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RESEARCH ARTICLE

# New Topological Operator over Intuitionistic Fuzzy Sets

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**Abstract:** Intuitionistic fuzzy sets (IFSs) are one of the fuzzy set extensions. Over them, a lot of operations, relations, and operators are introduced. A part of the operators defined over IFSs are topological ones. In this paper, a new topological operator is defined, some of its properties are studied, and some open problems related to it are given.

**Keywords:** intuitionistic fuzzy operator, intuitionistic fuzzy set, topology

## 1. Introduction

The concept of an intuitionistic fuzzy set (IFS) was introduced in 1983 in Atanassov, 2016 as an extension of the fuzzy set of Zadeh (1965). During these approximately 40 years, the theory of the IFSs was developed in different directions. One of them is the research over intuitionistic fuzzy topological operators, introduced in 1985 by Atanassov (1985) and described in more details in Atanassov (1986). About 10 years ago, Dogan Çoker published first research over intuitionistic fuzzy topology (Çoker, 1996; Çoker, 2000; Çoker & Demirci, 1995; Çoker & Haydar, 1995) giving another interpretation of the two intuitionistic fuzzy topological operators from Atanassov (1985) Atanassov (1986). They were described in details in Atanassov (1999) and only two years later, they were extended in Atanassov (2001). These and three other extended intuitionistic fuzzy topological operators were described in details in Atanassov (2012). Now, there are a lot of research of different authors from different countries, devoted to the intuitionistic fuzzy topology, but the curious fact is that all this research is related only to the first two intuitionistic fuzzy topological operators.

The definitions of all older intuitionistic fuzzy topological operators will be given in Section 2.

Starting with the first paper over IFS (Atanassov, 2016), a lot of modal type of intuitionistic fuzzy operators are introduced, too. The last two ones are given in Atanassov (2022). In Section 3, we will use them in a combination of the first two intuitionistic fuzzy topological operators with aim to construct the region in which all intuitionistic fuzzy evaluations of a given universe are placed. This region will be a basis for introduction of a new intuitionistic fuzzy topological operator in Section 4. In the conclusion, some open problems are formulated.

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We will not comment the results of the colleagues, working in the area of intuitionistic fuzzy topology, because they are a lot and a lot of them are very serious and it will be well if they are an object of special research, while, as we mentioned above, the aim of the present research is directed to the introduction of a new operator.

## 2. Preliminaries

Initially, we give some basic definitions, related to the IFSs, following Atanassov (2021).

Let a set $E$ be fixed. An IFS $A$ in $E$ is an object of the following form:

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in E\},$$

where the functions $\mu_A : E \to [0, 1]$ and $\nu_A : E \to [0, 1]$ define the degree of membership and the degree of non-membership of the element $x \in E$, respectively, and for every $x \in E$:

$$0 \leq \mu_A(x) + \nu_A(x) \leq 1.$$

Let for every $x \in E$:

$$\pi_A(x) = 1 - \mu_A(x) - \nu_A(x).$$

Therefore, function $\pi$ determines the degree of uncertainty.

Obviously, for every ordinary fuzzy set $\pi_A(x) = 0$ for each $x \in E$ and these sets have the form:

$$\{\langle x, \mu_A(x), 1 - \mu_A(x)\rangle | x \in E\}.$$

Following Atanassov (1999) and Atanassov (2012), for two IFSs $A$ and $B$ we define

$$A \subset B \text{ if and only if } (\forall x \in E)((\mu_A(x) \leq \mu_B(x) \ \& \ \nu_A(x) > \nu_B(x))$$

$$\vee\,(\mu_A(x) < \mu_B(x)\,\&\,\nu_A(x) \geq \nu_B(x))$$
$$\vee\,(\mu_A(x) < \mu_B(x)\,\&\,\nu_A(x) > \nu_B(x))).$$

Let a universe $E$ be given. One of the geometrical interpretations of the IFSs uses figure $F$ in Figure 1.

The first two intuitionistic fuzzy topological operators are defined over the IFSs by:

$$\mathcal{C}(A) = \{\langle x, K, L\rangle | x \in E\},$$

$$\mathcal{I}(A) = \{\langle x, k, l\rangle | x \in E\},$$

where

$$K = \sup_{y \in E} \mu_A(y),$$

$$L = \inf_{y \in E} \nu_A(y),$$

$$k = \inf_{y \in E} \mu_A(y),$$

$$l = \sup_{y \in E} \nu_A(y).$$

These two operators have geometrical interpretations as shown in Figures 2 and 3. They are called *"closure"* ($\mathcal{C}$) and *"interior"* ($\mathcal{I}$) and they are analogs of the respective topological operators (see e.g., Kuratowski, 1966; Yosida, 1995).

**Figure 1**
**A geometrical interpretation of an IFS**

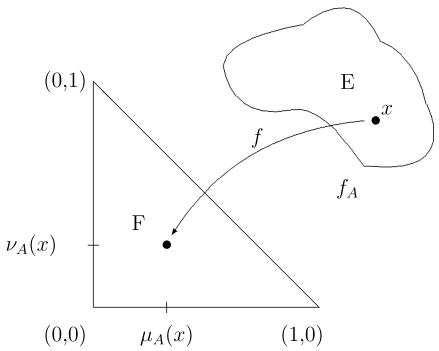



**Figure 2**
**Operator $\mathcal{C}$**

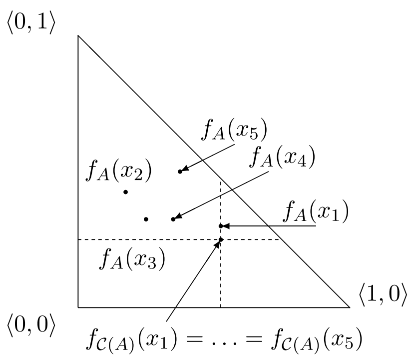
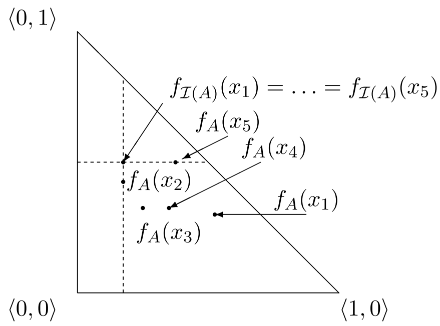



**Figure 3**
**Operator $\mathcal{I}$**




The first two intuitionistic fuzzy topological operators are generalized to the following six new ones (see Atanassov, 2001; Atanassov, 2010; Atanassov, 2012):

$$\mathcal{C}_\mu(A) = \{\langle x, K, \min(1-K, \nu_A(x))\rangle | x \in E\};$$
$$\mathcal{C}_\nu(A) = \{\langle x, \mu_A(x), L\rangle | x \in E\};$$
$$\mathcal{I}_\mu(A) = \{\langle x, k, \nu_A(x)\rangle | x \in E\};$$
$$\mathcal{I}_\nu(A) = \{\langle x, \min(1-l, \mu_A(x)), l\rangle | x \in E\};$$
$$\mathcal{C}^*_\mu(A) = \{\langle x, \min(K, 1-\nu_A(x)), \min(1-K, \nu_A(x))\rangle | x \in E\};$$
$$\mathcal{I}^*_\nu(A) = \{\langle x, \min(1-l, \mu_A(x)), \min(l, 1-\mu_A(x))\rangle | x \in E\},$$

where $K, L, k, l$ are as defined above.

The geometrical interpretations of these operators applied to the IFS $A$ are shown in Figures 4, 5, 6, 7, 8 and 9.

The *"weight-center operator"* over a given IFS $A$ was introduced in Atanassov and Ban (2000) by:

$$W(A) = \left\{ \left\langle x, \frac{\sum\limits_{y\in E} \mu_A(y)}{card(E)}, \frac{\sum\limits_{y\in E} \nu_A(y)}{card(E)} \right\rangle \middle| x \in E \right\},$$

where $card(E)$ is the number of the elements of a finite set $E$. For the continuous case, the "summations" may be replaced by integrals over $E$.

**Figure 4**
**Operator $\mathcal{C}_\mu$**

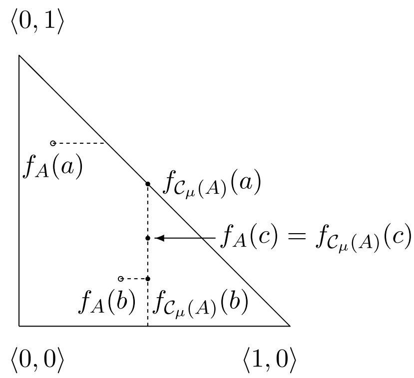


**Figure 5**
**Operator $\mathcal{C}_\nu$**

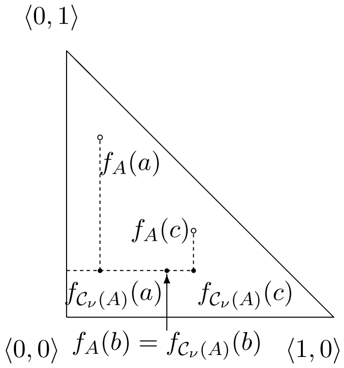



**Figure 6**
**Operator $\mathcal{I}_\mu$**

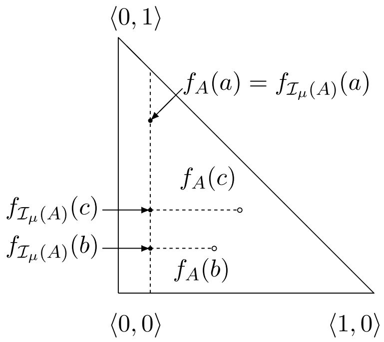



**Figure 7**
**Operator $\mathcal{I}_\nu$**

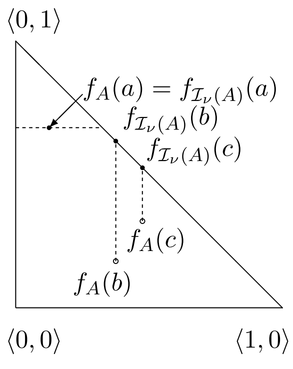



**Figure 8**
**Operator $\mathcal{C}^*_\mu$**

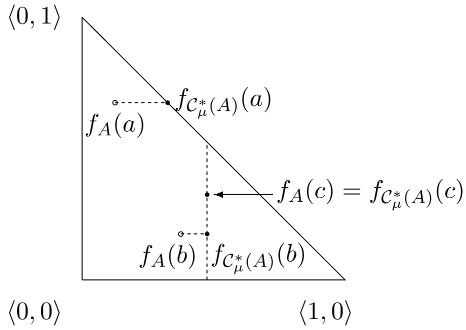



**Figure 9**
**Operator $\mathcal{I}^*_\nu$**

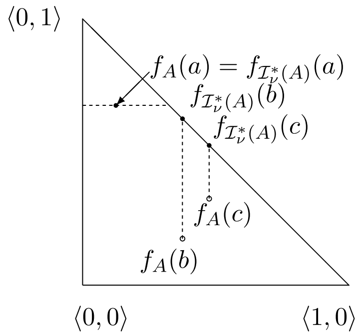
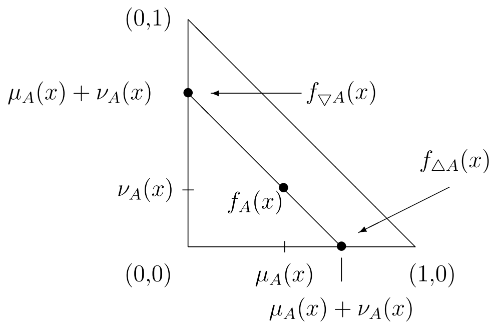



**Figure 10**
**Operators $\nabla$ and $\Delta$**




## 3. The Region of the Intuitionistic Fuzzy Evaluations of a Given Universe

In Atanassov (2021), the following two modal operators are introduced:

$$\Delta A = \Delta\{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in E\} = \{\langle x, \mu_A(x) + \nu_A(x), 0\rangle | x \in E\},$$

$$\nabla x = \nabla\{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in E\} = \{\langle x, 0, \mu_A(x) + \nu_A(x)\rangle | x \in E\}.$$

The geometrical interpretation of both operators is shown in Figure 10. for the element $x \in E$.

Now, we will combine these two operators together with the first two intuitionistic fuzzy topological operators.

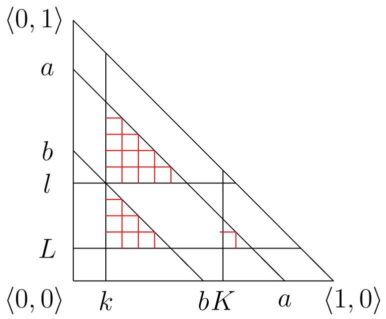
Let everywhere below:

$$a = 1 - \inf_{y \in E}(\mu_A(y) + \nu_A(y)),$$

$$b = 1 - \sup_{y \in E}(\mu_A(y) + \nu_A(y)),$$

while $K, L, k, l$ are as above. In Figure 11, the red regions within the intuitionistic fuzzy interpretation triangle represent the regions where evaluations of the elements of the universe cannot exist.

**Figure 11**
**Impossible case**




Let $x \in E$ and let $x = \langle \mu, \nu \rangle$.

If we assume that $x$ lies in the upper red-colored triangle, then we see that

$$k \leq \mu \leq K,$$

$$b \leq 1 - \mu - \nu \leq a,$$

but $\nu > l$ that is impossible. If $x$ lies in the lower-left red-colored triangle, then we see that

$$k \leq \mu \leq K,$$

$$L \leq \nu \leq l,$$

but $\mu + \nu < b$, while if $x$ lies in the lower-right red-colored triangle, then we see that

$$L \leq \nu \leq l,$$

$$b \leq 1 - \mu - \nu \leq a,$$

but $\mu + \nu > a$.

Now, we will discuss the forms of the different regions that can be generated by the two topological and two modal operators exhausting all possible cases. There are six cases, shown in Figures 12–30. For each separate case, we give the relations among parameters $a, b, K, k, L, l$.
Everywhere below $a > b$.

Therefore, each IFS generates a region with one of the above forms, which contains all evaluations of the elements of the given IFS.

**Figure 12**
**Case 1: $a = b = K + L, K = k, L = l$**

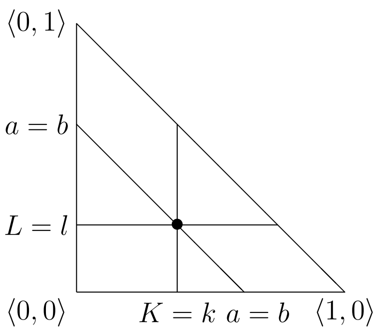



**Figure 13**
**Case 2.1: $a = b = K + L = k + l$**

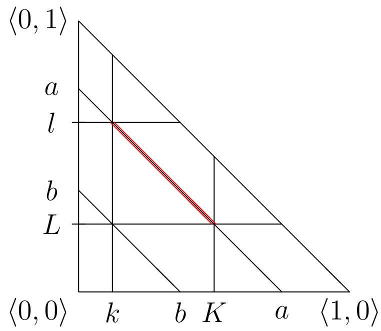



**Figure 14**
**Case 2.2: $L = l$, $a = K + L$, $b = k + l$**

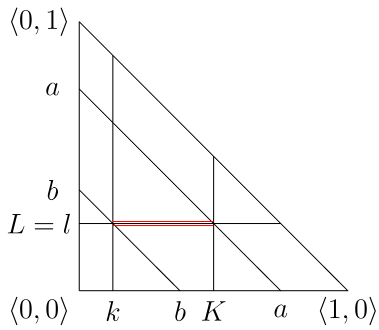



**Figure 15**
**Case 2.3: $K = k$, $a = K + l$, $b = k + L$**

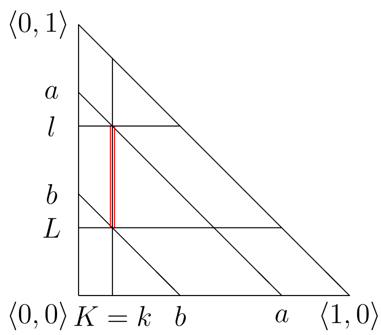



**Figure 16**
**Case 3.1: $a = K + L = k + l, b = k + L$**

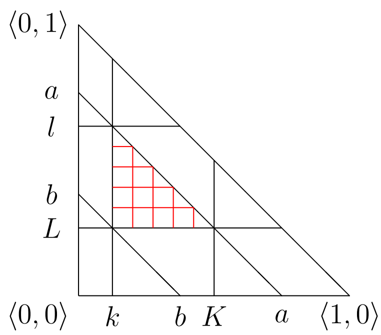


**Figure 17**
**Case 3.2: $\boldsymbol{a = K + l}$, $\boldsymbol{b = K + L = k + l}$**

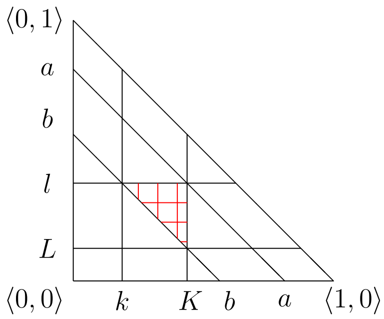



**Figure 18**
**Case 4.1: $\boldsymbol{a = K + l, b = k + L}$**

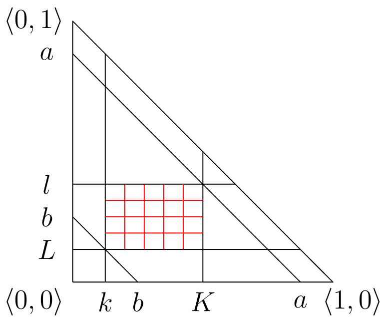



**Figure 19**
**Case 4.2: $\boldsymbol{a = K + L, b = k + L}$**

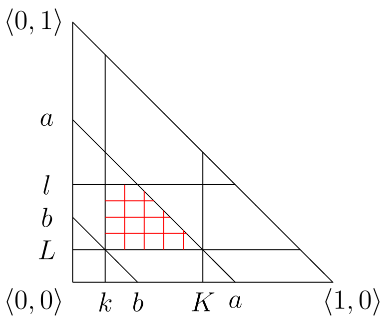



**Figure 20**
**Case 4.3: $\boldsymbol{a = k + l, b = k + L}$**

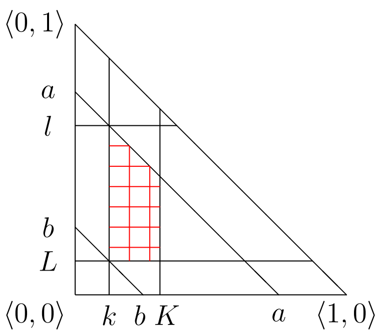



**Figure 21**
**Case 4.4: $\boldsymbol{a = k + l, b = K + L}$**

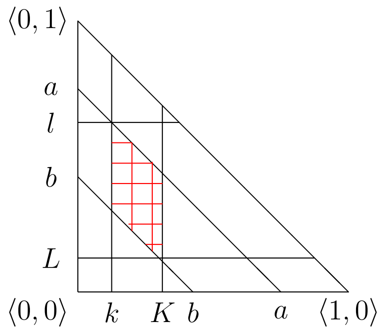



**Figure 22**
**Case 4.5: $\boldsymbol{a = K + L, b > k + L}$**

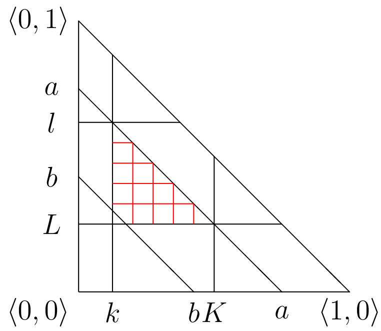



**Figure 23**
**Case 4.6: $\boldsymbol{a = K + L, b = k + l}$**

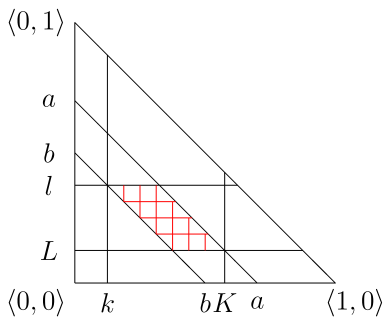



**Figure 24**
**Case 5.1: $\boldsymbol{K + L < a < K + l, b = k + l}$**

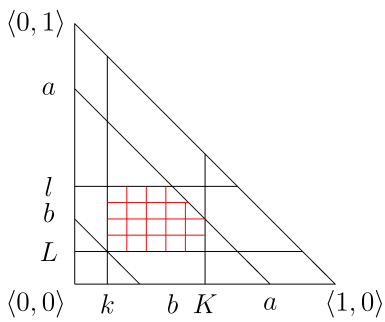


**Figure 25**
**Case 5.2: $a = K + l$, $K + L > b > k + L$**

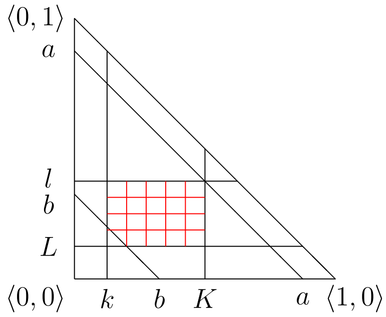



**Figure 26**
**Case 5.3: $a = k + l$, $k + L < b < K + L$**

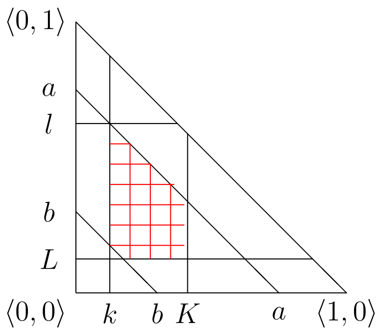



**Figure 27**
**Case 5.4: $a = K + L$, $k + l > b > k + L$**

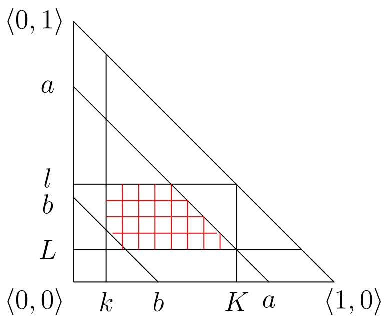



**Figure 28**
**Case 5.5: $K + L < a < K + l$, $b = k + l$**

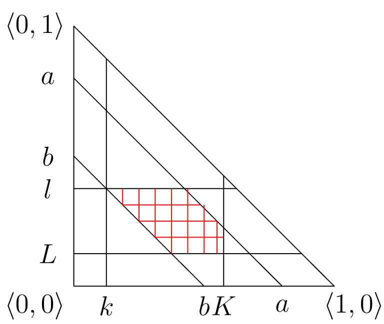



**Figure 29**
**Case 5.6: $a = k + l$, $b = K + L$**

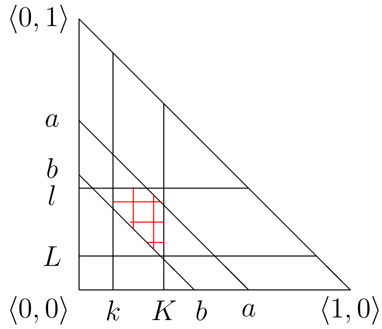
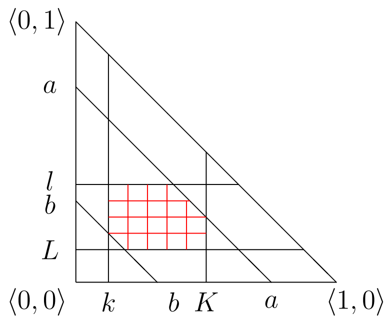



**Figure 30**
**Case 6: $K + l > a > K + L$, $k + l > b > k + L$**




## 4. A New Intuitionistic Fuzzy Topological Operator

First let us determine the coordinates of the vertices of the regions shown in Figures 12–30, starting with the lower left vertex and going counter-clockwise.

- Figure 12: $\langle K, L \rangle$
- Figure 13: $\langle a - L, L \rangle$, $\langle k, l \rangle$
- Figure 14: $\langle k, L \rangle$, $\langle K, l \rangle$
- Figure 15: $\langle k, L \rangle$, $\langle k, l \rangle$
- Figure 16: $\langle k, L \rangle$, $\langle K, L \rangle$, $\langle k, l \rangle$
- Figure 17: $\langle K, L \rangle$, $\langle K, L \rangle$, $\langle k, l \rangle$
- Figure 18: $\langle k, L \rangle$, $\langle K, L \rangle$, $\langle K, l \rangle$, $\langle k, l \rangle$
- Figure 19: $\langle k, L \rangle$, $\langle K, L \rangle$, $\langle a - l, l \rangle$, $\langle k, l \rangle$
- Figure 20: $\langle k, L \rangle$, $\langle K, L \rangle$, $\langle K, a - K \rangle$, $\langle k, l \rangle$
- Figure 21: $\langle K, L \rangle$, $\langle K, a - K \rangle$, $\langle k, l \rangle$, $\langle k, b - k \rangle$
- Figure 22: $\langle b - L, L \rangle$, $\langle K, L \rangle$, $\langle k, l \rangle$, $\langle k, b - k \rangle$
- Figure 23: $\langle b - L, L \rangle$, $\langle K, L \rangle$, $\langle a - l, l \rangle$, $\langle k, l \rangle$
- Figure 24: $\langle k, L \rangle$, $\langle K, L \rangle$, $\langle K, a - K \rangle$, $\langle a - l, l \rangle$, $\langle k, l \rangle$
- Figure 25: $\langle b - L, L \rangle$, $\langle K, L \rangle$, $\langle K, l \rangle$, $\langle k, l \rangle$, $\langle k, b - k \rangle$
- Figure 26: $\langle b - L, L \rangle$, $\langle K, L \rangle$, $\langle K, a - K \rangle$, $\langle k, l \rangle$, $\langle k, b - k \rangle$
- Figure 27: $\langle b - L, L \rangle$, $\langle K, L \rangle$, $\langle a - l, l \rangle$, $\langle k, l \rangle$, $\langle k, b - k \rangle$
- Figure 28: $\langle b - L, L \rangle$, $\langle K, L \rangle$, $\langle K, a - K \rangle$, $\langle a - l, l \rangle$, $\langle k, l \rangle$
- Figure 29: $\langle K, L \rangle$, $\langle K, a - K \rangle$, $\langle a - l, l \rangle$, $\langle k, l \rangle$, $\langle k, b - k \rangle$
- Figure 30: $\langle b - L, L \rangle$, $\langle K, L \rangle$, $\langle K, a - K \rangle$, $\langle a - l, l \rangle$, $\langle k, l \rangle$, $\langle k, b - k \rangle$.

Let the vertices of the region that corresponds to the IFS $A$ be $\langle \mu_{A,1}, \nu_{A,1} \rangle, \ldots, \langle \mu_{A,s(A)}, \nu_{A,s(A)} \rangle$, where $s(A)$ is the number of the vertices of this region.

Now, by analogy with operator $W$ from Section 2, we can introduce the following operator:

$$V(A) = \left\{ \left\langle x, \frac{\sum_{i=1}^{s(A)} \mu_{A,i}}{s(A)}, \frac{\sum_{i=1}^{s(A)} \nu_{A,i}}{s(A)} \right\rangle | x \in E \right\}.$$

Therefore, if the regions from Figures 12–30 are generated by the IFSs $A_1, \ldots, A_{19}$, respectively, then we calculate

$$V(A_1) = \{\langle x, K, L\rangle | x \in E\},$$

$$V(A_2) = \left\{\left\langle x, \frac{K+k}{2}, \frac{L+l}{2}\right\rangle | x \in E\right\},$$

$$V(A_3) = \left\{\left\langle x, \frac{K+k}{2}, L\right\rangle | x \in E\right\},$$

$$V(A_4) = \left\{\left\langle x, K, \frac{L+l}{2}\right\rangle | x \in E\right\},$$

$$V(A_5) = \left\{\left\langle x, \frac{K+2k}{3}, \frac{2L+l}{3}\right\rangle | x \in E\right\},$$

$$V(A_6) = \left\{\left\langle x, \frac{2K+k}{3}, \frac{L+2l}{3}\right\rangle | x \in E\right\},$$

$$V(A_7) = \left\{\left\langle x, \frac{K+k}{2}, \frac{L+l}{2}\right\rangle | x \in E\right\},$$

$$V(A_8) = \left\{\left\langle x, \frac{a+K+2k-l}{4}, \frac{L+l}{2}\right\rangle | x \in E\right\},$$

$$V(A_9) = \left\{\left\langle x, \frac{K+k}{2}, \frac{a-K+k+2L+l}{4}\right\rangle | x \in E\right\},$$

$$V(A_{10}) = \left\{\left\langle x, \frac{K+k}{2}, \frac{a+b-K-k+L+l}{4}\right\rangle | x \in E\right\},$$

$$V(A_{12}) = \left\{\left\langle x, \frac{b+K+2k-L}{4}, \frac{b-k+2L+l}{2}\right\rangle | x \in E\right\},$$

$$V(A_{12}) = \left\{\left\langle x, \frac{a+b+K+k-L-l}{4}, \frac{L+l}{2}\right\rangle | x \in E\right\},$$

$$V(A_{13}) = \left\{\left\langle x, \frac{a+2K+2k-l}{5}, \frac{a-K+2L+2l}{5}\right\rangle | x \in E\right\},$$

$$V(A_{14}) = \left\{\left\langle x, \frac{b+2K+2k-L}{5}, \frac{a+b-K+2L+2l}{5}\right\rangle | x \in E\right\},$$

$$V(A_{15}) = \left\{\left\langle x, \frac{b+2K+2k-L}{5}, \frac{b-K+2L+l}{5}\right\rangle | x \in E\right\},$$

$$V(A_{16}) = \left\{\left\langle x, \frac{a+b+2K+k-L-l}{5}, \frac{b-k+2L+2l}{5}\right\rangle | x \in E\right\},$$

$$V(A_{17}) = \left\{\left\langle x, \frac{a+b+2K+k-L-l}{5}, \frac{a-K+2L+2l}{5}\right\rangle | x \in E\right\}$$

$$V(A_{18}) = \left\{\left\langle x, \frac{a+2K+2k-l}{5}, \frac{a+b-K-k+L+2l}{5}\right\rangle | x \in E\right\}$$

$$V(A_{19}) = \left\{\left\langle x, \frac{a+b+2K+2k-L-l}{6}, \frac{a+b-K-k+2L+2l}{6}\right\rangle | x \in E\right\}.$$

From the above definitions, it follows the validity of the following:

**Theorem.** For each IFS $A_i$, for $i = 1, 2, \ldots, 19$

- $V(V(A_i)) = V(A_i)$,
- $V(C(A_i)) = C(A_i)$,
- $C(V(A_i)) = V(A_i)$,
- $V(I(A_i)) = I(A_i)$,
- $I(V(A_i)) = V(A_i)$,
- $V(W(A_i)) = W(A_i)$,
- $W(V(A_i)) = V(A_i)$.

**Proof.** Let for $i = 1, 2, \ldots, 19$, $A_i$ be an IFS. Then, for example, for $i = 19$, for (a) we obtain

$$\begin{aligned} V(V(A_{19})) &= V\left(\left\{\left\langle x, \frac{a+b+2K+2k-L-l}{6}, \frac{a+b-K-k+2L+2l}{6}\right\rangle | x \in E\right\}\right) \\ &= V(A_{19}), \end{aligned}$$

because all elements of the set $V(A_{19})$ have equal degrees, that is, it has a form, similar to the form of set $A_1$. For (c) we obtain

$$\begin{aligned} C(V(A_{19})) &= C\left(\left\{\left\langle x, \frac{a+b+2K+2k-L-l}{6}, \frac{a+b-K-k+2L+2l}{6}\right\rangle | x \in E\right\}\right) \\ &= \left(\left\{\left\langle x, \sup_{y \in E} \frac{a+b+2K+2k-L-l}{6}, \inf_{y \in E} \frac{a+b-K-k+2L+2l}{6}\right\rangle | x \in E\right\}\right) \\ &= \left(\left\{\left\langle x, \frac{a+b+2K+2k-L-l}{6}, \frac{a+b-K-k+2L+2l}{6}\right\rangle | x \in E\right\}\right) = V(A_{19}). \end{aligned}$$

The validity of assertions (b), (d) – (g) is checked by the same manner, because we see again that all elements of the sets $C(A_i), I(A_i), W(A_i)$ have equal degrees, and in each one of these cases, we obtain a set from $A_i$-type.

## 5. A Property of Operators $V$ and $W$

In Atanassov et al. (2013), the ordered pair $\langle a, b\rangle$ is called an intuitionistic fuzzy pair (IFP) when $a, b, a+b \in [0, 1]$. For two IFPs $\langle a, b\rangle$ and $\langle c, d\rangle$, we define

$$\langle a, b\rangle \le \langle c, d\rangle \text{ if and only if } a \le c \;\&\; b \ge d.$$

We can tell that IFP $\langle a, b\rangle$ is more positive than IFP $\langle c, d\rangle$ if and only if

$$\langle c, d\rangle \le \langle a, b\rangle.$$

Now, we can define: the IFS $A$ is

- *positive* if and only if (see Figure 31)

$$W(A) \subset V(A);$$

- *negative* if and only if (see Figure 32)

$$V(A) \subset W(A);$$

- *indeterminate* otherwise (see Figures 33 and 34).

**Figure 31**
**The IFS $A$ is positive**

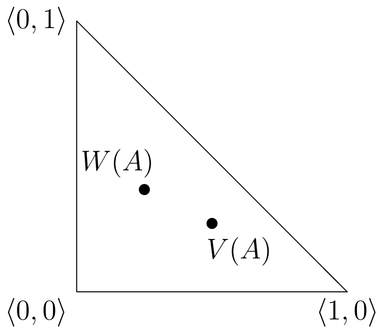



**Figure 32**
**The IFS $A$ is negative**

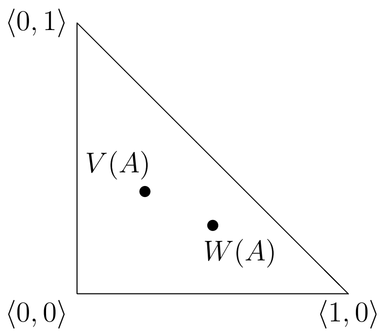



**Figure 33**
**The IFS $A$ is indeterminate**

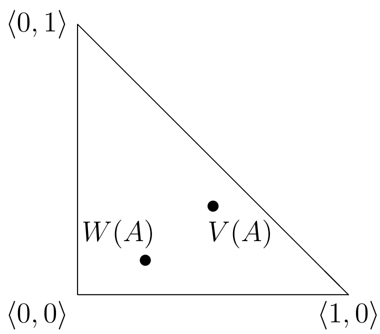
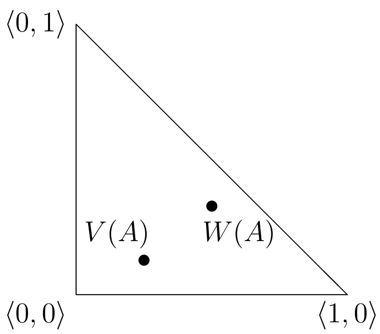



**Figure 34**
**The IFS $A$ is indeterminate**




## 6. Conclusion

In this paper, the possible regions that contain all evaluations of a given IFS elements are described and some of their properties are discussed. A new operator of topological type is introduced.

The present research opens a new direction for research in the area of the intuitionistic fuzziness and generates a lot of **open problems**. Four of them are the following.

- If $\circ$ is some intuitionistic fuzzy operation (for more details on them see, e.g., Atanassov, 2012), then what is the relation between the IFSs $V(A \circ B)$ and $V(A) \circ B(A)$?
- If $T$ is some one of the six intuitionistic fuzzy topological operators given in Section 2 (different from these, mentioned in the theorem), for which of them the equalities $T(V(A)) = V(A)$ and $V(T(A)) = T(A)$ are valid?
- If $M$ is some intuitionistic fuzzy modal operator (for them see, e.g., Atanassov, 2012), then for which of them the equality $M(V(A)) = V(M(A))$ is valid?
- If $L$ is some intuitionistic fuzzy level operator (for them see, e.g., Atanassov, 2012), then for which of them the equality $L(V(A)) = V(L(A))$ is valid?

The new operator can obtain application in different decision-making procedures, in interpretation of the results from intercriteria analysis, etc.

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## Conflicts of Interest

Krassimir T. Atanassov is an associate editor for *Journal of Computational and Cognitive Engineering*, and was not involved in the editorial review or the decision to publish this article. The author declares that he has no conflicts of interest to this work.

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