

RESEARCH ARTICLE



Lattice Ordered Neutrosophic Soft Set and Its Usage for a Large-Scale Company in Adopting Best NGO to Deliver Funds During COVID-19

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Abstract: Soft sets, fuzzy sets, lattices, and their generalisations have always been significant for research on uncertainties. By combining the parameterization tool and neutrosophic logic, neutrosophic soft sets are developed to overcome uncertainty in terms of falsity, indeterminacy and truth membership values. In this study, we generalised the concept of lattice(anti-lattice) ordered neutrosophic soft set and some results are demonstrated by using restricted union, restricted intersection, extended union, and extended intersection. Using this theory, an application is developed to assist decision makers in selecting an NGO to use the Covid-19 fund of a large scale company where some order exists among the elements of parameter sets.

Keywords: neutrosophic soft sets, lattice ordered neutrosophic soft sets, antilattice ordered neutrosophic soft sets, decision making

1. Introduction

Molodtsov (1999) made known the definition of soft sets, a contemporary mathematical way to vagueness. The soft sets have been exercised to varied fields with great boom. In detail, Maji et al. (2003) studied the idea of soft sets and conferred an application. Broumi and Smarandache (2013) introduces the neutrosophic set (NS) comprising inconsistent, indeterminacy, and indefinite data. The properties and applications have been developed increasingly by Aslam et al. (2019), Ajay and Chellamani (2022), Broumi and Smarandache (2013), Broumi (2013) and Karaaslan (2015). Maji (2013) fused the above notions by formulating neutrosophic soft set with many applications. Fuzzy logic was first coined Zadeh (1965). Many researchers have worked on the extension of fuzzy set and soft set which gave rise to a new concept called complex intuitionistic fuzzy soft set (Garg et al., 2022).

Birkhoff (1967) initiated the notion called lattice. Lattices are dominant mathematical tool that has been used nicely to solve many essential problems in computer science, mainly in the fields of combinatorial optimizations, cryptography, and decision making. Ali et al. (2015) and Mahmood et al. (2018) proffered soft sets with order in parameters and proved some important theorems of lattice ordered soft sets. Further lattice ordered structure to various concepts and their applications in myriad of science fields were studied in detail by Khan et al. (2019), Li and Li, (2013), Preethi et al. (2020), Rajareega et al. (2020), Smarandache (2005) and Tripathy et al. (2016).

Keeping in view the importance of neutrosophic soft sets and its generalizations, our purpose is to launch lattice ordered neutrosophic

soft sets. And also the basic union, basic intersection, restricted union, restricted intersection, extended union, and extended intersection on neutrosophic soft sets and lattice ordered neutrosophic soft sets are analyzed. A group decision-making problem has been handled by utilizing these concepts.

2. Preliminaries

Throughout this article, \mathcal{U} denotes the universal set, \mathcal{E} represents the set of parameters, and $\bar{A} \subseteq \mathcal{E}$.

Definition 2.1. Ahmad and Kharal (2009) and (Garg et al., 2022) Let $FP(\mathcal{U})$ denote the set of all fuzzy sets on \mathcal{U} and $\bar{A} \subseteq \mathcal{E}$. Then a pair (\mathcal{F}, \bar{A}) is known as fuzzy soft set over \mathcal{U} , where \mathcal{F} is a mapping given by $\mathcal{F} : \bar{A} \rightarrow FP(\mathcal{U})$.

Definition 2.2. Ali et al. (2015) and Aygun and Aygunoglu (2009) Let (\mathcal{F}, \bar{A}) be a fuzzy soft set over \mathcal{U} . Then it is known to be lattice(antilattice) ordered fuzzy soft set over \mathcal{U} , where $\mathcal{F} : \bar{A} \rightarrow FP(\mathcal{U})$, if $\varepsilon_1 \leq \varepsilon_2$, then $\mathcal{F}(\varepsilon_1) \subseteq \mathcal{F}(\varepsilon_2)$ ($\mathcal{F}(\varepsilon_2) \subseteq \mathcal{F}(\varepsilon_1)$), for every $\varepsilon_1, \varepsilon_2 \in \bar{A}$.

Definition 2.3. Maji (2013) A NS \bar{A} in \mathcal{U} is distinguished by a truth-membership \mathcal{T}_A function, an indeterminacy membership \mathcal{I}_A function, and a falsity-membership \mathcal{F}_A function, where $\mathcal{T}_A, \mathcal{I}_A$, and \mathcal{F}_A are in $[0, 1]$. It can be represented as

$$\bar{A} = \{(x, (\mathcal{T}_A(x), \mathcal{I}_A(x), \mathcal{F}_A(x))) : x \in \mathcal{U} \text{ and } \mathcal{T}_A, \mathcal{I}_A, \mathcal{F}_A \in]^{-0}, 1^{+}[\}$$

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There is no restriction on the sum of $\mathcal{T}_{\mathcal{A}}(\bar{u})$, $\mathcal{I}_{\mathcal{A}}(\bar{u})$, and $\mathcal{F}_{\mathcal{A}}(\bar{u})$ and so

$$0^- \leq \mathcal{T}_{\mathcal{A}}(\bar{u}) + \mathcal{I}_{\mathcal{A}}(\bar{u}) + \mathcal{F}_{\mathcal{A}}(\bar{u}) \leq 3^+$$

Definition 2.4. Maji (2013) Let $NS(\mathcal{U})$ denote the set of all NSSs of \mathcal{U} . Then a pair $(\mathcal{F}, \bar{\mathcal{A}})$ is termed to be a neutrosophic soft set (NSS) over \mathcal{U} where $\mathcal{F} : \bar{\mathcal{A}} \rightarrow NS(\mathcal{U})$.

That is, $(\mathcal{F}, \bar{\mathcal{A}}) = \{\langle u, \mathcal{T}_{\mathcal{F}(\bar{\varepsilon})}(\bar{u}), \mathcal{I}_{\mathcal{F}(\bar{\varepsilon})}(\bar{u}), \mathcal{F}_{\mathcal{F}(\bar{\varepsilon})}(\bar{u}) \rangle : \bar{\varepsilon} \in \bar{\mathcal{A}}, \bar{u} \in \mathcal{U} \text{ and } \mathcal{T}_{\mathcal{F}(\bar{\varepsilon})}(\bar{u}), \mathcal{I}_{\mathcal{F}(\bar{\varepsilon})}(\bar{u}), \mathcal{F}_{\mathcal{F}(\bar{\varepsilon})}(\bar{u}) \in [0, 1]\}$

There is no restriction on the sum $\mathcal{T}_{\mathcal{F}(\bar{\varepsilon})}(\bar{u})$, $\mathcal{I}_{\mathcal{F}(\bar{\varepsilon})}(\bar{u})$ and $\mathcal{F}_{\mathcal{F}(\bar{\varepsilon})}(\bar{u})$ and so

$$0^- \leq \mathcal{T}_{\mathcal{F}(\bar{\varepsilon})}(\bar{u}) + \mathcal{I}_{\mathcal{F}(\bar{\varepsilon})}(\bar{u}) + \mathcal{F}_{\mathcal{F}(\bar{\varepsilon})}(\bar{u}) \leq 3^+$$

Definition 2.5. Maji (2013) Let $(\mathcal{F}, \bar{\mathcal{A}}), (\mathcal{G}, \bar{\mathcal{B}}) \in NSS(\mathcal{U})$. Then $(\mathcal{F}, \bar{\mathcal{A}})$ is a neutrosophic soft subset of $(\mathcal{G}, \bar{\mathcal{B}})$ if

- (i) $\bar{\mathcal{A}} \subseteq \bar{\mathcal{B}}$ and
- (ii) $\mathcal{T}_{\mathcal{F}(\bar{\varepsilon})}(\bar{u}) \leq \mathcal{T}_{\mathcal{G}(\bar{\varepsilon})}(\bar{u})$, $\mathcal{I}_{\mathcal{F}(\bar{\varepsilon})}(\bar{u}) \leq \mathcal{I}_{\mathcal{G}(\bar{\varepsilon})}(\bar{u})$ and $\mathcal{F}_{\mathcal{F}(\bar{\varepsilon})}(\bar{u}) \leq \mathcal{F}_{\mathcal{G}(\bar{\varepsilon})}(\bar{u})$, for every $\bar{\varepsilon} \in \bar{\mathcal{A}}$ and $\bar{u} \in \mathcal{U}$.

We denote it by $(\mathcal{F}, \bar{\mathcal{A}}) \subseteq (\mathcal{G}, \bar{\mathcal{B}})$.

3. Lattice Ordered Neutrosophic Soft Set

Definition 3.1. Let $(\mathcal{F}, \bar{\mathcal{A}})$ be a neutrosophic soft set over \mathcal{U} . Then it is known as lattice ordered neutrosophic soft set over \mathcal{U} ($\mathcal{LONSS}(\mathcal{U})$), where \mathcal{F} is a mapping defined by $\mathcal{F} : \bar{\mathcal{A}} \rightarrow NS(\mathcal{U})$, if $\bar{\varepsilon}_i, \bar{\varepsilon}_j \in \bar{\mathcal{A}}$ such that $\bar{\varepsilon}_i \leq \bar{\varepsilon}_j$, then $\mathcal{F}(\bar{\varepsilon}_i) \subseteq \mathcal{F}(\bar{\varepsilon}_j)$.

(i.e.), $\mathcal{T}_{\mathcal{F}(\bar{\varepsilon}_i)}(\bar{u}) \leq \mathcal{T}_{\mathcal{F}(\bar{\varepsilon}_j)}(\bar{u})$, $\mathcal{I}_{\mathcal{F}(\bar{\varepsilon}_i)}(\bar{u}) \leq \mathcal{I}_{\mathcal{F}(\bar{\varepsilon}_j)}(\bar{u})$ and $\mathcal{F}_{\mathcal{F}(\bar{\varepsilon}_i)}(\bar{u}) \leq \mathcal{F}_{\mathcal{F}(\bar{\varepsilon}_j)}(\bar{u})$, for all $\bar{u} \in \mathcal{U}$.

Example 1. Let $\mathcal{U} = \{h_1, h_2, h_3, h_4, h_5\}$ be the set of houses. Consider

$\mathcal{E} = \{\text{wooden, costly, in bad repair, beautiful, very costly, green surroundings, in good repair, moderate, cheap, expensive}\}$

Let $\bar{\mathcal{A}} = \{\bar{\varepsilon}_1(\text{moderate}), \bar{\varepsilon}_2(\text{beautiful}), \bar{\varepsilon}_3(\text{costly}) \text{ and } \bar{\varepsilon}_4(\text{very costly})\} \subseteq \mathcal{E}$.

The order among the elements of $\bar{\mathcal{A}}$ is $\bar{\varepsilon}_1 \leq \bar{\varepsilon}_2 \leq \bar{\varepsilon}_4$ and $\bar{\varepsilon}_1 \leq \bar{\varepsilon}_3 \leq \bar{\varepsilon}_4$ Table 1.

Clearly $\mathcal{F}(\bar{\varepsilon}_1) \subseteq \mathcal{F}(\bar{\varepsilon}_2) \subseteq \mathcal{F}(\bar{\varepsilon}_4)$ and $\mathcal{F}(\bar{\varepsilon}_1) \subseteq \mathcal{F}(\bar{\varepsilon}_3) \subseteq \mathcal{F}(\bar{\varepsilon}_4)$.

Definition 3.2. Let $(\mathcal{F}, \bar{\mathcal{A}}), (\mathcal{G}, \bar{\mathcal{B}}) \in \mathcal{LONSS}(\mathcal{U})$. Then their restricted union is denoted and is defined by $(\mathcal{F}, \bar{\mathcal{A}}) \cup_{RES} (\mathcal{G}, \bar{\mathcal{B}}) = (\mathcal{H}, \bar{\mathcal{C}})$, where $\bar{\mathcal{C}} = \bar{\mathcal{A}} \cap \bar{\mathcal{B}}$ and $\forall \bar{\varepsilon} \in \bar{\mathcal{C}}, \bar{u} \in \mathcal{U}, \mathcal{H}(\bar{\varepsilon}) = \mathcal{F}(\bar{\varepsilon}) \cup \mathcal{G}(\bar{\varepsilon})$

$$\mathcal{T}_{\mathcal{H}(\bar{\varepsilon})}(\bar{u}) = \text{Max}\{\mathcal{T}_{\mathcal{F}(\bar{\varepsilon})}(\bar{u}), \mathcal{T}_{\mathcal{G}(\bar{\varepsilon})}(\bar{u})\}$$

$$\mathcal{I}_{\mathcal{H}(\bar{\varepsilon})}(\bar{u}) = \text{Min}\{\mathcal{I}_{\mathcal{F}(\bar{\varepsilon})}(\bar{u}), \mathcal{I}_{\mathcal{G}(\bar{\varepsilon})}(\bar{u})\}$$

$$\mathcal{F}_{\mathcal{H}(\bar{\varepsilon})}(\bar{u}) = \text{Min}\{\mathcal{F}_{\mathcal{F}(\bar{\varepsilon})}(\bar{u}), \mathcal{F}_{\mathcal{G}(\bar{\varepsilon})}(\bar{u})\}$$

Proposition 3.3. Let $(\mathcal{F}, \bar{\mathcal{A}}), (\mathcal{G}, \bar{\mathcal{B}}) \in \mathcal{LONSS}(\mathcal{U})$. Then $(\mathcal{F}, \bar{\mathcal{A}}) \cup_{RES} (\mathcal{G}, \bar{\mathcal{B}}) \in \mathcal{LONSS}(\mathcal{U})$.

Proof. Let $(\mathcal{F}, \bar{\mathcal{A}}), (\mathcal{G}, \bar{\mathcal{B}}) \in \mathcal{LONSS}(\mathcal{U})$. Then by Definition 3.2

$$\mathcal{F}(\bar{\varepsilon}) \cup \mathcal{G}(\bar{\varepsilon}) = \mathcal{H}(\bar{\varepsilon}), \text{ where } \bar{\varepsilon} \in \bar{\mathcal{C}} = \bar{\mathcal{A}} \cap \bar{\mathcal{B}}.$$

If $\bar{\mathcal{A}} \cap \bar{\mathcal{B}} = \emptyset$, then the result is trivial.

Now for $\bar{\mathcal{A}} \cap \bar{\mathcal{B}} \neq \emptyset$, since $\bar{\mathcal{A}}, \bar{\mathcal{B}} \subseteq \mathcal{E}$, \therefore for any $\bar{\varepsilon}_i \in \bar{\mathcal{A}} \cap \bar{\mathcal{B}}$

$$\mathcal{F}(\bar{\varepsilon}_i) \subseteq \mathcal{F}(\bar{\varepsilon}_j), \forall \bar{\varepsilon}_i, \bar{\varepsilon}_j \in \bar{\mathcal{A}}$$

and for any $\eta_i \leq_B \eta_j, \mathcal{G}(\eta_i) \subseteq \mathcal{G}(\eta_j), \forall \eta_i, \eta_j \in \bar{\mathcal{B}}$

Now for any $\gamma_i, \gamma_j \in \mathcal{C}$ and $\gamma_i \leq \gamma_j$

$$\Rightarrow \gamma_i, \gamma_j \in \bar{\mathcal{A}} \cap \bar{\mathcal{B}}$$

$$\Rightarrow \gamma_i, \gamma_j \in \bar{\mathcal{A}} \text{ and } \gamma_i, \gamma_j \in \bar{\mathcal{B}}$$

$$\Rightarrow \mathcal{F}(\gamma_i) \subseteq \mathcal{F}(\gamma_j) \text{ and } \mathcal{G}(\gamma_i) \subseteq \mathcal{G}(\gamma_j) \text{ whenever } \gamma_i \leq_{\bar{\mathcal{A}}} \gamma_j, \gamma_i \leq_{\bar{\mathcal{B}}} \gamma_j$$

\Rightarrow

$$\mathcal{T}_{\mathcal{F}(\gamma_i)}(\bar{u}) \leq \mathcal{T}_{\mathcal{F}(\gamma_j)}(\bar{u}), \mathcal{T}_{\mathcal{G}(\gamma_i)}(\bar{u}) \leq \mathcal{T}_{\mathcal{G}(\gamma_j)}(\bar{u})$$

$$\mathcal{I}_{\mathcal{F}(\gamma_i)}(\bar{u}) \leq \mathcal{I}_{\mathcal{F}(\gamma_j)}(\bar{u}), \mathcal{I}_{\mathcal{G}(\gamma_i)}(\bar{u}) \leq \mathcal{I}_{\mathcal{G}(\gamma_j)}(\bar{u})$$

$$\mathcal{F}_{\mathcal{F}(\gamma_i)}(\bar{u}) \leq \mathcal{F}_{\mathcal{F}(\gamma_j)}(\bar{u}), \mathcal{F}_{\mathcal{G}(\gamma_i)}(\bar{u}) \leq \mathcal{F}_{\mathcal{G}(\gamma_j)}(\bar{u})$$

\Rightarrow

$$\text{Max}\{\mathcal{T}_{\mathcal{F}(\gamma_i)}(\bar{u}), \mathcal{T}_{\mathcal{G}(\gamma_i)}(\bar{u})\} \leq \text{Max}\{\mathcal{T}_{\mathcal{F}(\gamma_j)}(\bar{u}), \mathcal{T}_{\mathcal{G}(\gamma_j)}(\bar{u})\}$$

$$\text{Min}\{\mathcal{I}_{\mathcal{F}(\gamma_i)}(\bar{u}), \mathcal{I}_{\mathcal{G}(\gamma_i)}(\bar{u})\} \leq \text{Min}\{\mathcal{I}_{\mathcal{F}(\gamma_j)}(\bar{u}), \mathcal{I}_{\mathcal{G}(\gamma_j)}(\bar{u})\}$$

$$\text{Min}\{\mathcal{F}_{\mathcal{F}(\gamma_i)}(\bar{u}), \mathcal{F}_{\mathcal{G}(\gamma_i)}(\bar{u})\} \leq \text{Min}\{\mathcal{F}_{\mathcal{F}(\gamma_j)}(\bar{u}), \mathcal{F}_{\mathcal{G}(\gamma_j)}(\bar{u})\}$$

\Rightarrow

$$\mathcal{T}_{\mathcal{F}(\gamma_i) \cup \mathcal{G}(\gamma_i)}(\bar{u}) \leq \mathcal{T}_{\mathcal{F}(\gamma_j) \cup \mathcal{G}(\gamma_j)}(\bar{u})$$

$$\mathcal{I}_{\mathcal{F}(\gamma_i) \cup \mathcal{G}(\gamma_i)}(\bar{u}) \leq \mathcal{I}_{\mathcal{F}(\gamma_j) \cup \mathcal{G}(\gamma_j)}(\bar{u})$$

$$\mathcal{F}_{\mathcal{F}(\gamma_i) \cup \mathcal{G}(\gamma_i)}(\bar{u}) \leq \mathcal{F}_{\mathcal{F}(\gamma_j) \cup \mathcal{G}(\gamma_j)}(\bar{u})$$

\Rightarrow

$$\mathcal{T}_{(\mathcal{F} \cup \mathcal{G})(\gamma_i)}(\bar{u}) \leq \mathcal{T}_{(\mathcal{F} \cup \mathcal{G})(\gamma_j)}(\bar{u})$$

$$\mathcal{I}_{(\mathcal{F} \cup \mathcal{G})(\gamma_i)}(\bar{u}) \leq \mathcal{I}_{(\mathcal{F} \cup \mathcal{G})(\gamma_j)}(\bar{u})$$

$$\mathcal{F}_{(\mathcal{F} \cup \mathcal{G})(\gamma_i)}(\bar{u}) \leq \mathcal{F}_{(\mathcal{F} \cup \mathcal{G})(\gamma_j)}(\bar{u})$$

Table 1
Table shows the example of $\mathcal{LONSS}(\mathcal{U})$

$(\mathcal{F}, \bar{\mathcal{A}})$	h_1	h_2	h_3	h_4	h_5
$\bar{\varepsilon}_1$	(0.4, 0.7, 0.9)	(0.5, 0.9, 0.8)	(0.5, 0.6, 0.7)	(0.3, 0.8, 0.7)	(0.7, 0.5, 0.9)
$\bar{\varepsilon}_2$	(0.5, 0.4, 0.6)	(0.7, 0.5, 0.4)	(0.6, 0.2, 0.5)	(0.5, 0.2, 0.6)	(0.8, 0.3, 0.5)
$\bar{\varepsilon}_3$	(0.7, 0.6, 0.8)	(0.6, 0.4, 0.3)	(0.8, 0.2, 0.6)	(0.6, 0.3, 0.4)	(0.8, 0.3, 0.7)
$\bar{\varepsilon}_4$	(0.8, 0.3, 0.4)	(0.8, 0.3, 0.2)	(0.9, 0.1, 0.4)	(0.7, 0.1, 0.3)	(0.9, 0.2, 0.4)

$$\Rightarrow \mathcal{T}_{\mathcal{H}(\gamma_i)}(\bar{u}) \leq \mathcal{T}_{\mathcal{H}(\gamma_j)}(\bar{u})$$

$$\mathcal{I}_{\mathcal{H}(\gamma_j)}(\bar{u}) \leq \mathcal{I}_{\mathcal{H}(\gamma_i)}(\bar{u})$$

$$\mathcal{F}_{\mathcal{H}(\gamma_j)}(\bar{u}) \leq \mathcal{F}_{\mathcal{H}(\gamma_i)}(\bar{u})$$

\Rightarrow

$$\mathcal{H}(\gamma_i) \subseteq \mathcal{H}(\gamma_j) \text{ for } \gamma_i \leq \gamma_j$$

\Rightarrow

$$(\mathcal{F}, \bar{\mathcal{A}}) \cup_{RES} (\mathcal{G}, \bar{\mathcal{B}}) \in \mathcal{LONSS}(\mathcal{U}). \quad \square$$

Example 2. Suppose $\mathcal{U} = \{u_1, u_2, u_3\}$ is a set of cars and $\mathcal{E} = \{\bar{\varepsilon}_1(\text{Color}), \bar{\varepsilon}_2(\text{Price}), \bar{\varepsilon}_3(\text{Tax}), \bar{\varepsilon}_4(\text{Speed})\}$ is the parameter set and $\bar{\mathcal{A}}, \bar{\mathcal{B}} \subseteq \mathcal{E}$, $\bar{\mathcal{A}} = \{\bar{\varepsilon}_1, \bar{\varepsilon}_2, \bar{\varepsilon}_3\}$, $\bar{\mathcal{B}} = \{\bar{\varepsilon}_2, \bar{\varepsilon}_3, \bar{\varepsilon}_4\}$ and $\bar{\mathcal{A}} \cap \bar{\mathcal{B}} = \{\bar{\varepsilon}_2, \bar{\varepsilon}_3\}$.

The order among the elements of $\bar{\mathcal{A}}$ is $\bar{\varepsilon}_1 \leq_{\bar{\mathcal{A}}} \bar{\varepsilon}_2 \leq_{\bar{\mathcal{A}}} \bar{\varepsilon}_3$.

$(\mathcal{F}, \bar{\mathcal{A}})$	u_1	u_2	u_3
$\bar{\varepsilon}_1$	(0.2, 0.3, 0.6)	(0.4, 0.3, 0.6)	(0.5, 0.4, 0.7)
$\bar{\varepsilon}_2$	(0.4, 0.2, 0.2)	(0.8, 0.2, 0.4)	(0.6, 0.2, 0.5)
$\bar{\varepsilon}_3$	(0.6, 0.1, 0.1)	(0.9, 0.1, 0.2)	(0.7, 0.1, 0.3)

Clearly $\mathcal{F}(\bar{\varepsilon}_1) \subseteq \mathcal{F}(\bar{\varepsilon}_2) \subseteq \mathcal{F}(\bar{\varepsilon}_3)$.

The order among the elements of $\bar{\mathcal{B}}$ is $\bar{\varepsilon}_2 \leq_{\bar{\mathcal{B}}} \bar{\varepsilon}_3 \leq_{\bar{\mathcal{B}}} \bar{\varepsilon}_4$.

$(\mathcal{G}, \bar{\mathcal{B}})$	u_1	u_2	u_3
$\bar{\varepsilon}_2$	(0.3, 0.6, 0.7)	(0.4, 0.6, 0.9)	(0.5, 0.8, 0.3)
$\bar{\varepsilon}_3$	(0.4, 0.2, 0.4)	(0.5, 0.3, 0.7)	(0.6, 0.6, 0.2)
$\bar{\varepsilon}_4$	(0.8, 0.1, 0.3)	(0.6, 0.2, 0.5)	(0.7, 0.4, 0.1)

Clearly $\mathcal{G}(\bar{\varepsilon}_2) \subseteq \mathcal{G}(\bar{\varepsilon}_3) \subseteq \mathcal{G}(\bar{\varepsilon}_4)$.

Then $(\mathcal{F}, \bar{\mathcal{A}}) \cup_{RES} (\mathcal{G}, \bar{\mathcal{B}}) = (\mathcal{H}, \bar{\mathcal{C}})$ is the restricted union of two

$(\mathcal{H}, \bar{\mathcal{C}})$	u_1	u_2	u_3
$\bar{\varepsilon}_2$	(0.4, 0.2, 0.2)	(0.8, 0.2, 0.4)	(0.6, 0.2, 0.3)
$\bar{\varepsilon}_3$	(0.6, 0.1, 0.1)	(0.9, 0.1, 0.2)	(0.7, 0.1, 0.2)

\mathcal{LONSS} over \mathcal{U} .

$\Rightarrow \mathcal{H}(\bar{\varepsilon}_2) \subseteq \mathcal{H}(\bar{\varepsilon}_3)$, for $\bar{\varepsilon}_2 \leq \bar{\varepsilon}_3$.

$\Rightarrow (\mathcal{F}, \bar{\mathcal{A}}) \cup_{RES} (\mathcal{G}, \bar{\mathcal{B}}) \in \mathcal{LONSS}(\mathcal{U})$.

Definition 3.4. Let $(\mathcal{F}, \bar{\mathcal{A}}), (\mathcal{G}, \bar{\mathcal{B}}) \in \mathcal{LONSS}(\mathcal{U})$. Then their restricted intersection is denoted and is defined by $(\mathcal{F}, \bar{\mathcal{A}}) \cap_{RES} (\mathcal{G}, \bar{\mathcal{B}}) = (\mathcal{H}, \bar{\mathcal{C}})$, where $\bar{\mathcal{C}} = \bar{\mathcal{A}} \cap \bar{\mathcal{B}}$ and $\forall \bar{\varepsilon} \in \bar{\mathcal{C}}, \bar{u} \in \mathcal{U}$, $\mathcal{H}(\bar{\varepsilon}) = \mathcal{F}(\bar{\varepsilon}) \cap \mathcal{G}(\bar{\varepsilon})$

$$\mathcal{T}_{\mathcal{H}(\bar{\varepsilon})}(\bar{u}) = \text{Min}\{\mathcal{T}_{\mathcal{F}(\bar{\varepsilon})}(\bar{u}), \mathcal{T}_{\mathcal{G}(\bar{\varepsilon})}(\bar{u})\}$$

$$\mathcal{I}_{\mathcal{H}(\bar{\varepsilon})}(\bar{u}) = \text{Max}\{\mathcal{I}_{\mathcal{F}(\bar{\varepsilon})}(\bar{u}), \mathcal{I}_{\mathcal{G}(\bar{\varepsilon})}(\bar{u})\}$$

$$\mathcal{F}_{\mathcal{H}(\bar{\varepsilon})}(\bar{u}) = \text{Max}\{\mathcal{F}_{\mathcal{F}(\bar{\varepsilon})}(\bar{u}), \mathcal{F}_{\mathcal{G}(\bar{\varepsilon})}(\bar{u})\}$$

Proposition 3.5. Let $(\mathcal{F}, \bar{\mathcal{A}}), (\mathcal{G}, \bar{\mathcal{B}}) \in \mathcal{LONSS}(\mathcal{U})$. Then $(\mathcal{F}, \bar{\mathcal{A}}) \cap_{RES} (\mathcal{G}, \bar{\mathcal{B}}) \in \mathcal{LONSS}(\mathcal{U})$.

Example 3. Suppose $\mathcal{U} = \{u_1, u_2, u_3\}$ is a set of shoes and $\mathcal{E} = \{\bar{\varepsilon}_1(\text{Price}), \bar{\varepsilon}_2(\text{Color}), \bar{\varepsilon}_3(\text{Quality}), \bar{\varepsilon}_4(\text{Comfort})\}$ is the parameter set and $\bar{\mathcal{A}}, \bar{\mathcal{B}} \subseteq \mathcal{E}$, $\bar{\mathcal{A}} = \{\bar{\varepsilon}_1, \bar{\varepsilon}_2, \bar{\varepsilon}_3\}$, $\bar{\mathcal{B}} = \{\bar{\varepsilon}_2, \bar{\varepsilon}_3, \bar{\varepsilon}_4\}$ and $\bar{\mathcal{A}} \cap \bar{\mathcal{B}} = \{\bar{\varepsilon}_2, \bar{\varepsilon}_3\}$.

The order among the elements of $\bar{\mathcal{A}}$ is $\bar{\varepsilon}_1 \leq_{\bar{\mathcal{A}}} \bar{\varepsilon}_2 \leq_{\bar{\mathcal{A}}} \bar{\varepsilon}_3$.

$(\mathcal{F}, \bar{\mathcal{A}})$	u_1	u_2	u_3
$\bar{\varepsilon}_1$	(0.2, 0.3, 0.7)	(0.4, 0.3, 0.6)	(0.5, 0.4, 0.8)
$\bar{\varepsilon}_2$	(0.4, 0.2, 0.4)	(0.8, 0.2, 0.5)	(0.6, 0.2, 0.5)
$\bar{\varepsilon}_3$	(0.6, 0.1, 0.2)	(0.9, 0.1, 0.4)	(0.7, 0.1, 0.2)

Clearly $\mathcal{F}(\bar{\varepsilon}_1) \subseteq \mathcal{F}(\bar{\varepsilon}_2) \subseteq \mathcal{F}(\bar{\varepsilon}_3)$.

The order among the elements of $\bar{\mathcal{B}}$ is $\bar{\varepsilon}_2 \leq_{\bar{\mathcal{B}}} \bar{\varepsilon}_3 \leq_{\bar{\mathcal{B}}} \bar{\varepsilon}_4$.

$(\mathcal{G}, \bar{\mathcal{B}})$	u_1	u_2	u_3
$\bar{\varepsilon}_2$	(0.3, 0.6, 0.4)	(0.4, 0.6, 0.7)	(0.5, 0.6, 0.7)
$\bar{\varepsilon}_3$	(0.4, 0.2, 0.3)	(0.5, 0.3, 0.6)	(0.6, 0.5, 0.4)
$\bar{\varepsilon}_4$	(0.8, 0.1, 0.2)	(0.6, 0.2, 0.5)	(0.7, 0.4, 0.2)

Clearly $\mathcal{G}(\bar{\varepsilon}_2) \subseteq \mathcal{G}(\bar{\varepsilon}_3) \subseteq \mathcal{G}(\bar{\varepsilon}_4)$.

Then $(\mathcal{F}, \bar{\mathcal{A}}) \cap_{RES} (\mathcal{G}, \bar{\mathcal{B}}) = (\mathcal{H}, \bar{\mathcal{C}})$ is the restricted intersection of two \mathcal{LONSS} over \mathcal{U} .

$(\mathcal{H}, \bar{\mathcal{C}})$	u_1	u_2	u_3
$\bar{\varepsilon}_2$	(0.3, 0.6, 0.4)	(0.4, 0.6, 0.7)	(0.5, 0.6, 0.7)
$\bar{\varepsilon}_3$	(0.4, 0.2, 0.3)	(0.5, 0.3, 0.6)	(0.6, 0.5, 0.4)

$\Rightarrow \mathcal{H}(\bar{\varepsilon}_2) \subseteq \mathcal{H}(\bar{\varepsilon}_3)$, for $\bar{\varepsilon}_2 \leq \bar{\varepsilon}_3$.

$\Rightarrow (\mathcal{F}, \bar{\mathcal{A}}) \cap_{RES} (\mathcal{G}, \bar{\mathcal{B}}) \in \mathcal{LONSS}(\mathcal{U})$.

Definition 3.6. Let $(\mathcal{F}, \bar{\mathcal{A}}), (\mathcal{G}, \bar{\mathcal{B}}) \in \mathcal{LONSS}(\mathcal{U})$. Then their extended union is denoted and is defined by $(\mathcal{F}, \bar{\mathcal{A}}) \cup_{EXT} (\mathcal{G}, \bar{\mathcal{B}}) = (\mathcal{H}, \bar{\mathcal{C}})$ where $\bar{\mathcal{C}} = \bar{\mathcal{A}} \cup \bar{\mathcal{B}}$

$$(\mathcal{H}, \bar{\mathcal{C}}) = \begin{cases} \langle \mathcal{T}_{\mathcal{F}(\bar{\varepsilon})}(\bar{u}), \mathcal{I}_{\mathcal{F}(\bar{\varepsilon})}(\bar{u}), \mathcal{F}_{\mathcal{F}(\bar{\varepsilon})}(\bar{u}) \rangle & \text{if } \bar{\varepsilon} \in \bar{\mathcal{A}} - \bar{\mathcal{B}} \\ \langle \mathcal{T}_{\mathcal{G}(\bar{\varepsilon})}(\bar{u}), \mathcal{I}_{\mathcal{G}(\bar{\varepsilon})}(\bar{u}), \mathcal{F}_{\mathcal{G}(\bar{\varepsilon})}(\bar{u}) \rangle & \text{if } \bar{\varepsilon} \in \bar{\mathcal{B}} - \bar{\mathcal{A}} \\ \langle \text{Max}\{\mathcal{T}_{\mathcal{F}(\bar{\varepsilon})}(\bar{u}), \mathcal{T}_{\mathcal{G}(\bar{\varepsilon})}(\bar{u})\}, \text{Min}\{\mathcal{I}_{\mathcal{F}(\bar{\varepsilon})}(\bar{u}), \mathcal{I}_{\mathcal{G}(\bar{\varepsilon})}(\bar{u})\}, \\ \text{Min}\{\mathcal{F}_{\mathcal{F}(\bar{\varepsilon})}(\bar{u}), \mathcal{F}_{\mathcal{G}(\bar{\varepsilon})}(\bar{u})\} \rangle & \text{if } \bar{\varepsilon} \in \bar{\mathcal{A}} \cap \bar{\mathcal{B}} \end{cases}$$

Proposition 3.7. Let $(\mathcal{F}, \bar{\mathcal{A}}), (\mathcal{G}, \bar{\mathcal{B}}) \in \mathcal{LONSS}(\mathcal{U})$. Then $(\mathcal{F}, \bar{\mathcal{A}}) \cup_{EXT} (\mathcal{G}, \bar{\mathcal{B}}) = (\mathcal{H}, \bar{\mathcal{C}}) \in \mathcal{LONSS}(\mathcal{U})$, if one of them is a lattice ordered neutrosophic soft subset of other.

Proof. Let $(\mathcal{F}, \bar{\mathcal{A}}), (\mathcal{G}, \bar{\mathcal{B}}) \in \mathcal{LONSS}(\mathcal{U})$. Then by Definition 3.6

$$\mathcal{F}(\bar{\varepsilon}) \cup \mathcal{G}(\bar{\varepsilon}) = \mathcal{H}(\bar{\varepsilon}) \text{ where } \bar{\varepsilon} \in \bar{\mathcal{C}} = \bar{\mathcal{A}} \cup \bar{\mathcal{B}}$$

Suppose $(\mathcal{F}, \bar{\mathcal{A}}) \subseteq (\mathcal{G}, \bar{\mathcal{B}})$. Then $\bar{\mathcal{A}} \subseteq \bar{\mathcal{B}}$ and $\mathcal{T}_{\mathcal{F}(\bar{\varepsilon})}(\bar{u}) \leq \mathcal{T}_{\mathcal{G}(\bar{\varepsilon})}(\bar{u})$, $\mathcal{I}_{\mathcal{G}(\bar{\varepsilon})}(\bar{u}) \leq \mathcal{I}_{\mathcal{F}(\bar{\varepsilon})}(\bar{u})$ and $\mathcal{F}_{\mathcal{G}(\bar{\varepsilon})}(\bar{u}) \leq \mathcal{F}_{\mathcal{F}(\bar{\varepsilon})}(\bar{u})$, for every $\bar{\varepsilon} \in \bar{\mathcal{A}}$ and $\bar{u} \in \mathcal{U}$

Since $\bar{\mathcal{A}}, \bar{\mathcal{B}} \subseteq \mathcal{E}$, \therefore for any $\bar{\varepsilon}_i \leq_{\bar{\mathcal{A}}} \bar{\varepsilon}_j$

$$\mathcal{F}(\bar{\varepsilon}_i) \subseteq \mathcal{F}(\bar{\varepsilon}_j), \text{ for every } \bar{\varepsilon}_i, \bar{\varepsilon}_j \in \bar{\mathcal{A}}$$

for any $\eta_i \leq_{\bar{\mathcal{B}}} \eta_j$, $\mathcal{G}(\eta_i) \subseteq \mathcal{G}(\eta_j)$, for every $\eta_i, \eta_j \in \bar{\mathcal{B}}$

\therefore for any $\gamma_i, \gamma_j \in \bar{\mathcal{C}}$ and $\gamma_i \leq \gamma_j$

$\Rightarrow \gamma_i, \gamma_j \in \bar{\mathcal{A}} \cup \bar{\mathcal{B}}$

$\Rightarrow \gamma_i, \gamma_j \in \bar{A} \cap \bar{B}$ or $\gamma_i, \gamma_j \in \bar{B}$ and $\gamma_i, \gamma_j \notin \bar{A}$ because $\bar{A} \subseteq \bar{B}$

Now take $\gamma_i, \gamma_j \in \bar{A} \cap \bar{B}$

$\Rightarrow \gamma_i, \gamma_j \in \bar{A}$ and $\gamma_i, \gamma_j \in \bar{B}$

$\Rightarrow \mathcal{F}(\gamma_i) \subseteq \mathcal{F}(\gamma_j)$ and $\mathcal{G}(\gamma_i) \subseteq \mathcal{G}(\gamma_j)$ whenever $\gamma_i \leq_{\bar{A}} \gamma_j$ and $\gamma_i \leq_{\bar{B}} \gamma_j$

\Rightarrow

$$\mathcal{T}_{\mathcal{F}(\gamma_i)}(\bar{u}) \leq \mathcal{T}_{\mathcal{F}(\gamma_j)}(\bar{u}), \mathcal{T}_{\mathcal{G}(\gamma_i)}(\bar{u}) \leq \mathcal{T}_{\mathcal{G}(\gamma_j)}(\bar{u})$$

$$\mathcal{I}_{\mathcal{F}(\gamma_i)}(\bar{u}) \leq \mathcal{I}_{\mathcal{F}(\gamma_j)}(\bar{u}), \mathcal{I}_{\mathcal{G}(\gamma_i)}(\bar{u}) \leq \mathcal{I}_{\mathcal{G}(\gamma_j)}(\bar{u})$$

$$\mathcal{F}_{\mathcal{F}(\gamma_i)}(\bar{u}) \leq \mathcal{F}_{\mathcal{F}(\gamma_j)}(\bar{u}), \mathcal{F}_{\mathcal{G}(\gamma_i)}(\bar{u}) \leq \mathcal{F}_{\mathcal{G}(\gamma_j)}(\bar{u})$$

\Rightarrow

$$\text{Max}\{\mathcal{T}_{\mathcal{F}(\gamma_i)}(\bar{u}), \mathcal{T}_{\mathcal{G}(\gamma_i)}(\bar{u})\} \leq \text{Max}\{\mathcal{T}_{\mathcal{F}(\gamma_j)}(\bar{u}), \mathcal{T}_{\mathcal{G}(\gamma_j)}(\bar{u})\}$$

$$\text{Min}\{\mathcal{I}_{\mathcal{F}(\gamma_i)}(\bar{u}), \mathcal{I}_{\mathcal{G}(\gamma_i)}(\bar{u})\} \leq \text{Min}\{\mathcal{I}_{\mathcal{F}(\gamma_j)}(\bar{u}), \mathcal{I}_{\mathcal{G}(\gamma_j)}(\bar{u})\}$$

$$\text{Min}\{\mathcal{F}_{\mathcal{F}(\gamma_i)}(\bar{u}), \mathcal{F}_{\mathcal{G}(\gamma_i)}(\bar{u})\} \leq \text{Min}\{\mathcal{F}_{\mathcal{F}(\gamma_j)}(\bar{u}), \mathcal{F}_{\mathcal{G}(\gamma_j)}(\bar{u})\}$$

\Rightarrow

$$\mathcal{T}_{\mathcal{F}(\gamma_i) \cup \mathcal{G}(\gamma_i)}(\bar{u}) \leq \mathcal{T}_{\mathcal{F}(\gamma_j) \cup \mathcal{G}(\gamma_j)}(\bar{u})$$

$$\mathcal{I}_{\mathcal{F}(\gamma_i) \cup \mathcal{G}(\gamma_i)}(\bar{u}) \leq \mathcal{I}_{\mathcal{F}(\gamma_j) \cup \mathcal{G}(\gamma_j)}(\bar{u})$$

$$\mathcal{F}_{\mathcal{F}(\gamma_i) \cup \mathcal{G}(\gamma_i)}(\bar{u}) \leq \mathcal{F}_{\mathcal{F}(\gamma_j) \cup \mathcal{G}(\gamma_j)}(\bar{u})$$

\Rightarrow

$$\mathcal{T}_{(\mathcal{F} \cup \mathcal{G})(\gamma_i)}(\bar{u}) \leq \mathcal{T}_{(\mathcal{F} \cup \mathcal{G})(\gamma_j)}(\bar{u})$$

$$\mathcal{I}_{(\mathcal{F} \cup \mathcal{G})(\gamma_i)}(\bar{u}) \leq \mathcal{I}_{(\mathcal{F} \cup \mathcal{G})(\gamma_j)}(\bar{u})$$

$$\mathcal{F}_{(\mathcal{F} \cup \mathcal{G})(\gamma_i)}(\bar{u}) \leq \mathcal{F}_{(\mathcal{F} \cup \mathcal{G})(\gamma_j)}(\bar{u})$$

\Rightarrow

$$\mathcal{T}_{\mathcal{H}(\gamma_i)}(\bar{u}) \leq \mathcal{T}_{\mathcal{H}(\gamma_j)}(\bar{u})$$

$$\mathcal{I}_{\mathcal{H}(\gamma_i)}(\bar{u}) \leq \mathcal{I}_{\mathcal{H}(\gamma_j)}(\bar{u})$$

$$\mathcal{F}_{\mathcal{H}(\gamma_i)}(\bar{u}) \leq \mathcal{F}_{\mathcal{H}(\gamma_j)}(\bar{u})$$

\Rightarrow

$$\mathcal{H}(\gamma_i) \subseteq \mathcal{H}(\gamma_j) \text{ for } \gamma_i \leq \gamma_j$$

Thus, $(\mathcal{F}, \bar{A}) \cup_{EXT} (\mathcal{G}, \bar{B}) = (\mathcal{H}, \bar{C}) \in \mathcal{LONSS}(U)$ if $\gamma_i, \gamma_j \in \bar{A} \cap \bar{B}$

Now suppose for any $\gamma_i, \gamma_j \in B$ and $\gamma_i, \gamma_j \notin \bar{A}$ and $\gamma_i \leq_{\bar{B}} \gamma_j$

$\Rightarrow \mathcal{G}(\gamma_i) \subseteq \mathcal{G}(\gamma_j)$ whenever $\gamma_i \leq_{\bar{B}} \gamma_j$

implies this is also a \mathcal{LONSS} .

Hence $(\mathcal{F}, \bar{A}) \cup_{EXT} (\mathcal{G}, \bar{B}) = (\mathcal{H}, \bar{C}) \in \mathcal{LONSS}(U)$ for both cases.

$\Rightarrow (\mathcal{F}, \bar{A}) \cup_{EXT} (\mathcal{G}, \bar{B}) = (\mathcal{H}, \bar{C}) \in \mathcal{LONSS}(U)$, if one of them is a lattice ordered neutrosophic soft subset of other. \square

Example 4. Let $U = \{u_1, u_2, u_3\}$ be the set of men and $\mathcal{E} = \{\bar{\varepsilon}_1(\text{educated}), \bar{\varepsilon}_2(\text{businessman}), \bar{\varepsilon}_3(\text{smart}), \bar{\varepsilon}_4(\text{government employee}), \bar{\varepsilon}_5(\text{bank balance})\}$ is the parameter set $\bar{A}, \bar{B} \subseteq E$ and $\bar{A} \subseteq \bar{B}$, $\bar{A} = \{\bar{\varepsilon}_2, \bar{\varepsilon}_3, \bar{\varepsilon}_4\}$, $\bar{B} = \{\bar{\varepsilon}_1, \bar{\varepsilon}_2, \bar{\varepsilon}_3, \bar{\varepsilon}_4, \bar{\varepsilon}_5\}$.

The order among the elements of \bar{A} is $\bar{\varepsilon}_2 \leq_{\bar{A}} \bar{\varepsilon}_3 \leq_{\bar{A}} \bar{\varepsilon}_4$.

(\mathcal{F}, \bar{A})	u_1	u_2	u_3
$\bar{\varepsilon}_2$	(0.1, 0.9, 0.7)	(0.1, 0.7, 0.6)	(0.3, 0.7, 0.9)
$\bar{\varepsilon}_3$	(0.2, 0.7, 0.6)	(0.3, 0.6, 0.5)	(0.4, 0.6, 0.8)
$\bar{\varepsilon}_4$	(0.6, 0.3, 0.5)	(0.6, 0.4, 0.4)	(0.5, 0.4, 0.7)

Clearly $\mathcal{F}(\bar{\varepsilon}_2) \subseteq \mathcal{F}(\bar{\varepsilon}_3) \subseteq \mathcal{F}(\bar{\varepsilon}_4)$.

The order among the elements of \bar{B} is $\bar{\varepsilon}_1 \leq_{\bar{B}} \bar{\varepsilon}_2 \leq_{\bar{B}} \bar{\varepsilon}_3 \leq_{\bar{B}} \bar{\varepsilon}_4 \leq_{\bar{B}} \bar{\varepsilon}_5$.

(\mathcal{G}, \bar{B})	u_1	u_2	u_3
$\bar{\varepsilon}_1$	(0.1, 0.9, 0.7)	(0.1, 0.9, 0.6)	(0.3, 0.7, 0.8)
$\bar{\varepsilon}_2$	(0.2, 0.6, 0.5)	(0.3, 0.4, 0.5)	(0.5, 0.5, 0.7)
$\bar{\varepsilon}_3$	(0.5, 0.4, 0.4)	(0.5, 0.3, 0.4)	(0.6, 0.3, 0.5)
$\bar{\varepsilon}_4$	(0.7, 0.2, 0.3)	(0.7, 0.2, 0.3)	(0.7, 0.2, 0.4)
$\bar{\varepsilon}_5$	(0.8, 0.1, 0.2)	(0.9, 0.1, 0.2)	(0.9, 0.1, 0.3)

Clearly $\mathcal{G}(\bar{\varepsilon}_1) \subseteq \mathcal{G}(\bar{\varepsilon}_2) \subseteq \mathcal{G}(\bar{\varepsilon}_3) \subseteq \mathcal{G}(\bar{\varepsilon}_4) \subseteq \mathcal{G}(\bar{\varepsilon}_5)$ and $(\mathcal{F}, \bar{A}) \subseteq (\mathcal{G}, \bar{B})$ and extended union is defined as

(\mathcal{H}, \bar{C})	u_1	u_2	u_3
$\bar{\varepsilon}_1$	(0.1, 0.9, 0.7)	(0.1, 0.9, 0.6)	(0.3, 0.7, 0.8)
$\bar{\varepsilon}_2$	(0.2, 0.6, 0.5)	(0.3, 0.4, 0.5)	(0.5, 0.5, 0.7)
$\bar{\varepsilon}_3$	(0.5, 0.4, 0.4)	(0.5, 0.3, 0.4)	(0.6, 0.3, 0.5)
$\bar{\varepsilon}_4$	(0.7, 0.2, 0.3)	(0.7, 0.2, 0.3)	(0.7, 0.2, 0.4)
$\bar{\varepsilon}_5$	(0.8, 0.1, 0.2)	(0.9, 0.1, 0.2)	(0.9, 0.1, 0.3)

$\Rightarrow H(\bar{\varepsilon}_1) \subseteq H(\bar{\varepsilon}_2) \subseteq H(\bar{\varepsilon}_3) \subseteq H(\bar{\varepsilon}_4) \subseteq H(\bar{\varepsilon}_5)$, for $\bar{\varepsilon}_1 \leq \bar{\varepsilon}_2 \leq \bar{\varepsilon}_3 \leq \bar{\varepsilon}_4 \leq \bar{\varepsilon}_5$
 $\Rightarrow (\mathcal{F}, \bar{A}) \cup_{EXT} (\mathcal{G}, \bar{B}) = (\mathcal{H}, \bar{C}) \in \mathcal{LONSS}(U)$.

Definition 3.8. Let $(\mathcal{F}, \bar{A}) \in \mathcal{LONSS}(U)$. Then complement of (\mathcal{F}, \bar{A}) is denoted by $(\mathcal{F}, \bar{A})^c$ and is

$$(\mathcal{F}, \bar{A})^c = \{(u, \mathcal{F}_{\mathcal{F}(\bar{\varepsilon})}(\bar{u}), 1 - \mathcal{I}_{\mathcal{F}(\bar{\varepsilon})}(\bar{u}), \mathcal{T}_{\mathcal{F}(\bar{\varepsilon})}(\bar{u})) : \bar{\varepsilon} \in \bar{A} \text{ and } \bar{u} \in U\}$$

Definition 3.9. Let $(\mathcal{F}, \bar{A}) \in \mathcal{LONSS}(U)$.

If $\mathcal{T}_{\mathcal{F}(\bar{\varepsilon})}(\bar{u}) = 0$ and $\mathcal{I}_{\mathcal{F}(\bar{\varepsilon})}(\bar{u}) = \mathcal{F}_{\mathcal{F}(\bar{\varepsilon})}(\bar{u}) = 1, \forall \bar{\varepsilon} \in \bar{A}$ and $\bar{u} \in U$, then (\mathcal{F}, \bar{A}) is called a relative null \mathcal{LONSS} and denoted by $\Phi_{\bar{A}}$

Similarly, the relative null \mathcal{LONSS} is the null \mathcal{LONSS} with respect to \mathcal{E} and is indicated by Φ .

Definition 3.10. Let $(\mathcal{F}, \bar{A}) \in \mathcal{LONSS}(U)$.

If $\mathcal{T}_{\mathcal{F}(\bar{\varepsilon})}(\bar{u}) = 1$ and $\mathcal{I}_{\mathcal{F}(\bar{\varepsilon})}(\bar{u}) = \mathcal{F}_{\mathcal{F}(\bar{\varepsilon})}(\bar{u}) = 0 \forall \bar{\varepsilon} \in \bar{A}$ and for all $\bar{u} \in U$, then (\mathcal{F}, \bar{A}) is called a relative universal \mathcal{LONSS} and denoted by $\bar{U}_{\bar{A}}$.

Similarly, the relative universal neutrosophic soft set with respect to the set of parameters E is called universal \mathcal{LONSS} and denoted by \bar{U} .

Proposition 3.11. Let $(\mathcal{F}, \bar{A}) \in \mathcal{LONSS}(U)$. Then

- $(\mathcal{F}, \bar{A}) \cap_{RES} (\mathcal{F}, \bar{A}) = (\mathcal{F}, \bar{A})$
- $(\mathcal{F}, \bar{A}) \cup_{RES} (\mathcal{F}, \bar{A}) = (\mathcal{F}, \bar{A})$
- $(\mathcal{F}, \bar{A}) \cap_{RES} \Phi_{\bar{A}} = \Phi_{\bar{A}}$
- $(\mathcal{F}, \bar{A}) \cup_{RES} \Phi_{\bar{A}} = (\mathcal{F}, \bar{A})$.

Proof. Straightforward. □

Definition 3.12. Let $(\mathcal{F}, \bar{\mathcal{A}}) \in \text{NSS}(\mathcal{U})$. Then it is known to be an antilattice ordered neutrosophic soft set (\mathcal{ALONSS}) over \mathcal{U} , where \mathcal{F} is a mapping defined by $\mathcal{F} : \bar{\mathcal{A}} \rightarrow \text{NS}(\mathcal{U})$, if $\bar{\varepsilon}_i, \bar{\varepsilon}_j \in \bar{\mathcal{A}}$ such that $\bar{\varepsilon}_i \leq \bar{\varepsilon}_j$, then

$$\mathcal{F}(\bar{\varepsilon}_j) \subseteq \mathcal{F}(\bar{\varepsilon}_i)$$

i.e. $\mathcal{T}_{\mathcal{F}(\bar{\varepsilon}_j)}(\bar{u}) \leq \mathcal{T}_{\mathcal{F}(\bar{\varepsilon}_i)}(\bar{u})$, $\mathcal{I}_{\mathcal{F}(\bar{\varepsilon}_j)}(\bar{u}) \leq \mathcal{I}_{\mathcal{F}(\bar{\varepsilon}_i)}(\bar{u})$ and $\mathcal{F}_{\mathcal{F}(\bar{\varepsilon}_j)}(\bar{u}) \leq \mathcal{F}_{\mathcal{F}(\bar{\varepsilon}_i)}(\bar{u})$

Proposition 3.13. Let $(\mathcal{F}, \bar{\mathcal{A}}) \in \mathcal{LONSS}(\mathcal{U})$. Then complement of $(\mathcal{F}, \bar{\mathcal{A}})$ is an \mathcal{ALONSS} over \mathcal{U} .

Proof. Given that $(\mathcal{F}, \bar{\mathcal{A}}) \in \mathcal{LONSS}(\mathcal{U})$.

For $\bar{\varepsilon}_i \leq_{\bar{\mathcal{A}}} \bar{\varepsilon}_j$
 $\Rightarrow \mathcal{F}(\bar{\varepsilon}_i) \subseteq \mathcal{F}(\bar{\varepsilon}_j)$

\Rightarrow

$$\mathcal{T}_{\mathcal{F}(\bar{\varepsilon}_i)}(\bar{u}) \leq \mathcal{T}_{\mathcal{F}(\bar{\varepsilon}_j)}(\bar{u})$$

$$\mathcal{I}_{\mathcal{F}(\bar{\varepsilon}_i)}(\bar{u}) \leq \mathcal{I}_{\mathcal{F}(\bar{\varepsilon}_j)}(\bar{u}) \text{ and}$$

$$\mathcal{F}_{\mathcal{F}(\bar{\varepsilon}_i)}(\bar{u}) \leq \mathcal{F}_{\mathcal{F}(\bar{\varepsilon}_j)}(\bar{u})$$

\Rightarrow

$$\mathcal{T}_{\mathcal{F}^c(\bar{\varepsilon}_i)}(\bar{u}) \leq \mathcal{T}_{\mathcal{F}^c(\bar{\varepsilon}_j)}(\bar{u})$$

$$1 - \mathcal{I}_{\mathcal{F}(\bar{\varepsilon}_i)}(\bar{u}) \leq 1 - \mathcal{I}_{\mathcal{F}(\bar{\varepsilon}_j)}(\bar{u}) \text{ and}$$

$$\mathcal{F}_{\mathcal{F}^c(\bar{\varepsilon}_i)}(\bar{u}) \leq \mathcal{F}_{\mathcal{F}^c(\bar{\varepsilon}_j)}(\bar{u})$$

\Rightarrow

$$\mathcal{T}_{\mathcal{F}^c(\bar{\varepsilon}_i)}(\bar{u}) \leq \mathcal{T}_{\mathcal{F}^c(\bar{\varepsilon}_j)}(\bar{u})$$

$$\mathcal{I}_{\mathcal{F}^c(\bar{\varepsilon}_i)}(\bar{u}) \leq \mathcal{I}_{\mathcal{F}^c(\bar{\varepsilon}_j)}(\bar{u}) \text{ and}$$

$$\mathcal{F}_{\mathcal{F}^c(\bar{\varepsilon}_i)}(\bar{u}) \leq \mathcal{F}_{\mathcal{F}^c(\bar{\varepsilon}_j)}(\bar{u})$$

\Rightarrow

$$\mathcal{F}^c(\bar{\varepsilon}_j) \subseteq \mathcal{F}^c(\bar{\varepsilon}_i) \text{ whenever } \bar{\varepsilon}_i \leq_{\bar{\mathcal{A}}} \bar{\varepsilon}_j$$

$\Rightarrow (\mathcal{F}, \bar{\mathcal{A}})^c$ is an \mathcal{ALONSS} . □

Proposition 3.14. Let $(\mathcal{F}, \bar{\mathcal{A}}) \in \mathcal{LONSS}(\mathcal{U})$. Then $((\mathcal{F}, \bar{\mathcal{A}})^c)^c = (\mathcal{F}, \bar{\mathcal{A}})$.

Proof. Let $(\mathcal{F}, \bar{\mathcal{A}}) \in \mathcal{LONSS}(\mathcal{U})$.

Then the complement of $(\mathcal{F}, \bar{\mathcal{A}})$ is

$$\mathcal{T}_{\mathcal{F}^c(\bar{\varepsilon})}(\bar{u}) = \mathcal{F}_{\mathcal{F}(\bar{\varepsilon})}(\bar{u}),$$

$$\mathcal{I}_{\mathcal{F}^c(\bar{\varepsilon})}(\bar{u}) = 1 - \mathcal{I}_{\mathcal{F}(\bar{\varepsilon})}(\bar{u}) \text{ and}$$

$$\mathcal{F}_{\mathcal{F}^c(\bar{\varepsilon})}(\bar{u}) = \mathcal{T}_{\mathcal{F}(\bar{\varepsilon})}(\bar{u}), \text{ where } \bar{\varepsilon} \in \bar{\mathcal{A}}$$

Now the complement of $(\mathcal{F}, \bar{\mathcal{A}})^c$ is

$$\mathcal{T}_{(\mathcal{F}^c)^c(\bar{\varepsilon})}(\bar{u}) = \mathcal{F}_{\mathcal{F}^c(\bar{\varepsilon})}(\bar{u}) = \mathcal{T}_{\mathcal{F}(\bar{\varepsilon})}(\bar{u})$$

$\mathcal{I}_{(\mathcal{F}^c)^c(\bar{\varepsilon})}(\bar{u}) = 1 - \mathcal{I}_{\mathcal{F}^c(\bar{\varepsilon})}(\bar{u}) = 1 - \{1 - \mathcal{I}_{\mathcal{F}(\bar{\varepsilon})}(\bar{u})\} = \mathcal{I}_{\mathcal{F}(\bar{\varepsilon})}(\bar{u})$, and

$$\mathcal{F}_{(\mathcal{F}^c)^c(\bar{\varepsilon})}(\bar{u}) = \mathcal{T}_{\mathcal{F}^c(\bar{\varepsilon})}(\bar{u}) = \mathcal{F}_{\mathcal{F}(\bar{\varepsilon})}(\bar{u}), \text{ where } \bar{\varepsilon} \in \bar{\mathcal{A}}$$

$\Rightarrow ((\mathcal{F}, \bar{\mathcal{A}})^c)^c = (\mathcal{F}, \bar{\mathcal{A}})$. □

Definition 3.15. Let $(\mathcal{F}, \bar{\mathcal{A}}), (\mathcal{G}, \bar{\mathcal{B}}) \in \mathcal{LONSS}(\mathcal{U})$. Then $(\mathcal{F}, \bar{\mathcal{A}}) \vee (\mathcal{G}, \bar{\mathcal{B}}) = (\mathcal{H}, \bar{\mathcal{C}})$ is known to be basic union of two \mathcal{LONSS} s over \mathcal{U} , where $\bar{\mathcal{C}} = \bar{\mathcal{A}} \times \bar{\mathcal{B}}$ and define $\mathcal{H}(\bar{\varepsilon}_i, \bar{\varepsilon}_j) = \mathcal{F}(\bar{\varepsilon}_i) \cup_{\text{RES}} \mathcal{G}(\bar{\varepsilon}_j)$ and

$$\mathcal{T}_{\mathcal{H}(\bar{\varepsilon}_i, \bar{\varepsilon}_j)}(\bar{u}) = \text{Max}\{\mathcal{T}_{\mathcal{F}(\bar{\varepsilon}_i)}(\bar{u}), \mathcal{T}_{\mathcal{G}(\bar{\varepsilon}_j)}(\bar{u})\}$$

$$\mathcal{I}_{\mathcal{H}(\bar{\varepsilon}_i, \bar{\varepsilon}_j)}(\bar{u}) = \text{Min}\{\mathcal{I}_{\mathcal{F}(\bar{\varepsilon}_i)}(\bar{u}), \mathcal{I}_{\mathcal{G}(\bar{\varepsilon}_j)}(\bar{u})\}$$

$\mathcal{F}_{\mathcal{H}(\bar{\varepsilon}_i, \bar{\varepsilon}_j)}(\bar{u}) = \text{Min}\{\mathcal{F}_{\mathcal{F}(\bar{\varepsilon}_i)}(\bar{u}), \mathcal{F}_{\mathcal{G}(\bar{\varepsilon}_j)}(\bar{u})\}$ for all $(\bar{\varepsilon}_i, \bar{\varepsilon}_j) \in \bar{\mathcal{C}}, \bar{u} \in \mathcal{U}$

Proposition 3.16. Let $(\mathcal{F}, \bar{\mathcal{A}}), (\mathcal{G}, \bar{\mathcal{B}}) \in \mathcal{LONSS}(\mathcal{U})$. Then $(\mathcal{F}, \bar{\mathcal{A}}) \vee (\mathcal{G}, \bar{\mathcal{B}}) \in \mathcal{LONSS}(\mathcal{U})$.

Proof. Suppose $(\mathcal{F}, \bar{\mathcal{A}}), (\mathcal{G}, \bar{\mathcal{B}}) \in \mathcal{LONSS}(\mathcal{U})$. Then by Definition 3.15

$$(\mathcal{F}, \bar{\mathcal{A}}) \vee (\mathcal{G}, \bar{\mathcal{B}}) = (\mathcal{H}, \bar{\mathcal{C}}), \text{ where } \bar{\mathcal{C}} = \bar{\mathcal{A}} \times \bar{\mathcal{B}}$$

Since $\bar{\mathcal{A}}, \bar{\mathcal{B}} \subseteq \mathcal{E}$, so both $\bar{\mathcal{A}}$ and $\bar{\mathcal{B}}$ inherit the partial order from \mathcal{E} also

$$\mathcal{H}(\varepsilon, \eta) = \mathcal{F}(\varepsilon) \vee \mathcal{G}(\eta) = \mathcal{F}(\varepsilon) \cup_{\text{RES}} \mathcal{G}(\eta)$$

Now, $\bar{\varepsilon}_i \leq_{\bar{\mathcal{A}}} \bar{\varepsilon}_j$ we have $\mathcal{F}(\bar{\varepsilon}_i) \subseteq \mathcal{F}(\bar{\varepsilon}_j)$, for every $\bar{\varepsilon}_i, \bar{\varepsilon}_j \in \bar{\mathcal{A}}$ and also for $\bar{\eta}_i \leq_{\bar{\mathcal{B}}} \bar{\eta}_j$ we have $\mathcal{G}(\bar{\eta}_i) \subseteq \mathcal{G}(\bar{\eta}_j)$, for every $\bar{\eta}_i, \bar{\eta}_j \in \bar{\mathcal{B}}$ Now for any $(\bar{\varepsilon}_i, \bar{\eta}_i), (\bar{\varepsilon}_j, \bar{\eta}_j) \in \bar{\mathcal{C}}$ and \leq is partial order on $\bar{\mathcal{C}}$ which is induced by partial orders on $\bar{\mathcal{A}}$ and $\bar{\mathcal{B}}$

The order on $\bar{\mathcal{A}} \times \bar{\mathcal{B}}$ is $(\bar{\varepsilon}_i, \bar{\eta}_i) \leq (\bar{\varepsilon}_j, \bar{\eta}_j)$, whenever $\bar{\varepsilon}_i \leq_{\bar{\mathcal{A}}} \bar{\varepsilon}_j$ and $\bar{\eta}_i \leq_{\bar{\mathcal{B}}} \bar{\eta}_j$
 $\Rightarrow \mathcal{F}(\bar{\varepsilon}_i) \subseteq \mathcal{F}(\bar{\varepsilon}_j)$ and $\mathcal{G}(\bar{\eta}_i) \subseteq \mathcal{G}(\bar{\eta}_j)$

\Rightarrow

$$\mathcal{T}_{\mathcal{F}(\bar{\varepsilon}_i)}(\bar{u}) \leq \mathcal{T}_{\mathcal{F}(\bar{\varepsilon}_j)}(\bar{u}), \mathcal{T}_{\mathcal{G}(\bar{\eta}_i)}(\bar{u}) \leq \mathcal{T}_{\mathcal{G}(\bar{\eta}_j)}(\bar{u})$$

$$\mathcal{I}_{\mathcal{F}(\bar{\varepsilon}_i)}(\bar{u}) \leq \mathcal{I}_{\mathcal{F}(\bar{\varepsilon}_j)}(\bar{u}), \mathcal{I}_{\mathcal{G}(\bar{\eta}_i)}(\bar{u}) \leq \mathcal{I}_{\mathcal{G}(\bar{\eta}_j)}(\bar{u})$$

$$\mathcal{F}_{\mathcal{F}(\bar{\varepsilon}_i)}(\bar{u}) \leq \mathcal{F}_{\mathcal{F}(\bar{\varepsilon}_j)}(\bar{u}), \mathcal{F}_{\mathcal{G}(\bar{\eta}_i)}(\bar{u}) \leq \mathcal{F}_{\mathcal{G}(\bar{\eta}_j)}(\bar{u})$$

\Rightarrow

$$\text{Max}\{\mathcal{T}_{\mathcal{F}(\bar{\varepsilon}_i)}(\bar{u}), \mathcal{T}_{\mathcal{G}(\bar{\eta}_i)}(\bar{u})\} \leq \text{Max}\{\mathcal{T}_{\mathcal{F}(\bar{\varepsilon}_j)}(\bar{u}), \mathcal{T}_{\mathcal{G}(\bar{\eta}_j)}(\bar{u})\}$$

$$\text{Min}\{\mathcal{I}_{\mathcal{F}(\bar{\varepsilon}_i)}(\bar{u}), \mathcal{I}_{\mathcal{G}(\bar{\eta}_i)}(\bar{u})\} \leq \text{Min}\{\mathcal{I}_{\mathcal{F}(\bar{\varepsilon}_j)}(\bar{u}), \mathcal{I}_{\mathcal{G}(\bar{\eta}_j)}(\bar{u})\}$$

$$\text{Min}\{\mathcal{F}_{\mathcal{F}(\bar{\varepsilon}_i)}(\bar{u}), \mathcal{F}_{\mathcal{G}(\bar{\eta}_i)}(\bar{u})\} \leq \text{Min}\{\mathcal{F}_{\mathcal{F}(\bar{\varepsilon}_j)}(\bar{u}), \mathcal{F}_{\mathcal{G}(\bar{\eta}_j)}(\bar{u})\}$$

\Rightarrow

$$\mathcal{T}_{\mathcal{F}(\bar{\varepsilon}_i) \vee \mathcal{G}(\bar{\eta}_i)}(\bar{u}) \leq \mathcal{T}_{\mathcal{F}(\bar{\varepsilon}_j) \vee \mathcal{G}(\bar{\eta}_j)}(\bar{u})$$

$$\mathcal{I}_{\mathcal{F}(\bar{\varepsilon}_i) \vee \mathcal{G}(\bar{\eta}_i)}(\bar{u}) \leq \mathcal{I}_{\mathcal{F}(\bar{\varepsilon}_j) \vee \mathcal{G}(\bar{\eta}_j)}(\bar{u})$$

$$\mathcal{F}_{\mathcal{F}(\bar{\varepsilon}_i) \vee \mathcal{G}(\bar{\eta}_i)}(\bar{u}) \leq \mathcal{F}_{\mathcal{F}(\bar{\varepsilon}_j) \vee \mathcal{G}(\bar{\eta}_j)}(\bar{u})$$

Table 2
Parameter data table

U	$\bar{\varepsilon}_1$	$\bar{\varepsilon}_2$	$\bar{\varepsilon}_3$	$\bar{\varepsilon}_4$	$\bar{\varepsilon}_5$	$\bar{\varepsilon}_6$
Priority	0.4	0.3	-0.15	0.05	0.1	0
Parameter rank	1	2	3	5	4	6

$$\Rightarrow \text{score} = \mathcal{T}_{\mathcal{F}}(1 + \mathcal{I}_{\mathcal{F}}) \tag{1}$$

$$\mathcal{T}_{\mathcal{H}(\varepsilon_i, \eta_i)}(\bar{u}) \leq \mathcal{T}_{\mathcal{H}(\varepsilon_j, \eta_j)}(\bar{u})$$

$$\mathcal{I}_{\mathcal{H}(\varepsilon_j, \eta_j)}(\bar{u}) \leq \mathcal{I}_{\mathcal{H}(\varepsilon_i, \eta_i)}(\bar{u})$$

$$\mathcal{F}_{\mathcal{H}(\varepsilon_j, \eta_j)}(\bar{u}) \leq \mathcal{F}_{\mathcal{H}(\varepsilon_i, \eta_i)}(\bar{u})$$

$\Rightarrow \mathcal{H}(\varepsilon_i, \eta_i) \subseteq \mathcal{H}(\varepsilon_j, \eta_j)$, for every $(\varepsilon_i, \eta_i) \leq (\varepsilon_j, \eta_j)$
 Therefore, $(\mathcal{F}, \bar{A}) \vee (\mathcal{G}, \bar{B}) \in \mathcal{LONSS}(\mathcal{U})$. □

Definition 3.17. Let $(\mathcal{F}, \bar{A}), (\mathcal{G}, \bar{B}) \in \mathcal{LONSS}(\mathcal{U})$. Then $(\mathcal{F}, \bar{A}) \wedge (\mathcal{G}, \bar{B}) = (\mathcal{H}, \bar{C})$ is known to be basic intersection of two \mathcal{LONSS} s over \mathcal{U} , where $\bar{C} = \bar{A} \times \bar{B}$ and define $\mathcal{H}(\bar{\varepsilon}_i, \bar{\varepsilon}_j) = \mathcal{F}(\bar{\varepsilon}_i) \cap_{RES} \mathcal{G}(\bar{\varepsilon}_j)$ and

$$\mathcal{T}_{\mathcal{H}(\bar{\varepsilon}_i, \bar{\varepsilon}_j)}(\bar{u}) = \text{Min}\{\mathcal{T}_{\mathcal{F}(\bar{\varepsilon}_i)}(\bar{u}), \mathcal{T}_{\mathcal{G}(\bar{\varepsilon}_j)}(\bar{u})\}$$

$$\mathcal{I}_{\mathcal{H}(\bar{\varepsilon}_i, \bar{\varepsilon}_j)}(\bar{u}) = \text{Max}\{\mathcal{I}_{\mathcal{F}(\bar{\varepsilon}_i)}(\bar{u}), \mathcal{I}_{\mathcal{G}(\bar{\varepsilon}_j)}(\bar{u})\}$$

$$\mathcal{F}_{\mathcal{H}(\bar{\varepsilon}_i, \bar{\varepsilon}_j)}(\bar{u}) = \text{Max}\{\mathcal{F}_{\mathcal{F}(\bar{\varepsilon}_i)}(\bar{u}), \mathcal{F}_{\mathcal{G}(\bar{\varepsilon}_j)}(\bar{u})\} \text{ for all } (\bar{\varepsilon}_i, \bar{\varepsilon}_j) \in \mathcal{C}, \bar{u} \in \mathcal{U}$$

Proposition 3.18. Suppose $(\mathcal{F}, \bar{A}), (\mathcal{G}, \bar{B}) \in \mathcal{LONSS}(\mathcal{U})$. Then $(\mathcal{F}, \bar{A}) \wedge (\mathcal{G}, \bar{B}) \in \mathcal{LONSS}(\mathcal{U})$.

Proof. The proof follows from Definition 3.17 and Proposition 3.16. □

4. Application

A large-scale company intends to contribute funds. The objective of the fund is to recover the people’s family whose livelihood is affected in COVID-19 pandemic. In order to carry out this project, the company tends to seek suitable non-governmental organization (NGO). The parameters are considered as

- (i) Positive parameter \Rightarrow Value \propto Preference,
- (ii) Negative parameter \Rightarrow Value $\propto \frac{1}{\text{Preference}}$.

The priority value lies in $[-1, 1]$.
 If the priority value,

- (i) Does not affect the expert decision \Rightarrow Priority is 0,
- (ii) Affects positively the expert decision \Rightarrow Priority is $(0, 1]$,
- (iii) Affects negatively the expert decision \Rightarrow Priority is $[-1, 0)$,
- (iv) Does not given \Rightarrow Priority is 0 (This can be eliminated).

If there is more than one object, we keep only one object (same values for all parameters).

In fuzzy soft set, equation (1) reduces to membership score only. The algorithm formulated by Tripathy et al. (2016) is considered to compute the following decision making.

Let U be a set of NGOs to carry out the project given by $U = \{n_1, n_2, n_3, n_4, n_5, n_6\}$ and the parameter set $E = \{\bar{\varepsilon}_1, \bar{\varepsilon}_2, \bar{\varepsilon}_3, \bar{\varepsilon}_4, \bar{\varepsilon}_5, \bar{\varepsilon}_6\}$ for the parameters “good track of record, number of volunteers, service experience, transparency, office network, familiarity,” respectively.

There are three experts (E_1, E_2, E_3) to analyze the skills and features of the NGOs. For each parameter, the experts analyze the priority values. As per priority values, the parameters are ranked. The highest absolute value parameter has more priority and it holds the highest rank and so on.

If the priority value is same for more than one parameter, then the expert can choose the rank among the parameters.

Clearly, the order among the parameters is

$$\bar{\varepsilon}_1 \succeq \bar{\varepsilon}_2 \succeq \bar{\varepsilon}_3 \succeq \bar{\varepsilon}_5 \succeq \bar{\varepsilon}_4 \succeq \bar{\varepsilon}_6$$

The NGO rankings based on each expert are represented as \mathcal{LONSS} in the tabular form Table 2.

Tables 3, 4, and 5 represent the ratings as per experts $E_1, E_2,$ and E_3 , respectively. Each NGO will get a rank from every expert. Because of null priority there is no column for $\bar{\varepsilon}_6$.

The priority Tables 6, 7, and 8 for each expert can be formulated by multiplying the values in Tables 3, 4, and 5 with respective values fixed by the expert. Parameter “Transfer fee” is having negative priority means negative parameter.

Calculate the entries as differences of each row sum in priority tables with those of all other rows and compute row sum in each table to create the corresponding comparison tables.

Tables 9, 10, and 11 are the comparison tables for the experts $n_4, n_5,$ and n_6 , respectively.

Tables 12, 13, and 14 by using the equation (1), the decision can be developed and rank is given. If there is same score for more than one NGO, then the NGO having top score in top ranked priority and so on. Similarly, all the decision tables are obtained.

As illustrated in Table 15, the rank table can be produced by adding the ranks assigned to NGOs by each expert. If same rank sum obtained by more than one NGO, then the dispute can be solved by the similar way as in decision table formation.

Decision Making:

The top ranked NGO is the good one to choose. If more than one NGO is needed, then the next subsequent rank holders can be chosen.

Comparative study:

In day-to-day life, we stumble upon with linguistic terms having particular ranking among them. Here, the decision makers have

Table 3
 \mathcal{LONSS} for E_1

U	$\bar{\varepsilon}_1$	$\bar{\varepsilon}_2$	$\bar{\varepsilon}_3$	$\bar{\varepsilon}_4$	$\bar{\varepsilon}_5$
n_1	0.8	0.1	0.3	0.7	0.2
n_2	0.6	0.1	0.0	0.5	0.3
n_3	0.7	0.0	0.1	0.6	0.2
n_4	0.5	0.2	0.4	0.4	0.3
n_5	0.9	0.2	0.0	0.7	0.4
n_6	0.9	0.4	0.3	0.8	0.6

Table 4
CONSS for E_2

U	$\bar{\varepsilon}_1$			$\bar{\varepsilon}_2$			$\bar{\varepsilon}_3$			$\bar{\varepsilon}_4$			$\bar{\varepsilon}_5$		
n_1	0.6	0.1	0.2	0.5	0.2	0.3	0.3	0.4	0.5	0.0	0.6	0.8	0.2	0.5	0.7
n_2	0.9	0.4	0.3	0.8	0.6	0.5	0.7	0.6	0.8	0.4	0.9	0.8	0.5	0.8	0.7
n_3	0.7	0.1	0.2	0.5	0.1	0.4	0.4	0.2	0.6	0.2	0.5	0.8	0.3	0.4	0.7
n_4	0.6	0.5	0.2	0.4	0.6	0.5	0.3	0.7	0.6	0.0	0.9	0.8	0.1	0.8	0.7
n_5	0.9	0.1	0.2	0.7	0.3	0.4	0.4	0.5	0.6	0.0	0.9	0.8	0.2	0.7	0.6
n_6	0.8	0.1	0.3	0.5	0.2	0.4	0.4	0.3	0.5	0.1	0.5	0.8	0.3	0.4	0.7

Table 5
CONSS for E_3

U	$\bar{\varepsilon}_1$			$\bar{\varepsilon}_2$			$\bar{\varepsilon}_3$			$\bar{\varepsilon}_4$			$\bar{\varepsilon}_5$		
n_1	0.7	0.2	0.1	0.6	0.5	0.2	0.5	0.7	0.4	0.1	0.9	0.8	0.4	0.8	0.5
n_2	0.9	0.4	0.3	0.8	0.6	0.5	0.7	0.6	0.8	0.4	0.9	0.8	0.5	0.8	0.7
n_3	0.8	0.4	0.3	0.5	0.6	0.4	0.4	0.7	0.6	0.2	0.9	0.8	0.3	0.8	0.7
n_4	0.9	0.3	0.1	0.7	0.4	0.3	0.6	0.7	0.4	0.4	0.9	0.7	0.5	0.8	0.6
n_5	0.5	0.1	0.6	0.4	0.3	0.7	0.3	0.5	0.8	0.0	0.7	0.9	0.1	0.6	0.8
n_6	0.7	0.0	0.2	0.5	0.2	0.4	0.3	0.4	0.5	0.1	0.6	0.8	0.2	0.5	0.7

Table 6
Priority table for the expert E_1

U	$\bar{\varepsilon}_1$			$\bar{\varepsilon}_2$			$\bar{\varepsilon}_3$			$\bar{\varepsilon}_4$			$\bar{\varepsilon}_5$			$\sum \mathcal{T}_F$	$\sum \mathcal{I}_F$	$\sum \mathcal{F}_F$
	\mathcal{T}_F	\mathcal{I}_F	\mathcal{F}_F	\mathcal{T}_F	\mathcal{I}_F	\mathcal{F}_F	\mathcal{T}_F	\mathcal{I}_F	\mathcal{F}_F	\mathcal{T}_F	\mathcal{I}_F	\mathcal{F}_F	\mathcal{T}_F	\mathcal{I}_F	\mathcal{F}_F			
n_1	0.32	0.04	0.12	0.21	0.06	0.12	-0.09	-0.045	-0.075	0.01	0.035	0.045	0.04	0.06	0.08	0.49	0.15	0.29
n_2	0.24	0.04	0.0	0.15	0.09	0.03	-0.045	-0.075	-0.03	0.0	0.04	0.03	0.02	0.07	0.04	0.365	0.165	0.07
n_3	0.28	0.0	0.04	0.18	0.06	0.03	-0.045	-0.06	-0.03	0.005	0.045	0.035	0.02	0.07	0.05	0.44	0.115	0.125
n_4	0.2	0.08	0.16	0.12	0.09	0.15	-0.045	-0.06	-0.09	0.005	0.035	0.04	0.02	0.06	0.07	0.3	0.205	0.33
n_5	0.36	0.08	0.0	0.21	0.12	0.03	-0.09	-0.075	-0.03	0.015	0.04	0.03	0.04	0.07	0.05	0.535	0.235	0.08
n_6	0.36	0.16	0.12	0.24	0.18	0.15	-0.105	-0.09	-0.12	0.02	0.045	0.04	0.05	0.08	0.07	0.565	0.375	0.26

Table 7
Priority table for the expert E_2

U	$\bar{\varepsilon}_1$			$\bar{\varepsilon}_2$			$\bar{\varepsilon}_3$			$\bar{\varepsilon}_4$			$\bar{\varepsilon}_5$			$\sum \mathcal{T}_F$	$\sum \mathcal{I}_F$	$\sum \mathcal{F}_F$
	\mathcal{T}_F	\mathcal{I}_F	\mathcal{F}_F	\mathcal{T}_F	\mathcal{I}_F	\mathcal{F}_F	\mathcal{T}_F	\mathcal{I}_F	\mathcal{F}_F	\mathcal{T}_F	\mathcal{I}_F	\mathcal{F}_F	\mathcal{T}_F	\mathcal{I}_F	\mathcal{F}_F			
n_1	0.24	0.04	0.08	0.15	0.06	0.09	-0.045	-0.06	-0.075	0.0	0.03	0.04	0.02	0.05	0.07	0.365	0.12	0.205
n_2	0.36	0.16	0.12	0.24	0.18	0.15	-0.105	-0.09	-0.12	0.02	0.045	0.04	0.05	0.08	0.07	0.565	0.375	0.26
n_3	0.28	0.04	0.08	0.15	0.03	0.12	-0.06	-0.03	-0.09	0.01	0.025	0.04	0.03	0.04	0.07	0.41	0.105	0.22
n_4	0.24	0.20	0.08	0.12	0.18	0.15	-0.045	-0.105	-0.09	0.0	0.045	0.04	0.01	0.08	0.07	0.325	0.4	0.25
n_5	0.36	0.04	0.08	0.21	0.09	0.12	-0.06	-0.075	-0.09	0.0	0.045	0.04	0.02	0.07	0.06	0.53	0.17	0.21
n_6	0.32	0.04	0.12	0.15	0.06	0.12	-0.06	-0.045	-0.075	0.005	0.025	0.04	0.03	0.04	0.07	0.445	0.12	0.275

Table 8
Priority table for the expert E_3

U	$\bar{\varepsilon}_1$			$\bar{\varepsilon}_2$			$\bar{\varepsilon}_3$			$\bar{\varepsilon}_4$			$\bar{\varepsilon}_5$			$\sum \mathcal{T}_F$	$\sum \mathcal{I}_F$	$\sum \mathcal{F}_F$
	\mathcal{T}_F	\mathcal{I}_F	\mathcal{F}_F	\mathcal{T}_F	\mathcal{I}_F	\mathcal{F}_F	\mathcal{T}_F	\mathcal{I}_F	\mathcal{F}_F	\mathcal{T}_F	\mathcal{I}_F	\mathcal{F}_F	\mathcal{T}_F	\mathcal{I}_F	\mathcal{F}_F			
n_1	0.28	0.08	0.04	0.18	0.15	0.06	-0.075	-0.105	-0.06	0.005	0.045	0.04	0.04	0.08	0.05	0.43	0.25	0.13
n_2	0.36	0.16	0.12	0.24	0.18	0.15	-0.105	-0.09	-0.12	0.02	0.045	0.04	0.05	0.08	0.07	0.565	0.375	0.26
n_3	0.32	0.16	0.12	0.15	0.18	0.12	-0.06	-0.105	-0.09	0.01	0.045	0.04	0.03	0.08	0.07	0.45	0.36	0.26
n_4	0.36	0.12	0.04	0.21	0.12	0.09	-0.09	-0.105	-0.06	0.02	0.045	0.035	0.05	0.08	0.06	0.55	0.26	0.165
n_5	0.20	0.04	0.24	0.12	0.09	0.21	-0.045	-0.075	-0.12	0.0	0.035	0.045	0.01	0.06	0.08	0.285	0.15	0.455
n_6	0.28	0.0	0.08	0.15	0.06	0.12	-0.045	-0.06	-0.075	0.005	0.03	0.04	0.02	0.05	0.07	0.41	0.08	0.235

Table 9
Comparison table for the expert E_1

U	n_1			n_2			n_3			n_4			n_5			n_6			$\sum \mathcal{T}_F$	$\sum \mathcal{I}_F$	$\sum \mathcal{F}_F$
	\mathcal{T}_F	\mathcal{I}_F	\mathcal{F}_F	\mathcal{T}_F	\mathcal{I}_F	\mathcal{F}_F	\mathcal{T}_F	\mathcal{I}_F	\mathcal{F}_F	\mathcal{T}_F	\mathcal{I}_F	\mathcal{F}_F	\mathcal{T}_F	\mathcal{I}_F	\mathcal{F}_F	\mathcal{T}_F	\mathcal{I}_F	\mathcal{F}_F			
n_1	0	0	0	0.125	-0.015	0.22	0.05	0.035	0.165	0.19	-0.055	-0.04	-0.045	-0.085	0.21	-0.075	-0.225	0.03	0.245	-0.345	0.585
n_2	-0.125	0.015	-0.22	0	0	0	-0.075	0.05	-0.055	0.065	-0.04	-0.26	-0.17	-0.07	-0.01	-0.2	-0.21	-0.19	-0.505	-0.255	-0.735
n_3	-0.05	-0.035	-0.165	0.075	-0.05	0.055	0	0	0	0.14	-0.09	-0.205	-0.095	-0.12	0.045	-0.125	-0.26	-0.135	-0.055	-0.555	-0.405
n_4	-0.19	0.055	0.04	-0.065	0.04	0.26	-0.14	0.09	0.205	0	0	0	-0.235	-0.03	0.25	-0.265	-0.17	0.07	-0.895	-0.015	0.825
n_5	0.045	0.085	-0.21	0.17	0.07	0.01	0.095	0.12	-0.045	0.235	0.03	-0.25	0	0	0	-0.03	-0.14	-0.18	0.515	0.165	-0.675
n_6	0.075	0.225	-0.03	0.2	0.21	0.19	0.125	0.26	0.135	0.265	0.17	-0.07	0.03	0.14	0.18	0	0	0	0.695	1.005	0.405

Table 10
Comparison table for the expert E_2

U	n_1			n_2			n_3			n_4			n_5			n_6			$\sum \mathcal{T}_F$	$\sum \mathcal{I}_F$	$\sum \mathcal{F}_F$
	\mathcal{T}_F	\mathcal{I}_F	\mathcal{F}_F	\mathcal{T}_F	\mathcal{I}_F	\mathcal{F}_F	\mathcal{T}_F	\mathcal{I}_F	\mathcal{F}_F	\mathcal{T}_F	\mathcal{I}_F	\mathcal{F}_F	\mathcal{T}_F	\mathcal{I}_F	\mathcal{F}_F	\mathcal{T}_F	\mathcal{I}_F	\mathcal{F}_F			
n_1	0	0	0	-0.2	-0.255	-0.055	-0.045	0.015	-0.015	0.04	-0.28	-0.045	-0.165	-0.05	-0.005	-0.08	0	-0.07	-0.45	-0.57	-0.19
n_2	0.2	0.255	0.055	0	0	0	0.155	0.27	0.04	0.24	-0.025	0.01	0.035	0.205	0.05	0.12	0.255	-0.015	0.75	0.96	0.14
n_3	0.045	-0.015	0.015	-0.155	-0.27	-0.04	0	0	0	0.085	-0.295	-0.03	-0.12	-0.065	0.01	-0.035	-0.015	-0.055	-0.18	-0.66	-0.1
n_4	-0.04	0.28	0.045	-0.24	0.025	-0.01	-0.085	0.295	0.03	0	0	0	-0.205	0.23	0.04	-0.12	0.28	-0.025	-0.69	1.11	0.08
n_5	0.165	0.05	0.005	-0.035	-0.205	-0.05	0.12	0.065	-0.01	0.205	-0.23	-0.04	0	0	0	0.085	0.05	-0.065	0.54	-0.27	-0.16
n_6	0.08	0.0	0.07	-0.12	-0.255	0.015	0.035	0.015	0.055	0.12	-0.28	0.025	-0.085	-0.05	0.065	0	0	0	0.03	-0.57	0.23

Table 11
Comparison table for the expert E_3

U	n_1			n_2			n_3			n_4			n_5			n_6			$\sum \mathcal{T}_F$	$\sum \mathcal{I}_F$	$\sum \mathcal{F}_F$
	\mathcal{T}_F	\mathcal{I}_F	\mathcal{F}_F	\mathcal{T}_F	\mathcal{I}_F	\mathcal{F}_F	\mathcal{T}_F	\mathcal{I}_F	\mathcal{F}_F	\mathcal{T}_F	\mathcal{I}_F	\mathcal{F}_F	\mathcal{T}_F	\mathcal{I}_F	\mathcal{F}_F	\mathcal{T}_F	\mathcal{I}_F	\mathcal{F}_F			
n_1	0	0	0	-0.135	-0.125	-0.13	-0.02	-0.11	-0.13	-0.12	-0.01	-0.035	0.145	0.1	-0.325	0.02	0.17	-0.105	-0.11	0.025	-0.725
n_2	0.135	0.125	0.13	0	0	0	0.115	0.015	0	0.015	0.115	0.095	0.28	0.225	-0.195	0.155	0.295	0.025	0.7	0.775	0.055
n_3	0.02	0.11	0.13	-0.115	-0.015	0	0	0	0	-0.1	0.1	0.095	0.165	0.21	-0.195	0.04	0.28	0.025	0.01	0.685	0.055
n_4	0.12	0.01	0.035	-0.015	-0.115	-0.095	0.1	-0.1	-0.095	0	0	0	0.265	0.11	-0.29	0.14	0.18	-0.07	0.61	0.085	-0.515
n_5	-0.145	-0.1	0.325	-0.28	-0.225	0.195	-0.165	-0.21	0.195	-0.265	-0.11	0.29	0	0	0	-0.125	0.07	0.22	-0.98	-0.575	1.225
n_6	-0.02	-0.17	0.105	-0.155	-0.295	-0.025	-0.04	-0.28	-0.025	-0.14	-0.18	0.07	0.125	-0.07	-0.22	0	0	0	-0.23	-0.995	-0.095

Table 12
Decision table for expert E_1

	\mathcal{T}_F	\mathcal{I}_F	\mathcal{F}_F	Score	Rank R_1
n_1	0.245	-0.345	0.585	0.160475	3
n_2	-0.505	-0.255	-0.735	-0.376225	5
n_3	-0.055	-0.555	-0.405	-0.024475	4
n_4	-0.895	-0.015	0.825	-0.881575	6
n_5	0.515	0.165	-0.675	0.599975	2
n_6	0.695	1.005	0.405	1.393475	1

Table 13
Decision table for expert E_2

	\mathcal{T}_F	\mathcal{I}_F	\mathcal{F}_F	Score	Rank R_2
n_1	-0.45	-0.57	-0.19	-0.1935	5
n_2	0.75	0.96	0.14	1.47	1
n_3	-0.18	-0.66	-0.1	-0.0612	4
n_4	-0.69	1.11	0.08	-1.4559	6
n_5	0.54	-0.27	-0.16	0.3942	2
n_6	0.03	-0.57	0.23	0.0129	3

Table 14
Decision table for expert E_3

	\mathcal{T}_F	\mathcal{I}_F	\mathcal{F}_F	Score	Rank R_3
n_1	-0.11	0.025	-0.725	-0.11275	5
n_2	0.7	0.775	0.055	1.2425	1
n_3	0.01	0.685	0.055	0.01685	3
n_4	0.61	0.085	-0.515	0.66185	2
n_5	-0.98	-0.575	1.225	-0.4165	6
n_6	-0.23	-0.995	-0.095	-0.00115	4

Table 15
Rank table

	E_1	E_2	E_3	Normalized Score	Final-Rank
n_1	3	5	5	0.111111111	5
n_2	5	1	1	0.244444444	1
n_3	4	4	3	0.155555556	4
n_4	6	6	2	0.088888889	6
n_5	2	2	6	0.177777778	3
n_6	1	3	4	0.222222222	2

given an order of importance to the elements of parameters. Hence, the lattice ordered neutrosophic soft sets are more helpful to deal with decision-making problems involving linguistic phrases. Thus, the results obtained by using \mathcal{LONSS} are taken into account.

5. Conclusion

The idea of lattice ordered neutrosophic soft sets is proposed. Also the effects of lattice ordered neutrosophic soft sets and antilattice ordered neutrosophic soft sets on restricted union, restricted intersection, extended union, extended intersection, basic union, and basic intersection are familiarized. A group decision-making problem is solved using these notions to

demonstrate the importance of the proposed theory. We can build the lattice ordered neutrosophic hypersoft set theory in the future by generalizing the soft set to the hypersoft set and finding applications in various areas of medicine.

Conflicts of Interest

The authors declare that they have no conflicts of interest to this work.

Informed Consent

Informed consent was obtained from all individual participants included in the study.

References

Ahmad, B., & Kharal, A. (2009). On fuzzy soft sets. *Advances in Fuzzy Systems*, 2009. <https://doi.org/10.1155/2009/586507>

Ali, M. I., Mahmood, T., Rehman, M. M. U., & Aslam, M. F. (2015). On lattice ordered soft set. *Applied Soft Computing*, 36, 499–505. <https://doi.org/10.1016/j.asoc.2015.05.052>

Aslam, M. F., Ali, M. I., Mahmood, T., Rehman, M. M. U., & Sarfraz, N. (2019). Study of fuzzy soft sets with some order on set of parameters. *International Journal of Algebra and Statistics*, 8(1), 50–65.

Ajay, D., & Chellamani, P. (2022). Pythagorean neutrosophic soft sets and their application to decision-making scenario. In *Intelligent and Fuzzy Techniques for Emerging Conditions and Digital Transformation: Proceedings of the INFUS 2021 Conference, 2021(2)*, 552–560. https://doi.org/10.1007/978-3-030-85577-2_65.

Aygun, H., & Aygunoglu, A. (2009). Introduction to fuzzy soft groups. *Computers & Mathematics with Applications*, 58(6), 1279–1286. <https://doi.org/10.1016/j.camwa.2009.07.047>

Birkhoff, G. (1967). *Lattice theory*. USA: American Mathematical Society.

Broumi, S., & Smarandache, F. (2013). Intuitionistic neutrosophic soft set. *Journal of Information and Computing Science*, 8(2), 130–140.

Broumi, S. (2013). Generalized neutrosophic soft set. *International Journal of Computer Science, Engineering and Information Technology*, 3(2), 17–30. <https://doi.org/10.12785/amis/080610>

Garg, H., Vimala, J., Rajareega, S., Preethi, D., & Perez-Dominguez, L. (2022). Complex intuitionistic fuzzy soft SWARA - COPRAS approach: An application of ERP software selection. *AIMS Mathematics*, 7(4), 5895–5909. <https://doi.org/10.3934/math.2022327>

Karaaslan, F. (2015). Neutrosophic soft sets with applications in decision making. *International Journal of Information Science and Intelligent System*, 4, 1–20. <https://doi.org/10.5281/zenodo.23151>.

Khan, M., Bakhat, T., & Iftikhar, M. (2019). Some results on lattice (anti-lattice) ordered double framed soft sets. *Journal of New Theory*, 29, 74–86.

Li, Z., & Li, S. (2013). Lattice structures of intuitionistic fuzzy soft sets. *Annals of Fuzzy Mathematics and Informatics*, 6(3), 467–477.

Maji, P. K. (2013). Neutrosophic soft set. *Annals of Fuzzy Mathematics and Informatics*, 5, 157–168.

Maji, P. K., Biswas, R., & Roy, A. R. (2003). Soft set theory. *Computers & Mathematics with Applications*, 45(4-5), 555–562. [https://doi.org/10.1016/S0898-1221\(03\)00016-6](https://doi.org/10.1016/S0898-1221(03)00016-6)

- Mahmood, T., Ali, M. I., Malik, M. A., & Ahmed, W. (2018). On lattice ordered intuitionistic fuzzy soft sets. *International Journal of Algebra and Statistics*, 7(1-2), 46–61. <https://doi.org/10.20454/ijas.2018.1434>
- Molodtsov, D. (1999). Soft set theory-first result. *Computers & Mathematics with Applications*, 37(4-5), 19–31. [https://doi.org/10.1016/S0898-1221\(99\)00056-5](https://doi.org/10.1016/S0898-1221(99)00056-5)
- Preethi, D., Vimala, J., & Rajareega, S. (2020). A systematic study in the applications of fuzzy hyperlattice. *AIMS Mathematics*, 6(2), 1695–1705. <https://doi.org/10.3934/math.2021100>
- Rajareega, S., Vimala, J., & Preethi, D. (2020). Complex intuitionistic fuzzy soft lattice ordered group and its weighted distance measures. *Mathematics*, 8(5), 705. <https://doi.org/10.3390/math8050705>
- Smarandache, F. (2005). Neutrosophic set-A generalization of the intuitionistic fuzzy set. *International Journal of Pure and Applied Mathematics*, 24(3), 287–297.
- Tripathy, B. K., Mohanty, R. K., & Sooraj, T. R. (2016). On intuitionistic fuzzy soft set and its application in group decision making. In *Proceedings of the International Conference on Signal, Networks, Computing, and Systems*, 67–73.
- Zadeh, L. A. (1965). Fuzzy sets. *Information and Control*, 8(3), 338–353. [https://doi.org/10.1016/S0019-9958\(65\)90241-X](https://doi.org/10.1016/S0019-9958(65)90241-X)

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