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Analysis of Digital Green Innovation Based on Schweizer–Sklar Prioritized Aggregation Operators for Interval-Valued Picture Fuzzy Supply Chain Management

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Abstract: Digital green innovation economics and management for Industry 5.0 was not widely recognized, and the commonly referenced industry paradigm was Industry 4.0. Furthermore, digital green innovation is part of the above information referring to the integration of digital technologies with environmentally sustainable practices to develop innovative solutions that evaluate ecological challenges. In this manuscript, we evaluate the Schweizer–Sklar operational laws based on interval-valued picture fuzzy (IVPF) values. Further, we investigate prioritized aggregation operators based on Schweizer–Sklar operational laws for IVPF information, called IVPF Schweizer–Sklar prioritized averaging operator, IVPF Schweizer–Sklar prioritized geometric operator, IVPF Schweizer–Sklar prioritized weighted averaging operator, and IVPF Schweizer–Sklar prioritized weighted geometric operator. Some properties for the above-initiated operators are also derived. Additionally, we analyze the digital green innovation with the help of multi-attribute decision-making technique for initiated operators to show the reliability and supremacy of the proposed theory. Finally, we demonstrate examples for addressing the comparison among the initiated theory and existing ideas to improve the worth of the derived theory.

Keywords: interval-valued picture fuzzy sets, Schweizer–Sklar prioritized aggregation operators, supply chain management, decision-making problems

1. Introduction

After a long assessment, we observed that the technique of digital green innovation (Yin & Yu, 2022) is widely used in many fields because they received a lot of attention from different scholars. Further, digital technologies are used in various areas, for instance, sustainability, agriculture, and environmental conservation (Yin et al., 2022). Additionally, the multi-attribute decision-making (MADM) technique involves the investigation of a finite family of alternatives and ranking information in terms of how reliable they are to experts when all the attribute is selected continuously. In the evaluation of these techniques, most people have given information in a crisp form (Dai et al., 2022). However, because of complications and complexity in the system, we are unable to cope with vague and complicated information because of limited options that are zero and one (Elbanna, 2006; Wang & Ruhe, 2007). For this reason, Zadeh (1965) developed the fuzzy set (FS), the range of the FS is unit interval, and the shape of truth grade in FS is computed in the real from like 0.2, 0.3, and between unit intervals. Furthermore, in the case of single-valued information instead of interval-valued information, we have a lot of chances to lose information, but if we have given information in the shape of an interval, then it is possible we will get correct results because of the wide range. For this, Zadeh (1975) exposed the interval-valued FSs (IVFSs), where the truth grade in IVFS is computed in the shape of a subinterval of the unit interval.

Further, Atanassov (1986) initiated the intuitionistic FS (IFS) because the truth grade is not enough for coping with uncertain and vague information in genuine life problems due to falsity information. Negativity is a major part of every decision-making procedure, and because of this reason, we have lost a lot of data. For this, the theory of IFSs is very flexible and dominant. The IFS has two grades, called truth and falsity information with a condition that the sum of both grades will be contained in the unit interval. Further, Atanassov and Gargov (1989) and Atanassov (1999) exposed the interval-valued IFS (IVIFS), where the truth grade and falsity grade in IVIFS are computed in the shape of the subintervals of the unit interval with a condition that the sum of the supremum of both grades will be contained in the unit interval. Furthermore, some applications of IFSs and IVIFSs are discussed in the shape, for instance, WASPAS technique and Aczel–Alsina operators for intuitionistic fuzzy soft sets (Albaity et al., 2023). Power operators based on Aczel–Alsina operational laws for IVIFSs (Shi et al., 2023) are also a very reliable technique for aggregating the collection of information into singleton sets, Davoudabadi et al. (2023) exposed the simulation approach for project evaluation under the consideration of IVIFSs, and Chen et al. (2023) presented coronavirus disease 2019 based on IVIFSs.

In the election campaign, many kinds of problems have occurred, for instance, yes, no, abstinence, and refusal, because many people have cast their vote in favor of someone or against someone, some people have

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abstained to cast his vote, and some people have refused his vote to cast it. For managing such kind of problems, FSs and IFSs are not enough, because they deal only with yes and no, but not with abstinence and refusal. For this, Cuong (2013) presented the picture FS (PFS), where the PFS contained the truth, falsity, abstinence, and refusal grades with a condition that the sum of all these grades will be equal to one. Further, Khalil et al. (2019) exposed the interval-valued PFS (IVPFS), where the truth, falsity, abstinence, and refusal grades are computed in the shape of the subinterval of the unit interval with the same condition for their supremum values.

Furthermore, Deschrijver and Kerre (2002) initiated the Schweizer–Sklar norms, algebraic norms, and Einstein norms for crisp set theory. Schweizer–Sklar power operators for IVIFSs were derived by Liu and Wang (2018). Mahmood et al. (2023) exposed the Frank operators and AHP technique for IVPFSs. Moreover, Kamaci et al. (2021) designed the dynamic operators and Einstein operators for interval-valued picture hesitant FSs. Additionally, Jabeen et al. (2023) exposed the Aczel–Alsina operators for IVPFSs. Also, Garg et al. (2021) initiated the interval-valued picture of uncertain linguistic generalized Hamacher operators and their application in decision support systems. The prioritized operators based on classical set theory were proposed by Yager (2008). Additionally, the utilization of the Schweizer–Sklar operators and PRO operator based on IVPFS is very complex and complicated because of their structure; therefore, the major investigation of this theory is listed below:

1) To derive the technique of IVPF Schweizer–Sklar operational laws.
2) To introduce the technique of the IVPF Schweizer–Sklar prioritized averaging (IVPFSSPROA) operator, IVPF Schweizer–Sklar prioritized geometric (IVPFSSPROG) operator, IVPF Schweizer–Sklar prioritized weighted averaging (IVPFSSPROWA) operator, and IVPF Schweizer–Sklar prioritized weighted geometric (IVPFSSPROWG) operator with three basic properties.
3) To analyze the digital green innovation with the help of the MADM technique for initiated operators to show the reliability and supremacy of the proposed theory.
4) To demonstrate examples for addressing the comparison among the initiated theory and existing ideas to improve the worth of the derived theory.

This manuscript is computed in the following shape: Section 2 stated the technique of IVPFS and their related operational laws. Further, we discussed the technique of the PRO averaging (PROA) operator, PRO geometric (PROG) operator, Schweizer–Sklar t-norm (SSTN), and Schweizer–Sklar t-conorm (SSTCN). Section 3 described the novel theory of IVPF Schweizer–Sklar operational laws with IVPFSSPROA operator, IVPFSSPROG operator, IVPFSSPROWA operator, and IVPFSSPROWG operator. Section 4 analyzed the digital green innovation with the help of the MADM technique for initiated operators to show the reliability and supremacy of the proposed theory. In Section 5, we demonstrated examples for addressing the comparison among the initiated theory and existing ideas to improve the worth of the derived theory in Section 6. Some concluding remarks are stated in Section 7.

2. Preliminaries

This section stated the technique of IVPFS and their related operational laws. Further, we discussed the technique of the PROA operator, PROG operator, SSTN, and SSTCN.

Definition 1: (Khalil et al., 2019) Let π be any fixed set. Thus, an IVPFS is demonstrated below:

$$\mathbb{RF} = \left\{ \begin{pmatrix} [\mathbb{TM}^-(\mathbb{x}), \mathbb{TM}^+(\mathbb{x})], \\ [\mathbb{AM}^-(\mathbb{x}), \mathbb{AM}^+(\mathbb{x})], \\ [\mathbb{FM}^-(\mathbb{x}), \mathbb{FM}^+(\mathbb{x})] \end{pmatrix} : \mathbb{x} \in \mathbb{X} \right\}$$

where $\mathbb{TM}$ represents the truth grade, $\mathbb{AM}$ represents the abstinence grade, and $\mathbb{FM}$ represents the falsity grade in the shape of an interval value, such as $[\mathbb{TM}^-(\mathbb{X}), \mathbb{TM}^+(\mathbb{X})]$, $[\mathbb{AM}^-(\mathbb{X}), \mathbb{AM}^+(\mathbb{X})]$, and $[\mathbb{FM}^-(\mathbb{X}), \mathbb{FM}^+(\mathbb{X})]$ with a condition that is as follows: $0 \leq \mathbb{TM}^+(\mathbb{X}) + \mathbb{AM}^+(\mathbb{X}) + \mathbb{FM}^+(\mathbb{X}) \leq 1$. Further, the simple shape of IVPFN is fixed in the following shape: $\mathbb{RF}^j = ([\mathbb{TM}_j^-, \mathbb{TM}_j^+], [\mathbb{AM}_j^-, \mathbb{AM}_j^+], [\mathbb{FM}_j^-, \mathbb{FM}_j^+]), j = 1, 2, \ldots, \gamma\gamma$.

Definition 2: (Khalil et al., 2019) Let $\mathbb{RF}^j = ([\mathbb{TMM}_j^-, \mathbb{TM}_j^+], [\mathbb{AMM}_j^-, \mathbb{AM}_j^+], [\mathbb{FMM}_j^-, \mathbb{FM}_j^+]), j = 1, 2,$ be any two IVPFNs. Then

$$SC(\mathbb{RF}^j) = \frac{\begin{pmatrix} \mathbb{TM}_j^- + \mathbb{TM}_j^+ + \mathbb{AM}_j^- + \\ \mathbb{AM}_j^+ - \mathbb{FM}_j^- - \mathbb{FM}_j^+ \end{pmatrix}}{3} \in [-1, 1]$$

$$AC(\mathbb{RF}^j) = \frac{\begin{pmatrix} \mathbb{TM}_j^- + \mathbb{TM}_j^+ + \mathbb{AM}_j^- + \\ \mathbb{AM}_j^+ + \mathbb{FM}_j^- + \mathbb{FM}_j^+ \end{pmatrix}}{3} \in [-1, 1]$$

called score and accuracy values with some properties, such as the following:

1) If $SC(\mathbb{RF}^1) > SC(\mathbb{RF}^2)$, then $\mathbb{RFF}^1 > \mathbb{RF}^2$.
2) If $SC(\mathbb{RF}^2) > SC(\mathbb{RF}^1)$, then $\mathbb{RF}^2 > \mathbb{RF}^1$.
3) If $SC(\mathbb{RF}^1) = SC(\mathbb{RF}^2)$, then
   i) If $AC(\mathbb{RF}^1) > AC(\mathbb{RF}^2)$, then $\mathbb{RF}^1 > \mathbb{RF}^2$.
   ii) If $AC(\mathbb{RF}^2) > AC(\mathbb{RF}^1)$, then $\mathbb{RF}^2 > \mathbb{RF}^1$.

Definition 3: (Khalil et al., 2019) Let $\mathbb{RF}^j = ([\mathbb{TM}_j^-, \mathbb{TM}_j^+], [\mathbb{AM}_j^-, \mathbb{AM}_j^+], [\mathbb{FM}_j^-, \mathbb{FM}_j^+]), j = 1, 2,$ be any two IVPFNs. Then

$$\mathbb{RF}^1 \oplus \mathbb{RF}^2 = \begin{pmatrix} \begin{bmatrix} \mathbb{TM}_1^- + \mathbb{TM}_2^- - \mathbb{TM}_1^- \mathbb{TM}_2^-, \\ \mathbb{TM}_1^+ + \mathbb{TM}_2^+ - \mathbb{TM}_1^+ \mathbb{TM}_2^+ \end{bmatrix}, \\ [\mathbb{AM}_1^- \mathbb{AM}_2^-, \mathbb{AM}_1^+ \mathbb{AM}_2^+], [\mathbb{FM}_1^- \mathbb{FM}_2^-, \mathbb{FM}_1^+ \mathbb{FM}_2^+] \end{pmatrix}$$

$$\mathbb{RF}^1 \otimes \mathbb{RF}^2 = \begin{pmatrix} [\mathbb{TM}_1^- \mathbb{TM}_2^-, \mathbb{TM}_1^+ \mathbb{TM}_2^+], \\ \begin{bmatrix} \mathbb{AM}_1^- + \mathbb{AM}_2^- - \mathbb{AM}_1^- \mathbb{AM}_2^-, \\ \mathbb{AM}_1^+ + \mathbb{AM}_2^+ - \mathbb{AM}_1^+ \mathbb{AM}_2^+ \end{bmatrix}, \\ \begin{bmatrix} \mathbb{FM}_1^- + \mathbb{FM}_2^- - \mathbb{FM}_1^- \mathbb{FM}_2^-, \\ \mathbb{FM}_1^+ + \mathbb{FM}_2^+ - \mathbb{FM}_1^+ \mathbb{FM}_2^+ \end{bmatrix} \end{pmatrix},$$

$$\#\mathbb{RF}^j = \begin{pmatrix} \left[1 - \left(1 - \mathbb{TM}_j^-\right)^{\#}, 1 - \left(1 - \mathbb{TM}_j^+\right)^{\#}\right], \\ \left[\left(\mathbb{AM}_j^-\right)^{\#}, \left(\mathbb{AM}_j^+\right)^{\#}\right], \left[\left(\mathbb{FM}_j^-\right)^{\#}, \left(\mathbb{FM}_j^+\right)^{\#}\right] \end{pmatrix}$$

$$(\mathbb{RF}^j)^{\sharp}=\begin{pmatrix}\left[\left(\mathbb{TM}_j^-\right)^{\sharp},\left(\mathbb{TM}_j^+\right)^{\sharp}\right],\\ \left[1-\left(1-\mathbb{AM}_j^-\right)^{\sharp},1-\left(1-\mathbb{AM}_j^+\right)^{\sharp}\right],\\ \left[1-\left(1-\mathbb{FM}_j^-\right)^{\sharp},1-\left(1-\mathbb{FM}_j^+\right)^{\sharp}\right]\end{pmatrix}$$

Definition 4: (Yager, 2008) Let $\mathbb{RF}^j, j=1,2,\ldots,\gamma\gamma$, be any collection of positive information. Then

$$PROA(\mathbb{RF}^1,\mathbb{RF}^2,\ldots,\mathbb{RF}^{\gamma\gamma})=\sum_{j=1}^{\gamma\gamma}\frac{\Xi_j}{\sum_{j=1}^{\gamma\gamma}\Xi_j}\mathbb{RF}^j$$

$$PROG(\mathbb{RF}^1,\mathbb{RF}^2,\ldots,\mathbb{RF}^{\gamma\gamma})=\prod_{j=1}^{\gamma\gamma}(\mathbb{RF}^j)^{\frac{\Xi_j}{\sum_{j=1}^{\gamma\gamma}\Xi_j}}$$

called PROA and PROG operators, where $\Xi_1=1$, and $\Xi_j=\prod_{k=1}^{j-1}SC(\mathbb{RF}^k)$.

Definition 5: (Deschrijver & Kerre, 2002) Let $\mathbb{RF}^j, j=1,2$, be any two positive information. Then

$$SSTN^{\mathbb{ll}}(\mathbb{RF}^1,\mathbb{RF}^2)=\left((\mathbb{RF}^1)^{\mathbb{ll}}+(\mathbb{RF}^2)^{\mathbb{ll}}-1\right)^{\frac{1}{\mathbb{ll}}}$$

$$SSTCN^{\mathbb{ll}}(\mathbb{RF}^1,\mathbb{RF}^2)=1-\begin{pmatrix}(1-\mathbb{RF}^1)^{\mathbb{ll}}+\\(1-\mathbb{RF}^2)^{\mathbb{ll}}-1\end{pmatrix}^{\frac{1}{\mathbb{ll}}}$$

called SSTN and SSTCN; if $\mathbb{ll}=0$, then, we have $SSTN^{\mathbb{ll}}(\mathbb{RF}^1,\mathbb{RF}^2)=\mathbb{RF}^1*\mathbb{RF}^2$ and $SSTCN^{\mathbb{ll}}(\mathbb{RF}^1,\mathbb{RF}^2)=\mathbb{RF}^1+\mathbb{RF}^2-\mathbb{RF}^1*\mathbb{RF}^2$.

3. IVPF Schweizer–Sklar PRO Operators

In this section, we introduce the novel theory of Schweizer–Sklar operational laws based on IVPFNs. Further, we derive the concept of the IVPFSSPROA operator, IVPFSSPROWA operator, IVPFSSPROG operator, and IVPFSSPROWG operator with some properties, called idempotency, monotonicity, and boundedness.

Definition 6: Let $\mathbb{RF}^j=([\mathbb{TM}_j^-,\mathbb{TM}_j^+],[\mathbb{AM}_j^-,\mathbb{AM}_j^+],[\mathbb{FM}_j^-,\mathbb{FM}_j^+]), j=1,2$, be any two IVPFNs. Then

$$\mathbb{RF}^1\oplus\mathbb{RF}^2=\begin{pmatrix}\begin{bmatrix}1-\left((1-\mathbb{TM}_1^-)^{\mathbb{ll}}+(1-\mathbb{TM}_2^-)^{\mathbb{ll}}-1\right)^{\frac{1}{\mathbb{ll}}},\\1-\left((1-\mathbb{TM}_1^+)^{\mathbb{ll}}+(1-\mathbb{TM}_2^+)^{\mathbb{ll}}-1\right)^{\frac{1}{\mathbb{ll}}}\end{bmatrix},\\ \begin{bmatrix}\left((\mathbb{AM}_1^-)^{\mathbb{ll}}+(\mathbb{AM}_2^-)^{\mathbb{ll}}-1\right)^{\frac{1}{\mathbb{ll}}},\\\left((\mathbb{AM}_1^+)^{\mathbb{ll}}+(\mathbb{AM}_2^+)^{\mathbb{ll}}-1\right)^{\frac{1}{\mathbb{ll}}}\end{bmatrix},\\ \begin{bmatrix}\left((\mathbb{FM}_1^-)^{\mathbb{ll}}+(\mathbb{FM}_2^-)^{\mathbb{ll}}-1\right)^{\frac{1}{\mathbb{ll}}},\\\left((\mathbb{FM}_1^+)^{\mathbb{ll}}+(\mathbb{FM}_2^+)^{\mathbb{ll}}-1\right)^{\frac{1}{\mathbb{ll}}}\end{bmatrix}\end{pmatrix}$$

$$\mathbb{RF}^1\otimes\mathbb{RF}^2=\begin{pmatrix}\begin{bmatrix}\left((\mathbb{TM}_1^-)^{\mathbb{ll}}+(\mathbb{TM}_2^-)^{\mathbb{ll}}-1\right)^{\frac{1}{\mathbb{ll}}},\\\left((\mathbb{TM}_1^+)^{\mathbb{ll}}+(\mathbb{TM}_2^+)^{\mathbb{ll}}-1\right)^{\frac{1}{\mathbb{ll}}}\end{bmatrix},\\ \begin{bmatrix}1-\left((1-\mathbb{AM}_1^-)^{\mathbb{ll}}+(1-\mathbb{AM}_2^-)^{\mathbb{ll}}-1\right)^{\frac{1}{\mathbb{ll}}},\\1-\left((1-\mathbb{AM}_1^+)^{\mathbb{ll}}+(1-\mathbb{AM}_2^+)^{\mathbb{ll}}-1\right)^{\frac{1}{\mathbb{ll}}}\end{bmatrix},\\ \begin{bmatrix}1-\left((1-\mathbb{FM}_1^-)^{\mathbb{ll}}+(1-\mathbb{FM}_2^-)^{\mathbb{ll}}-1\right)^{\frac{1}{\mathbb{ll}}},\\1-\left((1-\mathbb{FM}_1^+)^{\mathbb{ll}}+(1-\mathbb{FM}_2^+)^{\mathbb{ll}}-1\right)^{\frac{1}{\mathbb{ll}}}\end{bmatrix}\end{pmatrix}$$

$$\sharp\mathbb{RF}^1=\begin{pmatrix}\begin{bmatrix}1-\left(\sharp(1-\mathbb{TM}_1^-)^{\mathbb{ll}}-(\sharp-1)\right)^{\frac{1}{\mathbb{ll}}},\\1-\left(\sharp(1-\mathbb{TM}_1^+)^{\mathbb{ll}}-(\sharp-1)\right)^{\frac{1}{\mathbb{ll}}}\end{bmatrix},\\ \left[\left(\sharp(\mathbb{AM}_1^-)^{\mathbb{ll}}-(\sharp-1)\right)^{\frac{1}{\mathbb{ll}}},\left(\sharp(\mathbb{AM}_1^+)^{\mathbb{ll}}-(\sharp-1)\right)^{\frac{1}{\mathbb{ll}}}\right],\\ \left[\left(\sharp(\mathbb{FM}_1^-)^{\mathbb{ll}}-(\sharp-1)\right)^{\frac{1}{\mathbb{ll}}},\left(\sharp(\mathbb{FM}_1^+)^{\mathbb{ll}}-(\sharp-1)\right)^{\frac{1}{\mathbb{ll}}}\right]\end{pmatrix}$$

$$(\mathbb{RF}^1)^{\sharp}=\begin{pmatrix}\left[\left(\sharp(\mathbb{TM}_1^-)^{\mathbb{ll}}-(\sharp-1)\right)^{\frac{1}{\mathbb{ll}}},\left(\sharp(\mathbb{TM}_1^+)^{\mathbb{ll}}-(\sharp-1)\right)^{\frac{1}{\mathbb{ll}}}\right],\\ \begin{bmatrix}1-\left(\sharp(1-\mathbb{AM}_1^-)^{\mathbb{ll}}-(\sharp-1)\right)^{\frac{1}{\mathbb{ll}}},\\1-\left(\sharp(1-\mathbb{AM}_1^+)^{\mathbb{ll}}-(\sharp-1)\right)^{\frac{1}{\mathbb{ll}}}\end{bmatrix},\\ \begin{bmatrix}1-\left(\sharp(1-\mathbb{FM}_1^-)^{\mathbb{ll}}-(\sharp-1)\right)^{\frac{1}{\mathbb{ll}}},\\1-\left(\sharp(1-\mathbb{FM}_1^+)^{\mathbb{ll}}-(\sharp-1)\right)^{\frac{1}{\mathbb{ll}}}\end{bmatrix}\end{pmatrix}$$

where $\sharp 1$.

Definition 7: Let $\mathbb{RF}^j=([\mathbb{TM}_j^-,\mathbb{TM}_j^+],[\mathbb{AM}_j^-,\mathbb{AM}_j^+],[\mathbb{FM}_j^-,\mathbb{FM}_j^+]), j=1,2,\ldots,\gamma\gamma$, be any collection of IVPFNs. Then

$$IVPFSSPROA(\mathbb{RF}^1,\mathbb{RF}^2,\ldots,\mathbb{RF}^{\gamma\gamma})=\sum_{j=1}^{\gamma\gamma}\frac{\Xi_j}{\sum_{j=1}^{\gamma\gamma}\Xi_j}\mathbb{RF}^j$$

called IVPFSSPROA operator, where $\Xi_1=1$, and $\Xi_j=\prod_{k=1}^{j-1}SC(\mathbb{RF}^k)$.

Theorem 1: Prove that the aggregated information of the IVPFSSPROA operator is again an IVPFN, such as

$$IVPFSSPROA(\mathbb{RF}^1,\mathbb{RF}^2,\ldots,\mathbb{RF}^{\gamma\gamma})$$

$$=\begin{pmatrix}\begin{bmatrix}1-\left(\sum_{j=1}^{\gamma\gamma}\frac{\Xi_j}{\sum_{j=1}^{\gamma\gamma}\Xi_j}\left(1-\mathbb{TM}_j^-\right)^{\mathbb{ll}}\right)^{\frac{1}{\mathbb{ll}}},\\ 1-\left(\sum_{j=1}^{\gamma\gamma}\frac{\Xi_j}{\sum_{j=1}^{\gamma\gamma}\Xi_j}\left(1-\mathbb{TM}_j^+\right)^{\mathbb{ll}}\right)^{\frac{1}{\mathbb{ll}}}\end{bmatrix},\\ \left[\left(\sum_{j=1}^{\gamma\gamma}\frac{\Xi_j}{\sum_{j=1}^{\gamma\gamma}\Xi_j}\left(\mathbb{AM}_j^-\right)^{\mathbb{ll}}\right)^{\frac{1}{\mathbb{ll}}},\left(\sum_{j=1}^{\gamma\gamma}\frac{\Xi_j}{\sum_{j=1}^{\gamma\gamma}\Xi_j}\left(\mathbb{AM}_j^+\right)^{\mathbb{ll}}\right)^{\frac{1}{\mathbb{ll}}}\right],\\ \left[\left(\sum_{j=1}^{\gamma\gamma}\frac{\Xi_j}{\sum_{j=1}^{\gamma\gamma}\Xi_j}\left(\mathbb{FM}_j^-\right)^{\mathbb{ll}}\right)^{\frac{1}{\mathbb{ll}}},\left(\sum_{j=1}^{\gamma\gamma}\frac{\Xi_j}{\sum_{j=1}^{\gamma\gamma}\Xi_j}\left(\mathbb{FM}_j^+\right)^{\mathbb{ll}}\right)^{\frac{1}{\mathbb{ll}}}\right]\end{pmatrix}$$

Proof: Using the mathematical induction, we prove the above theory, for this, if we consider $\gamma\gamma = 2$; thus,

$$\frac{\Xi_1}{\sum_{j=1}^{2}\Xi_j}\mathrm{RF}^1 = \left(\begin{array}{c}\left[\begin{array}{c}1-\left(\frac{\Xi_1}{\sum_{j=1}^{2}\Xi_j}(1-\mathbb{TM}_1^-)^{\mathbb{ll}}-\left(\frac{\Xi_1}{\sum_{j=1}^{2}\Xi_j}-1\right)\right)^{\frac{1}{\mathbb{ll}}},\\ 1-\left(\frac{\Xi_1}{\sum_{j=1}^{2}\Xi_j}(1-\mathbb{TM}_1^+)^{\mathbb{ll}}-\left(\frac{\Xi_1}{\sum_{j=1}^{2}\Xi_j}-1\right)\right)^{\frac{1}{\mathbb{ll}}}\end{array}\right],\\ \left[\begin{array}{c}\left(\frac{\Xi_1}{\sum_{j=1}^{2}\Xi_j}(\mathbb{AM}_1^-)^{\mathbb{ll}}-\left(\frac{\Xi_1}{\sum_{j=1}^{2}\Xi_j}-1\right)\right)^{\frac{1}{\mathbb{ll}}},\\ \left(\frac{\Xi_1}{\sum_{j=1}^{2}\Xi_j}(\mathbb{AM}_1^+)^{\mathbb{ll}}-\left(\frac{\Xi_1}{\sum_{j=1}^{2}\Xi_j}-1\right)\right)^{\frac{1}{\mathbb{ll}}}\end{array}\right],\\ \left[\begin{array}{c}\left(\frac{\Xi_1}{\sum_{j=1}^{2}\Xi_j}(\mathbb{AM}_1^-)^{\mathbb{ll}}-\left(\frac{\Xi_1}{\sum_{j=1}^{2}\Xi_j}-1\right)\right)^{\frac{1}{\mathbb{ll}}},\\ \left(\frac{\Xi_1}{\sum_{j=1}^{2}\Xi_j}(\mathbb{AM}_1^+)^{\mathbb{ll}}-\left(\frac{\Xi_1}{\sum_{j=1}^{2}\Xi_j}-1\right)\right)^{\frac{1}{\mathbb{ll}}}\end{array}\right]\end{array}\right)$$

$$\frac{\Xi_2}{\sum_{j=1}^{2}\Xi_j}\mathbb{RF}^2 = \left(\begin{array}{c}\left[\begin{array}{c}1-\left(\frac{\Xi_2}{\sum_{j=1}^{2}\Xi_j}(1-\mathbb{TM}_2^-)^{\mathbb{ll}}-\left(\frac{\Xi_2}{\sum_{j=1}^{2}\Xi_j}-1\right)\right)^{\frac{1}{\mathbb{ll}}},\\ 1-\left(\frac{\Xi_2}{\sum_{j=1}^{2}\Xi_j}(1-\mathbb{TM}_2^+)^{\mathbb{ll}}-\left(\frac{\Xi_2}{\sum_{j=1}^{2}\Xi_j}-1\right)\right)^{\frac{1}{\mathbb{ll}}}\end{array}\right],\\ \left[\begin{array}{c}\left(\frac{\Xi_2}{\sum_{j=1}^{2}\Xi_j}(\mathbb{AM}_2^-)^{\mathbb{ll}}-\left(\frac{\Xi_2}{\sum_{j=1}^{2}\Xi_j}-1\right)\right)^{\frac{1}{\mathbb{ll}}},\\ \left(\frac{\Xi_2}{\sum_{j=1}^{2}\Xi_j}(\mathbb{AM}_2^+)^{\mathbb{ll}}-\left(\frac{\Xi_2}{\sum_{j=1}^{2}\Xi_j}-1\right)\right)^{\frac{1}{\mathbb{ll}}}\end{array}\right],\\ \left[\begin{array}{c}\left(\frac{\Xi_2}{\sum_{j=1}^{2}\Xi_j}(\mathbb{FM}_2^-)^{\mathbb{ll}}-\left(\frac{\Xi_2}{\sum_{j=1}^{2}\Xi_j}-1\right)\right)^{\frac{1}{\mathbb{ll}}},\\ \left(\frac{\Xi_2}{\sum_{j=1}^{2}\Xi_j}(\mathbb{FM}_2^+)^{\mathbb{ll}}-\left(\frac{\Xi_2}{\sum_{j=1}^{2}\Xi_j}-1\right)\right)^{\frac{1}{\mathbb{ll}}}\end{array}\right]\end{array}\right)$$

Then

$$IVPFSSPROA(\mathbb{RF}^1,\mathbb{RF}^2) = \frac{\Xi_1}{\sum_{j=1}^{2}\Xi_j}\mathbb{RF}^1 \oplus \frac{\Xi_2}{\sum_{j=1}^{2}\Xi_j}\mathbb{RF}^2$$

$$= \left(\begin{array}{c}\left[\begin{array}{c}1-\left(\frac{\Xi_1}{\sum_{j=1}^{2}\Xi_j}(1-\mathbb{TM}_1^-)^{\mathbb{ll}}-\left(\frac{\Xi_1}{\sum_{j=1}^{2}\Xi_j}-1\right)\right)^{\frac{1}{\mathbb{ll}}},\\ 1-\left(\frac{\Xi_1}{\sum_{j=1}^{2}\Xi_j}(1-\mathbb{TM}_1^+)^{\mathbb{ll}}-\left(\frac{\Xi_1}{\sum_{j=1}^{2}\Xi_j}-1\right)\right)^{\frac{1}{\mathbb{ll}}}\end{array}\right],\\ \left[\begin{array}{c}\left(\frac{\Xi_1}{\sum_{j=1}^{2}\Xi_j}(\mathbb{AM}_1^-)^{\mathbb{ll}}-\left(\frac{\Xi_1}{\sum_{j=1}^{2}\Xi_j}-1\right)\right)^{\frac{1}{\mathbb{ll}}},\\ \left(\frac{\Xi_1}{\sum_{j=1}^{2}\Xi_j}(\mathbb{AM}_1^+)^{\mathbb{ll}}-\left(\frac{\Xi_1}{\sum_{j=1}^{2}\Xi_j}-1\right)\right)^{\frac{1}{\mathbb{ll}}}\end{array}\right],\\ \left[\begin{array}{c}\left(\frac{\Xi_1}{\sum_{j=1}^{2}\Xi_j}(\mathbb{FM}_1^-)^{\mathbb{ll}}-\left(\frac{\Xi_1}{\sum_{j=1}^{2}\Xi_j}-1\right)\right)^{\frac{1}{\mathbb{ll}}},\\ \left(\frac{\Xi_1}{\sum_{j=1}^{2}\Xi_j}(\mathbb{FM}_1^+)^{\mathbb{ll}}-\left(\frac{\Xi_1}{\sum_{j=1}^{2}\Xi_j}-1\right)\right)^{\frac{1}{\mathbb{ll}}}\end{array}\right]\end{array}\right)$$

$$\oplus \left(\begin{array}{c}\left[\begin{array}{c}1-\left(\frac{\Xi_2}{\sum_{j=1}^{2}\Xi_j}(1-\mathbb{TM}_2^-)^{\mathbb{ll}}-\left(\frac{\Xi_2}{\sum_{j=1}^{2}\Xi_j}-1\right)\right)^{\frac{1}{\mathbb{ll}}},\\ 1-\left(\frac{\Xi_2}{\sum_{j=1}^{2}\Xi_j}(1-\mathbb{TM}_2^+)^{\mathbb{ll}}-\left(\frac{\Xi_2}{\sum_{j=1}^{2}\Xi_j}-1\right)\right)^{\frac{1}{\mathbb{ll}}}\end{array}\right],\\ \left[\begin{array}{c}\left(\frac{\Xi_2}{\sum_{j=1}^{2}\Xi_j}(\mathbb{AM}_2^-)^{\mathbb{ll}}-\left(\frac{\Xi_2}{\sum_{j=1}^{2}\Xi_j}-1\right)\right)^{\frac{1}{\mathbb{ll}}},\\ \left(\frac{\Xi_2}{\sum_{j=1}^{2}\Xi_j}(\mathbb{AM}_2^+)^{\mathbb{ll}}-\left(\frac{\Xi_2}{\sum_{j=1}^{2}\Xi_j}-1\right)\right)^{\frac{1}{\mathbb{ll}}}\end{array}\right],\\ \left[\begin{array}{c}\left(\frac{\Xi_2}{\sum_{j=1}^{2}\Xi_j}(\mathbb{FM}_2^-)^{\mathbb{ll}}-\left(\frac{\Xi_2}{\sum_{j=1}^{2}\Xi_j}-1\right)\right)^{\frac{1}{\mathbb{ll}}},\\ \left(\frac{\Xi_2}{\sum_{j=1}^{2}\Xi_j}(\mathbb{FM}_2^+)^{\mathbb{ll}}-\left(\frac{\Xi_2}{\sum_{j=1}^{2}\Xi_j}-1\right)\right)^{\frac{1}{\mathbb{ll}}}\end{array}\right]\end{array}\right)$$

$$= \left(\begin{array}{c}\left[\begin{array}{c}1-\left(\sum_{j=1}^{2}\frac{\Xi_j}{\sum_{j=1}^{2}\Xi_j}\left(1-\mathbb{TM}_j^-\right)^{\mathbb{ll}}\right)^{\frac{1}{\mathbb{ll}}},\\ 1-\left(\sum_{j=1}^{2}\frac{\Xi_j}{\sum_{j=1}^{2}\Xi_j}\left(1-\mathbb{TM}_j^+\right)^{\mathbb{ll}}\right)^{\frac{1}{\mathbb{ll}}}\end{array}\right],\\ \left[\left(\sum_{j=1}^{2}\frac{\Xi_j}{\sum_{j=1}^{2}\Xi_j}\left(\mathbb{AM}_j^-\right)^{\mathbb{ll}}\right)^{\frac{1}{\mathbb{ll}}},\left(\sum_{j=1}^{2}\frac{\Xi_j}{\sum_{j=1}^{2}\Xi_j}\left(\mathbb{AM}_j^+\right)^{\mathbb{ll}}\right)^{\frac{1}{\mathbb{ll}}}\right],\\ \left[\left(\sum_{j=1}^{2}\frac{\Xi_j}{\sum_{j=1}^{2}\Xi_j}\left(\mathbb{FM}_j^-\right)^{\mathbb{ll}}\right)^{\frac{1}{\mathbb{ll}}},\left(\sum_{j=1}^{2}\frac{\Xi_j}{\sum_{j=1}^{2}\Xi_j}\left(\mathbb{FM}_j^+\right)^{\mathbb{ll}}\right)^{\frac{1}{\mathbb{ll}}}\right]\end{array}\right)$$

For $\gamma\gamma = 2$, we have corrected. If we $\gamma\gamma = \gamma\gamma'$, then

$$IVPFSSPROA\left(\mathbb{RF}^1,\mathbb{RF}^2,\ldots,\mathbb{RF}^{\gamma\gamma'}\right)$$

$$= \left(\begin{array}{c}\left[\begin{array}{c}1-\left(\sum_{j=1}^{\gamma\gamma'}\frac{\Xi_j}{\sum_{j=1}^{\gamma\gamma'}\Xi_j}\left(1-\mathbb{TM}_j^-\right)^{\mathbb{ll}}\right)^{\frac{1}{\mathbb{ll}}},\\ 1-\left(\sum_{j=1}^{\gamma\gamma'}\frac{\Xi_j}{\sum_{j=1}^{\gamma\gamma'}\Xi_j}\left(1-\mathbb{TM}_j^+\right)^{\mathbb{ll}}\right)^{\frac{1}{\mathbb{ll}}}\end{array}\right],\\ \left[\left(\sum_{j=1}^{\gamma\gamma'}\frac{\Xi_j}{\sum_{j=1}^{\gamma\gamma'}\Xi_j}\left(\mathbb{AM}_j^-\right)^{\mathbb{ll}}\right)^{\frac{1}{\mathbb{ll}}},\left(\sum_{j=1}^{\gamma\gamma'}\frac{\Xi_j}{\sum_{j=1}^{\gamma\gamma'}\Xi_j}\left(\mathbb{AM}_j^+\right)^{\mathbb{ll}}\right)^{\frac{1}{\mathbb{ll}}}\right],\\ \left[\left(\sum_{j=1}^{\gamma\gamma'}\frac{\Xi_j}{\sum_{j=1}^{\gamma\gamma'}\Xi_j}\left(\mathbb{FM}_j^-\right)^{\mathbb{ll}}\right)^{\frac{1}{\mathbb{ll}}},\left(\sum_{j=1}^{\gamma\gamma'}\frac{\Xi_j}{\sum_{j=1}^{\gamma\gamma'}\Xi_j}\left(\mathbb{FM}_j^+\right)^{\mathbb{ll}}\right)^{\frac{1}{\mathbb{ll}}}\right]\end{array}\right)$$

Then, we prove it for $\gamma\gamma = \gamma\gamma' + 1$, such as

$$IVPFSSPROA\left(\mathbb{RF}^1,\mathbb{RF}^2,\ldots,\mathbb{RF}^{\gamma\gamma'+1}\right)$$
$$= \oplus_{j=1}^{\gamma\gamma'}\frac{\Xi_j}{\sum_{j=1}^{\gamma\gamma'+1}\Xi_j}\mathbb{RF}^j \oplus \frac{\Xi_{\gamma\gamma'+1}}{\sum_{j=1}^{\gamma\gamma'+1}\Xi_j}\mathbb{RF}^{\gamma\gamma'+1}$$

$$= \left(\begin{array}{c}\left[\begin{array}{c}1-\left(\sum_{j=1}^{\gamma\gamma'}\frac{\Xi_j}{\sum_{j=1}^{\gamma\gamma'}\Xi_j}\left(1-\mathbb{TM}_j^-\right)^{\mathbb{ll}}\right)^{\frac{1}{\mathbb{ll}}},\\ 1-\left(\sum_{j=1}^{\gamma\gamma'}\frac{\Xi_j}{\sum_{j=1}^{\gamma\gamma'}\Xi_j}\left(1-\mathbb{TM}_j^+\right)^{\mathbb{ll}}\right)^{\frac{1}{\mathbb{ll}}}\end{array}\right],\\ \left[\left(\sum_{j=1}^{\gamma\gamma'}\frac{\Xi_j}{\sum_{j=1}^{\gamma\gamma'}\Xi_j}\left(\mathbb{AM}_j^-\right)^{\mathbb{ll}}\right)^{\frac{1}{\mathbb{ll}}},\left(\sum_{j=1}^{\gamma\gamma'}\frac{\Xi_j}{\sum_{j=1}^{\gamma\gamma'}\Xi_j}\left(\mathbb{AM}_j^+\right)^{\mathbb{ll}}\right)^{\frac{1}{\mathbb{ll}}}\right],\\ \left[\left(\sum_{j=1}^{\gamma\gamma'}\frac{\Xi_j}{\sum_{j=1}^{\gamma\gamma'}\Xi_j}\left(\mathbb{FM}_j^-\right)^{\mathbb{ll}}\right)^{\frac{1}{\mathbb{ll}}},\left(\sum_{j=1}^{\gamma\gamma'}\frac{\Xi_j}{\sum_{j=1}^{\gamma\gamma'}\Xi_j}\left(\mathbb{FM}_j^+\right)^{\mathbb{ll}}\right)^{\frac{1}{\mathbb{ll}}}\right]\end{array}\right)$$
$$\oplus \frac{\Xi_{\gamma\gamma'+1}}{\sum_{j=1}^{\gamma\gamma'+1}\Xi_j}\mathbb{RF}^{\gamma\gamma'+1}$$

$$
= \left(\begin{bmatrix} 1-\left(\sum_{j=1}^{\gamma\gamma'} \frac{\Xi_j}{\sum_{j=1}^{\gamma\gamma'+1} \Xi_j}\left(1-\mathbb{TM}_j^-\right)^{\mathbb{ll}}\right)^{\frac{1}{\mathbb{ll}}}, \\ 1-\left(\sum_{j=1}^{\gamma\gamma'} \frac{\Xi_j}{\sum_{j=1}^{\gamma\gamma'+1} \Xi_j}\left(1-\mathbb{TM}_j^+\right)^{\mathbb{ll}}\right)^{\frac{1}{\mathbb{ll}}} \end{bmatrix}, \begin{bmatrix} \left(\sum_{j=1}^{\gamma\gamma'} \frac{\Xi_j}{\sum_{j=1}^{\gamma\gamma'+1} \Xi_j}\left(\mathbb{AM}_j^-\right)^{\mathbb{ll}}\right)^{\frac{1}{\mathbb{ll}}}, \\ \left(\sum_{j=1}^{\gamma\gamma'} \frac{\Xi_j}{\sum_{j=1}^{\gamma\gamma'+1} \Xi_j}\left(\mathbb{AM}_j^+\right)^{\mathbb{ll}}\right)^{\frac{1}{\mathbb{ll}}} \end{bmatrix}, \begin{bmatrix} \left(\sum_{j=1}^{\gamma\gamma'} \frac{\Xi_j}{\sum_{j=1}^{\gamma\gamma'+1} \Xi_j}\left(\mathbb{FM}_j^-\right)^{\mathbb{ll}}\right)^{\frac{1}{\mathbb{ll}}}, \\ \left(\sum_{j=1}^{\gamma\gamma'} \frac{\Xi_j}{\sum_{j=1}^{\gamma\gamma'+1} \Xi_j}\left(\mathbb{FM}_j^+\right)^{\mathbb{ll}}\right)^{\frac{1}{\mathbb{ll}}} \end{bmatrix} \right)
$$

$$
\oplus \left(\begin{bmatrix} 1-\begin{pmatrix} \frac{\Xi_{\gamma\gamma'+1}}{\sum_{j=1}^{\gamma\gamma'+1} \Xi_j}\left(1-\mathbb{TM}_{\gamma\gamma'+1}^-\right)^{\mathbb{ll}} \\ -\left(\frac{\Xi_{\gamma\gamma'+1}}{\sum_{j=1}^{\gamma\gamma'+1} \Xi_j}-1\right) \end{pmatrix}^{\frac{1}{\mathbb{ll}}}, \\ 1-\begin{pmatrix} \frac{\Xi_{\gamma\gamma'+1}}{\sum_{j=1}^{\gamma\gamma'+1} \Xi_j}\left(1-\mathbb{TM}_{\gamma\gamma'+1}^+\right)^{\mathbb{ll}} \\ -\left(\frac{\Xi_{\gamma\gamma'+1}}{\sum_{j=1}^{\gamma\gamma'+1} \Xi_j}-1\right) \end{pmatrix}^{\frac{1}{\mathbb{ll}}} \end{bmatrix}, \begin{bmatrix} \left(\frac{\Xi_{\gamma\gamma'+1}}{\sum_{j=1}^{\gamma\gamma'+1} \Xi}\left(\mathbb{AM}_{\gamma\gamma'+1}^-\right)^{\mathbb{ll}}-\left(\frac{\Xi_{\gamma\gamma'+1}}{\sum_{j=1}^{\gamma\gamma'+1} \Xi_j}-1\right)\right)^{\frac{1}{\mathbb{ll}}}, \\ \left(\frac{\Xi_{\gamma\gamma'+1}}{\sum_{j=1}^{\gamma\gamma'+1} \Xi_j}\left(\mathbb{AM}_{\gamma\gamma'+1}^+\right)^{\mathbb{ll}}-\left(\frac{\Xi_{\gamma\gamma'+1}}{\sum_{j=1}^{\gamma\gamma'+1} \Xi_j}-1\right)\right)^{\frac{1}{\mathbb{ll}}} \end{bmatrix}, \begin{bmatrix} \left(\frac{\Xi_{\gamma\gamma'+1}}{\sum_{j=1}^{\gamma\gamma'+1} \Xi_j}\left(\mathbb{FM}_{\gamma\gamma'+1}^-\right)^{\mathbb{ll}}-\left(\frac{\Xi_{\gamma\gamma'+1}}{\sum_{j=1}^{\gamma\gamma'+1} \Xi_j}-1\right)\right)^{\frac{1}{\mathbb{ll}}}, \\ \left(\frac{\Xi_{\gamma\gamma'+1}}{\sum_{j=1}^{\gamma\gamma'+1} \Xi_j}\left(\mathbb{FM}_{\gamma\gamma'+1}^+\right)^{\mathbb{ll}}-\left(\frac{\Xi_{\gamma\gamma'+1}}{\sum_{j=1}^{\gamma\gamma'+1} \Xi_j}-1\right)\right)^{\frac{1}{\mathbb{ll}}} \end{bmatrix} \right)
$$

$$
= \left(\begin{bmatrix} 1-\left(\sum_{j=1}^{\gamma\gamma'+1} \frac{\Xi_j}{\sum_{j=1}^{\gamma\gamma'+1} \Xi_j}\left(1-\mathbb{TM}_j^-\right)^{\mathbb{ll}}\right)^{\frac{1}{\mathbb{ll}}}, \\ 1-\left(\sum_{j=1}^{\gamma\gamma'+1} \frac{\Xi_j}{\sum_{j=1}^{\gamma\gamma'+1} \Xi_j}\left(1-\mathbb{TM}_j^+\right)^{\mathbb{ll}}\right)^{\frac{1}{\mathbb{ll}}} \end{bmatrix}, \begin{bmatrix} \left(\sum_{j=1}^{\gamma\gamma'+1} \frac{\Xi_j}{\sum_{j=1}^{\gamma\gamma'+1} \Xi_j}\left(\mathbb{AM}_j^-\right)^{\mathbb{ll}}\right)^{\frac{1}{\mathbb{ll}}}, \\ \left(\sum_{j=1}^{\gamma\gamma'+1} \frac{\Xi_j}{\sum_{j=1}^{\gamma\gamma'+1} \Xi_j}\left(\mathbb{AM}_j^+\right)^{\mathbb{ll}}\right)^{\frac{1}{\mathbb{ll}}} \end{bmatrix}, \begin{bmatrix} \left(\sum_{j=1}^{\gamma\gamma'+1} \frac{\Xi_j}{\sum_{j=1}^{\gamma\gamma'+1} \Xi_j}\left(\mathbb{FM}_j^-\right)^{\mathbb{ll}}\right)^{\frac{1}{\mathbb{ll}}}, \\ \left(\sum_{j=1}^{\gamma\gamma'+1} \frac{\Xi_j}{\sum_{j=1}^{\gamma\gamma'+1} \Xi_j}\left(\mathbb{FM}_j^+\right)^{\mathbb{ll}}\right)^{\frac{1}{\mathbb{ll}}} \end{bmatrix} \right)
$$

Hence, the result is proved. Moreover, we aim to simplify the above technique with the help of some suitable examples; for this, we consider four attributes, such as $\mathbb{RF}^1 = ([0.1, 0.2], [0.2, 0.3], [0.3, 0.4])$, $\mathbb{RF}^2 = ([0.11, 0.21], [0.21, 0.31], [0.31, 0.41])$, $\mathbb{RF}^3 = ([0.12, 0.22], [0.22, 0.32], [0.32, 0.42])$, and $\mathbb{RF}^4 = ([0.13, 0.23], [0.23, 0.33], [0.33, 0.43])$, and then by using the technique of IVPFSSPROA operator, we have

$$
\begin{aligned} &IVPFSSPROA(\mathbb{RF}^1, \mathbb{RF}^2, \mathbb{RF}^3, \mathbb{RF}^4) \\ &\quad = ([0.1022, 0.2022], [0.2023, 0.3023], [0.3023, 0.4023]). \end{aligned}
$$

Further, we discussed some properties of the above-initiated theory, such as the following.

Property 1: Let $\mathbb{RF}^j = ([\mathbb{TM}_j^-, \mathbb{TM}_j^+], [\mathbb{AM}_j^-, \mathbb{AM}_j^+], [\mathbb{FM}_j^-, \mathbb{FM}_j^+]), j = 1, 2, \ldots, \gamma\gamma$, be any collection of IVPFNs. Then

1) Idempotency: If $\mathbb{RF}^j = \mathbb{RF}.j = 1, 2, \ldots, \gamma\gamma$

$$
IVPFSSPROA(\mathbb{RF}^1, \mathbb{RF}^2, \ldots, \mathbb{RF}^{\gamma\gamma}) = \mathbb{RF}
$$

2) Monotonicity: If $\mathbb{RF}^j \leq \mathbb{RF}^{\#}j$, then

$$
\begin{aligned} &IVPFSSPROA(\mathbb{RF}^1, \mathbb{RF}^2, \ldots, \mathbb{RF}^{\gamma\gamma}) \\ &\quad \leq IVPFSSPROA(\mathbb{RF}^{\#}1, \mathbb{RF}\boxtimes^2, \ldots, \mathbb{RF}\boxtimes^{\gamma\gamma}) \end{aligned}
$$

3) Boundedness: If $\mathbb{RF}^- = \min\{\mathbb{RF}^j\}$ and $\mathbb{RF}^+ = \max\{\mathbb{RF}^j\}$, thus

$$
\mathbb{RF}^- \leq IVPFSSPROA(\mathbb{RF}^1, \mathbb{RF}^2, \ldots, \mathbb{RF}^{\gamma\gamma}) \leq \mathbb{RF}^+
$$

Proof: Omitted.

Definition 8: Let $\mathbb{RF}^j = ([\mathbb{TM}_j^-, \mathbb{TM}_j^+], [\mathbb{AM}_j^-, \mathbb{AM}_j^+], [\mathbb{FM}_j^-, \mathbb{FM}_j^+]), j = 1, 2, \ldots, \gamma\gamma$, be any collection of IVPFNs. Then

$$
IVPFSSPROWA(\mathbb{RF}^1, \mathbb{RF}^2, \ldots, \mathbb{RF}^{\gamma\gamma}) = \sum_{j=1}^{\gamma\gamma} \frac{\#_j \Xi_j}{\sum_{j=1}^{\gamma\gamma} \#_j \Xi_j} \mathbb{RF}^j
$$

called IVPFSSPROWA operator, where $\Xi_1 = 1$, and $\Xi_j = \prod_{k=1}^{j-1} SC(\mathbb{RF}^k)$.

Theorem 2: Prove that the aggregated information of the IVPFSSPROWA operator is again an IVPFN, such as

$$
\begin{aligned} &IVPFSSPROWA(\mathbb{RF}^1, \mathbb{RF}^2, \ldots, \mathbb{RF}^{\gamma\gamma}) \\ &= \left(\begin{bmatrix} 1-\left(\sum_{j=1}^{\gamma\gamma} \frac{\#_j \Xi_j}{\sum_{j=1}^{\gamma\gamma} \#_j \Xi_j}\left(1-\mathbb{TM}_j^-\right)^{\mathbb{ll}}\right)^{\frac{1}{\mathbb{ll}}}, \\ 1-\left(\sum_{j=1}^{\gamma\gamma} \frac{\#_j \Xi_j}{\sum_{j=1}^{\gamma\gamma} \#_j \Xi_j}\left(1-\mathbb{TM}_j^+\right)^{\mathbb{ll}}\right)^{\frac{1}{\mathbb{ll}}} \end{bmatrix}, \begin{bmatrix} \left(\sum_{j=1}^{\gamma\gamma} \frac{\#_j \Xi_j}{\sum_{j=1}^{\gamma\gamma} \#_j \Xi_j}\left(\mathbb{AM}_j^-\right)^{\mathbb{ll}}\right)^{\frac{1}{\mathbb{ll}}}, \\ \left(\sum_{j=1}^{\gamma\gamma} \frac{\#_j \Xi_j}{\sum_{j=1}^{\gamma\gamma} \#_j \Xi_j}\left(\mathbb{AM}_j^+\right)^{\mathbb{ll}}\right)^{\frac{1}{\mathbb{ll}}} \end{bmatrix}, \begin{bmatrix} \left(\sum_{j=1}^{\gamma\gamma} \frac{\#_j \Xi_j}{\sum_{j=1}^{\gamma\gamma} \#_j \Xi_j}\left(\mathbb{FM}_j^-\right)^{\mathbb{ll}}\right)^{\frac{1}{\mathbb{ll}}}, \\ \left(\sum_{j=1}^{\gamma\gamma} \frac{\#_j \Xi_j}{\sum_{j=1}^{\gamma\gamma} \#_j \Xi_j}\left(\mathbb{FM}_j^+\right)^{\mathbb{ll}}\right)^{\frac{1}{\mathbb{ll}}} \end{bmatrix} \right) \end{aligned}
$$

Proof: Omitted.

Moreover, we aim to simplify the above technique with the help of some suitable examples; for this, we consider four attributes, such as $\mathbb{RF}^1 = ([0.1, 0.2], [0.2, 0.3], [0.3, 0.4]), \mathbb{RF}^2 = ([0.11, 0.21], [0.21, 0.31], [0.31, 0.41]), \mathbb{RF}^3 = ([0.12, 0.22], [0.22, 0.32], [0.32, 0.42])$, and $\mathbb{RF}^4 = ([0.13, 0.23], [0.23, 0.33], [0.33, 0.43])$, and then by using the technique of the IVPFSSPROWA operator, we have

$$\begin{aligned}&\mathit{IVPFSSPROWA}(\mathbb{RF}^1, \mathbb{RF}^2, \mathbb{RF}^3, \mathbb{RF}^4)\\&\quad = ([0.1029, 0.2028], [0.2030, 0.3030], [0.3030, 0.4030]).\end{aligned}$$

Further, we discussed some properties of the above-initiated theory, such as the following.

Property 2: Let $\mathbb{RF}^j = ([\mathbb{TM}_j^-, \mathbb{TM}_j^+], [\mathbb{AM}_j^-, \mathbb{AM}_j^+], [\mathbb{FM}_j^-, \mathbb{FM}_j^+]), j = 1, 2, \ldots, \gamma\gamma$, be any collection of IVPFNs. Then

1) Idempotency: If $\mathbb{RF}^j = \mathbb{RF}.j = 1, 2, \ldots, \gamma\gamma$

$$\mathit{IVPFSSPROWA}(\mathbb{RF}^1, \mathbb{RF}^2, \ldots, \mathbb{RF}^{\gamma\gamma}) = \mathbb{RF}$$

2) Monotonicity: If $\mathbb{RF}^j \leq \mathbb{RF}^{\#j}$, then

$$\begin{aligned}&\mathit{IVPFSSPROWA}(\mathbb{RF}^1, \mathbb{RF}^2, \ldots, \mathbb{RF}^{\gamma\gamma})\\&\quad \leq \mathit{IVPFSSPROWA}(\mathbb{RF}^{\#}1, \mathbb{RF}^{\#2}, \ldots, \mathbb{RF}^{\#\gamma\gamma})\end{aligned}$$

3) Boundedness: If $\mathbb{RF}^- = \min\{\mathbb{RF}^j\}$ and $\mathbb{RF}^+ = \max\{\mathbb{RF}^j\}$, thus

$$\mathbb{RF}^- \leq \mathit{IVPFSSPROWA}(\mathbb{RF}^1, \mathbb{RF}^2, \ldots, \mathbb{RF}^{\gamma\gamma}) \leq \mathbb{RF}^+$$

Proof: Omitted.

Definition 9: Let $\mathbb{RF}^j = ([\mathbb{TM}_j^-, \mathbb{TM}_j^+], [\mathbb{AM}_j^-, \mathbb{AM}_j^+], [\mathbb{FM}_j^-, \mathbb{FM}_j^+]), j = 1, 2, \ldots, \gamma\gamma$, be any collection of IVPFNs. Then

$$\mathit{IVPFSSPROG}(\mathbb{RF}^1, \mathbb{RF}^2, \ldots, \mathbb{RF}^{\gamma\gamma}) = \otimes_{j=1}^{\gamma\gamma} (\mathbb{RF}^j)^{\frac{\Xi_j}{\sum_{j=1}^{\gamma\gamma} \Xi_j}}$$

called IVPFSSPROG operator, where $\Xi_1 = 1$, and $\Xi_j = \prod_{k=1}^{j-1} SC(\mathbb{RF}^k)$.

Theorem 3: Prove that the aggregated information of the IVPFSSPROG operator is again an IVPFN, such as

$$\mathit{IVPFSSPROG}(\mathbb{RF}^1, \mathbb{RF}^2, \ldots, \mathbb{RF}^{\gamma\gamma})$$
$$= \left(\begin{array}{c} \left[\begin{array}{c} \left(\sum_{j=1}^{\gamma\gamma} \frac{\Xi_j}{\sum_{j=1}^{\gamma\gamma} \Xi_j} \left(\mathbb{TM}_j^- \right)^{\mathbb{\Pi}} \right)^{\frac{1}{\mathbb{\Pi}}}, \\ \left(\sum_{j=1}^{\gamma\gamma} \frac{\Xi_j}{\sum_{j=1}^{\gamma\gamma} \Xi_j} \left(\mathbb{TM}_j^+ \right)^{\mathbb{\Pi}} \right)^{\frac{1}{\mathbb{\Pi}}} \end{array} \right], \\ \left[\begin{array}{c} 1 - \left(\sum_{j=1}^{\gamma\gamma} \frac{\Xi_j}{\sum_{j=1}^{\gamma\gamma} \Xi_j} \left(1 - \mathbb{AM}_j^- \right)^{\mathbb{\Pi}} \right)^{\frac{1}{\mathbb{\Pi}}}, \\ 1 - \left(\sum_{j=1}^{\gamma\gamma} \frac{\Xi_j}{\sum_{j=1}^{\gamma\gamma} \Xi_j} \left(1 - \mathbb{AM}_j^+ \right)^{\mathbb{\Pi}} \right)^{\frac{1}{\mathbb{\Pi}}} \end{array} \right], \\ \left[\begin{array}{c} 1 - \left(\sum_{j=1}^{\gamma\gamma} \frac{\Xi_j}{\sum_{j=1}^{\gamma\gamma} \Xi_j} \left(1 - \mathbb{FM}_j^- \right)^{\mathbb{\Pi}} \right)^{\frac{1}{\mathbb{\Pi}}}, \\ 1 - \left(\sum_{j=1}^{\gamma\gamma} \frac{\Xi_j}{\sum_{j=1}^{\gamma\gamma} \Xi_j} \left(1 - \mathbb{FM}_j^+ \right)^{\mathbb{\Pi}} \right)^{\frac{1}{\mathbb{\Pi}}} \end{array} \right] \end{array} \right)$$

Proof: Omitted.

Moreover, we aim to simplify the above technique with the help of some suitable examples; for this, we consider four attributes, such as $\mathbb{RF}^1 = ([0.1, 0.2], [0.2, 0.3], [0.3, 0.4]), \mathbb{RF}^2 = ([0.11, 0.21], [0.21, 0.31], [0.31, 0.41]), \mathbb{RF}^3 = ([0.12, 0.22], [0.22, 0.32], [0.32, 0.42])$, and $\mathbb{RF}^4 = ([0.13, 0.23], [0.23, 0.33], [0.33, 0.43])$, and then by using the technique of the IVPFSSPROG operator, we have

$$\begin{aligned}&\mathit{IVPFSSPROG}(\mathbb{RF}^1, \mathbb{RF}^2, \mathbb{RF}^3, \mathbb{RF}^4)\\&\quad = ([0.1025, 0.2023], [0.2022, 0.3022], [0.3022, 0.4022]).\end{aligned}$$

Further, we discussed some properties of the above-initiated theory, such as the following.

Property 3: Let $\mathbb{RF}^j = ([\mathbb{TM}_j^-, \mathbb{TM}_j^+], [\mathbb{AM}_j^-, \mathbb{AM}_j^+], [\mathbb{FM}_j^-, \mathbb{FM}_j^+]), j = 1, 2, \ldots, \gamma\gamma$, be any collection of IVPFNs. Then

1) Idempotency: If $\mathbb{RF}^j = \mathbb{RF}.j = 1, 2, \ldots, \gamma\gamma$

$$\mathit{IVPFSSPROG}(\mathbb{RF}^1, \mathbb{RF}^2, \ldots, \mathbb{RF}^{\gamma\gamma}) = \mathbb{RF}$$

2) Monotonicity: If $\mathbb{RF}^j \leq \mathbb{RF}\boxtimes^{j}$, then

$$\begin{aligned}&\mathit{IVPFSSPROG}(\mathbb{RF}^1, \mathbb{RF}^2, \ldots, \mathbb{RF}^{\gamma\gamma})\\&\quad \leq \mathit{IVPFSSPROG}(\mathbb{RF}^{\#1}, \mathbb{RF}^{\#2}, \ldots, \mathbb{RF}^{\#\gamma\gamma})\end{aligned}$$

3) Boundedness: If $\mathbb{RF}^- = \min\{\mathbb{RF}^j\}$ and $\mathbb{RF}^+ = \max\{\mathbb{RF}^j\}$, thus

$$\mathbb{RF}^- \leq \mathit{IVPFSSPROG}(\mathbb{RF}^1, \mathbb{RF}^2, \ldots, \mathbb{RF}^{\gamma\gamma}) \leq \mathbb{RF}^+$$

Proof: Omitted.

Definition 10: Let $\mathbb{RF}^j = ([\mathbb{TM}_j^-, \mathbb{TM}_j^+], [\mathbb{AM}_j^-, \mathbb{AM}_j^+], [\mathbb{FM}_j^-, \mathbb{FM}_j^+]), j = 1, 2, \ldots, \gamma\gamma$, be any collection of IVPFNs. Then

$$IVPFSSPROWG(\mathbb{RF}^1, \mathbb{RF}^2, \ldots, \mathbb{RF}^{\gamma\gamma}) = \otimes_{j=1}^{\gamma\gamma} (\mathbb{RF}^j)^{\frac{\sharp_j \Xi_j}{\sum_{j=1}^{\gamma\gamma} \sharp_j \Xi_j}}$$

called IVPFSSPROWG operator, where $\Xi_1 = 1$, and $\Xi_j = \prod_{k=1}^{j-1} SC(\mathbb{RF}^k)$.

Theorem 4: Prove that the aggregated information of the IVPFSSPROWG operator is again an IVPFN, such as the following.

$$IVPFSSPROWG(\mathbb{RF}^1, \mathbb{RF}^2, \ldots, \mathbb{RF}^{\gamma\gamma})$$
$$= \left(\begin{array}{c} \left[\left(\sum_{j=1}^{\gamma\gamma} \frac{\sharp_j \Xi_j}{\sum_{j=1}^{\gamma\gamma} \sharp_j \Xi_j} \left(\mathbb{TM}_j^- \right)^{\text{ll}} \right)^{\frac{1}{\text{ll}}}, \left(\sum_{j=1}^{\gamma\gamma} \frac{\sharp_j \Xi_j}{\sum_{j=1}^{\gamma\gamma} \sharp_j \Xi_j} \left(\mathbb{TM}_j^+ \right)^{\text{ll}} \right)^{\frac{1}{\text{ll}}} \right], \\ \left[1 - \left(\sum_{j=1}^{\gamma\gamma} \frac{\sharp_j \Xi_j}{\sum_{j=1}^{\gamma\gamma} \sharp_j \Xi_j} \left(1 - \mathbb{AM}_j^- \right)^{\text{ll}} \right)^{\frac{1}{\text{ll}}}, 1 - \left(\sum_{j=1}^{\gamma\gamma} \frac{\sharp_j \Xi_j}{\sum_{j=1}^{\gamma\gamma} \sharp_j \Xi_j} \left(1 - \mathbb{AM}_j^+ \right)^{\text{ll}} \right)^{\frac{1}{\text{ll}}} \right], \\ \left[1 - \left(\sum_{j=1}^{\gamma\gamma} \frac{\sharp_j \Xi_j}{\sum_{j=1}^{\gamma\gamma} \sharp_j \Xi_j} \left(1 - \mathbb{FM}_j^- \right)^{\text{ll}} \right)^{\frac{1}{\text{ll}}}, 1 - \left(\sum_{j=1}^{\gamma\gamma} \frac{\sharp_j \Xi_j}{\sum_{j=1}^{\gamma\gamma} \sharp_j \Xi_j} \left(1 - \mathbb{FM}_j^+ \right)^{\text{ll}} \right)^{\frac{1}{\text{ll}}} \right] \end{array} \right)$$

Proof: Omitted.

Moreover, we aim to simplify the above technique with the help of some suitable examples; for this, we consider four attributes, such as $\mathbb{RF}^1 = ([0.1, 0.2], [0.2, 0.3], [0.3, 0.4])$, $\mathbb{RF}^2 = ([0.11, 0.21], [0.21, 0.31], [0.31, 0.41])$, $\mathbb{RF}^3 = ([0.12, 0.22], [0.22, 0.32], [0.32, 0.42])$, and $\mathbb{RF}^4 = ([0.13, 0.23], [0.23, 0.33], [0.33, 0.43])$, and then by using the technique of the IVPFSSPROWG operator, we have

$$IVPFSSPROWG(\mathbb{RF}^1, \mathbb{RF}^2, \mathbb{RF}^3, \mathbb{RF}^4)$$
$$= ([0.1032, 0.2030], [0.2028, 0.3028], [0.3028, 0.4028]).$$

Further, we discussed some properties of the above-initiated theory, such as the following.

Property 4: Let $\mathbb{RF}^j = ([\mathbb{TM}_j^-, \mathbb{TM}_j^+], [\mathbb{AM}_j^-, \mathbb{AM}_j^+], \left[\mathbb{FM}_j^-, \mathbb{FM}_j^+\right]), j = 1, 2, \ldots, \gamma\gamma$, be any collection of IVPFNs. Then

1) Idempotency: If $\mathbb{RF}^j = \mathbb{RF}.j = 1, 2, \ldots, \gamma\gamma$

$$IVPFSSPROWG(\mathbb{RF}^1, \mathbb{RF}^2, \ldots, \mathbb{RF}^{\gamma\gamma}) = \mathbb{RF}$$

2) Monotonicity: If $\mathbb{RF}^j \leq \mathbb{RF}^{\sharp j}$, then

$$IVPFSSPROWG(\mathbb{RF}^1, \mathbb{RF}^2, \ldots, \mathbb{RF}^{\gamma\gamma})$$
$$\leq IVPFSSPROWG(\mathbb{RF}^{\sharp 1}, \mathbb{RF}^{\sharp 2}, \ldots, \mathbb{RF}^{\sharp \gamma\gamma})$$

3) Boundedness: If $\mathbb{RF}^- = \min\{\mathbb{RF}^j\}$ and $\mathbb{RF}^+ = \max\{\mathbb{RF}^j\}$, thus

$$\mathbb{RF}^- \leq IVPFSSPROWG(\mathbb{RF}^1, \mathbb{RF}^2, \ldots, \mathbb{RF}^{\gamma\gamma}) \leq \mathbb{RF}^+$$

Proof: Omitted.

4. MADM Method Based on Proposed Operators

In this section, we compute the supremacy and validity of the derived operators, called IVPFSSPWA operator and IVPFSSPWG operator, by using the technique of MADM problems to enhance the worth of the exposed information.

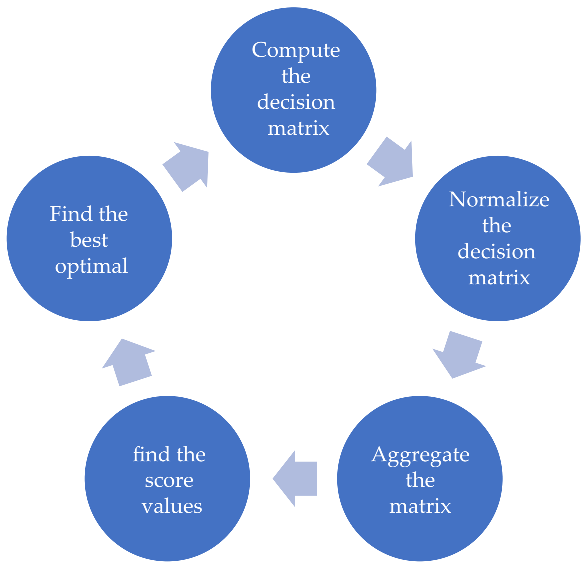
Consider the collection of alternatives $\mathbb{RF}^1, \mathbb{RF}^2, \ldots, \mathbb{RF}^{\gamma\gamma}$ with some attributes $A^1, A^2, \ldots, A^n$. Further, for each attribute, we have the collection of weight vector, where the order of weight vector and attributes must be equal, such as $\sharp_j \in [0, 1]$, with a strong condition $\sum_{j=1}^{n} \sharp_j = 1$. Additionally, we derive the matrix by including the IVPFNs, where $\mathbb{TM}$ represents the truth grade, $\mathbb{AM}$ represents the abstinence grade, and $\mathbb{FM}$ represents the falsity grade in the shape of interval value, such as $[\mathbb{TM}^-(\mathbb{X}), \mathbb{TM}^+(\mathbb{X})]$, $[\mathbb{AM}^-(\mathbb{X}), \mathbb{AM}^+(\mathbb{X})]$, and $[\mathbb{FM}^-(\mathbb{X}), \mathbb{FM}^+(\mathbb{X})]$ with a condition that is as follows: $0 \leq \mathbb{TM}^+(\mathbb{X}) + \mathbb{AM}^+(\mathbb{X}) + \mathbb{FM}^+(\mathbb{X}) \leq 1$. Further, the simple shape of IVPFN is fixed in the following shape: $\mathbb{RF}^j = ([\mathbb{TM}_j^-, \mathbb{TM}_j^+], [\mathbb{AM}_j^-, \mathbb{AM}_j^+], [\mathbb{FM}_j^-, \mathbb{FM}_j^+]), j = 1, 2, , \gamma\gamma$. The geometrical interpretation of the proposed theory is listed in Figure 1.

Finally, we compute the procedure of the MADM technique for evaluating the real-life problems, such as the following:

Step 1: Compute the decision matrix. Further, normalize the matrix if the matrix covers the cost type of information, such as

$$N = \begin{cases} \left(\begin{array}{c} \left[\mathbb{TM}_j^-, \mathbb{TM}_j^+\right], \left[\mathbb{AM}_j^-, \mathbb{AM}_j^+\right], \\ \left[\mathbb{FM}_j^-, \mathbb{FM}_j^+\right] \end{array} \right) & \textit{benefit} \\ \left(\begin{array}{c} \left[\mathbb{FM}_j^-, \mathbb{FM}_j^+\right], \left[\mathbb{AM}_j^-, \mathbb{AM}_j^+\right], \\ \left[\mathbb{TM}_j^-, \mathbb{TM}_j^+\right] \end{array} \right) & \textit{cost} \end{cases}$$

Figure 1
Geometrical representation of the proposed algorithm



Do not normalize the matrix if the matrix covers the benefit type of information.

Step 2: Aggregate the matrix according to the theory of IVPFSSPWA operator and IVPFSSPWG operator, such as

$$
\begin{aligned}
&IVPFSSPROA(\mathbb{RF}^{1},\mathbb{R}^{2},\ldots,\mathbb{RF}^{\gamma\gamma})\\
&=\begin{pmatrix}
\begin{bmatrix}
1-\left(\sum_{j=1}^{\gamma\gamma}\frac{\Xi_j}{\sum_{j=1}^{\gamma\gamma}\Xi_j}\left(1-\mathbb{TM}_j^{-}\right)^{\Pi}\right)^{\frac{1}{\Pi}},\\
1-\left(\sum_{j=1}^{\gamma\gamma}\frac{\Xi_j}{\sum_{j=1}^{\gamma\gamma}\Xi_j}\left(1-\mathbb{TM}_j^{+}\right)^{\Pi}\right)^{\frac{1}{\Pi}}
\end{bmatrix}\\
\left[\left(\sum_{j=1}^{\gamma\gamma}\frac{\Xi_j}{\sum_{j=1}^{\gamma\gamma}\Xi_j}\left(\mathbb{AM}_j^{-}\right)^{\Pi}\right)^{\frac{1}{\Pi}},\left(\sum_{j=1}^{\gamma\gamma}\frac{\Xi_j}{\sum_{j=1}^{\gamma\gamma}\Xi_j}\left(\mathbb{AM}_j^{+}\right)^{\Pi}\right)^{\frac{1}{\Pi}}\right],\\
\left[\left(\sum_{j=1}^{\gamma\gamma}\frac{\Xi_j}{\sum_{j=1}^{\gamma\gamma}\Xi_j}\left(\mathbb{FM}_j^{-}\right)^{\Pi}\right)^{\frac{1}{\Pi}},\left(\sum_{j=1}^{\gamma\gamma}\frac{\Xi_j}{\sum_{j=1}^{\gamma\gamma}\Xi_j}\left(\mathbb{FM}_j^{+}\right)^{\Pi}\right)^{\frac{1}{\Pi}}\right]
\end{pmatrix}
\end{aligned}
$$

$$
\begin{aligned}
&IVPFSSPROWA(\mathbb{RF}^{1},\mathbb{RF}^{2},\ldots,\mathbb{RF}^{\gamma\gamma})\\
&=\begin{pmatrix}
\begin{bmatrix}
1-\left(\sum_{j=1}^{\gamma\gamma}\frac{\#_j\Xi_j}{\sum_{j=1}^{\gamma\gamma}\#_j\Xi_j}\left(1-\mathbb{TM}_j^{-}\right)^{\Pi}\right)^{\frac{1}{\Pi}},\\
1-\left(\sum_{j=1}^{\gamma\gamma}\frac{\#_j\Xi_j}{\sum_{j=1}^{\gamma\gamma}\#_j\Xi_j}\left(1-\mathbb{TM}_j^{+}\right)^{\Pi}\right)^{\frac{1}{\Pi}}
\end{bmatrix},\\
\begin{bmatrix}
\left(\sum_{j=1}^{\gamma\gamma}\frac{\#_j\Xi_j}{\sum_{j=1}^{\gamma\gamma}\#_j\Xi_j}\left(\mathbb{AM}_j^{-}\right)^{\Pi}\right)^{\frac{1}{\Pi}},\\
\left(\sum_{j=1}^{\gamma\gamma}\frac{\#_j\Xi_j}{\sum_{j=1}^{\gamma\gamma}\#_j\Xi_j}\left(\mathbb{AM}_j^{+}\right)^{\Pi}\right)^{\frac{1}{\Pi}}
\end{bmatrix},\\
\begin{bmatrix}
\left(\sum_{j=1}^{\gamma\gamma}\frac{\#_j\Xi_j}{\sum_{j=1}^{\gamma\gamma}\#_j\Xi_j}\left(\mathbb{FM}_j^{-}\right)^{\Pi}\right)^{\frac{1}{\Pi}},\\
\left(\sum_{j=1}^{\gamma\gamma}\frac{\#_j\Xi_j}{\sum_{j=1}^{\gamma\gamma}\#_j\Xi_j}\left(\mathbb{FM}_j^{+}\right)^{\Pi}\right)^{\frac{1}{\Pi}}
\end{bmatrix}
\end{pmatrix}
\end{aligned}
$$

$$
\begin{aligned}
&IVPFSSPROG(\mathbb{RF}^{1},\mathbb{RF}^{2},\ldots,\mathbb{RF}^{\gamma\gamma})\\
&=\begin{pmatrix}
\begin{bmatrix}
\left(\sum_{j=1}^{\gamma\gamma}\frac{\Xi_j}{\sum_{j=1}^{\gamma\gamma}\Xi_j}\left(\mathbb{TM}_j^{-}\right)^{\Pi}\right)^{\frac{1}{\Pi}},\\
\left(\sum_{j=1}^{\gamma\gamma}\frac{\Xi_j}{\sum_{j=1}^{\gamma\gamma}\Xi_j}\left(\mathbb{TM}_j^{+}\right)^{\Pi}\right)^{\frac{1}{\Pi}}
\end{bmatrix},\\
\begin{bmatrix}
1-\left(\sum_{j=1}^{\gamma\gamma}\frac{\Xi_j}{\sum_{j=1}^{\gamma\gamma}\Xi_j}\left(1-\mathbb{AM}_j^{-}\right)^{\Pi}\right)^{\frac{1}{\Pi}},\\
1-\left(\sum_{j=1}^{\gamma\gamma}\frac{\Xi_j}{\sum_{j=1}^{\gamma\gamma}\Xi_j}\left(1-\mathbb{AM}_j^{+}\right)^{\Pi}\right)^{\frac{1}{\Pi}}
\end{bmatrix},\\
\begin{bmatrix}
1-\left(\sum_{j=1}^{\gamma\gamma}\frac{\Xi_j}{\sum_{j=1}^{\gamma\gamma}\Xi_j}\left(1-\mathbb{FM}_j^{-}\right)^{\Pi}\right)^{\frac{1}{\Pi}},\\
1-\left(\sum_{j=1}^{\gamma\gamma}\frac{\Xi_j}{\sum_{j=1}^{\gamma\gamma}\Xi_j}\left(1-\mathbb{FM}_j^{+}\right)^{\Pi}\right)^{\frac{1}{\Pi}}
\end{bmatrix}
\end{pmatrix}
\end{aligned}
$$

$$
\begin{aligned}
&IVPFSSPROWG(\mathbb{RF}^{1},\mathbb{RF}^{2},\ldots,\mathbb{RF}^{\gamma\gamma})\\
&=\begin{pmatrix}
\begin{bmatrix}
\left(\sum_{j=1}^{\gamma\gamma}\frac{\#_j\Xi_j}{\sum_{j=1}^{\gamma\gamma}\#_j\Xi_j}\left(\mathbb{TM}_j^{-}\right)^{\Pi}\right)^{\frac{1}{\Pi}},\\
\left(\sum_{j=1}^{\gamma\gamma}\frac{\#\Xi_j}{\sum_{j=1}^{\gamma\gamma}\#_j\Xi_j}\left(\mathbb{TM}_j^{+}\right)^{\Pi}\right)^{\frac{1}{\Pi}}
\end{bmatrix},\\
\begin{bmatrix}
1-\left(\sum_{j=1}^{\gamma\gamma}\frac{\#_j\Xi_j}{\sum_{j=1}^{\gamma\gamma}\#_j\Xi_j}\left(1-\mathbb{AM}_j^{-}\right)^{\Pi}\right)^{\frac{1}{\Pi}},\\
1-\left(\sum_{j=1}^{\gamma\gamma}\frac{\#_j\Xi_j}{\sum_{j=1}^{\gamma\gamma}\#_j\Xi_j}\left(1-\mathbb{AM}_j^{+}\right)^{\Pi}\right)^{\frac{1}{\Pi}}
\end{bmatrix},\\
\begin{bmatrix}
1-\left(\sum_{j=1}^{\gamma\gamma}\frac{\#_j\Xi_j}{\sum_{j=1}^{\gamma\gamma}\#_j\Xi_j}\left(1-\mathbb{FM}_j^{-}\right)^{\Pi}\right)^{\frac{1}{\Pi}},\\
1-\left(\sum_{j=1}^{\gamma\gamma}\frac{\#_j\Xi_j}{\sum_{j=1}^{\gamma\gamma}\#_j\Xi_j}\left(1-\mathbb{FM}_j^{+}\right)^{\Pi}\right)^{\frac{1}{\Pi}}
\end{bmatrix}
\end{pmatrix}
\end{aligned}
$$

Step 3: Calculate the score values, such as

$$
\begin{aligned}
SC(\mathbb{RF}^{j}) &= \frac{\left(\mathbb{TM}_j^{-}+\mathbb{TM}_j^{+}+\mathbb{AM}_j^{-}+\mathbb{AM}_j^{+}-\mathbb{FM}_j^{-}-\mathbb{FM}_j^{+}\right)}{3}\\
&\in[-1,1]
\end{aligned}
$$

Step 4: Rank all alternatives and find the best one.

Based on the above procedure, we aim to evaluate the problem of digital green innovation based on the proposed supply chain management to enhance the worth and effectiveness of the proposed information.

5. Analysis of Digital Green Innovation Based on Proposed Supply Chain Management

In this section, we discuss the problem of digital green innovation based on the proposed theory, called IVPFSSPROWA operator and IVPFSSPROWG operator, to enhance the worth of the proposed theory. The main theme of this application is to find the best and worst aspects among the collection of five major aspects of digital green innovation. For this, we consider the following five alternatives, such as

1) Precision agriculture "$\mathbb{RF}^{1}$."
2) Smart farming practices "$\mathbb{RF}^{2}$."
3) Environmental monitoring "$\mathbb{RF}^{3}$."
4) Blockchain in agriculture "$\mathbb{RF}^{4}$."
5) Climate-smart agriculture "$\mathbb{RF}^{5}$."

Further, we have the following weight vector $(0.2, 0.3, 0.2, 0.3)^{T}$ for each criterion, where the criteria are followed as growth analysis, social impact, political impact, and environmental impact. Finally, we compute the procedure of the MADM technique for evaluating the real-life problems, such as the following:

Step 1: Compute the decision matrix; see Table 1. Further, normalize the matrix if the matrix covers cost type of information, such as

$$N = \begin{cases} \left(\left[\mathrm{TM}_j^-, \mathrm{TM}_j^+\right], \left[\mathrm{AM}_j^-, \mathrm{AM}_j^+\right], \left[\mathrm{FM}_j^-, \mathrm{FM}_j^+\right]\right) \textit{benefit} \\ \left(\left[\mathrm{FM}_j^-, \mathrm{FM}_j^+\right], \left[\mathrm{AM}_j^-, \mathrm{AM}_j^+\right], \left[\mathrm{TM}_j^-, \mathrm{TM}_j^+\right]\right) \textit{cost} \end{cases}$$

Do not normalize the matrix if the matrix covers the benefit type of information. In Table 1, we have the benefit type of data.

Step 2: Aggregate the matrix according to the theory of IVPFSSPWA operator and IVPFSSPWG operator; see Table 2.

Step 3: Calculate the score values; see Table 3.

Step 4: Rank all alternatives and find the best one; see Table 4.

The best optimal is $\mathbb{RF}^2$ based on the IVPFSSPWA operator and IVPFSSPWG operator. Further, we simplify the supremacy and validity of the proposed operators by making a comparison between proposed and existing techniques.

6. Comparative Analysis

In this section, we discuss the comparison between proposed and existing techniques by using the information in Table 1. For comparison, we have needed some existing techniques, for instance, Schweizer–Sklar power operators for IVIFSs were derived by Liu and Wang (2018). Mahmood et al. (2023) exposed the Frank operators and AHP technique for IVPFSs. Moreover, Kamaci et al. (2021) designed the dynamic operators and Einstein operators for interval-valued picture hesitant FSs. Additionally, Jabeen et al. (2023) exposed the Aczel–Alsina operators for IVPFSs. Further, Garg et al. (2021) initiated the interval-valued picture of uncertain linguistic generalized Hamacher operators and their application in decision support systems. Further, the prioritized operators based on classical set theory were proposed by Yager (2008). Therefore, using the information in Table 1, the comparison techniques are stated in Table 5.

The best optimal is $\mathbb{RF}^2$ based on the IVPFSSPWA operator and IVPFSSPWG operator. Further, the existing techniques are also given the same ranking techniques, but the existing techniques,

Table 1
IVPF decision matrix

	A^1	A^2	A^3	A^4
$\mathbb{RF}^1$	([0.1, 0.2], [0.2, 03], [0.3, 0.4])	([0.11, 0.21], [0.21, 0.31], [0.31, 0.41])	([0.12, 0.22], [0.22, 0.32], [0.32, 0.42])	([0.13, 0.23], [0.23, 0.33], [0.33, 0.43])
$\mathbb{RF}^2$	([0.2, 0.4], [0.1, 0.2], [0.1, 0.2])	([0.21, 0.41], [0.11, 0.21], [0.11, 0.21])	([0.22, 0.42], [0.12, 0.22], [0.12, 0.22])	([0.23, 0.43], [0.13, 0.23], [0.13, 0.23])
$\mathbb{RF}^3$	([0.2, 0.3], [0.2, 0.3], [0.1, 0.2])	([0.21, 0.31], [0.21, 0.31], [0.11, 0.12])	([0.22, 0.32], [0.22, 0.32], [0.12, 0.22])	([0.23, 0.33], [0.23, 0.33], [0.13, 0.23])
$\mathbb{RF}^4$	([0.3, 0.4], [0.2, 0.3], [0.1, 0.2])	([0.31, 0.41], [0.21, 0.31], [0.11, 0.21])	([0.32, 0.42], [0.22, 0.32], [0.12, 0.22])	([0.33, 0.43], [0.23, 0.33], [0.13, 0.23])
$\mathbb{RF}^5$	([0.1, 0.2], [0.1, 0.2], [0.1, 0.2])	([0.11, 0.21], [0.11, 0.21], [0.11, 0.21])	([0.12, 0.22], [0.12, 0.22], [0.12, 0.22])	([0.13, 0.23], [0.13, 0.23], [0.13, 0.23])

Table 2
IVPF aggregated values

	IVPFSSPROWA operator	IVPFSSPROWG operator
$\mathbb{RF}^1$	([0.1029, 0.2028], [0.2030, 0.3030], [0.3030, 0.4030])	([0.1032, 0.2030], [0.2028, 0.3028], [0.3028, 0.4028]).
$\mathbb{RF}^2$	([0.2090, 0.4090], [0.1097, 0.2094], [0.1097, 0.2094])	([0.2094, 0.4093], [0.1090, 0.2090], [0.1090, 0.2090])
$\mathbb{RF}^3$	([0.2098, 0.3098], [0.2102, 0.3101], [0.1105, 0.2102])	([0.2102, 0.3101], [0.2098, 0.3098], [0.1098, 0.2098])
$\mathbb{RF}^4$	([0.311, 0.4109], [0.2113, 0.3113], [0.1117, 0.2114])	([0.3113, 0.4112], [0.2110, 0.311], [0.1110, 0.2110])
$\mathbb{RF}^5$	([0.1060, 0.2060], [0.1066, 0.2063], [0.1066, 0.2063])	([0.1066, 0.2063], [0.1060, 0.2060], [0.1060, 0.2060])

Table 3
IVPF score information

	IVPFSSPROWA operator	IVPFSSPROWG operator
$\mathbb{RF}^1$	−0.30213	−0.30174
$\mathbb{RF}^2$	.0068	−0.00584
$\mathbb{RF}^3$	−0.10717	−0.1063
$\mathbb{RF}^4$	−0.04133	−0.04048
$\mathbb{RF}^5$	−0.10465	−0.10373

Table 4
Representation of the ranking values

Methods	Ranking values	Best optimal
IVPFSSPWA	$\mathbb{RF}^2 > \mathbb{RF}^4 > \mathbb{RF}^5 > \mathbb{RF}^3 > \mathbb{RF}^1$	$\mathbb{RF}^2$
IVPFSSPWG	$\mathbb{RF}^2 > \mathbb{RF}^4 > \mathbb{RF}^5 > \mathbb{RF}^3 > \mathbb{RF}^1$	$\mathbb{RF}^2$

Table 5
Comparative information of the data in Table 1

Methods	Ranking techniques	Best optimal
Liu and Wang (2018)	$\mathbb{RF}^2 > \mathbb{RF}^4 > \mathbb{RF}^5 > \mathbb{RF}^3 > \mathbb{RF}^1$	$\mathbb{RF}^2$
Mahmood et al. (2023)	$\mathbb{RF}^2 > \mathbb{RF}^4 > \mathbb{RF}^5 > \mathbb{RF}^3 > \mathbb{RF}^1$	$\mathbb{RF}^2$
Kamaci et al. (2021)	$\mathbb{RF}^2 > \mathbb{RF}^4 > \mathbb{RF}^5 > \mathbb{RF}^3 > \mathbb{RF}^1$	$\mathbb{RF}^2$
Jabeen et al. (2023)	$\mathbb{RF}^2 > \mathbb{RF}^4 > \mathbb{RF}^5 > \mathbb{RF}^3 > \mathbb{RF}^1$	$\mathbb{RF}^2$
Garg et al. (2021)	Limited features	Limited features
Yager (2008)	Limited features	Limited features
IVPFSSPWA	$\mathbb{RF}^2 > \mathbb{RF}^4 > \mathbb{RF}^5 > \mathbb{RF}^3 > \mathbb{RF}^1$	$\mathbb{RF}^2$
IVPFSSPWG	$\mathbb{RF}^2 > \mathbb{RF}^4 > \mathbb{RF}^5 > \mathbb{RF}^3 > \mathbb{RF}^1$	$\mathbb{RF}^2$

which were proposed by Garg et al. (2021) and Yager (2008), have not been working effectively because these are the special cases of the initiated techniques. Hence, thewhihc initiated techniques are novel and superior than existing information.

7. Conclusion

In this manuscript, we derived the technique of Schweizer–Sklar operational laws based on IVPF values. Further, we investigated the IVPFSSPA operator, IVPFSSPG operator, IVPFSSPWA operator, and IVPFSSPWG operator with some reliable properties, called idempotency, monotonicity, and boundedness. Additionally, we analyzed the digital green innovation with the help of the MADM technique for initiated operators to show the reliability and supremacy of the proposed theory. Finally, we demonstrated examples for addressing the comparison among the initiated theory and existing ideas to improve the worth of the derived theory. The initiated technique based on IVPFSs is very dominant and reliable for depicting vague and uncertain information in genuine life problems.

Further, the technique of IVPFSs is superior and dominant than many existing techniques, but in some situations, they fail, for instance, if someone provides the value of truth, abstinence, and falsity grades in the shape of collection instead of one value, then the theory of IVPFS has failed; for this, we needed to use the technique of interval-valued picture hesitant FSs and their extensions.

In the future, we aim to utilize some operators, called Dombi operators (Akram et al., 2021; Yu & Xu, 2013), Hamacher operators (Chen et al., 2014), and Einstein operators (Rahman et al., 2020) based on IVPFSs, and also discuss their application in neural networks, decision-making, genetic algorithms (Sahoo et al., 2023), supply chain management (Adegbola, 2023; Verma et al., 2023), and engineering sciences to enhance the worth of the initiated techniques.

Ethical Statement

This study does not contain any studies with human or animal subjects performed by any of the authors.

Conflicts of Interest

The authors declare that they have no conflicts of interest to this work.

Data Availability Statement

Data are available from the corresponding author upon reasonable request.

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