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RESEARCH ARTICLE

Reliability Analysis of Multi-Hardware–Software System with Failure Interaction

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Abstract: The effect of hardware and software plays a critical role in achieving high device reliability. Failure to communicate with hardware and software can be responsible for a short life and low system operation. The failure of the corresponding hardware is one factor contributing to the program's failure and vice versa. Hardware and software reliability research activities and failure encounters are being undertaken, and hardware and software reliability models are being proposed. The reliability study of a multi-hardware–software system with hardware–software–hardware, hardware–software–software, software–hardware–software, and software–hardware–software failure interaction receives less attention. This paper analyzes the reliability of a multi-hardware–software system whose failure is classified into hardware–software–hardware, hardware–software–software, software–hardware–software, and software–hardware–hardware. The failure and repair time of the running hardware–software and standby hardware–software is assumed to be exponentially distributed. Differential difference equations are built and resolved to derive explicit formulations for profit, mean time to failure, and steady-state availability. In addition, some significant results are presented in the graphs and tables.

Key words: reliability, availability, mean time to failure, hardware, software, failure interaction

1. Introduction

Communication devices have been utilized in various settings, including educational institutions, banks, military systems, aviation systems, and industrial settings, to simplify data storage, distribution, interchange, and dissemination to various regions, departments, units, and sections. Devices undertaking such tasks are systems themselves which are bound to fail. Such systems are seen as distributed systems. A distributed system is made up of a network of hardware and software elements. The distributed system is a cluster of computers in which each client can assist in executing various functions. A computer network and a middleware distribution connect the devices that make up a distributed system. These devices aid in delivering high-performance services, performance guarantees, fault tolerance, and security to the system. Each distributed system has applications operating on multiple computers connected to the network, which have become highly complex and challenging to master.

Reliability is one of the essential concerns for users and service providers in distributed system environments (Ahmed & Wu, 2013). This is evident in recent times when systems are susceptible to failure due to several factors, such as security, software, hardware, and human. A critical activity in reliability engineering is the assessment of the accuracy and availability of the established system or product. Suppose there is a system failure, resources and time will be lost, and there could be a tragedy. Components are designed to be highly dependable and durable, in the sense that they rarely fail unexpectedly, allowing for reliable system functioning. The engineers use the redundant strategy to ensure the system's reliability and availability or enhance those features. As a result of this, over the last few decades, numerous standby systems have been developed and evaluated.

In general, the redundancy approach can be in two good ways: active and up-to-date. All units are inactive redundancy simultaneously, with one of the redundant units switching to standby redundancy automatically if the functional unit fails. Standby redundancy, cold and warm, is divided into two parts. However, some systems and subsystems are prone to failure due to incorrect production methods, wrong design, inadequate maintenance, unskilled operational expertise, overload, maintenance delays, and even human error. Systems are expected to run failure free over time for efficient operation and achieve high performance and good quality. A detailed behavioral analysis of the system and empiric maintenance would be beneficial.

Many scholars have presented their work and efforts to improve system efficiency by designing complicated repairable structures that can withstand various types of failure and repair delivery distributions. Gahlot et al. (2018) examined the dependability of a complex system made up of two subsystems, subsystem-1 and

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subsystem-2, in a series configuration under the policies of 2-out-of-3: F and 1-out-of-2: G; copula and general repair policies; and full and partial failure types. Gahlot et al. (2020) investigated the performance of a complex system with two subsystems in sequence under the 2-out-of-3: G and 1-out-of-2: G policies. A human operator is attached to the system to keep it up and to run, and the system entirely fails due to human error. Chopra and Ram (2019) study the system's availability and reliability in a parallel network with two distinct units under copula. Ram and Goyal (2018) developed a stochastic model that included repair impact, failure modes, time trend variation, and coverage factor. Sha (2021) investigated the working unit dependency using Clayton copula functions, and Farlie–Gumbel–Morgenstern established models for parallel–series and series–parallel. Also, an investigation of the dependability of complex systems controlled by a human operator, with three units operating under the super-priority, priority, and ordinary policies, and a preemptive resume repair policy has been reported. Yusuf et al. (2020) have shown the effectiveness of a multi-computer system consisting of three subsystems in series using the Copula repair technique. Singh et al. (2020) provided an improved model for detecting flaws in previously published models and evaluating performance for various types of failure and repair, claiming that the system performs better than once assessed systems. The Copula technique has been studied to achieve a probabilistic assessment of a complex repairable system with two subsystems arranged in series, multiple types of failure, and two types of repairs.

However, many hardware and software reliability models have been proposed to tackle hardware and software failures in recent decades. For instance, Garg (2019) provided a method for resolving heterogeneity in server-client systems utilizing Remote Procedure Call (RPC). Potapov et al. (2019) used the Markov process to construct mathematical models for analyzing a system with a client-server architecture. The dependability characteristics of a computer network system comprised of load balancers, distributed database servers, and a centralized server arranged as a series–parallel system with three subsystems are discussed in Yusuf et al. (2020). With an architecture-based self-adaptive framework, Modibbo et al. (2021) proposed two concepts for estimating reliability functions based on Maximum Likelihood Estimators (MLE) and Uniformly Minimum Variance Unbiased Estimators (UMVUE) techniques. They used the proposed method to estimate some lifetime distribution parameters and employed the optimization technique to solve an engineering problem relating to system component cost optimization.

Similarly, Khan et al. (2022) applied a nonlinear optimization model to the reliability theory's bi-level selective maintenance allocation problem. Osemwengie et al. (2022) designed a cost-efficient network for running computer internet by optimizing the computer hardware and solar panel introduction. It was achieved by using more wireless access points and the lesser running cost of different computer topologies. Sanusi and Yusuf (2022) studied the resilience of a dispersed data center network topology with three components. Kumar and Lather (2018) used existing data to assess the reliability of a robotic system. Yi et al. (2017) proposed a new form of reliability analysis for repairable systems based on a goal-oriented methodology with multifunction modes. For accelerated life tests, Rodríguez-Borbón et al. (2017) studied reliability estimation based on a Cox proportional hazard model with an error effect. Zhu and Pham (2019) proposed a novel system reliability modeling of hardware, software, and hardware and software interactions. Zeng et al. (2019) studied the empirical approach of hardware–software co-design device reliability analysis. Zhu and Pham (2018) presented a model of software reliability integrating the martingale method with environmental factors distributed by gamma. Huang et al. (2011) presented a study on hardware error likelihood induced by the operation of the software. Song et al. (2014) studied the reliability analysis for multi-component systems subject to multiple dependent competing failure processes. Gao et al. (2019) provided a novel framework for the reliability modeling of repairable multistate complex mechanical systems considering propagation relationships. Park and Baik (2015) provided the combination of an improving software reliability prediction through multi-criteria-based dynamic model selection. Lung et al. (2016) suggested improving software performance and reliability in a distributed and concurrent environment with an architecture-based self-adaptive framework.

The present study is inspired by the work of Zhu and Pham (2019). They divided hardware interaction failures in their work into two types, that is, software-induced hardware failures and hardware-induced software failures. In the light of the above, this study proposes a new computer networking device model. This model is a system of multi-hardware applications categorized into hardware–software–hardware, hardware–software–software, software–hardware–software, and software–hardware–hardware. The study aims to examine the system's reliability to determine its performance.

The remainder of the paper is structured as follows. The notations used in the analysis are given in Section 2. The definition and the state of the system are given in Section 3. Section 4 deals with formulating the model and solutions. Section 5 presents the results of our numerical simulations and discussion, and Section 6 concludes the article.

2. Notations

S_i: State of the system $i = 0, 1, 2, \ldots, 10$.

$h(t)$: Row vector probability.

$h_i(t)$: The chance that the system sojourn in state i at time $t \geq 0$.

$A_v(\infty)$: Steady-state availability.

$P_f(\infty)$: Profit function.

η_1/η_2: Rate of units belonging to subsystem A/subsystem B.

δ_1/δ_2: Rate of failure of units belonging to subsystem A/subsystem B.

B_{T1}/B_{T3}: Stand for the chance that repairman is busy performing corrective maintenance due to partial failure of units' subsystem A/subsystem B.

B_{T2}/B_{T4}: Stand for the chance that the repairman is busy performing corrective maintenance due to complete failure of units in subsystem A/subsystem B.

C_0: stand for revenue gathered.

C_1/C_3: stand for cost paid due to service of partial failure of units' subsystem A/subsystem B.

C_2/C_4: stand for cost paid due to repair of complete failure of units' subsystem A/subsystem B.

3. System Description

Two or more hardware can run similar software simultaneously as it is connected to a network. This is very popular in distributed systems. Distributed systems may, however, appear to the user as a single coherent system but consist of different autonomous computing elements; autonomous computing elements or nodes may be software or hardware (Van Steen & Tanenbaum, 2002). Middleware (Figure 1) offers the means for programs to be distributed on various devices. Distributed architectures, such as peer-to-peer (P2P) and cluster, can consist of multiple networked computers interacting and communicating via distributed software. The P2P distributed software framework enables various computers (peers) to work together, interact and share resources over a network. Typical examples include online streaming systems such as BitTorrent, Skype, Zoom, and SopCast. In cluster computing, the cluster (master–slave) architecture consists of some computing nodes under the controller node power. The controller server performs load balancing and scheduling operations. In contrast, the agent server executes (answers) user requests, or, in other words, the master maintains queue requests and handles job assignments for the execution of nodes (Bai et al., 2015; Van Steen & Tanenbaum, 2002). Under the above, the master is fitted with the middleware functions required for scheduling and load balancing and the middleware functions for communication running. Table 1 gives brief description of the states of the system.

4. Formulation and Solution of Mathematical Model

The chance that the system at $t \geq 0$ is within the state s_i is defined as $h_i(t), i = 0, 1, 2, 3, \ldots\ldots\ldots, 10$. Define $h(t) = [h_1(t), h_2(t), \ldots, h_{10}(t)]$ at time t to be the row probabilities vector. In this analysis, the initial condition is

$$h_i(0) = \begin{cases} 1, i = 0 \\ 0, i = 1, 2, 3, \ldots, 10 \end{cases} \quad (1)$$

Figure 1
The system's reliability block diagram

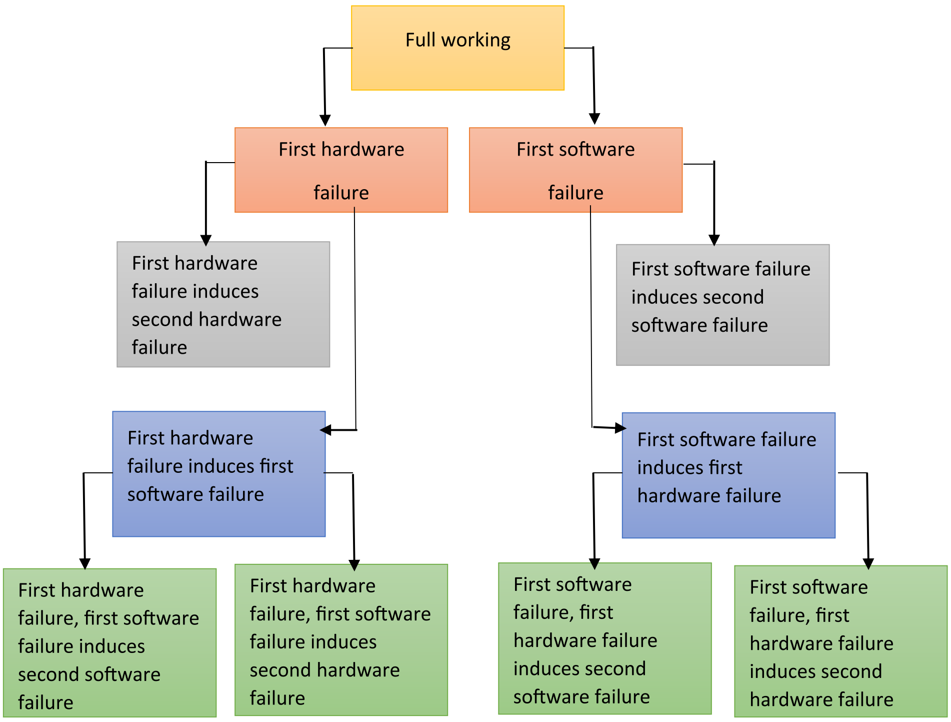


Table 1
States of the system

	Subsystem 1		Subsystem 2		
State	Hardware I	Hardware II	Software I	Software II	System status
S_0	Good	Standby	Good	Standby	Operational
S_1	Failed first	Good	Good	Standby	Operational
S_2	Good	Standby	Failed first	Good	Operational
S_3	Failed first	Good	Failed second	Good	Operational
S_4	Failed second	Good	Failed first	Good	Operational
S_5	Failed first	Failed second	Suspended	Standby	Down
S_6	Suspended	Standby	Failed first	Failed second	Down
S_7	Failed first	Suspended	Failed second	Failed last	Down
S_8	Failed second	Suspended	Failed first	Failed last	Down
S_9	Failed first	Failed last	Failed second	Suspended	Down
S_{10}	Failed second	Failed last	Failed first	Suspended	Down

The equations of differential difference derived from the configuration of the system are given by:

$$\frac{d}{dt}h_0(t) = -(\delta_1+\delta_2)h_0(t) + \eta_1 h_1(t) + \eta_2 h_2(t)$$

$$\frac{d}{dt}h_1(t) = -(\eta_1+\delta_1+\delta_2)h_1(t) + \delta_1 h_0(t) + \eta_2 h_3(t) + \eta_1 h_5(t)$$

$$\frac{d}{dt}h_2(t) = -(\eta_2+\delta_1+\delta_2)h_2(t) + \delta_2 h_0(t) + \eta_1 h_4(t) + \eta_2 h_6(t)$$

$$\frac{d}{dt}h_3(t) = -(\eta_2+\delta_1+\delta_2)h_3(t) + \delta_2 h_1(t) + \eta_2 h_7(t) + \eta_1 h_9(t)$$

$$\frac{d}{dt}h_4(t) = -(\eta_1+\delta_1+\delta_2)h_4(t) + \delta_1 h_2(t) + \eta_2 h_8(t) + \eta_1 h_{10}(t)$$

$$\frac{d}{dt}h_5(t) = -\eta_1 h_5(t) + \delta_1 h_1(t) \tag{2}$$

$$\frac{d}{dt}h_6(t) = -\eta_2 h_6(t) + \delta_2 h_2(t)$$

$$\frac{d}{dt}h_7(t) = -\eta_2 h_7(t) + \delta_2 h_3(t)$$

$$\frac{d}{dt}h_8(t) = -\eta_2 h_8(t) + \delta_2 h_4(t)$$

$$\frac{d}{dt}h_9(t) = -\eta_1 h_9(t) + \delta_1 h_3(t)$$

$$\frac{d}{dt}h_{10}(t) = -\eta_1 h_{10}(t) + \delta_1 h_4(t)$$

$$\begin{pmatrix} h'_0(t) \\ h'_1(t) \\ h'_2(t) \\ h'_3(t) \\ h'_4(t) \\ h'_5(t) \\ h'_6(t) \\ h'_7(t) \\ h'_8(t) \\ h'_9(t) \\ h'_{10}(t) \end{pmatrix} = \begin{pmatrix} -y_0 & \eta_1 & \eta_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \delta_1 & -y_1 & 0 & \eta_2 & 0 & \eta_1 & 0 & 0 & 0 & 0 & 0 \\ \delta_2 & 0 & -y_2 & 0 & \eta_1 & 0 & \eta_2 & 0 & 0 & 0 & 0 \\ 0 & \delta_2 & 0 & -y_2 & 0 & 0 & 0 & \eta_2 & 0 & \eta_1 & 0 \\ 0 & 0 & \delta_1 & 0 & -y_1 & 0 & 0 & 0 & \eta_2 & 0 & \eta_1 \\ 0 & \delta_1 & 0 & 0 & 0 & -\eta_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \delta_2 & 0 & 0 & 0 & -\eta_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \delta_2 & 0 & 0 & 0 & -\eta_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \delta_2 & 0 & 0 & 0 & -\eta_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\eta_1 & 0 \\ 0 & 0 & 0 & 0 & \delta_1 & 0 & 0 & 0 & 0 & 0 & -\eta_1 \end{pmatrix} \begin{pmatrix} h_0(t) \\ h_1(t) \\ h_2(t) \\ h_3(t) \\ h_4(t) \\ h_5(t) \\ h_6(t) \\ h_7(t) \\ h_8(t) \\ h_9(t) \\ h_{10}(t) \end{pmatrix} \tag{3}$$

where $y_0 = (\delta_1+\delta_2)$, $y_1 = (\eta_1+\delta_1+\delta_2)$, and $y_2 = (\eta_2+\delta_1+\delta_2)$.

The reliability models needed to compute the profit are provided by:

$$A_V(\infty) = p_0(\infty) + p_1(\infty) + p_2(\infty) + p_3(\infty) + p_4(\infty)$$

$$B_{P1}(\infty) = p_1(\infty) + p_4(\infty)$$

$$B_{P2}(\infty) = p_2(\infty) + p_3(\infty) \tag{4}$$

$$B_{P3}(\infty) = p_5(\infty) + p_9(\infty) + p_{10}(\infty)$$

$$B_{P4}(\infty) = p_6(\infty) + p_7(\infty) + p_8(\infty)$$

(3) are set equal to zero in steady state; thus, equation (3) becomes

$$\begin{pmatrix} -y_0 & \eta_1 & \eta_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \delta_1 & -y_1 & 0 & \eta_2 & 0 & \eta_1 & 0 & 0 & 0 & 0 & 0 \\ \delta_2 & 0 & -y_2 & 0 & \eta_1 & 0 & \eta_2 & 0 & 0 & 0 & 0 \\ 0 & \delta_2 & 0 & -y_2 & 0 & 0 & 0 & \eta_2 & 0 & \eta_1 & 0 \\ 0 & 0 & \delta_1 & 0 & -y_1 & 0 & 0 & 0 & \eta_2 & 0 & \eta_1 \\ 0 & \delta_1 & 0 & 0 & 0 & -\eta_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \delta_2 & 0 & 0 & 0 & -\eta_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \delta_2 & 0 & 0 & 0 & -\eta_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \delta_2 & 0 & 0 & 0 & -\eta_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\eta_1 & 0 \\ 0 & 0 & 0 & 0 & \delta_1 & 0 & 0 & 0 & 0 & 0 & -\eta_1 \end{pmatrix} \begin{pmatrix} h_0(\infty) \\ h_1(\infty) \\ h_2(\infty) \\ h_3(\infty) \\ h_4(\infty) \\ h_5(\infty) \\ h_6(\infty) \\ h_7(\infty) \\ h_8(\infty) \\ h_9(\infty) \\ h_{10}(\infty) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \tag{5}$$

In this problem, the normalizing condition is defined as:

$$\sum_{i=0}^{11} h_i(\infty) = 1 \tag{6}$$

Combining (5) and (6), we get

$$\begin{pmatrix} -y_0 & \eta_1 & \eta_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \delta_1 & -y_1 & 0 & \eta_2 & 0 & \eta_1 & 0 & 0 & 0 & 0 & 0 \\ \delta_2 & 0 & -y_2 & 0 & \eta_1 & 0 & \eta_2 & 0 & 0 & 0 & 0 \\ 0 & \delta_2 & 0 & -y_2 & 0 & 0 & 0 & \eta_2 & 0 & \eta_1 & 0 \\ 0 & 0 & \delta_1 & 0 & -y_1 & 0 & 0 & 0 & \eta_2 & 0 & \eta_1 \\ 0 & \delta_1 & 0 & 0 & 0 & -\eta_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \delta_2 & 0 & 0 & 0 & -\eta_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \delta_2 & 0 & 0 & 0 & -\eta_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \delta_2 & 0 & 0 & 0 & -\eta_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\eta_1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} h_0(\infty) \\ h_1(\infty) \\ h_2(\infty) \\ h_3(\infty) \\ h_4(\infty) \\ h_5(\infty) \\ h_6(\infty) \\ h_7(\infty) \\ h_8(\infty) \\ h_9(\infty) \\ h_{10}(\infty) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \tag{7}$$

Solving (7) to obtain $h_i(t)$, the reliability models are given in equation (8).

$$p_0(\infty) = \frac{\eta_1^2\eta_2^2}{2\eta_2\delta_1^2\delta_2 + \eta_2^2\delta_1^2 + 2\eta_1\delta_1\delta_2^2 + 2\eta_1\eta_2\delta_1\delta_2 + \eta_1\eta_2^2\delta_1 + \eta_1^2\delta_2^2 + \eta_1^2\eta_2\delta_2 + \eta_1^2\eta_2^2}$$

$$p_1(\infty) = \frac{\eta_1\eta_2^2\delta_1}{2\eta_2\delta_1^2\delta_2 + \eta_2^2\delta_1^2 + 2\eta_1\delta_1\delta_2^2 + 2\eta_1\eta_2\delta_1\delta_2 + \eta_1\eta_2^2\delta_1 + \eta_1^2\delta_2^2 + \eta_1^2\eta_2\delta_2 + \eta_1^2\eta_2^2}$$

$$p_2(\infty) = \frac{\eta_1^2\eta_2\delta_2}{2\eta_2\delta_1^2\delta_2 + \eta_2^2\delta_1^2 + 2\eta_1\delta_1\delta_2^2 + 2\eta_1\eta_2\delta_1\delta_2 + \eta_1\eta_2^2\delta_1 + \eta_1^2\delta_2^2 + \eta_1^2\eta_2\delta_2 + \eta_1^2\eta_2^2}$$

$$p_3(\infty) = \frac{\eta_1\eta_2\delta_1\delta_2}{2\eta_2\delta_1^2\delta_2 + \eta_2^2\delta_1^2 + 2\eta_1\delta_1\delta_2^2 + 2\eta_1\eta_2\delta_1\delta_2 + \eta_1\eta_2^2\delta_1 + \eta_1^2\delta_2^2 + \eta_1^2\eta_2\delta_2 + \eta_1^2\eta_2^2}$$

$$p_4(\infty) = \frac{\eta_1\eta_2\delta_1\delta_2}{2\eta_2\delta_1^2\delta_2 + \eta_2^2\delta_1^2 + 2\eta_1\delta_1\delta_2^2 + 2\eta_1\eta_2\delta_1\delta_2 + \eta_1\eta_2^2\delta_1 + \eta_1^2\delta_2^2 + \eta_1^2\eta_2\delta_2 + \eta_1^2\eta_2^2} \tag{8}$$

$$p_5(\infty) = \frac{\eta_2^2\delta_2^2}{2\eta_2\delta_1^2\delta_2 + \eta_2^2\delta_1^2 + 2\eta_1\delta_1\delta_2^2 + 2\eta_1\eta_2\delta_1\delta_2 + \eta_1\eta_2^2\delta_1 + \eta_1^2\delta_2^2 + \eta_1^2\eta_2\delta_2 + \eta_1^2\eta_2^2}$$

$$p_6(\infty) = \frac{\eta_1^2\delta_2^2}{2\eta_2\delta_1^2\delta_2 + \eta_2^2\delta_1^2 + 2\eta_1\delta_1\delta_2^2 + 2\eta_1\eta_2\delta_1\delta_2 + \eta_1\eta_2^2\delta_1 + \eta_1^2\delta_2^2 + \eta_1^2\eta_2\delta_2 + \eta_1^2\eta_2^2}$$

$$p_7(\infty) = \frac{\eta_1\delta_1\delta_2^2}{2\eta_2\delta_1^2\delta_2 + \eta_2^2\delta_1^2 + 2\eta_1\delta_1\delta_2^2 + 2\eta_1\eta_2\delta_1\delta_2 + \eta_1\eta_2^2\delta_1 + \eta_1^2\delta_2^2 + \eta_1^2\eta_2\delta_2 + \eta_1^2\eta_2^2}$$

$$p_8(\infty) = \frac{\eta_1\delta_1\delta_2^2}{2\eta_2\delta_1^2\delta_2 + \eta_2^2\delta_1^2 + 2\eta_1\delta_1\delta_2^2 + 2\eta_1\eta_2\delta_1\delta_2 + \eta_1\eta_2^2\delta_1 + \eta_1^2\delta_2^2 + \eta_1^2\eta_2\delta_2 + \eta_1^2\eta_2^2}$$

$$p_9(\infty) = \frac{\eta_2\delta_1^2\delta_2}{2\eta_2\delta_1^2\delta_2 + \eta_2^2\delta_1^2 + 2\eta_1\delta_1\delta_2^2 + 2\eta_1\eta_2\delta_1\delta_2 + \eta_1\eta_2^2\delta_1 + \eta_1^2\delta_2^2 + \eta_1^2\eta_2\delta_2 + \eta_1^2\eta_2^2}$$

$$p_{10}(\infty) = \frac{\eta_2\delta_1^2\delta_2}{2\eta_2\delta_1^2\delta_2 + \eta_2^2\delta_1^2 + 2\eta_1\delta_1\delta_2^2 + 2\eta_1\eta_2\delta_1\delta_2 + \eta_1\eta_2^2\delta_1 + \eta_1^2\delta_2^2 + \eta_1^2\eta_2\delta_2 + \eta_1^2\eta_2^2}$$

$$A_V(\infty) = \frac{\eta_1^2\eta_2^2 + \eta_1\eta_2^2\delta_1 + \eta_1^2\eta_2\delta_2 + 2\eta_1\eta_2\delta_1\delta_2}{2\eta_2\delta_1^2\delta_2 + \eta_2^2\delta_1^2 + 2\eta_1\delta_1\delta_2^2 + 2\eta_1\eta_2\delta_1\delta_2 + \eta_1\eta_2^2\delta_1 + \eta_1^2\delta_2^2 + \eta_1^2\eta_2\delta_2 + \eta_1^2\eta_2^2}$$

$$B_{P1}(\infty) = \frac{\eta_1\eta_2^2\delta_1 + \eta_1\eta_2\delta_1\delta_2}{2\eta_2\delta_1^2\delta_2 + \eta_2^2\delta_1^2 + 2\eta_1\delta_1\delta_2^2 + 2\eta_1\eta_2\delta_1\delta_2 + \eta_1\eta_2^2\delta_1 + \eta_1^2\delta_2^2 + \eta_1^2\eta_2\delta_2 + \eta_1^2\eta_2^2}$$

$$B_{P2}(\infty) = \frac{\eta_1^2\eta_2\delta_2 + \eta_1\eta_2\delta_1\delta_2}{2\eta_2\delta_1^2\delta_2 + \eta_2^2\delta_1^2 + 2\eta_1\delta_1\delta_2^2 + 2\eta_1\eta_2\delta_1\delta_2 + \eta_1\eta_2^2\delta_1 + \eta_1^2\delta_2^2 + \eta_1^2\eta_2\delta_2 + \eta_1^2\eta_2^2}$$

$$B_{P3}(\infty) = \frac{\eta_2^2\delta_2^2 + 2\eta_2\delta_1^2\delta_2}{2\eta_2\delta_1^2\delta_2 + \eta_2^2\delta_1^2 + 2\eta_1\delta_1\delta_2^2 + 2\eta_1\eta_2\delta_1\delta_2 + \eta_1\eta_2^2\delta_1 + \eta_1^2\delta_2^2 + \eta_1^2\eta_2\delta_2 + \eta_1^2\eta_2^2}$$

$$B_{P4}(\infty) = \frac{\eta_1^2\delta_2^2 + 2\eta_1\delta_1\delta_2^2}{2\eta_2\delta_1^2\delta_2 + \eta_2^2\delta_1^2 + 2\eta_1\delta_1\delta_2^2 + 2\eta_1\eta_2\delta_1\delta_2 + \eta_1\eta_2^2\delta_1 + \eta_1^2\delta_2^2 + \eta_1^2\eta_2\delta_2 + \eta_1^2\eta_2^2}$$

The units are prone to partial and complete failure, so the repairman is busy conducting maintenance measures on failed items. Let C_0, C_1, C_2, C_3, and C_4 as the income produced when the system is in working condition and no income when it is in failed condition, the cost of each repair due to partial failure and complete failure, respectively. The estimated overall system benefit per unit time incurred in a steady state is the expected total system profit per unit time and is given by:

$$\begin{aligned} P_T(\infty) = & C_0 * A_V(\infty) - C_1 * B_{P1}(\infty) - C_2 * B_{P2}(\infty) \\ & - C_3 * B_{P3}(\infty) - C_4 * B_{P4}(\infty) \end{aligned} \tag{9}$$

The explicit expression for the mean time to failure (MTTF) is computed using:

$$MTTF = P(0)(-Q^{-1})[1, 1, 1, 1, 1]^T \tag{10}$$

Thus, the *MTTF* expression for the system is

$$MTTF = \frac{N_1 + N_2 + \delta_1 N_3 + \delta_2 N_4 + \delta_1\delta_2(N_5 + N_6)}{D_1 + D_2} \tag{11}$$

where

$$Q = \begin{pmatrix} -(\delta_1 + \delta_2) & \delta_1 & \delta_2 & 0 & 0 \\ \eta_1 & -(\eta_1 + \delta_1 + \delta_2) & 0 & \delta_2 & 0 \\ \eta_2 & 0 & -(\eta_2 + \delta_1 + \delta_2) & 0 & \delta_1 \\ 0 & \eta_2 & 0 & -(\eta_2 + \delta_1 + \delta_2) & 0 \\ 0 & 0 & \eta_1 & 0 & -(\eta_1 + \delta_1 + \delta_2) \end{pmatrix} \tag{12}$$

obtained by deleting the rows and columns of failure states of the matrix M and

$$M = \begin{pmatrix} -y_0 & \eta_1 & \eta_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \delta_1 & -y_1 & 0 & \eta_2 & 0 & \eta_1 & 0 & 0 & 0 & 0 & 0 \\ \delta_2 & 0 & -y_2 & 0 & \eta_1 & 0 & \eta_2 & 0 & 0 & 0 & 0 \\ 0 & \delta_2 & 0 & -y_2 & 0 & 0 & 0 & \eta_2 & 0 & \eta_1 & 0 \\ 0 & 0 & \delta_1 & 0 & -y_1 & 0 & 0 & 0 & \eta_2 & 0 & \eta_1 \\ 0 & \delta_1 & 0 & 0 & 0 & -\eta_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \delta_2 & 0 & 0 & 0 & -\eta_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \delta_2 & 0 & 0 & 0 & -\eta_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \delta_2 & 0 & 0 & 0 & -\eta_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\eta_1 & 0 \\ 0 & 0 & 0 & 0 & \delta_1 & 0 & 0 & 0 & 0 & 0 & -\eta_1 \end{pmatrix} \tag{13}$$

$$\begin{aligned} N_1 = & (\delta_1 + \delta_2)^4 + \delta_1^3(\eta_1 + 2\eta_2) + \delta_2^3(2\eta_1 + \eta_2) + 3\eta_1\eta_2(\delta_1^2 + \delta_2^2) \\ & + \delta_1\delta_2(\eta_1^2 + \eta_2^2) \end{aligned}$$

$$\begin{aligned} N_2 = & \eta_1\eta_2^2(\delta_1 + \delta_2) + \delta_1\delta_2^2(4\eta_2 + 5\eta_1) + \delta_1^2\delta_2(4\eta_1 + 5\eta_2) \\ & + (\eta_2\delta_1 + \eta_1\delta_2)^2 + 5\eta_1\eta_2\delta_1\delta_2 \end{aligned}$$

$$\begin{aligned} N_3 = & (\delta_1 + \delta_2)^3 + \delta_1^2(\eta_1 + 2\eta_2) + \eta_2^2(\delta_1 + \delta_2) + 2\eta_2\delta_1(\delta_1 + 2\delta_2) \\ & + \eta_1\delta_2(\delta_1 + 2\eta_2) + \eta_1\eta_2(\eta_2 + \delta_1) \end{aligned}$$

$$\begin{aligned} N_4 = & (\delta_1 + \delta_2)^3 + \delta_1^2(2\eta_1 + \eta_2) + \eta_1^2(\delta_1 + \delta_2) + \eta_1(\eta_1\eta_2 + 2\delta_2^2) \\ & + \eta_2\delta_1(2\eta_1 + \delta_2) + \eta_1\delta_2(4\delta_1 + \eta_2) \end{aligned}$$

$$N_5 = ((\delta_1 + \delta_2)^2 + \eta_2(\delta_1 + \delta_2) + \eta_1(\eta_2 + \delta_2)) \tag{14}$$

$$N_6 = ((\delta_1 + \delta_2)^2 + \eta_1(\delta_1 + \delta_2) + \eta_2(\eta_1 + \delta_2))$$

$$\begin{aligned} D_1 = & \eta_1\eta_2^2\delta_1^2 + \eta_2^2\delta_1^2\delta_2 + \eta_2^2\delta_1^3 + \eta_1^2\eta_2\delta_2^2 + \eta_1\eta_2\delta_1^3 + \eta_1\eta_2\delta_2^3 \\ & + 4\eta_1\eta_2\delta_1^2\delta_2 + 4\eta_1\eta_2\delta_2^2\delta_1 + 6\eta_2\delta_1^3\delta_2 + 2\eta_2\delta_1^4 \end{aligned}$$

$$\begin{aligned} D_2 = & (\delta_1 + \delta_2)^5 + 6\eta_2\delta_1^2\delta_2^2 + 2\eta_2\delta_1\delta_2^3 + \eta_1^2\delta_2^3 + \eta_1^2\delta_1\delta_2^2 + 6\eta_1\delta_1^2\delta_2^2 \\ & + 2\eta_1\delta_1^3\delta_2 + 6\eta_1\delta_1\delta_2^3 + 2\eta_1\delta_2^4 \end{aligned}$$

5. Numerical Simulation

In this section, we present numerical simulations with respect to availability, MTTF, and benefit function for the established models. The following set of parameter values is fixed for consistency in the simulations: $d1 = 0.3$; $n1 = 0.9$; $d2 = 0.2$; $n2 = 0.6$; $C0 = 25{,}000{,}000$, $C1 = 550$, $C2 = 250$, $C3 = 1250$, $C4 = 700$.

Figure 2 shows the availability patterns for the system against the rates of failure (δ_1) and repair (η_1), respectively. As the repair rate (η_1) increases, availability increases, while availability decreases with an increase in the rate of failure (δ_1). This means that preventive and substantial maintenance is essential in optimizing the system's availability. Concerning variance in failure (δ_1) and repair (η_1) rates, Figure 3 yields the MTTF of the system. The variation in MTTF equal to the rate of repair (η_1) is greater than the variation in MTTF equivalent to the rate of failure (δ_1). This analysis indicates that the failure rate is more responsible for the system's efficient functioning.

Figure 4 presents the consequences of the failure rate (δ_1) and repair rate (η_1) on profit function. It can be seen from this statistic

Figure 2
Surface plot of availability against η_1 and δ_1

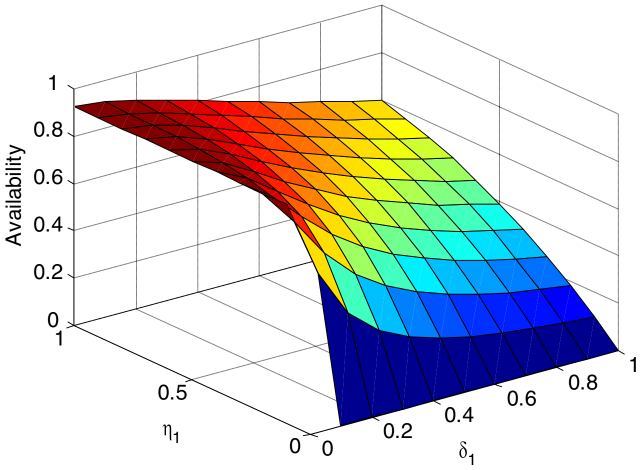


Figure 4
Surface plot of profit against η_1 and δ_1

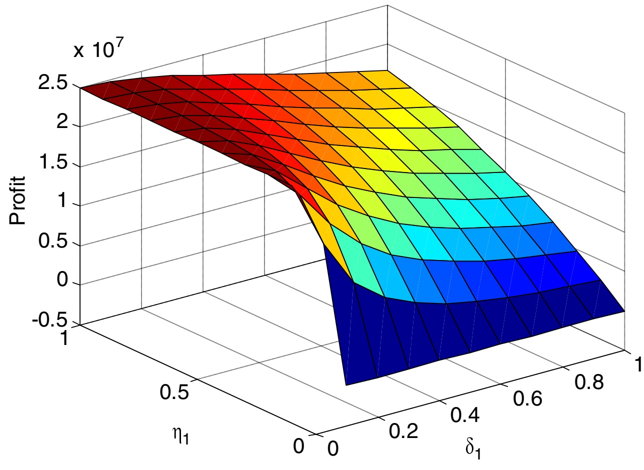


Figure 3
MTTF sensitivity against η_1 and δ_1

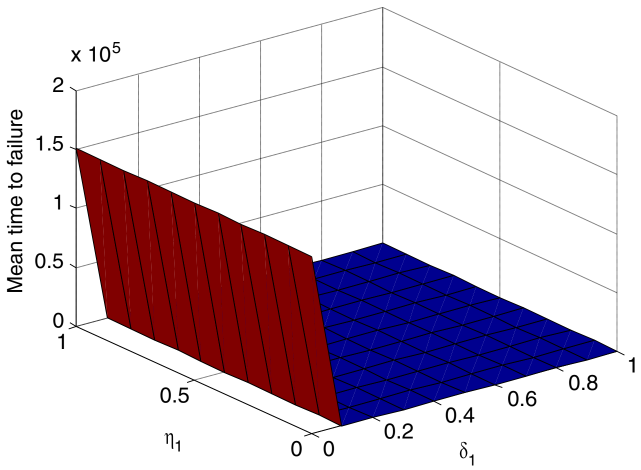
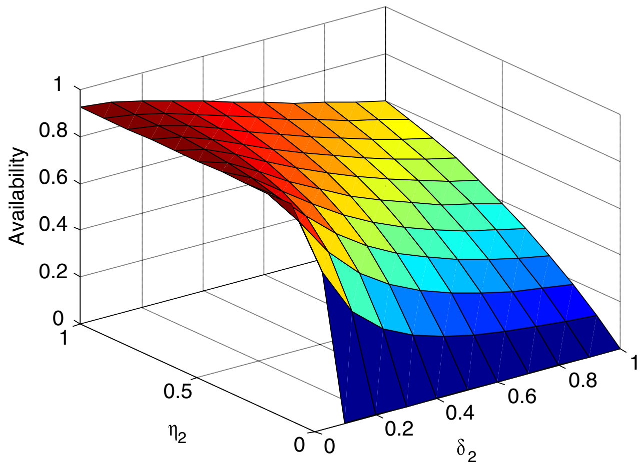


Figure 5
Surface plot of availability against η_2 and δ_2



that profit shows a growing trend with an increase in repair rate (η_1) values and a declining trend with an increase in failure rate (δ_1) values. This means that routine preventive maintenance or perfect system repair would help to achieve optimum performance and benefit.

Figures 5 and 7, respectively, show the patterns in the availability of the system and the profit against the failure (δ_2) and repair (η_2) rates for different rate support values. From these figures, it is evident that system availability and profit show growing trends with η_2 and decreasing trends with δ_2 for various device rate support values, respectively. This sensitivity analysis shows that the failure of the supporting device affects reliability metrics such as availability and profit. As both δ_1 and δ_2 increase, the differences between the curves in the figures broaden. This suggests that significant maintenance of the units, supporting devices, and the whole system should be invoked to enhance and optimize the availability of the design, production performance,

Figure 6
MTTF sensitivity against η_2 and δ_2

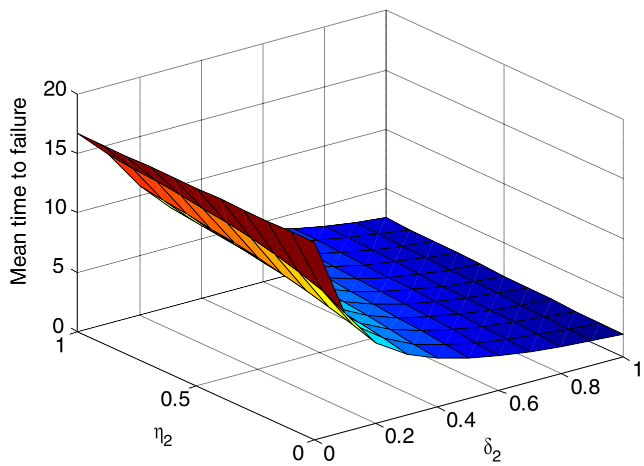
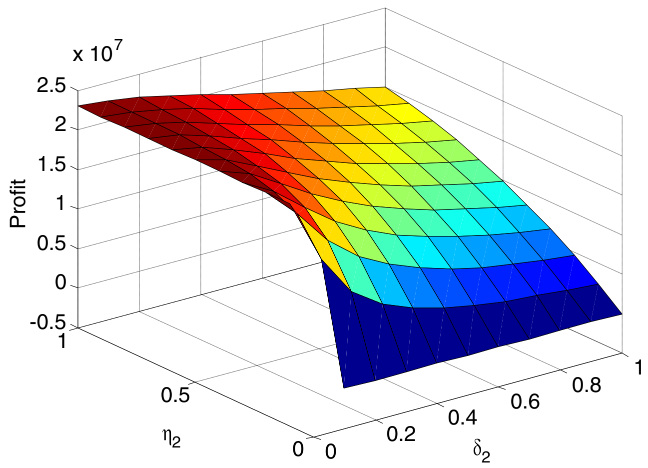


Figure 7
Surface plot of profit against η_2 and δ_2



and profitability. In terms of failure and repair rates of the supporting unit, Figure 6 shows the difference in MTTF. As the repair rate values increase, the system's MTTF increases as it decreases with an increase in the failure rate values.

Tables 2, 3, 4, and 5 demonstrate the sensitivity analysis of the study. From the simulation shown in Table 2, it is observed that availability, MTTF, and benefit function decrease with an increase in failure rate (δ_1) values. As shown in Table 3, the availability, MTTF, and profit function significantly increases with an increase in the repair rate (η_1) values. A sensitivity analysis concerning the failure rate of supporting unit (δ_2) is shown in Table 4. From Table 4, it is clear that availability, MTTF, and benefit function decrease drastically with an increase in values of failure rate (δ_2). However, on the other hand, with an improvement in the repair rate values of the supporting units, the availability, MTTF, and benefit function increase significantly. This is apparent in Table 5. This illustrates that inability to maintain the unit implies the reliability measures such as availability and profit.

Table 2
Sensitivity assessment concerning δ_1 for $\eta_1(0.2, 0.6, 1.0)$

	$A_V(\infty)$			$MTTF$			$P_F(\infty) * 10^7$		
δ_1	$\eta_1 = 0.2$	$\eta_1 = 0.6$	$\eta_1 = 1.0$	$\eta_1 = 0.2$	$\eta_1 = 0.6$	$\eta_1 = 1.0$	$\eta_1 = 0.2$	$\eta_1 = 0.6$	$\eta_1 = 1.0$
0	0.9880	0.9880	0.9880	151.0204	151.0204	151.0204	2.4699	2.4699	2.4699
0.1	0.8378	0.9619	0.9774	30.6664	49.7736	63.6395	2.0945	2.4047	2.4435
0.2	0.6484	0.9053	0.9522	13.4785	20.7312	27.2557	1.6210	2.2633	2.3806
0.3	0.5118	0.8378	0.9180	8.3572	11.9777	15.4183	1.2795	2.0945	2.2950
0.4	0.4174	0.7698	0.8789	5.9978	8.1451	10.2293	1.0434	1.9244	2.1972
0.5	0.3503	0.7061	0.8378	4.6590	6.0756	7.4649	0.8756	1.7651	2.0945
0.6	0.3008	0.6484	0.7966	3.8015	4.8049	5.7947	0.7519	1.6210	1.9916
0.7	0.2631	0.5971	0.7566	3.2071	3.9547	4.4649	0.6577	1.4928	1.8914
0.8	0.2335	0.5518	0.7183	2.7717	3.3501	3.9241	0.5837	1.3795	1.7958
0.9	0.2098	0.5118	0.6822	2.4394	2.9002	3.3582	0.5244	1.2795	1.7055

Table 3
Sensitivity assessment concerning η_1 for $\delta_1(0.1, 0.4, 0.7)$

	$A_V(\infty)$			$MTTF$			$P_F(\infty) * 10^7$		
η_1	$\delta_1 = 0.1$	$\delta_1 = 0.4$	$\delta_1 = 0.7$	$\delta_1 = 0.1$	$\delta_1 = 0.4$	$\delta_1 = 0.7$	$\delta_1 = 0.1$	$\delta_1 = 0.4$	$\delta_1 = 0.7$
0	0	0	0	18.0960	4.8995	2.8305	−0.0001	−0.0001	−0.0001
0.1	0.6484	0.2335	0.1385	24.6995	5.4508	3.0191	1.6210	0.5837	0.3462
0.2	0.8378	0.4174	0.2631	30.6664	5.9978	3.2071	2.0945	1.0434	0.6577
0.3	0.9053	0.5518	0.3703	36.0910	6.5407	3.3947	2.2633	1.3795	0.9257
0.4	0.9360	0.6484	0.4604	41.0460	7.0795	3.5819	2.3399	1.6210	1.1509
0.5	0.9522	0.7183	0.5352	45.5904	7.6143	3.7685	2.3806	1.7958	1.3380
0.6	0.9619	0.7698	0.5971	49.7736	8.1451	3.9547	2.4047	1.9244	1.4928
0.7	0.9681	0.8083	0.6484	53.6372	8.6719	4.1405	2.4201	2.0208	1.6210
0.8	0.9722	0.8378	0.6910	57.2165	9.1949	4.3257	2.4306	2.0945	1.7276
0.9	0.9752	0.8607	0.7267	60.5419	9.7140	4.5105	2.4380	2.1518	1.8166

Table 4
Sensitivity assessment concerning δ_2 for $\eta_2(0.1, 0.5, 0.9)$

	$A_V(\infty)$			$MTTF$			$P_F(\infty) * 10^7$		
δ_2	$\eta_2 = 0.1$	$\eta_2 = 0.5$	$\eta_2 = 0.9$	$\eta_2 = 0.1$	$\eta_2 = 0.5$	$\eta_2 = 0.9$	$\eta_2 = 0.1$	$\eta_2 = 0.5$	$\eta_2 = 0.9$
0	0.9231	0.9231	0.9231	16.6667	16.6667	16.6667	2.3077	2.3077	2.3077
0.1	0.6000	0.8824	0.9066	10.9931	13.0457	13.9995	1.5000	2.2059	2.2666
0.2	0.3925	0.8108	0.8753	7.4578	9.3686	10.6177	0.9813	2.0270	2.1881
0.3	0.2864	0.7343	0.8361	5.5640	6.9942	8.0952	0.7160	1.8356	2.0901
0.4	0.2243	0.6630	0.7937	4.4174	5.4760	6.3648	0.5607	1.6574	1.9842
0.5	0.1839	0.6000	0.7510	3.6552	4.4569	5.1646	0.4598	1.5000	1.8775
0.6	0.1557	0.5455	0.7097	3.1137	3.7373	4.3056	0.3892	1.3636	1.7742
0.7	0.1349	0.4985	0.6705	2.7100	3.2071	3.6697	0.3373	1.2462	1.6762
0.8	0.1190	0.4580	0.6339	2.3977	2.8024	3.1847	0.2975	1.1450	1.5848
0.9	0.1064	0.4230	0.6000	2.1491	2.4847	2.8050	0.2660	1.0574	1.5000

Table 5
Sensitivity assessment concerning η_2 for $\delta_2(0.2, 0.4, 0.6)$

	$A_V(\infty)$			$MTTF$			$P_F(\infty) * 10^7$		
η_2	$\delta_2 = 0.2$	$\delta_2 = 0.4$	$\delta_2 = 0.6$	$\delta_2 = 0.2$	$\delta_2 = 0.4$	$\delta_2 = 0.6$	$\delta_2 = 0.2$	$\delta_2 = 0.4$	$\delta_2 = 0.6$
0	0	0	0	6.8069	4.1200	2.9481	−0.0001	−0.0001	−0.0001
0.1	0.3925	0.2243	0.1557	7.4578	4.4174	3.1137	0.9813	0.5607	0.3892
0.2	0.6000	0.3925	0.2864	8.0255	4.7007	3.2753	1.5000	0.9813	0.7160
0.3	0.7097	0.5134	0.3925	8.5258	4.9710	3.4330	1.7742	1.2834	0.9813
0.4	0.7723	0.6000	0.4775	8.9705	5.2292	3.5869	1.9307	1.5000	1.1937
0.5	0.8108	0.6630	0.5455	9.3686	5.4760	3.7373	2.0270	1.6574	1.3636
0.6	0.8361	0.7097	0.6000	9.7272	5.7124	3.8842	2.0901	1.7742	1.5000
0.7	0.8535	0.7450	0.6441	10.0520	5.9389	4.0279	2.1337	1.8625	1.6102
0.8	0.8660	0.7723	0.6801	10.3475	6.1562	4.1683	2.1649	1.9307	1.7001
0.9	0.8753	0.7937	0.7097	10.6177	6.3648	4.3056	2.1881	1.9842	1.7742

6. Conclusions through Result Discussion

The reliability metrics for various failure values and repair rates are critically analyzed to consider the system's performance. The basic expressions for system characteristics such as system availability, busy repairman time due to partial and complete failure, and benefit function were obtained and validated by numerical experiments. Based on the numerical results obtained for a specific case in Figures 2, 3, 4, 5, 6, and 7 and Tables 2, 3, 4, and 5, it is evident that the optimum availability of the system and benefit can be achieved when the entire system has been periodically repaired and supporting units have been invoked. It is a common understanding that system failure will decrease production performance, and there might even be a tragedy, as mentioned earlier.

In the interest of humanity, understanding availability metrics can assist engineers and designers in designing more critical systems. The current research will integrate partial and complete maintenance and replacement. In future, interested researchers can explore such areas.

Conflicts of Interest

The authors declare that they have no conflicts of interest to this work.

Informed Consent

Informed consent was obtained from all individual participants included in the study.

References

Ahmed, W., & Wu, Y. W. (2013). A survey on reliability in distributed systems. *Journal of Computer and System Sciences*, *79*, 1243–1255. https://doi.org/10.1016/j.jcss.2013.02.006

Bai, W. H., Xi, J. Q., Zhu, J. X., & Huang, S. W. (2015). Performance analysis of heterogeneous data centers in cloud computing using a complex queuing model. *Mathematical Model in Engineering*, *2015*, 980945. https://doi.org/10.1155/2015/980945

Chopra, G., & Ram, M. (2019). Reliability measures of two dissimilar units' parallel system using Gumbel-Hougaard family copula. *International Journal of Mathematical, Engineering and Management Sciences*, *4*, 116–130. https://doi.org/10.33889/IJMEMS.2019.4.1-011

Garg, S. (2019). An approach to resolve heterogeneity using RPC in client server systems. *International Journal of Engineering Applied Sciences and Technology*, *4*, 301–305. https://doi.org/10.33564/IJEAST.2019.v04i04.049

Gahlot, M., Singh, V. V., Ayagi, H. I., & Abdullahi, I. (2020). Stochastic analysis of a two units' complex repairable system with Switch and Human failure using Copula approach. *Life Cycle Reliability and Safety Engineering*, *9*(1), 1–11. https://doi.org/10.1007/s41872-019-00103-1

Gahlot, M., Singh, V. V., Ayagi, H. I., & Goel, C. K (2018). Performance assessment of repairable system in series configuration under different types of failure and repair policies using copula linguistics. *International Journal of Reliability and Safety*, *12*, 367–374. https://doi.org/10.1504/IJRS.2018.10017231

Gao, X., Wang, R., Gao, J., Gao, Z., & Deng, W. (2019). A novel framework for the reliability modelling of repairable multistate complex mechanical systems considering propagation relationships. *Quality and Reliability Engineering International*, *35*, 84–98. https://doi.org/10.1002/qre.2382

Huang, B., Rodriguez, M., Li, M., Bernstein, J. B., & Smidts, C. S. (2011). Hardware error likelihood induced by the operation of software. *IEEE Transactions on Reliability*, *60*, 622–639. https://doi.org/10.1109/TR.2011.2161699

Khan, M. F., Modibbo, U. M., Ahmad, N., & Ali, I. (2022). Nonlinear optimization in bi-level selective maintenance allocation problem. *Journal of King Saud University-Science*, *34*, 101933. https://doi.org/10.1016/j.jksus.2022.101933

Kumar, N., & Lather, J. S. (2018). Reliability analysis of a robotic system using hybridized technique. *Journal of Industrial Engineering International, 14*, 443. https://doi.org/10.1007/s40092-017-0235-5

Lung, C. H., Zhang, X., & Rajeswaran, P. (2016). Improving software performance and reliability in a distributed and concurrent environment with an architecture-based self-adaptive framework. *Journal of Systems and Software*, *121*, 311–328. https://doi.org/10.1016/j.jss.2016.06.102

Modibbo, U. M., Arshad, M., Abdalghani, O., & Ali, I. (2021). Optimization and estimation in system reliability allocation problem. *Reliability Engineering & System Safety*, *212*, 107620. https://doi.org/10.1016/j.ress.2021.107620

Osemwengie, L., Jafari, F., & Karami, A. (2022). Designing a cost-efficient network for a small enterprise. In *Intelligent Computing: Proceedings of the 2021 Computing Conference*, *1*, 255–273. Springer International Publishing. https://doi.org/10.1007/978-3-030-80119-9_14

Park, J., & Baik, J. (2015). Improving software reliability prediction through multi-criteria based dynamic model selection and combination. *Journal of Systems and Software*, *101*, 236–244. https://doi.org/10.1016/j.jss.2014.12.029

Potapov, V. I., Shafeeva, O. P., Gritsay, A. S., Makarov, V. V., Kuznetsova, O. P., & Kondratukova, L. K. (2019). Reliability in the model of an information system with client-server architecture. *International Journal of Physics: Conference Series*, *1260*, 022007. https://doi.org/10.1088/1742-6596/1260/2/022007

Rodríguez-Borbón, M. I., Rodríguez-Medina, M. A., Rodríguez-Picón, L. A., Alvarado-Iniesta, A., & Sha, N. (2017). Reliability estimation for accelerated life tests based on a Cox proportional hazard model with error effect. *Quality and Reliability Engineering International*, *33*, 1407–1416. https://doi.org/10.1002/qre.2113

Ram, M., & Goyal, N. (2018). Bi-directional system analysis under copula-coverage approach. *Communications in Statistics-Simulation and Computation*, *47*(6), 1831–1844. https://doi.org/10.1080/03610918.2017.1327068

Sanusi, A., & Yusuf, I. (2022). Reliability assessment and profit analysis of distributed data center network topology. *Life Cycle Reliability and Safety Engineering*, *11*(1), 75–86. https://doi.org/10.1007/s41872-022-00186-3

Sha, N. (2021). A copula approach of reliability analysis for hybrid systems. *Reliability: Theory & Applications*, *16*, 231–242. https://doi.org/10.24412/1932-2321-2021-161-231-242

Song, S., Coit, D.W., Feng, Q. & Peng, H. (2014). Reliability analysis for multi-component systems subject to multiple dependent competing failure processes. *IEEE Transactions on Reliability*, *63*, 331–345. https://doi.org/10.1109/TR.2014.2299693

Yi, X. J., Shi, J., Dhillon, B. S., Hou, P., & Lai, Y. H. (2017). A new reliability analysis method for repairable systems with multifunction modes based on goal-oriented methodology. *Quality and Reliability Engineering International*, *33*, 2215–2237. https://doi.org/10.1002/qre.2180

Yusuf, I., Ismail, A. L., Singh, V. V., Ali, U. A., & Sufi, N. A. (2020). Performance analysis of multi-computer system consisting of three subsystems in series configuration using copula repair policy. *SN Computer Science*, 1(5), 1–11.

Van Steen, M., & Tanenbaum, A. (2002). Distributed systems principles and paradigms. *Network*, *2* 1.

Zhu, M., & Pham, H. (2019). A novel system reliability modeling of hardware, software, and interactions of hardware and software. *Mathematics*, *7*, 1049. https://doi.org/10.3390/math7111049

Zhu, M., & Pham, H. (2018). A software reliability model incorporating martingale process with gamma-distributed environmental factors. *Annals of Operations Research*, 1–22. https://doi.org/10.1007/s10479-018-2951-7

Zeng, Y., Xing, L., Zhang, Q., & Jia, X. (2019). An analytical method for reliability analysis of hardware-software co-design system. *Quality and Reliability Engineering International*, *35*, 165–178. https://doi.org/10.1002/qre.2389

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