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Optimization in Business Trade by Using Fuzzy Incidence Graphs

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Abstract: In classical graph theory, set of vertices in which every edge is incident to at least one of the vertex is called a vertex covering set and the problems of vertex covering (*VC*) are known as *VC* problems. The classical graphs do not reflect the impact of vertex on the edge and such impact is reflected in fuzzy incidence graphs (*FIGs*). We used optimization technique in a trade using clique covering (*CC*) in *FIGs*. This article includes some basic definitions such as strength of clique, strength of path, strength of minimum clique (*SMC*), acceptable strength of path and acceptable strength of clique, robust number, minimum covering number of the clique, close measure, some examples and theorems in *FIGs*, and new strategy of optimization to characterize and solve facility point problems by adopting the *CC* application of *FIGs* for a business alliance to get maximum total achievement by reducing the transportation cost. This procedure will be helpful to develop sustainable economic developmental goal of the world. Some already existing theorems in fuzzy graphs are also checked in *FIGs*. A mathematical model is defined and a real-life problem from a trade is solved keeping the fuzziness of the objects related to FIG.

Keywords: fuzzy incidence graphs, strength of clique, clique covering number, robust number and optimization in fuzzy incidence graph

1. Introduction

A graph helps us to express the information of data including the relationship between distinct objects. The objects are shown by nodes and their relationships are described by edges. Zadeh (1965) and Zadeh (2008) introduced the fuzzy sets (*FSs*). *FSs* do talk about the membership function and handle the uncertainty-related mathematical problems. Rosenfeld (1975) was inspired by the work of Zadeh and gave the idea of fuzzy graphs (*FGs*). He also introduced many concepts related to *FGs* like path, cycle, and connectedness. Yeh and Bang (1975) also did their work independently in the field of *FG*. They introduced new idea of clustering analysis using the connectedness in *FGs*.

Crnkovic (2004) gave the detail study on path graphs of incidence graphs. Dinesh (2016) introduced the fuzzy incidence graph (*FIGs*). *FIGs* are the generalization of crisp graphs and show the impact of vertices on the edges. This was the main reason to introduce the idea of *FIG*. Consider an example, if vertices are represented by public schools and edges show the roads which connect all schools, we can have an *FG* reflecting the extent of traffic from one school to another school. The school having the maximum students will have the maximum ramps in the school. So, if S_1 and S_2 are two schools and S_1S_2 is a road which connects these schools, then (S_1, S_1S_2) can represent the ramp system from the road S_1S_2 to school S_1. In un-weighted graphs, both S_1 and S_2 will have an impact of 1 on S_1S_2. In directed graphs, the influence of S_1 on S_1S_2 given by (S_1, S_1S_2) is 1, whereas (S_2, S_1S_2) is 0. This idea is generalized by the *FIG*. After that, Mathew and Mordeson (2017) gave distinct ideas regarding connectivity perceptions in *FIGs*. They also described a strong path between any node arc pair of *FIG*. Later on, Binu et al. (2020) studied the dielectric synthesis and FIG. Mordeson et al. (2018) also gave the application of *FGs* in immigration. Nazeer et al. (2021a); Nazeer et al. (2021b) also gave the applications of *FIGs* in human trafficking, etc. Akram et al. (2018) gave the detailed study regarding the neutrosophic incidence graphs with application.

In the literature of operational research, the set covering problems are briefly studied. Hakimi (1965) gave the idea of covering problems. From the fundamental concept of graph theory, one is clique. Pullman (1983) also worked on clique covering (*CC*) of a graph. Nair and Cheng (2001) were the first to introduce the idea of fuzzy clique. Later on, Sun et al. (2016) gave the concept regarding the clique and clique covers in the *FGs*. Bhattacharya and Pal (2021) gave the idea regarding optimization using the *CC* in *FGs*.

The main and finest ability of human being is to make a decision. The decision makers like to pursue many targets or they suppose many factors. The facility point or vertex is a continuation of operational research in order to establish minimum one new facility between already available facilities, so as to optimize (minimize or maximize) with minimum one objective function.

This research work includes *FIGs* to be supposed to symbolize a fuzzy business alliance and is completely investigated in accordance with optimization sense for the better provision of economic facilities around the globe. The fuzzy incidence clique covering has proved to be an essential type of covering to optimize the reliance on the *FIG* of this research work.

Now, if the vertices have fuzzy values, then we can make edge values and incidence values as well. The question is, how will a decision maker takes a decision for the optimization to have

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minimum transportation cost and to get maximum gain or profit. In this article, per dose cost will be considered as vertex value and transportation cost will be the edge value. Then using the vertex covering modeling and optimization modeling, mathematically results are found to optimize the business alliance.

In this article, a facility position problem is assumed and two types of corona vaccines are circulated to demand points from facility points in a fuzzy background. In the *FIG*, a source vertex is fixed and it is considered that in the beginning items are circulated to facilities from source vertex.

So, the problem is to locate some facilities with the following objectives:

(i) **Objective 1:** Minimize the dependency between the facility points.
(ii) **Objective 2:** Maximize the demand ratio (The ratio between saturated demand and total demand)
(iii) **Objective 3:** Minimize the total transportation cost.
(iv) **Objective 4:** Maximize the total gain of the system.

Moreover, idea for finding the optimization in the *FIG* is discussed. Our contribution in this work in the summarized form is given as follows:

(i) To define the strength of clique (*SC*) in the *FIG* which is used in the mathematical programming.
(ii) To find the robust number and minimum covering number of the clique (*MCNC*) in the *FIG* which will be helpful to find the other parameters of the optimization.
(iii) A mathematical model is discussed in which we get maximum gain by reducing the transportation cost.

There are some restrictions/ assumptions in this model:

(i) The product of total cost for each item and a special factor for demand should be greater than or equal to the total demand (*TD*).
(ii) The sum of all demands of all demand points should be less than or equal to *TD*.

This article is described as follows: Section 2 deals with the provision of primary definitions and some results of *FIGs* which are helpful for the expansion of the content. In Section 3, we discuss auxiliary results of *FIGs*. Section 4 explains mathematical formulation of *FIGs* and optimization in trades using *CC* of *FIG* and its characteristics. Section 5 describes real-life problem in business trade of corona vaccination in distinct countries of the world.

2. Preliminaries

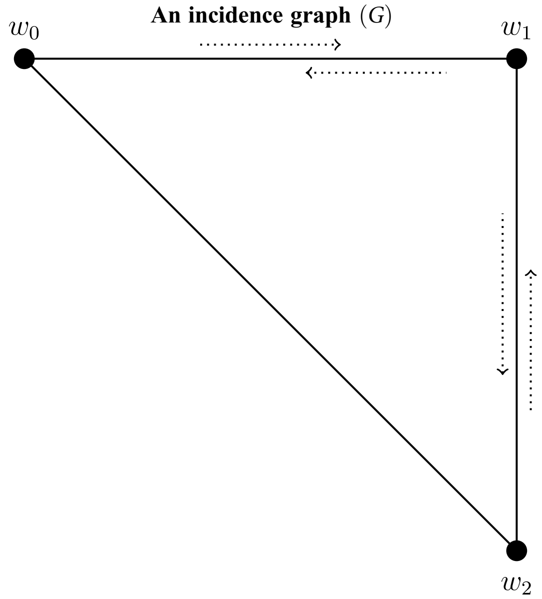
It is of pivotal importance to explain some of the basic terms and definitions used in this article. A graph is an interesting and simple way to describe the data and correspondence between the sets. The weighted graphs are used to reflect the relational strength between the nodes and *FIGs* are also weighted graphs. In this article, *V*, *E*, and *I* represent the set of vertices, set of edges, and set of incidence pair in the graph *G*.

Definition 2.1. [**Rosenfeld, 1975**] *Let B be a set of vertices. A FG* $G = (b, \beta)$ *is a pair, where b is a fuzzy subset of the set B and β is a fuzzy subset of* $B \times B$ *such that* $\beta(x, y) \leq b(x) \wedge b(y)$ *for all* $x, y \in B$.

Here, in this article maximum and minimum are expressed as $\vee$ and $\wedge$, respectively.

Definition 2.2. [**Mathew & Mordeson, 2017**] *Let* $G = (V, E, I)$ *be the incidence graph of a crisp graph G, where* $I \subseteq V \times E$*. Here, I represents the set of incidence values in the graph G. If* (w_0, w_0w_1) *is in I, then* (w_0, w_0w_1) *is known as an incidence pair of simply pair (see Figure 1).*

Figure 1
An incidence graph (G)



Example 2.3. *Let* $G = (V, E, I)$ *be the incidence graph having three vertices and three edges as shown in Figure 1.*

Now, we consider two arcs w_0w_1 and w_1w_2 as the vertices in the incidence graph, which are called adjacent if all the four pairs (w_0, w_0w_1), (w_1, w_1w_0), (w_1, w_1w_2), and (w_2, w_2w_1) are in *I*. According to Figure 1, w_0w_1 and w_1w_2 are adjacent, whereas w_0w_2 and w_1w_2 are not adjacent.

The incidence path is a sequence presented as w_0, (w_0, w_0w_1), w_0w_1, (w_1, w_0w_1), $w_1, \ldots, w_{n-1}$ $(w_{n-1}, w_{n-1}w_n)$, $w_{n-1}w_n$, $(w_n, w_{n-1}w_n)$, w_n.

Definition 2.4. [**Zadeh, 1965**] *Let A be a FS of a space X and described with a membership value* $\mu_A(x)$ *which related with every point of X a real number in the interval* [0,1]*, with the value of* $\mu_A(x)$ *at x representing the "grade of membership" of x in A.*

Definition 2.5. [**Mathew & Mordeson, 2017**] *Let* $G = (V, E, I)$ *be a FG and α and β be the fuzzy subsets of V and* $V \times E$*, respectively. If* $I(v^*, e^*) \leq \alpha(v^*) \wedge \beta(e^*)$ *for every* $v^* \in V$ *and* $e^* \in E$*, then I is called a fuzzy incidence of graph G.*

Definition 2.6. [**Mathew & Mordeson, 2017**] *Let* $G = (V, E, I)$ *be a FIG, then fuzzy incidence strength of a FIG is given as* $SP(G) = min\{I(w_i, w_iw_j) | \forall (w_i, w_iw_j) \in I > 0\}$.

Definition 2.7. [**Pullman, 1983**] *An undirected crisp graph* $G = (V, E)$ *has a clique C subset of V such that each of the two vertices of C is adjacent. This clique will be maximal if the clique cannot be expanded with the addition of more adjacent vertices.*

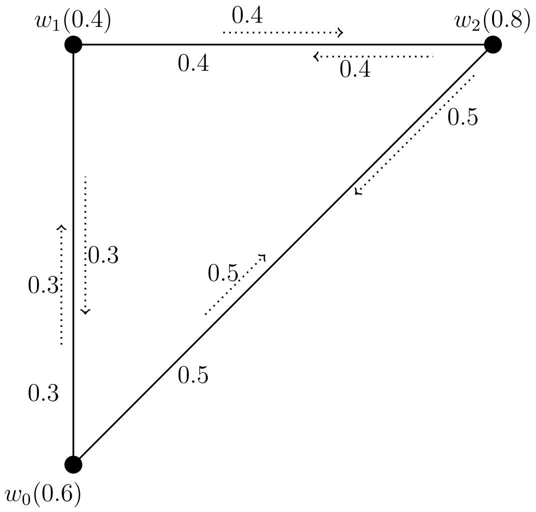
Example 2.8. *Consider a FIG G.*

The path between w_0 and w_2 is given as $P_1 = \{w_0, w_1, w_2\}$ and $P_2 = \{w_0, w_2\}$ in Figure 2.

So, the strength of path (*SP*) is given as $SP(G) = 0.3$

Definition 2.9. [**Nazeer, Rashid, Hussain, et al., 2021a**] *Let* $G = (V, E, I)$ *be the FIG, then the complement of FIG is denoted by* $\bar{G} = (\bar{V}, \bar{E}, \bar{I})$ *and is defined as*

Figure 2
***SP* in *FIG* (*G*)**



- $\bar{V} = V, \forall v_i \in V$.
- $\bar{E}(v_i, v_j) = min(v_i, v_j) - E(v_i, v_j), \forall v_i v_j \in E$.
- $\bar{I}(v_i, v_i v_j) = min(v_i, v_j) - I(v_i, v_i v_j), \forall v_i \in V, v_i v_j \in E$.

Example 2.10. *FIG (G) and its complement ($\bar{G}$) are calculated as follows:*

It is very clear that the complement of $\bar{G}$ is always again G.

3. Covering Clique in Fuzzy Incidence Graph

In this section, it is necessary to identify the weight of *FIG*, *SC*, acceptable strength of path (*ASP*), acceptable strength of clique (*ASC*), strongest or robust covering number of the clique, and closeness measure in the *FIG*. The formal definitions and some examples are as follows:

Definition 3.1. *Let* $G = (V, E, I)$ be a *FIG*, then the weight of incidence of the graph G is the sum of its fuzzy incidence values. It is represented by $W(G)$ and is classified as $W(G) = \sum_{v \in V(G)} I(w_i, w_i w_j)$.

Proposition 3.2. *Let C be a clique of a FIG* $G = (V, E, I)$*, then the complement of* $\bar{C}$ *will always be the same clique with same order and size.*

Definition 3.3. *Let* $G = (V, E, I)$ *be a FIG, then the weight of a clique C is the sum of its fuzzy incidence values present in it. It is indicated by* $W(C)$ *and is classified as* $W(C) = \sum_{(w_i, w_i w_j) \in C} I(w_i, w_i w_j)$.

Definition 3.4. *Let C be any clique of the FIG. A total clique cover of FIG is the set of C which involves every single vertex of incidence graph. It is represented by* $C(IG)$.

Definition 3.5. *In a FIG* $G = (V, E, I)$*, a cover of clique is a collection of the cliques which involves maximum vertices of FIG and minimizing the covering number of the clique by ignoring 1-order cliques. It is represented by* $CC(G)$.

Definition 3.6. *Let C be a clique of a FIG. The SC is represented by* $SC(G)$ *and described as*

$$SC(G) = \frac{\sum_{(w_i, w_i w_j) \in V(G) I(w_i, w_i w_j)}}{min\{I(w_i, w_i w_j) : (w_i, w_i w_j) \in Supp(I)\}}.$$

Among all the cliques, the weakest *SC* of the *FIG* is represented by $SMC(G)$.

Example 3.7. *Suppose the FIG G*

From Figure 3, one can observe the fuzzy incidence clique $C = \{w_0, w_1, w_2\}$.

Therefore, $SC(G) = \frac{1.2}{0.3} = 4$ is the *SC* of *FIG*.

Definition 3.8. *Let G be a FIG, then the acceptable strength for incidence path is represented by* $\tilde{A}S(P)$ *and is described as*

$$\tilde{A}S(P) = min\{min\{I(w_i, w_i w_j) : (w_i, w_i w_j) \in I > 0\}\}.$$

Example 3.9. *Let G be the FIG as it is shown in Figure 4. Now, the fuzzy paths which are obtained are given as:*

Figure 3
(a) *FIG* (*G*) and (b) complement of *G*

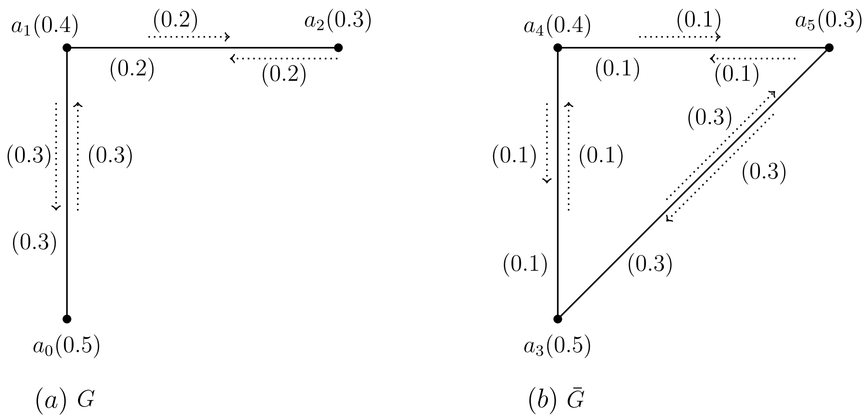
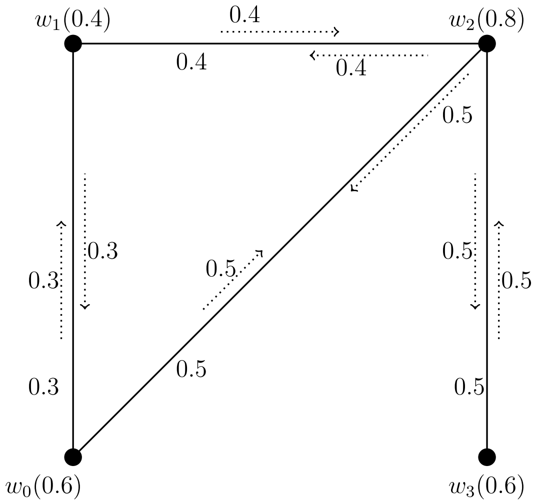


Figure 4
***SC* in *FIG* (*G*)**



$P_1 = \{w_1, w_2\},\ P_2 = \{w_1, w_0\},\ P_3 = \{w_2, w_0\},$
$P_4 = \{w_2, w_3\}, P_5 = \{w_1, w_2, w_0\}, P_6 = \{w_1, w_0, w_2\},$
$P_7 = \{w_1, w_2, w_3\}, P_8 = \{w_0, w_2, w_3\}, P_9 = \{w_0, w_1, w_2, w_3\},$

Hence, the strengths of these paths are given as:

$SP_1(G_3) = 0.4,\ SP_2(G_3) = 0.3,\ SP_3(G_3) = 0.5,\ SP_4(G_3) = 0.5,$
$SP_5(G_3) = 0.4,\ SP_6(G_3) = 0.3,\ SP_7(G_3) = 0.4,\ SP_8(G_3) = 0.5,$
$SP_9(G_3) = 0.3.$

The acceptable strength of *FIG* is given as:

$$\tilde{A}S(P) = min_{P(G)\in G}\{min\{SP_1(G_3), SP_2(G_3), SP_3(G_3), SP_4(G_3), SP_5(G_3), SP_6(G_3), SP_7(G_3), SP_8(G_3), SP_9(G_3)\}\},$$

$$\tilde{A}S(P) = min_{P(G)\in G}\{min\{0.4, 0.3, 0.5, 0.5, 0.4, 0.3, 0.4, 0.5, 0.3\}\},$$
$$\tilde{A}S(P) = 0.3.$$

Definition 3.10. *Let G be the FIG, then the clique's acceptable incidence strength is represented by* $\tilde{A}S(C)$ and is described as:

$$\tilde{A}S(C) = \frac{1}{2}[max_{C\in G}(SC(G)) + min_{C\in G}(SC(G))].$$

Example 3.11. *Let G be a FIG. The following are the cliques in Figure 5.*

$C_1 = \{w_0, w_1, w_2\},\ C_2 = \{w_1, w_2, w_3\},\ C_3 = \{w_2, w_3, w_4\},\ C_4 = w_0, w_1, w_2, w_3\}$ and lastly $C_5 = \{w_0, w_1, w_3\},\ C_6 = \{w_0, w_2, w_3\}$.

So, the strengths of these cliques are given as:

$SC_1(G_4) = 0.3,\ SC_2(G_4) = 0.3, SC_3(G_4) = 0.4,$
$SC_4(G_4) = 0.3,$ and $SC_5(G_4) = 3$

Now, for the clique the acceptable strength of *FIG* using the definition is given as:

$$\tilde{A}S(C) = \frac{1}{2}[min_{C\in G}\{SC(G)\} + max_{C\in G}\{SC(G)\}] = \frac{0.3 + 3}{2} = 1.65$$

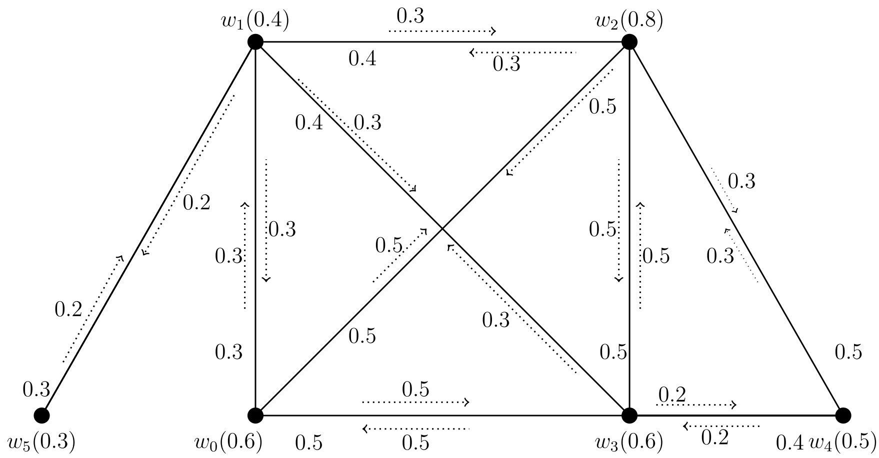
.

There are three new types of cliques in *FIG* which are described using the concept of strength, *SP* and *ASP* and *ASC* of *FIG*:

Definition 3.12. *Let C be any clique of FIG. Then the clique C is known as*

(i) λ-robust if the acceptable incidence strength of *FIG* is less than the clique's strength, that is $\tilde{A}S, (C) < SC(G)$.
(ii) μ-robust if the acceptable incidence strength of FIG is equal to the clique's strength, that is, $\tilde{A}S(C) = SC(G)$.
(iii) ν-robust if the acceptable incidence strength of *FIG* is greater than the clique's strength, that is, $\tilde{A}S(C) > SC(G)$.

Figure 5
***FIG* (*G*) for cliques**



Example 3.13. *Let G be a FIG as given in Figure 5. According to the definition 3.9, we can find out that C_3 is λ-robust clique in FIG and on the other hand C_1, C_2, and C_4 are ν-robust cliques.*

Definition 3.14. *Let $\lambda_{CC}(G)$ be the MCNC in a connected FIG $G = (V, E, I)$, and it can be described as strength of minimum covering number of the covering clique multiplied with their weight in a FIG.*

$$\lambda_{CC}(G) = \sum\nolimits_{C \in CC(G)} W(C) \times SMC(CC(G)).$$

Example 3.15. *Let $G_3 = (V, E, I)$ be a FIG as shown in Figure 5. One can find out, $CC(G) = \{C_3, C_5\}$. So, using the example of acceptable incidence strengths of FIG for fuzzy incidence cliques, we can find out that $SMC(CC(G)) = 3.67$, and by applying the definition of clique's weight in G, we can calculate $W(C_3) = 1$ and $W(C_5) = 1.1$. So, using the formula of CC number,*

$$\lambda_{CC}(G) = 7.7.$$

Definition 3.16. *Let $\lambda_{SC}(G)$ be the robust covering clique of FIG, then it is defined as follows:*

$$\lambda_{SC}(G) = \sum_{i=1}^{N} \{SC_i(CC(G))\}.$$

Example 3.17. *Assume a FIG as given in Figure 5. We have obtained $CC(G) = \{C_3, C_5\}$. So, $\lambda_{SC}(G) = 3.67 + 5 = 8.67$.*

Proposition 3.18. *Let $G = (V, E, I)$ be a FIG. Then $\lambda_{SC}(G) \geq \lambda_{CC}(G)$ holds.*

Proof. In any *FIG*, it is clear that SC is always greater than or equal to the weight times strength of the minimum CC of the *FIG*. So,

$$SC(CC(G)) \geq W(C) \times SMC(CC(G)),$$

then the sum of *SC* of CC of the *FIG* is also greater than or equal to the weight times *SMCC* of *FIG*.

$$\sum SC_i(CC(G)) \geq \sum W(C) \times SMC(CC(G)),$$

which gives the result as,

$$\lambda_{SC}(G) \geq \lambda_{CC}(G).$$

Example 3.19. *Using Figure 5. We have already calculated*

$\lambda_{CC}(G) = 7.7$ and $\lambda_{SC}(G) = 3.67 + 5 = 8.67$ as given in example 3.12 and 3.14.

So, it is clear that $\lambda_{SC}(G) \geq \lambda_{CC}(G)$.

Definition 3.20. *Let a FIG have two different vertices w_i and w_j. The closeness measure between these vertices is denoted by $close(w_i, w_j)$* and it is defined as:

$$close(w_i, w_j) = \frac{l}{2 \times 10^{l}};$$

where *l* denotes the length of the fuzzy incidence that joins w_i and w_j.

The maximum and minimum measure of closeness in a *FIG* G_2 are written as $close_{max}(w_i, w_j)$ and $close_{min}(w_i, w_j)$, respectively.

Example 3.21. *Consider a FIG as shown in Figure 4, as the values of fuzzy incidence are applied. So, the closeness measure for different vertices is given as:*

$$close(w_0, w_1) = \frac{0.3}{2 \times 10^{0.3}} = 0.075;$$

$$close(w_0, w_2) = \frac{0.5}{2 \times 10^{0.5}} = 0.079;$$

$$close(w_1, w_2) = \frac{0.4}{2 \times 10^{0.4}} = 0.079;$$

$$close(w_2, w_3) = \frac{0.5}{2 \times 10^{0.5}} = 0.079;$$

Proposition 3.22. *In a FIG, $CN(G) \times |CC(G)| \times SMC(G) \geq \lambda_{CC}(G)$ holds.*

Here, $CN(G)$, $|CC(G)|$, and $SMC(G)$ represent the total number of cliques, collection of cliques which involve maximum vertices covering *FIG*, and strength of weakest clique in *FIG*.

Proof. It is given that

$$W(C) = \sum\nolimits_{(w_i, w_i w_j) \in C} I(w_i, w_i w_j),$$

also

$$W(C) \leq CN(G) \times |CC(G)|,$$

because the product of maximum number of cliques of any possible order and collection of cliques is always greater than or equal to the weight of the clique. So,

$$W(C) \times SMC(G) \leq CN(G) \times |CC(G)| \times SMC(G),$$

$$\lambda_{CC}(G) \leq CN(G) \times |CC(G)| \times SMC(G).$$

Example 3.23. *Consider Figure 5. We have already obtained,*

Here $CC(G) = \{C_3 = \{w_2, w_3, w_4\}, C_5 = \{w_0, w_1, w_3\}\}$, $SMC(C_C(G)) = 3.67$, $|CC(G)| = 2$, $CN(G) = 13$ and $\lambda_{CC}(G) = 7.7$.

Therefore, using these values we can easily verify that $\lambda_{CC}(G) \leq CN(G) \times |CC(G)| \times SMC(G)$.

Example 3.24. *Consider Figure 6.*

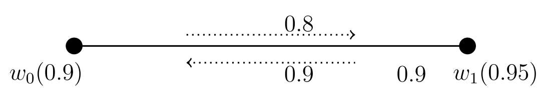
Here, it is clear that, $CC(G) = \{C_1 = \{w_0, w_1\}\}$, So, $|CC(G)| = 1$, $CN(G) = 1$ and $W(C) = 0.8$.

So, it is obvious that

$$W(C) \leq CN(G) \times |CC(G)|,$$

Therefore, $\lambda_{CC}(G) \leq CN(G) \times |CC(G)| \times SMC(G)$.

Figure 6
An *FIG G*



4. Mathematical Model

In this section, for the fuzzy incidence system, we obtain all fuzzy objects involved in the system. The whole trade system shows the relations between facility points and demand points which are described in Figure 7. The business alliance companies of different countries are considered as vertices and distances between them are considered as edge values in this FIG. The geodesic distance between the points where facilities are available and the points where facilities are required must be less than or equal to covering radius. The value of the total demand (*TD*) and population of the *FIG* is already provided. The value of the (*TD*) relies on the value of the population. In this article, our main purpose is

(i) **Minimize the reliance by minimizing the total strength of *FIG*.**

After finding the fuzzy incidence values and covering number of the robust clique, we will use the following objective function with some constraints to minimize the strength of the *FIG* and membership values of edges and their incidence values because reducing the strength of the objective function means reducing the reliance of the vertices in *FIG*.

$$Min(SG) = \lambda_{SC}(G) + \sum(I(w_i, w_iw_j))$$

with the following constraints:

$$\begin{aligned} &min\{min\{I_i : I_i \in C_a\}, min\{I_j : I_j \in C_b\} : C_a \cap C_b = \{\}\} \\ &\le min\{I_k, I_l\} \le max\{min\{I_i : I_i \in C_a\}, min\{I_j : I_j \in C_b\} \\ &: C_a \cap C_b = \{\}\}. \end{aligned}$$

Here, *I* represents the fuzzy incidence value.

(ii) **Maximize the demand ratio**.

Here, the objective function will give us the demand ratio of the demand vertices. We will maximize this objective function for the saturation of the demand of the objects in the *FIG*.

$$Max(DR) = \frac{\{x_1q_1 + x_2q_2\}TD}{T_{c_p}},$$

with the following limitations:

$$T_{c_p} \le TD \le P(nbd(I_s))xq,$$

$$C_p \ge \frac{x_1q_1 + x_2q_2}{T_{c_p}},$$

Here, x and q represent the cost of the vaccine and the number of vaccines accordingly. The total demand of all the demand points is represented by *TD*. T_{c_p} and C_p represent the total cost price and cost price, respectively. By applying the above objective function and constraints, we will find the demand ratio, number of items, and cost per item.

(iii) **Minimize the delivery cost**.

Now, using the following objective function and constraints, we will minimize the transportation cost using the geodesic distance. By minimizing this objective function, we will be able to minimize the cost of the transportation in this *FIG*.

$$Min(T_c) = q_1 \times \{|P_1(w_s, w_d)|\} + q_2 \times \{|P_2(w_s, w_d)|\},$$

with the following limitations:

$$m + n \le q_1 + q_2,$$

$$close(w_s, w_d) \le \{|P_1(w_s, w_d)|\} + \{|P_2(w_s, w_d)|\}.$$

Here, $|P_1(w_s, w_d)|$ and $|P_2(w_s, w_d)|$ represent the shortest paths from the demand points. Lastly, the closeness measure must be less than or equal to the sum of the shortest distances for the better impact and smooth running of the system of transportation.

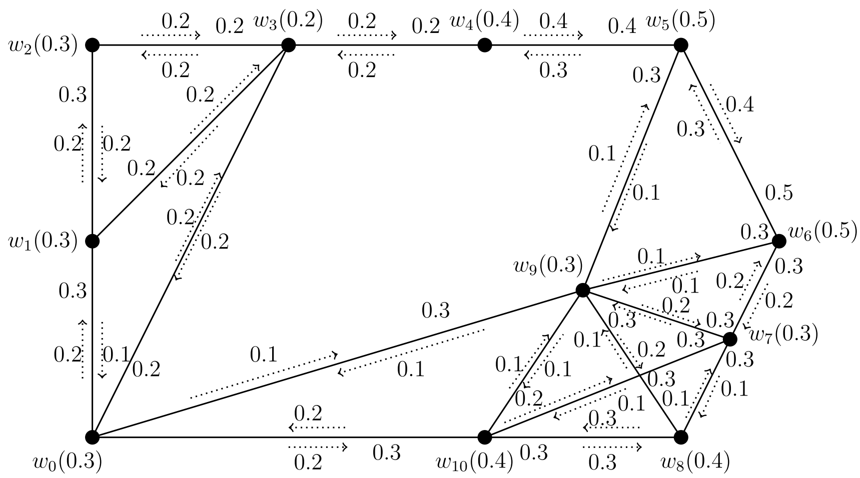
(iv) **Maximize the total efficiency of the system**.

Here, we maximize the total gain.

$$Max(G) = \frac{SC}{P_c + T_c},$$

with the constraint,

Figure 7
The network of the model using *FIG* (G)



$$SC \geq x_1q_1 + x_2q_2.$$

Here, G represents the total gain of the system.

5. Case Study

In this section, we discuss the system of covid vaccinations (Astrazeneca, Sinovac) for 11 different countries. For this vaccine, we have four targets which are mentioned in the mathematical model.

First of all, to minimize the reliance by minimizing the total strength of the *FIG*, we will obtain all the maximal fuzzy incidence cliques of the network defined in Figure 7.

$$\{w_1, w_2, w_3\},\ \{w_0, w_1, w_3\},\ \{w_5, w_6, w_9\},\ \{w_6, w_7, w_9\},\ \{w_7, w_8, w_{10}, w_9\}, \{w_7, w_8, w_9\},\ \{w_8, w_9, w_{10}\},\ \{w_7, w_9, w_{10}\},\ \{w_0, w_9, w_{10}\}.$$

After this, we can obtain the fuzzy incidence CC set of the fuzzy incidence network $CC(G)$.

$$CC(G) = \{C_1 = \{w_0, w_1, w_3\},\ C_2 = \{w_1, w_2, w_3\},\ C_3 = \{w_5, w_6, w_9\},\ C_4 = \{w_7, w_8, w_9\},\ C_5 = \{w_0, w_9, w_{10}\}\}.$$

Now, to keep the system connected and to minimize the dependency of the vertices we consider the edges between the cliques C_1 and C_3 as cliques are disjoint.

Here, the *CSs* involved in the reduced covering number of the clique in *FIG* network given in Figure 6 are given as:

$$SC_1(CC(G)) = 5,\ SC_2(CC(G)) = 3,\ SC_3(CC(G)) = 5,\ SC_4(CC(G)) = 4,\ SC_5(CC(G)) = 4.$$

Hence, the $\lambda_{SC}(G) = 21$.

Here, to minimize the reliance by minimizing the total strength of the *FIG*, we will solve the model described above.

$$Min(SG) = 21 + I(w_3, w_3w_4) + I(w_4, w_4w_5),$$

with the following constraints:

$$0.2 \leq I(w_3, w_3w_4) \leq 0.4.$$

$$0.3 \leq I(w_5, w_5w_4) \leq 0.4.$$

Now, using mathematical tool "LINGO," we obtained

$$I(w_3, w_3w_4) = 0.2,\ I(w_5, w_5w_4) = 0.3 \text{ and SG} = 21.5$$

In the second step, we maximize the demand ratio and saturate the demand in order to get the maximum profit.

Let the membership value of the dealers be $T_{cp} = 10$ dollar per dose and the total demand per dose in dollar purely depends on the population density which is treated as special parameter $P(nbd(I_s)) = 475$ approximately. So, the function and constraints are

$$Max(DR) = \frac{\{x_1q_1 + x_2q_2\}TD}{10},$$

$$10 \leq TD \leq 475(x_1q_1 + x_2q_2),$$

$$(8.5)(10) \geq x_1q_1 + x_2q_2.$$

Using the mathematical tool "LINGO," we obtained the demand ratio as $DR = 343187.6$ with the values $TD = 40375$, cost per item as $x_1 = 0.5$, $x_2 = 0.1$, and the number of items as $q_1 = 80$ and $q_2 = 450$.

Our third aim is to minimize the transportation cost of the defined network system. Consider the transportation of cost per dose in dollar are $T_c^1 = 80$ and $T_c^2 = 450$, respectively. The objective function T_c gives the total transportation cost of the FIG.

Consider the objective function

$$Min(T_c) = q_1 \times \{|P_1(w_s, w_d)|\} + q_2 \times \{|P_2(w_s, w_d)|\},$$

with the following constraints:

$$m + n \leq q_1 + q_2,$$

$$close(w_s, w_d) \leq \{|P_1(w_s, w_d)|\} + \{|P_2(w_s, w_d)|\}.$$

Using the mathematical tool "LINGO," we can obtain $T_c = 6.32$, $|P_1(w_s, w_d)| = 0.01$ and $\{|P_2(w_s, w_d)|\} = 0.00005$. Here, $T_c = 6.32$ is the transportation cost per dose in dollar. But, if a country needs 2×10^6 or more doses according to the population of that country, then we will multiply this cost and the population of that country.

Lastly, we get the maximum gain by applying the optimization. Consider the $P_c = 70$ per dose in dollars:

$$Max(G) = \frac{SC}{76.32},$$

with the constraint,

$$SC \geq 85.$$

Using mathematical tool "LINGO," we obtained the total gain $Max(G) = 1.1137$ and the sailing cost is $SC = 85$.

Hence, we arrived at the final point by applying the optimization in a business trade involving the fuzzy parameters and maximized the total gain by minimizing the transportational cost as well.

6. Conclusion

In this article, we can say that from the analysis of business trade using the optimization in the *FIG*, we deduce better result on different parameters of business trade such as strength of *FIG*, system's demand ratio, total cost, and total gain satisfying all the presumptions and limitations on network is deducted using the analytical analysis through the mode of optimization method. In order to achieve vaccination goals, this application is very important. Thus, for the economic point of view, this analysis is of pivotal importance for the better health care improvement of our society as well.

Moreover, the covering number of robust clique due to the covering set of clique using optimal fuzzy incidence clique of a *FIG* is also analyzed and explained. As for as future prospects of the researcher are concerned, the focus will be to work on *FIG* and new type of covering concepts have been analyzed in detail.

The model held in obeyance is altogether an innovative model which has been synthesized by associating clique covers of *FIGs* with the concept of optimization programming problems. In the earlier findings, only the operations field research uses developed programming problems, and in *FIG* these clique covers were never utilized in selecting the constraints of the programming problems in such modeling as well as not in constructing the objective functions.

Conflicts of Interest:

The authors declare that they have no conflicts of interest to this work.

Data Availability Statement:

All the required data are available in the manuscript.

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