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RESEARCH ARTICLE

A Combined Use of Soft Sets and Grey Numbers in Decision Making

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Abstract: Decision making is the process of evaluating multiple alternatives to choose the one satisfying in the best way the required goals. The classical methods for decision making cannot always be applied due to the various forms of existing uncertainty in the corresponding problems. As a result, various methods for decision making under fuzzy conditions have been proposed. In this work, a new parametric decision-making method is introduced using a combination of soft sets and grey numbers as tools, which modifies a method with soft sets proposed by Maji, Roy, and Biswas. The importance of this modification is illustrated by an application concerning the decision for the purchase of a car satisfying in the best possible way the goals put by the candidate buyer.

Keywords: decision making, fuzzy logic, soft sets, grey numbers, intelligent computing, soft computing

1. Introduction

The types of uncertainty that frequently exists in real-word situations, in everyday life and in science, depend on the form of the corresponding problems. Thus, uncertainties may appear due to randomness, imprecise data, vague information, etc. (Klir, 1995). Probability has been proved sufficient for tackling the cases of uncertainty which are due to randomness, whereas several other tools have been proposed during the last 50–60 years for tackling effectively the other forms of uncertainty. Those tools include fuzzy sets (Zadeh, 1965), intuitionistic fuzzy sets (Atanassov, 1986), neutrosophic sets (Smarandache, 1998), rough sets (Pawlak, 1991), soft sets (Molodtsov, 1999), grey systems (Deng, 1982), and several others (Dubois & Prade, 2005; Mendel, 2001; Ramot et al., 2002; Torra & Narukawa, 2009; Yager, 2013, etc.).

Decision making is the process of evaluating, with the help of suitable criteria, multiple alternatives to choose the one satisfying in the best way the required goals. In decision making under fuzzy conditions, the boundaries of the goals and/or of the constraints are not sharply defined. Several methods for decision making under fuzzy conditions have been proposed (Alcantud, 2020; Bellman & Zadeh, 1970; Fahad et al., 2021, Khan et al., 2022, Zhu & Ren, 2022, etc.). Maji et al. (2002) used a tabular form of soft sets in the form of a binary matrix to introduce a type of parametric decision-making method. Their method, however, replaces the characterizations of the elements of the set of the discourse (houses in their example) by the corresponding parameters (e.g., beautiful) with the binary elements (truth values) 0, 1. In other words, although their method starts from a fuzzy basis (soft sets), it uses bivalent logic for making the required decision (e.g., beautiful or not beautiful)! Consequently, this methodology could lead to a wrong decision, when some (or all) of the parameters have not a bivalent texture, for example, the parameter "wooden" characterizing a house has a bivalent texture but not the parameter "beautiful."

In this work, in order to tackle this problem, we modify the method of Maji et al. (2002) by using grey numbers instead of the binary elements 0, 1. The rest of the paper is organized as follows: in Section 2, the basic information about grey numbers and soft sets is given which is necessary for the understanding of the paper. In Section 3, the modified decision-making method is presented and compared with the method of Maji et al. (2002) through a suitable example. The paper closes with the final conclusions and some hints for future research contained in Section 4.

2. Preliminaries

2.1. Grey numbers

The existing in the contemporary world complexity makes necessary the frequent use of approximate data for describing the function of a system. Fuzzy sets and the related to them generalizations or alternative theories mentioned in Section 1 are usually the tools used by the specialists to handle such kind of data. The grey system theory, introduced by Deng (1982), which uses the grey numbers as tools for calculations, is one of the most popular methods for this purpose. It is recalled that any system that lacks proper information about its structure and/or its other characteristics can be characterized as a grey system. The grey systems theory has found already various practical applications (Deng, 1989).

A *grey number* is understood to be a number whose position is unknown within its known boundaries. The natural way to represent

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a grey number is by using real intervals. In fact, if x and y are the lower and upper boundaries, respectively, of a grey number G, then we can write $G \in [x, y]$.

The grey number G, however, may differ from the real interval $[x, y]$ by defining a *whitenization function* f: $[x, y] \rightarrow [0, 1]$, such that the closer is $f(g)$ to 1, the better $g \in [x, y]$ approximates G. If no such function has been defined, we usually consider as the crisp representative of G the real number

$$W(G) = \frac{x+y}{2} \tag{1}$$

The basic arithmetic operations between grey numbers (Liu & Lin, 2010) are defined with the help of the arithmetic of the real intervals (Moore et al., 2009). In this paper, we make use only of the addition of grey numbers, which is defined as follows:

Let $G_1 \in [x_1, y_1]$ and $G \in [x_2, y_2]$ be two grey numbers, then their sum is the grey number

$$G_1 + G_2 \in [x_1 + y_1,\ x_2 + y_2] \tag{2}$$

The sum of n grey numbers, with n a positive integer, $n \geq 2$, is defined inductively with the help of (2). Further, the scalar product rG_1, with r a positive number, is equal to the grey number

$$rG_1 \in [rx_1,\ ry_1] \tag{3}$$

In earlier works, the present author has used grey numbers as tools for assessment under fuzzy conditions (e.g. Voskoglou, 2019, Section 6.3).

2.2. Soft sets

Molodtsov (1999) introduced the concept of *soft set* for tackling the uncertainty created by a set of parameters characterizing the elements of the set of the discourse.

Let P be a set of parameters, let Q be a subset of P, and let g: $Q \rightarrow \Delta(V)$ be a map from Q to the power set $\Delta(V)$ of the universal set V. Then the soft set on V defined with the help of g and Q and denoted by (g, Q) is the set of ordered pairs:

$$(g,\ Q) = \{(q,\ g(q)) : q \in Q\} \tag{4}$$

The characterization "soft" is related to the fact that the form of the set (g, Q) depends on the parameters.

For example, let $V = \{C_1, C_2, C_3, C_4, C_5, C_6\}$ be a set of cars and let $P = \{p_1, p_2, p_3, p_4, p_5\}$ be the set of the parameters p_1 = high speed, p_2 = automatic (gear-box), p_3 = hybrid (petrol and electric power), $p_4 = 4 \times 4$, and p_5 = cheap. Consider the subset $Q = \{p_1, p_2, p_3, p_5\}$ of P and assume that C_1, C_2, C_6 are the high speed, C_2, C_3, C_5, C_6 are the automatic, C_3, C_5 are the hybrid cars, and C_4 is the unique cheap car. Then a map g: $Q \rightarrow \Delta(V)$ is defined by $g(p_1) = \{C_1, C_2, C_6\}$, $g(p_2) = \{C_2, C_3, C_5, C_6\}$, $g(p_3) = \{C_3, C_5\}$, $g(p_5) = \{C_4\}$ and the soft set

$$(g, Q) = \{(p_1, \{C_1, C_2, C_6\}), (p_2, \{C_2, C_3, C_5, C_6\}), (p_3, \{C_3, C_5\}), (p_5, \{C_4\})\} \tag{5}$$

A fuzzy set on V, with membership function $y = \mu(x)$, is also a soft set $(g, [0, 1])$ on V with $g(a)=\{x \in V: \mu(x) \geq a\}$, for each a in [0, 1]. The use of soft sets instead of fuzzy sets has, among others, the advantage that, by using the parameters, one overpasses the existing difficulty of defining properly the membership function of a fuzzy set. In fact, the definition of the membership function is not unique, depending on the observer's subjective criteria (Klir & Folger, 1988). In case of determining the fuzzy set of the cheap houses under sale in a certain area of a city, for example, the membership function could be defined in several ways, according to the financial power of each candidate buyer.

Table 1
Tabular representation of the soft set (*g*, *Q*)

	p_1	p_2	p_3	p_5
C_1	1	0	0	0
C_2	1	1	0	0
C_3	0	1	1	0
C_4	0	0	0	1
C_5	0	1	1	0
C_6	1	1	0	0

Maji et al. (2002) introduced a tabular representation of soft sets in the form of a binary matrix in order to be stored easily in a computer's memory. For example, the tabular representation of the soft set (5) is given in Table 1.

Soft sets have found important applications to several sectors of the human activity like decision making, parameter reduction, data clustering and data dealing with incompleteness, etc. (Tripathy & Arun, 2016). One of the most important steps for the theory of soft sets was to define mappings on soft sets, which was achieved by Kharal and Ahmad (2011) and was applied to the problem of medical diagnosis in medical expert systems. But fuzzy mathematics has also significantly developed at the theoretical level providing important insights even into branches of classical mathematics like algebra, analysis, geometry, topology, etc. For example, one can extend the concept of topological space, the most general category of mathematical spaces, to fuzzy structures and in particular can define soft topological spaces and generalize the concepts of convergence, continuity, and compactness within such kind of spaces (Shabir & Naz, 2011). The present author has recently used them as tools in assessment problems (Voskoglou, 2022).

3. The Hybrid Decision-Making Method

3.1. The method of Maji et al. (2002)

Maji et al. (2002) used the tabular form of a soft set described in Section 2.2 as a tool for decision making in a parametric manner. Here, we use the example of Section 2.2 to highlight their method.

For this, assume that one is interested to buy a high-speed, automatic, hybrid, and cheap car by choosing among the six cars of the previous example. Forming the tabular representation of the soft set (g, Q) (Table 1) the *choice value* of each car is calculated by adding the binary elements of the corresponding row of it. The cars C_1 and C_4 have, therefore, choice value 1 and all the others have choice value 2. Consequently, the buyer must choose one of the cars C_2, C_3, C_5, or C_6, which is not a very helpful decision.

3.2. The new method

Observe now that the parameters p_1 and p_5 have not a bivalent texture. In fact, the high speed (or not) of a car depends on the subjective opinion of each person, whereas its low or high price depends on the financial ability of the buyer. In other words, the previous decision could not be the best one.

Table 2
Revised tabular representation of the soft set (g, Q)

	p_1	p_2	p_3	p_5
C_1	G_1	0	0	G_3
C_2	G_1	1	0	G_5
C_3	G_3	1	1	G_3
C_4	G_4	0	0	G_1
C_5	G_4	1	1	G_3
C_6	G_1	1	0	G_4

To tackle this problem, we replace in Table 1 the binary elements 0, 1 for the parameters p_1 and p_5 by the grey numbers $G_1 \in [0.85, 1]$, $G_2 \in [0.75, 0.84]$, $G_3 \in [0.6, 0.74]$, $G_4 \in [0.5, 0.59]$, and $G_5 \in [0, 0.49]$.

Assume further that the candidate buyer, after studying the existing information about the six cars, decided to use Table 2 for making the final decision.

From Table 2, we calculate the choice value V_i of the car C_i, $i = 1, 2, 3, 4, 5, 6$ as follows:

$V_1 = W(G_1 + G_3)$, or by (2) $V_1 = W([0.85 + 0.6, 1 + 0.74])$ and finally by (1) $V_1 = \frac{1.45+1.74}{2} = 1.595$. Similarly $V_2 = 1 + W(G_1 + G_5) = 1 + \frac{0.85+1.49}{2} = 2.17$, $V_3 = 2 + W(G_3 + G_3) = 3.34$, $V_4 = W(G_4 + G_1) = 1.47$, $V_5 = 2 + W(G_4 + G_1) = 3.215$, $V_6 = 1 + W(G_1 + G_4) = 2.47$.

Therefore, the right decision is to buy the car C_3.

3.3. Remarks

(I) The choice of the grey numbers G_i, $i = 1, 2, 2, 4, 5$, was made according to the accepted standards for the linguistic grades excellent (A), very good (B), good (C), almost good (D), and unsatisfactory (F). Their choice however is not unique and could slightly differ from case to case according to the goals of the decision maker.

(II) If a parameter takes the value 0 (or 1) for all elements of the set of the discourse, then it can be withdrawn from the tabular form of the corresponding soft set, since it does not affect the final decision. In our example, for instance, this could happen, if the parameter p_4 was included in the soft set and the candidate buyer was not interested (or was interested) to buy a 4 × 4 car. The resulting table in those cases is called a *reduct table* of the soft set. When all the parameters having the previous property have been withdrawn, then the resulting reduct table is called the *core* of the soft set (Maji et al., 2002). Obviously, the core of a soft set is contained in all reduct tables of it.

(III) An analogous to the previous one decision-making method could be introduced by using *triangular fuzzy numbers* (Voskoglou, 2019, Section 5.1) instead of grey numbers.

3.4. Weighted decision making

Frequently, the goals put by a decision maker have not the same importance. In this case, and in order to make the proper decision, weight coefficients must be assigned to each parameter.

For instance, assume that in the previous example the candidate car buyer has assigned the weight coefficients 0.9 to p_1, 0.7 to p_2, 0.6 to p_3, and 0.5 to the parameter p_5. Then, the weighted choice values of the cars are calculated with the help of Table 2 as follows:

$V_1 = W(0.9G_1 + 0.5G_3)$, or by (2) and (3) $V_1 = W([1.65, 1.27]) = 1.46$. Similarly $V_2 = 0.7 + W(0.9G_1 + 0.5G_5) = 0.7 + W([0.765, 1.145]) = 1.655$, $V_3 = 0.7 + 0.6 + W(0.9G_3 + 0.5G_3) = 1.3 + W([0.84, 1.036]) = 2.238$, $V_4 = W(0.9G_4 + 0.5G_1) = W([0.875, 1.031]) = 0.953$, $V_5 = 0.7 + 0.6 + W(0.9G_4 + 0.5G_3) = 1.3 + W([0.75, 0.901]) = 2.1255$, $V_6 = 0.7 + W(0.9G_1 + 0.5G_4) = 0.7 + W([1.015, 1.195]) = 1.805$.

Consequently, the right decision is again to buy the car C_3.

4. Discussion and Conclusions

In this work, a parametric decision-making method was introduced by using grey numbers and soft sets as tools. As it became evident from the given example, this hybrid approach is very useful in cases where one or more parameters have not a bivalent texture (fast and cheap cars in our example), because it enables one to make the best decision, which is frequently impossible to be succeeded by using only soft sets, as the method of Maji et al. (2002) suggests.

The combination of two or more of the theories that have been developed for tackling efficiently the various forms of the existing in real world, everyday life, and science uncertainty, appears in general to be an effective tool for obtaining better results, not only for decision making but also for assessment and other human activities. Consequently, this is a promising area for future research.

Conflicts of Interest

The author declares that he has no conflicts of interest to this work.

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