grave{n}_2) \in \wp$ and $\grave{n}_3 \in \wp[\grave{n}_1]$ $\wp(\grave{n}_3, \grave{n}_1) \in \wp$.

According to the condition, an IVCP-equivalence-FR is an IVCP-transitive-FR, so

$$(\grave{n}_3, \grave{n}_2) \in \wp \Rightarrow \grave{n}_3 \in \wp[\grave{n}_2].$$

Thus,

$$\wp[\grave{n}_1] \subseteq \wp[\grave{n}_2] \tag{3}$$

As $(\grave{n}_1, \grave{n}_2) \in \wp$, an IVCP-equivalence-FR is also an IVCP-symmetric-FR, then $(\grave{n}_2, \grave{n}_1) \in \wp$.

Additionally, suppose this $\grave{n}_3 \in \wp[\grave{n}_2] \Rightarrow (\grave{n}_3, \grave{n}_2) \in \wp$.

Again, an IVCP-equivalence-FR is an IVCP-transitive-FR too,

Then

$$(\grave{n}_3, \grave{n}_1) \in \wp \Rightarrow \grave{n}_3 \in \wp[\grave{n}_1].$$

Thus,

$$\wp[\grave{n}_2] \subseteq \wp[\grave{n}_1] \tag{4}$$

Therefore, (3) and (4) imply that

$$\wp[\grave{n}_2] = \wp[\grave{n}_1]$$

Conversely, suppose that $\wp[\grave{n}_2] = \wp[\grave{n}_1]$,

$\grave{n}_3 \in \wp[\grave{n}_1]$ and $\grave{n}_3 \in \wp[\grave{n}_2] \Rightarrow (\grave{n}_3, \grave{n}_2) \in \wp$ and $(\grave{n}_3, \grave{n}_1) \in \wp$.

Now, provided that an IVCP-equivalence-FR is an IVCP-symmetric-FR, so

$$(\grave{n}_3, \grave{n}_1) \in \wp \Rightarrow (\grave{n}_1, \grave{n}_3) \in \wp$$

According to an IVCP-transitive-FR,

$$(\grave{n}_1, \grave{n}_3) \in \wp \text{ and } (\grave{n}_3, \grave{n}_2) \in \wp \Rightarrow (\grave{n}_1, \grave{n}_2) \in \wp,$$

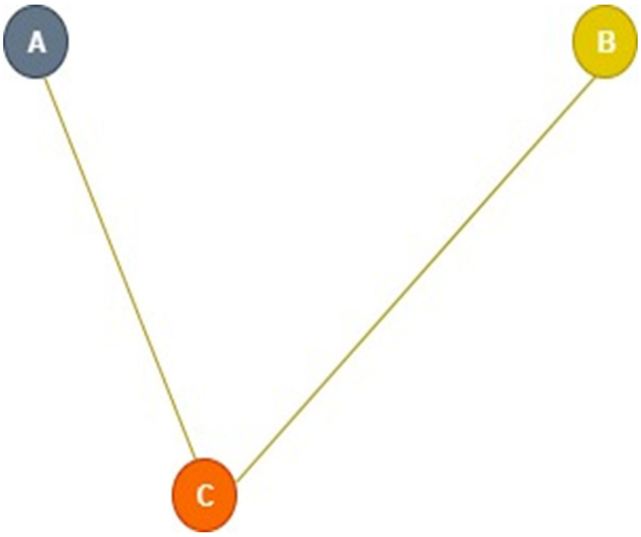
Hence proved.

4. Hasse Diagram

This part defines the Hasse diagram for IVCPFR. Additionally, a number of important ideas of Hasse diagram are described.

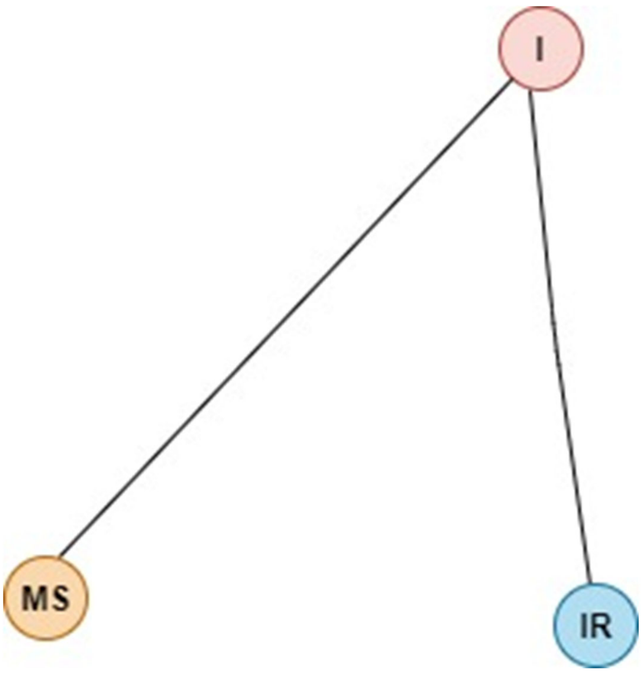
Direct relationships are given below in Figures 1, 2, 3, and 4:

Figure 1
Hasse diagram (direct relation)



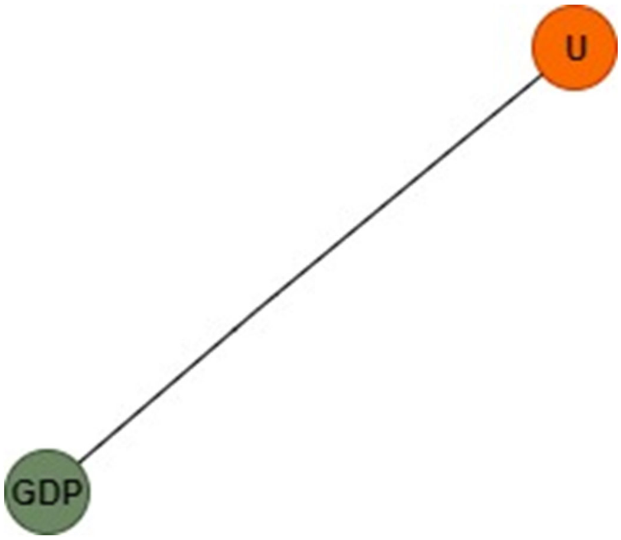
A	Price	$([0.2, 0.4]e^{[0,0.2]2\pi i}, [0, 0.3]e^{[0.1,0.5]2\pi i})$
B	Supply	$([0.1, 0.5]e^{[0.3,0.6]2\pi i}, [0.1, 0.4]e^{[0.2,0.5]2\pi i})$
C	Demand	$([0, 0.2]e^{[0.1,0.6]2\pi}, [0.2, 0.7]e^{[0,0.4]2\pi i})$

Figure 2
Hasse diagram (direct relation)



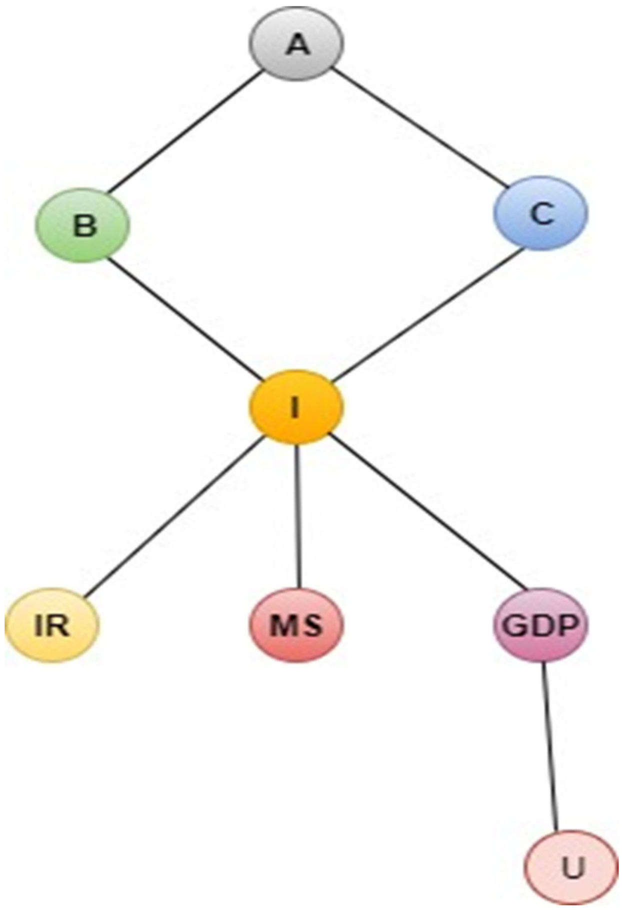
I	Investment	$([0.1, 0.8]e^{[0.01,0.05]2\pi i}, [0, 0.3]e^{[0.04,0.07]2\pi i})$,
IR	Interest Rate	$([0.1, 0.9]e^{[0.05,0.35]2\pi i}, [0.3, 0.7]e^{[0.13,0.17]2\pi i})$
MS	Money Supply	$([0, 0.2]e^{[0.06,0.75]2\pi i}, [0.2, 0.6]e^{[0.25,0.35]2\pi i})$

Figure 3
Hasse diagram (direct relation)



U	Unemployment	$([0.1, 0.3]e^{[0.1,0.5]2\pi i}, [0.45, 0.51]e^{[0.2,0.4]2\pi i})$
GDP	Gross Domestic Product	$([0.1, 0.8]e^{[0.15,0.25]2\pi i}, [0, 0.1]e^{[0.03,0.75]2\pi i})$

Figure 4
Hasse diagram (direct and indirect relations)



Indirect and direct relationships are given below:

5. Application

This section presents the application of IVCPFS and IVCPFR. We implemented IVCPFS and IVCPFR on economic relations of demand, supply, and price with interest rate, investment, and money supply.

5.1. Economic relationships

Success of a republic mainly relies on its economic power. Social sciences deal with the study of relationships of wealth with humans, termed as economics. The manner of production, distribution, and consumption of goods and services is defined in economics. It is the major factor that proceeds financial development in the economy. Financial growth performs a very important role in the development of quality of life as it produces high-salary jobs. The economic developers expand the business of organizations by connecting them with different organizations and partners. The growth of business in highly diverse economies produces excessive tax revenues. Therefore, the quality of life improved as plenty of job opportunities available.

Some basic economic relationships are given in Figure 5.

So, the interest rate is reversed to the quantity of money available. Printing the foreign money notes has awful effect at the financial system. This phenomenon is deliberated in Figure 6.

Figure 6
Relation between interest rate, investment, money supply

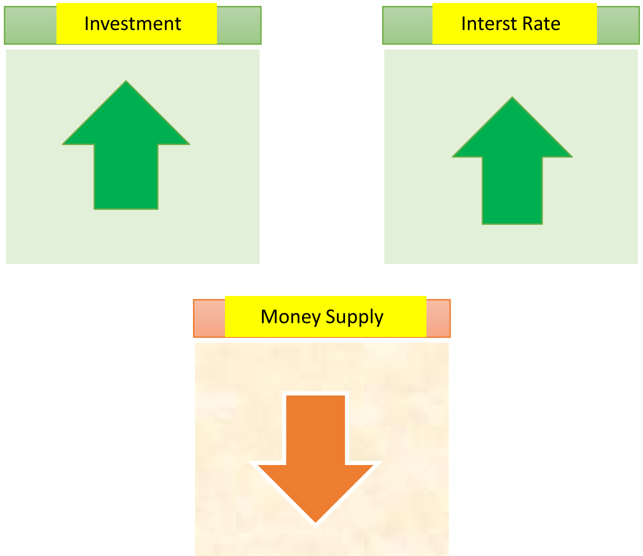
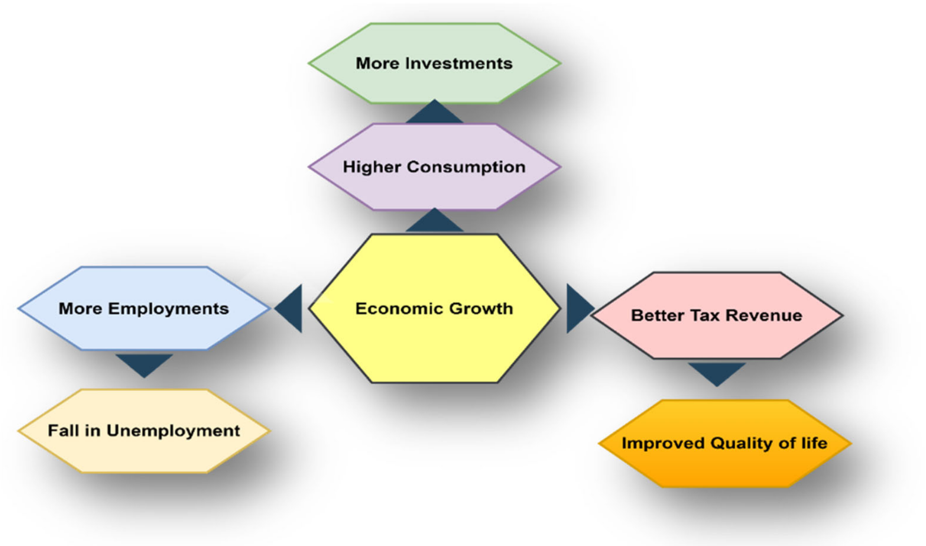


Figure 5
Effect of economic growth



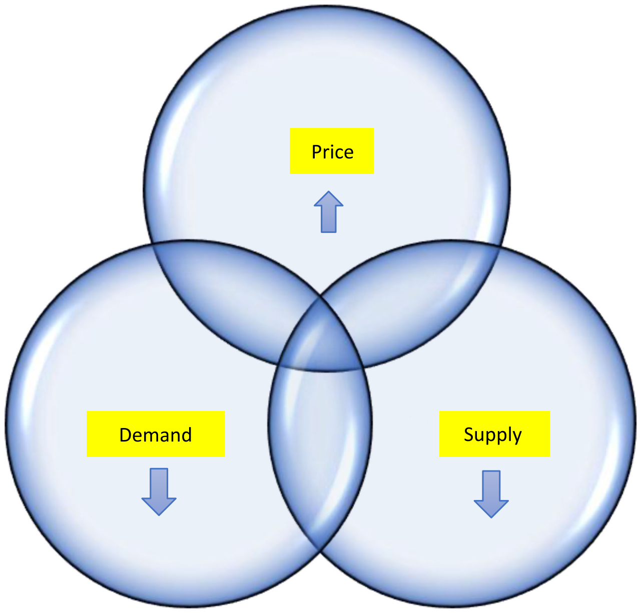
5.2. Investment, money supply, and interest rates

The increase of business and the financial system of a country depends upon the investments and investments rely on the interest rates. Since the traders usually observe the better returns, in order that they prefer the industry with better interest rates. The better interest rates fascinate the traders to serve their money in the business. The prices of the CPs are commonly determined by the amount of the goods in the market. In a financial structure, when the amount of money grows up the price of money reduces. Consequently, the greater money supply decreases interest rates.

5.3. Price, demand and supply

Prices, demand, and supply are the products and ministrations which are associated in direct relation. Whenever an organization will increase the expenses of its products and services, the income of products and services drops down due to the fact that the buyers select inexpensive products. As the expenses increase, less individuals will be able to bear the cost of the products. So, the demand of the goods drops down. Besides, when the goods sale at better rates rather than an increase in supply, extra earnings may be generated. Whenever the supply of the products increases, the prices fall down and the demand increases. When the demand exceeds the supply, the expenses generally increase. The pictorial view is given in Figure 7.

Figure 7
Relation of price, demand, and supply



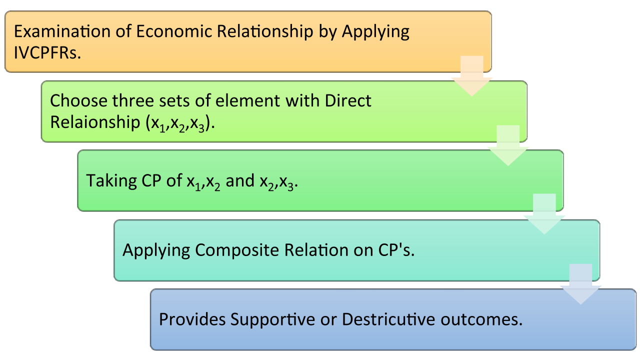
5.4. Economic growth and unemployment

Financing is a very important part in deciding the economy, and GDP is an excellent unit. The quantity of the money determines the interest rate which determines the investments made.

The value of money spent at the expenditure, trades, and government services termed as GDP. For example, if the expenditure of goods is not always sufficient, then it should stop manufacturing of products. On the opposite way, if the spending is extra, then it is required for extra manufacturing. So, extra manufacturing needs more people and create activity possibilities that reduced unemployment. There is a link among GDP and unemployment. The measurement of financial increase is the conversion of GDP from one year to other.

Following Figure 8 contains a step-by-step methodology of the proposed application. This application discusses the effect on economic relationship by using IVCPFR.

Figure 8
Steps of proposed model



Now the IVCPFRs were utilized to examine the financial and economic relationships. Reshaping this problem by using the theory of IVCPFSs and IVCPFRs will now no longer simply help to set up the effect of one element at the other, additionally modifying the grades of supportive results and destructive results with respect to the time.

Let A, B, and C symbolize the price, supply, and demand, respectively. And I, IR, MS, and U express the investment, interest rate, money supply, and unemployment, respectively. Then the set of factors are given below:

$$\mathcal{F}=\left\{\begin{array}{c}\left(A,[0.2,0.4]e^{[0,0.2]2\pi i},[0,0.3]e^{[0.1,0.5]2\pi i}\right),\\ \left(B,[0.1,0.5]e^{[0.3,0.6]2\pi i},[0.1,0.4]e^{[0.2,0.5]2\pi i}\right),\\ \left(C,[0,0.2]e^{[0.1,0.6]2\pi i},[0.2,0.7]e^{[0,0.4]2\pi i}\right),\\ \left(I,[0.1,0.8]e^{[0.01,0.05]2\pi i},[0,0.3]e^{[0.04,0.07]2\pi i}\right),\\ \left(IR,[0.1,0.9]e^{[0.05,0.35]2\pi i},[0.3,0.7]e^{[0.13,0.17]2\pi i}\right),\\ \left(MS,[0,0.2]e^{[0.06,0.75]2\pi i},[0.2,0.6]e^{[0.25,0.35]2\pi i}\right),\\ \left(GDP,[0.1,0.8]e^{[0.15,0.25]2\pi i},[0,0.1]e^{[0.03,0.75]2\pi i}\right),\\ \left(U,[0.1,0.3]e^{[0.1,0.5]2\pi i},[0.45,0.51]e^{[0.2,0.4]2\pi i}\right)\end{array}\right\}$$

Even so, these elements require to be grouped on the basis of direct relationships. Consequently, the subsequent three sets χ_1, χ_2, χ_3 constructed, which are directly associated with each other. Given below

$$\chi_1=\left\{\begin{array}{c}\left(A,[0.2,0.4]e^{[0,0.2]2\pi i},[0,0.3]e^{[0.1,0.5]2\pi i}\right),\\ \left(B,[0.1,0.5]e^{[0.3,0.6]2\pi i},[0.1,0.4]e^{[0.2,0.5]2\pi i}\right),\\ \left(C,[0,0.2]e^{[0.1,0.6]2\pi i},[0.2,0.7]e^{[0,0.4]2\pi i}\right)\end{array}\right\}$$

$$\chi_2=\left\{\begin{array}{c}\left(I,[0.1,0.8]e^{[0.01,0.05]2\pi i},[0,0.3]e^{[0.04,0.07]2\pi i}\right),\\ \left(IR,[0.1,0.9]e^{[0.05,0.35]2\pi i},[0.3,0.7]e^{[0.13,0.17]2\pi i}\right),\\ \left(MS,[0,0.2]e^{[0.06,0.75]2\pi i},[0.2,0.6]e^{[0.25,0.35]2\pi i}\right)\end{array}\right\}$$

$$\chi_3=\left\{\begin{array}{c}\left(GDP,[0.1,0.8]e^{[0.15,0.25]2\pi i},[0,0.1]e^{[0.03,0.75]2\pi i}\right),\\ \left(U,[0.1,0.3]e^{[0.1,0.5]2\pi i},[0.45,0.51]e^{[0.2,0.4]2\pi i}\right)\end{array}\right\}$$

The CP of $X_1\times X_2$ is

$$\chi_1\times\chi_2=\left\{\begin{array}{c}\left((A,I),[0.1,0.4]e^{[0,0.05]2\pi i},[0,0.3]e^{[0.1,0.5]2\pi i}\right),\\ \left((A,),[0.1,0.4]e^{[0,0.2]2\pi i},[0.3,0.7]e^{[0.13,0.17]2\pi i}\right),\\ \left((A,MS),[0,0.2]e^{[0,0.2]2\pi i},[0.2,0.6]e^{[0.25,0.5]2\pi i}\right),\\ \left((B,I),[0.1,0.5]e^{[0.01,0.05]2\pi i},[0.1,0.4]e^{[0.2,0.5]2\pi i}\right),\\ \left((B,IR),[0.1,0.5]e^{[0.05,0.35]2\pi i},[0.3,0.7]e^{[0.13,0.17]2\pi i}\right),\\ \left((B,MS),[0,0.2]e^{[0.06,0.6]2\pi i},[0.2,0.6]e^{[0.25,0.5]2\pi i}\right),\\ \left((C,I),[0,0.2]e^{[0.01,0.05]2\pi i},[0.2,0.7]e^{[0.04,0.4]2\pi i}\right),\\ \left((C,IR),[0,0.2]e^{[0.05,0.35]2\pi i},[0.3,0.7]e^{[0.13,0.17]2\pi i}\right),\\ \left((C,MS),[0,0.2]e^{[0.06,0.6]2\pi i},[0.2,0.7]e^{[0.25,0.4]2\pi i}\right)\end{array}\right\}$$

The relationship of $\chi_1\times\chi_2$ is

$$\wp_{\chi_1\times\chi_2}=\left\{\begin{array}{c}\left((A,I),[0.1,0.4]e^{[0,0.05]2\pi i},[0,0.3]e^{[0.1,0.5]2\pi i}\right),\\ \left((A,MS),[0,0.2]e^{[0,0.2]2\pi i},[0.2,0.6]e^{[0.25,0.5]2\pi i}\right),\\ \left((B,I),[0.1,0.5]e^{[0.01,0.05]2\pi i},[0.1,0.4]e^{[0.2,0.5]2\pi i}\right),\\ \left((B,IR),[0.1,0.5]e^{[0.05,0.35]2\pi i},[0.3,0.7]e^{[0.13,0.17]2\pi i}\right),\\ \left((C,I),[0,0.2]e^{[0.01,0.05]2\pi i},[0.2,0.7]e^{[0.04,0.4]2\pi i}\right)\end{array}\right\}$$

The IVCPFR $\wp_{\chi_1}\times\chi_2$ includes the most selective activities that need to be examined, so the unused elements of the CP are eliminated. In $\wp_{\chi_1}\times\chi_2$, the event $\left((A,I),[0.1,0.4]e^{[0,0.05]2\pi i},[0,0.3]e^{[0.1,0.5]2\pi i}\right)$

allocates the data about the influence of the cost of materials and ministrations on the finance. The membership grade from 0.1 to 0.4 suggests that how lots of the prices help the finance and that in short time period, since the phase term in the exponent show the time period from 0 to 0.05 year and the non-membership grade from 0 to 0.3 decide that the charges specify the minimal dispirit on investment with the time period from 0.1 to 0.5.

The CP of $\chi_2 \times \chi_3$ is

$$\chi_2 \times \chi_3 = \left\{ \begin{array}{c} \left((I, GDP), [0.1, 0.8]e^{[0.01,0.05]2\pi i}, [0, 0.3]e^{[0.04,0.75]2}\right), \\ \left((I, U), [0.1, 0.3]e^{[0.01,0.05]2\pi i}, [0.45, 0.51]e^{[0.2,0.4]2\pi i}\right), \\ \left((IR, GDP), [0.1, 0.8]e^{[0.05,0.25]2\pi i}, [0.3, 0.7]e^{[0.13,0.75]2\pi i}\right), \\ \left((IR, U), [0.1, 0.3]e^{[0.05,0.35]2\pi i}, [0.45, 0.51]e^{[0.13,0.17]2\pi i}\right), \\ \left((MS, GDP), [0, 0.2]e^{[0.06,0.25]2\pi i}, [0.2, 0.6]e^{[0.25,0.75]2\pi i}\right), \\ \left((MS, U), [0, 0.2]e^{[0.06,0.5]2\pi i}, [0.45, 0.51]e^{[0.25,0.4]2\pi i}\right) \end{array} \right\}$$

In the same manner, some other IVCPFR $\wp_{\chi_2} \times \chi_3$ describe the relationship of χ_2 and χ_3. $\wp_{\chi_2} \times \chi_3$ is a subset of the CP $\chi_2 \times \chi_3$, that is

$$\wp_{\chi_2 \times \chi_3} = \left\{ \begin{array}{c} \left((I, GDP), [0.1, 0.8]e^{[0.01,0.05]2\pi i}, [0, 0.3]e^{[0.04,0.75]2\pi i}\right), \\ \left((I, U), [0.1, 0.3]e^{[0.01,0.05]2\pi i}, [0.45, 0.51]e^{[0.2,0.4]2\pi i}\right), \\ \left((IR, GDP), [0.1, 0.8]e^{[0.05,0.25]2\pi i}, [0.3, 0.7]e^{[0.13,0.75]2\pi i}\right), \\ \left((MS, GDP), [0, 0.2]e^{[0.06,0.25]2\pi i}, [0.2, 0.6]e^{[0.25,0.75]2\pi i}\right) \end{array} \right\}$$

In the earlier relation, each event narrates the beneficial effects and detrimental effects of one variable on the other variable. For example, $\left((I, GDP), [0.1, 0.8]e^{[0.01,0.05]2\pi i}, [0, 0.3]e^{[0.04,0.75]2\pi i}\right)$ explain the impact of expenditure on the GDP. Then membership grade from 0.1 to 0.8 and the non-membership grades from 0 to 0.3 represent the better investment helps GDP in the short duration of time. In the case of detrimental effects, GDP declines very sluggishly because of finance.

The IVCP-composite-FR is used to locate the relationships among the elements of set χ_1 and χ_3. The effect of prices on the investment and effect of finance on GDP have been described. In chain relationship, the composite relation enables in referring prices to GDP. The composite relation is given

$$\begin{aligned} \wp &= \left(\wp_{\chi_1 \times \chi_2}\right) \circ \left(\wp_{\chi_2 \times \chi_3}\right) \\ &= \left\{ \begin{array}{c} \left((A, GDP), [0.1, 0.4]e^{[0,0.2]2\pi i}, [0, 0.3]e^{[0.1,0.75]2\pi i}\right), \\ \left((A, U), [0.1, 0.3]e^{[0,0.2]2\pi i}, [0.45, 0.51]e^{[0.2,0.4]2\pi i}\right), \\ \left((B, GDP), [0.1, 0.5]e^{[0.15,0.25]2\pi i}, [0.1, 0.4]e^{[0.2,0.75]2\pi i}\right), \\ \left((C, GDP), [0, 0.2]e^{[0.15,0.25]2\pi i}, [0.2, 0.7]e^{[0.03,0.75]2\pi i}\right) \end{array} \right\} \end{aligned}$$

The event $\left((A, GDP), [0.1, 0.4]e^{[0,0.2]2\pi i}, [0, 0.3]e^{[0.1,0.75]2\pi i}\right)$ gives the details approximately the effects of prices on GDP. It suggests that prices play a vital role in the growth of economy.

6. Comparative Analysis

In this section, the correlation between presented method and existing method was described. IVCPFSs and IVCPFRs are tremendous of all above ideas and methods to manipulate the fuzziness. In these sets, we talk about the membership grade and non-membership grade for fixing problems. The major benefit of IVCPFR over FR, CPFR, and IVPFR is an interval-valued complex membership and non-membership grades. Construction of IVCPFR is made out of the intervals of amplitude term and phase term, which permit it to design the conditions with phase alteration and periodicity. Alternatively, FR and IVPFRs lack the intervals of multidimensional phase terms; accordingly, they are limited. Moreover, the structure of FR and related basic models are assumed from complex numbers and the simple intervals, accordingly comprises of amplitude and phase terms intervals.

In this application, we discuss about the good influences and bad influences of first component to the second, represented by membership and non-membership levels in sequence. Since CIFRs and CPFRs are supplied with complex-valued membership and non-membership levels, it is far enough to check them on the introduced methodology.

1. **FR vs IVCPFR**
   In the comparison of FR with IVCPFR, FR depicts only the membership level of any value even so the IVCPFR is a stepped forward form of FR that characterizes the intervals of membership and non-membership level with complex values.

2. **IFR vs IVCPFR**
   IFR describes the membership and non-membership levels while the addition of membership and non-membership levels must be in [0,1] if it exceeds then the operation fails to clear up any problem.

   But on the other hand, in IVCPFR there we are able to take squares of membership and non-membership levels and add them solution may be in unit interval [0,1]. Therefore, it is better and more preferable than IFR.

3. **CIFR vs IVCPFR**
   CIFR determines the membership and non-membership levels of complex values. Whether IVCPFR is an advanced shape as opposed to CIFR, IVCPFR depicts the intervals of complex values and produces more accurate results than CIFR. IVCPFR is used to determine multidimensional problems and intervals of complexities rather than CIFRs.

4. **IVCIFR vs IVCPFR**
   IVCIFR expresses the intervals of membership and non-membership grades of complex values. Although IVCPFR is an improved form of IVCIFR, IVCPFR eliminates the restrictions of IVCIFR. In IVCIFR, only limited problems can be solved because it has some limitations otherwise it fails.

5. **PFR vs IVCPFR**
   PFR expresses the membership and non-membership grades for solving problem. But if the intervals of complex values are to be in question, then PFR fails to solve it as compared to IVCPFR. IVCPFR is an advanced technique instead of PFR. IVCPFR is time-consuming and produces better and exact outcomes quickly and rapidly.

6. **CPFR vs IVCPFR**
   IVCPFR is a high-level methodology in preference to CPFR. IVCPFR provides intervals of membership and non-membership grades with amplitude and phase terms, individually. Additionally, it is better to apply than CPFR.

7. IVPFR vs IVCPFR

IVCPFR is an advanced approach rather than IVPFR. Because IVPFR applies to the intervals of simple problems, IVCPFR is used to solve the intervals of different complexities. Table 1 shows the comparison summary of the proposed structures with other predefined structures.

Table 1
Comparison of IVPFR with other predefined structures

Sets	Membership	Non-membership	Multi-dimensional	Remarks
FS (Zadeh, 1965)	Yes	No	No	Only discuss membership with single value
IFS (Atanassov, 1999)	Yes	Yes	No	Contain membership and non-membership levels
CIFS (Alkouri & Salleh, 2012)	Yes	Yes	Yes	Discuss multi-variable values
IVCIFS (Garg & Rani, 2019)	Yes	Yes	Yes	Membership and non-membership levels. Multi-variable values with intervals
PFS (Yager, 2013)	Yes	Yes	No	Membership and non-membership values Sum of the squares of membership and non-membership also in [0,1]
CPFS (Dick et al., 2015)	Yes	Yes	Yes	Membership and non-membership values. Discuss multi-variable values
IVPFS (Peng, 2019)	Yes	Yes	No	Intervals of membership and non-membership levels
IVCPFS (Ali et al., 2021)	Yes	Yes	Yes	Intervals of membership and non-membership values. Multi-variable values with intervals

7. Conclusion

In this research, the innovative theory of IVCPFR and its kinds are defined with related examples. These include IVCP-equivalence-FR, IVCP-partial order-FR, IVCP-total order-FR, IVCP-composite-FR, and so on. Additionally, a couple of valuable and fascinating characteristics and outcomes of IVCPFRs are explained deeply. Besides, a methodology of introduced technique is explained to examine the impacts of financial indicators of a nation. This examination produces the impact of uncertainty on economic growth and investments analyzed with the GDP. This article added the revolutionary ideas of IVCPFR and the CP among two IVCPFSs. The thoughts and considerations associated with the Hasse diagram have additionally been described. Forthcomings, ideas may be enlarged for establishment of FSs that will bring numerous interesting items with a large verity of methodologies.

Conflicts of interest

The authors declare that they have no conflicts of interest to this work.

References

Akram, M., & Naz, S. (2019). A novel decision-making approach under complex pythagorean fuzzy environment. *Mathematical and Computational Applications*, *24*(3), 73.

Ali, Z., Mahmood, T., Ullah, K., & Khan, Q. (2021). Einstein geometric aggregation operators using a novel complex interval-valued pythagorean fuzzy setting with application in green supplier chain management. *Reports in Mechanical Engineering*, *2*, 105–134.

Alkouri, A. M. D. J. S., & Salleh, A. R. (2012). Complex intuitionistic fuzzy sets. In *AIP Conference Proceedings*, *1482*(1) 464–470.

Atanassov, K. T. (1999). *Interval valued intuitionistic fuzzy sets*. Germany: Springer. https://doi.org/10.1007/978-3-7908-1870-3

Atanassov, K. T. (1999). *Intuitionistic fuzzy sets*. Germany: Springer. https://doi.org/10.1007/978-3-7908-1870-3

Bustince, H., & Burillo, P (2000). Mathematical analysis of interval-valued fuzzy relations: application to approximate reasoning. *Fuzzy Sets and Systems*, *113*(2), 205–219.

Chung, J. H., & Yang, K. (2012). New classes of optimal low-hit-zone frequency-hopping sequence sets by cartesian product. *IEEE Transactions on Information Theory*, *59*(1), 726–732.

Cornelis, C., Deschrijver, G., & Kerre, E. E. (2004). Implication in intuitionistic fuzzy and interval-valued fuzzy set theory: construction, classification, application. *International Journal of Approximate Reasoning*, *35*(1), 55–95.

De, S. K., Biswas, R., & Roy, A. R. (2001). An application of intuitionistic fuzzy sets in medical diagnosis. *Fuzzy Sets and Systems*, *117*(2), 209–213.

Dick, S., Yager, R. R., & Yazdanbakhsh, O. (2015). On pythagorean and complex fuzzy set operations. *IEEE Transactions on Fuzzy Systems*, *24*(5), 1009–1021.

Dinakaran, U. (2021). Analyzing online food delivery industries using pythagorean fuzzy relation and composition. *International Journal of Hospitality and Tourism Systems*, *14*(2), 36.

Garg, H. (2017). A new improved score function of an interval-valued pythagorean fuzzy set based TOPSIS method. *International Journal for Uncertainty Quantification*, *7*(5), 463–474.

Garg, H. (2018). Linguistic pythagorean fuzzy sets and its applications in multiattribute decision-making process. *International Journal of Intelligent Systems*, *33*(6), 1234–1263.

Garg, H., & Rani, D. (2019). Complex interval-valued intuitionistic fuzzy sets and their aggregation operators. *Fundamenta Informaticae*, *164*(1), 61–101.

Goguen Jr, J. A. (1974). Concept representation in natural and artificial languages: Axioms, extensions and applications for fuzzy sets. *International Journal of Man-Machine Studies*, *6*(5), 513–561.

Greenfield, S., Chiclana, F., & Dick, S. (2016). Interval-valued complex fuzzy logic. In *2016 IEEE International Conference on Fuzzy Systems*, 2014–2019.

Greenfield, S., Chiclana, F., & Dick, S. (2016). Join and meet operations for interval-valued complex fuzzy logic. In *2016*

Annual Conference of the North American Fuzzy Information Processing Society, 1–5.

Hur, K., Lee, J. G., & Choi, J. Y. (2009). Interval-valued fuzzy relations. *Journal of Korean Institute of Intelligent Systems*, *19*(3), 425–431.

Mendel, J. M. (1995). Fuzzy logic systems for engineering: A tutorial. *In Proceedings of the IEEE*, *83*(3), 345–377.

Nasir, A., Jan, N., Gumaei, A., & Khan, S. U. (2021). Medical diagnosis and life span of sufferer using interval valued complex fuzzy relations. *IEEE Access*, *9*, 93764–93780.

Nasir, A., Jan, N., Gumaei, A., Khan, S. U., & Albogamy, F. R. (2021). Cybersecurity against the loopholes in industrial control systems using interval-valued complex intuitionistic fuzzy relations. *Applied Sciences*, *11*(16), 7668.

Peng, X. (2019). New operations for interval-valued pythagorean fuzzy set. *Scientia Iranica*, *26*(2), 1049–1076.

Ramot, D., Milo, R., Friedman, M., & Kandel, A. (2002). Complex fuzzy sets. *IEEE Transactions on Fuzzy Systems*, *10*(2), 171–186.

Wang, L., & Wang, H. (2022). An integrated qualitative group decision-making method for assessing health-care waste treatment technologies based on linguistic terms with weakened hedges. *Applied Soft Computing*, *117*(10), 108435.

Yager, R. R. (2013). Pythagorean fuzzy subsets. In *2013 Joint IFSA World Congress and NAFIPS Annual Meeting*, 57–61.

Zadeh, L. A. (1975). The concept of a linguistic variable and its application to approximate reasoning—I. *Information Sciences*, *8*(3), 199–249.

Zadeh, L. A. (1965). Fuzzy sets. *Information and Control*, *8*(3), 338–353.

Zhang, F. A. S. (2011). Research of supplier evaluation approach based on interval-valued intuitionistic fuzzy relations. In *2011 Chinese Control and Decision Conference*, 1055–1059.

Zhang, G., Dillon, T. S., Cai, K., Ma, J., & Lu, J. (2010). Delta-equalities of complex fuzzy relations. In *2010 24th IEEE International Conference on Advanced Information Networking and Applications*, 1218–1224.

Zimmermann, H. J. (2011). *Fuzzy set theory—And its applications*. Germany: Springer Science & Business Media.

How to Cite: Yazbek, H. A., Surriya, F., Khan, S. U., Jan, N., & Marinkovic, D. (2023). A Novel Approach to Model the Economic Characteristics of an Organization by Interval-Valued Complex Pythagorean Fuzzy Information. *Journal of Computational and Cognitive Engineering* 2(1), 75–87, https://doi.org/10.47852/bonviewJCCE2202249