Received: 3 February 2022 | Revised: 3 March 2022 | Accepted: 7 March 2022 | Published online: 15 March 2022

Journal of Computational and Cognitive Engineering
2023, Vol. 2(1) 47–56
DOI: 10.47852/bonviewJCCE20235142025

BON VIEW PUBLISHING

RESEARCH ARTICLE

On the Application of a Lexicographic Method to Fuzzy Linear Programming Problems

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Abstract: The literature on fuzzy variable linear programming (FVLP) and fuzzy number linear programming (FNLP) is prolific in terms of the number of available solution methods. In FVLP problems, only the decision variables and right-hand side values of the constraints are fuzzy numbers. In FNLP problems, except for the decision variables, all parameters are fuzzy numbers. A widespread approach for solving problems in FVLP and FNLP is to use linear ranking functions in order to transform the fuzzy problems into conventional ones. Previous studies have shown that linear ranking functions do not guarantee uniqueness of the optimal fuzzy objective values. In this paper, we use a lexicographic method to find unique optimal fuzzy objective values of such problems and compare the results with those obtained via linear ranking function approaches. The paper also discusses applications of the lexicographic method in diet and time–cost trade-off problems in fuzzy environments.

Keywords: fuzzy linear programming, fuzzy number, fuzzy inequality, lexicographic method, diet problem, time–cost trade-off problem

1. Introduction

The concept of decision-making in fuzzy environments was first proposed by Bellman and Zadeh (1970) to account for real-world decision-making situations in which the goals, the constraints, and the outcomes of potential decisions are not known precisely. Their ideas were soon adopted by Tanaka et al. (1974) and Zimmermann (1976) who introduced fuzzy linear programming (FLP). Since then, tremendous research efforts have gone to the development of FLP, and novel methods are continuously proposed for solving FLP problems arising in many areas of management, science, and engineering.

FLP is classified according to how fuzziness appears in the FLP problem. Tanaka et al. (1974) and Zimmermann (1976) considered FLP problems in which decision-makers permit partial accomplishment of the constraints. Verdegay (1982) generalized their approaches through parametric linear programming and the representation theorem for fuzzy sets. Tanaka et al. (1984) considered two types of FLP problems: a fuzzy number linear programming (FNLP) problem, in which all objective function coefficients, technological coefficients, and right-hand side values of the constraints are fuzzy numbers, and the decision variables take on real values; and a fuzzy variable linear programming (FVLP) problem, in which only the decision variables and the right-hand side values of the constraints are assumed to be fuzzy numbers. Delgado et al. (1989) proposed a general model for FLP. The authors considered FLP problems with fuzzy numbers in the technological matrix and in the right-hand side values of the constraints, plus the assumption of partial accomplishment of the problem constraints.

A more general model for FLP is obtained when all parameters and decision variables are assumed as fuzzy numbers; this is the case of fully fuzzy linear programming (FFLP) introduced by Buckley and Feuring (2000) and later researched in Hashemi et al. (2006), Hosseinzadeh Lotfi et al. (2009), Kaur and Kumar (2012), Khan et al. (2013), Ezzati et al. (2015), and Das et al. (2017) among other studies. As the reader may have noticed, FLP is a vast research field. Recent and comprehensive surveys by Ebrahimnejad and Verdegay (2018) and Ghanbari et al. (2020) discuss on theoretical aspects of available models and solution methods along with some numerical examples.

A widespread approach for solving problems in FNLP, FVLP, and FFLP is to use linear ranking functions in order to transform FLP problems into conventional ones. Kaur and Kumar (2012) showed that the optimal fuzzy objective value of an FFLP problem obtained by using linear ranking functions is not necessarily unique; they therefore suggested to use lexicographic ranking criteria to guarantee uniqueness. In Kaur and Kumar (2016), the authors pointed out that there was no method to find unique optimal fuzzy objective values of FFLP problems with inequality constraints. Pérez-Cañedo and Concepción-Morales (2019) proposed such a method and compared it with several others showing its advantages.

In this paper, we shall focus on the application to FNLP and FVLP of the lexicographic method proposed in Pérez-Cañedo and Concepción-Morales (2019). Our contribution serves four main purposes:

- To exemplify the working of the lexicographic method in handling fuzzy inequality constraints.
- To solve FNLP and FVLP problems with inequality constraints via the lexicographic method.
- To compare the obtained results on a diet problem with those obtained via Ebrahimnejad's (2015) method.
- To solve a fuzzy time–cost trade-off problem formulated as a bi-objective FVLP problem.

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The rest of the paper is organized as follows: Section 2 presents basic definitions concerning fuzzy numbers. In Section 3, the mathematical models of the FVLP problem and the FNLP problem are presented. The lexicographic method for solving FNLP and FVLP problems is described in Section 4. Some advantages of the lexicographic method over existing ones are highlighted in Section 5. Illustrative examples regarding the working of the lexicographic method are provided in Section 6. Applications of the lexicographic method are discussed in Subsections 6.1 and 6.2, and a comparison with Ebrahimnejad's (2015) method is performed as well. Concluding remarks are given in Section 7.

2. Preliminaries

In this section, some basic definitions concerning fuzzy numbers are presented (Bellman & Zadeh, 1970; Carlsson & Fullér, 2005; Dubois & Prade, 1978).

Definition 1. Let $X = \{x\}$ denote a collection of objects (points) denoted generically by x. Then a fuzzy set $\widetilde{a}$ in X is a set of ordered pairs $\widetilde{a} = \{(x, \mu_{\widetilde{a}}(x))|x \in X\}$, where $\mu_{\widetilde{a}} : X \to [0,1]$ and $\mu_{\widetilde{a}}(x)$ is termed the grade of membership of x in $\widetilde{a}$.

Definition 2. A fuzzy set $\widetilde{a} = (m, n, \alpha, \beta)_{LR}$ is an LR fuzzy number if its membership function is given by:

$$\mu_{\widetilde{a}}(x) = \begin{cases} L\left(\frac{m-x}{\alpha}\right) & m-\alpha \le x \le m,\ \alpha > 0 \\ 1 & m \le x \le n \\ R\left(\frac{x-n}{\beta}\right) & n \le x \le n+\beta,\ \beta > 0 \\ 0 & \text{otherwise} \end{cases}$$

L and R are nonincreasing continuous functions $[0,1] \to [0,1]$, invertible on [0,1], that fulfill $L(0) = R(0) = 1$, $L(1) = R(1) = 0$, $[m, n]$ is the peak of $\widetilde{a}$, α, and β are the left and right spreads of $\widetilde{a}$, respectively. In cases where $m = n$, we shall write $\widetilde{a} = (m, \alpha, \beta)_{LR}$. The set of LR fuzzy numbers is denoted by $F(\mathbb{R})$.

Definition 3. Let $\widetilde{a}_1 = (m_1, n_1, \alpha_1, \beta_1)_{LR}$ and $\widetilde{a}_2 = (m_2, n_2, \alpha_2, \beta_2)_{LR}$, be any LR fuzzy numbers, then $\widetilde{a}_1 = \widetilde{a}_2$ if and only if $m_1 = m_2$, $n_1 = n_2$, $\alpha_1 = \alpha_2$ and $\beta_1 = \beta_2$.

Definition 4. An LR fuzzy number $\widetilde{a} = (m, n, \alpha, \beta)_{LR}$ is non-negative, denoted by $\widetilde{a} \ge 0$ (resp. non-positive, denoted by $\widetilde{a} \le 0$) if $m - \alpha \ge 0$ (resp. $n + \beta \le 0$).

Definition 5. Let $\widetilde{a}_1 = (m_1, n_1, \alpha_1, \beta_1)_{LR}$, $\widetilde{a}_2 = (m_2, n_2, \alpha_2, \beta_2)_{LR}$ be any LR fuzzy numbers and $\widetilde{a}_3 = (m_3, n_3, \alpha_3, \beta_3)_{RL}$ any RL fuzzy number. Then arithmetic operations of addition, subtraction, and multiplication by a scalar are defined as follows:

Addition: $\widetilde{a}_1 \oplus \widetilde{a}_2 = (m_1 + m_2, n_1 + n_2, \alpha_1 + \alpha_2, \beta_1 + \beta_2)_{LR}$

Subtraction: $\widetilde{a}_1 \ominus \widetilde{a}_3 = (m_1 - n_3, n_1 - m_3, \alpha_1 + \beta_3, \beta_1 + \alpha_3)_{LR}$

Multiplication by a scalar: $k\widetilde{a}_1 = \begin{cases} (km_1, kn_1, k\alpha_1, k\beta_1)_{LR}, & k \ge 0 \\ (kn_1, km_1, -k\beta_1, -k\alpha_1)_{RL}, & k < 0 \end{cases}$

2.1. Lexicographic ranking of LR fuzzy numbers

Let $\widetilde{a} = (m, n, \alpha, \beta)_{LR}$ be any LR fuzzy number and f_1, f_2, f_3, and f_4 be linear functions of the parameters of $\widetilde{a}$ ($f_k(\widetilde{a}) = w_{k1}m + w_{k2}n + w_{k3}\alpha + w_{k4}\beta$) with non-singular coefficient matrix $[w_{kr}]_{4\times 4}$. Then, it can be shown that the following lexicographic procedure defines a total order in the set of all LR fuzzy numbers of the same type.

Step 1. Find $f_1(\widetilde{a})$ and $f_1(\widetilde{b})$.
If $f_1(\widetilde{a}) >$ (**or** $<$)$f_1(\widetilde{b})$, **then** $\widetilde{a} \succ$ (**or** $\prec$)$\widetilde{b}$
If $f_1(\widetilde{a}) = f_1(\widetilde{b})$, **then go to Step 2**
Step 2. Find $f_2(\widetilde{a})$ and $f_2(\widetilde{b})$.
If $f_2(\widetilde{a}) >$ (**or** $<$)$f_2(\widetilde{b})$, **then** $\widetilde{a} \succ$ (**or** $\prec$)$\widetilde{b}$
If $f_2(\widetilde{a}) = f_2(\widetilde{b})$, **then go to Step 3**
Step 3. Find $f_3(\widetilde{a})$ and $f_3(\widetilde{b})$.
If $f_3(\widetilde{a}) >$ (**or** $<$)$f_3(\widetilde{b})$, **then** $\widetilde{a} \succ$ (**or** $\prec$)$\widetilde{b}$
If $f_3(\widetilde{a}) = f_3(\widetilde{b})$, **then go to Step 4**
Step 4. Find $f_4(\widetilde{a})$ and $f_4(\widetilde{b})$.
If $f_4(\widetilde{a}) >$ (**or** $<$)$f_4(\widetilde{b})$, **then** $\widetilde{a} \succ$ (**or** $\prec$)$\widetilde{b}$
If $f_4(\widetilde{a}) = f_4(\widetilde{b})$, **then** $\widetilde{a} = \widetilde{b}$

In practical applications, f_1, f_2, f_3, and f_4 must be carefully selected to capture, as accurately as possible, a decision-maker's ranking criterion. Special cases are the Rank-Mode-Divergence-Left spread (RMDS) ranking criterion proposed by Kaur and Kumar (2016) $f_1(\widetilde{a}) := \frac{1}{2}\left(m+n+\beta\int_0^1 R^{-1}(\lambda)d\lambda - \alpha\int_0^1 L^{-1}(\lambda)d\lambda\right)$, $f_2(\widetilde{a}) := \frac{1}{2}(m+n)$, $f_3(\widetilde{a}) := n-m+\beta\int_0^1 R^{-1}(\lambda)d\lambda + \alpha\int_0^1 L^{-1}(\lambda)d\lambda$ and $f_4(\widetilde{a}) := \alpha\int_0^1 L^{-1}(\lambda)d\lambda$; Farhadinia's (2009) ranking criterion $f_1(\widetilde{a}) := m$, $f_2(\widetilde{a}) := m - \alpha$, $f_3(\widetilde{a}) := n - m + \alpha + \beta$ and $f_4(\widetilde{a}) := \int \mu_{\widetilde{a}}(x)dx$; and Wang et al.'s (2005) criterion $f_1(\widetilde{a}) := \frac{1}{2}(m+n)$, $f_2(\widetilde{a}) := \frac{1}{2}(n-m) + \beta$, $f_3(\widetilde{a}) := \int \mu_{\widetilde{a}}(x)dx$ and $f_4(\widetilde{a}) := n - m$.

3. Mathematical Models of FVLP and FNLP

We consider the following FLP problem:

$$\begin{aligned} \max & \sum_{j=1}^{n} c_j\widetilde{x}_j \\ \text{s.t.} & \sum_{j=1}^{n} a_{ij}\widetilde{x}_j = \widetilde{b}_i, \text{ for } i \in I_1 := \{1, 2, \ldots, m_1\} \\ & \sum_{j=1}^{n} a_{ij}\widetilde{x}_j \preceq \widetilde{b}_i, \text{ for } i \in I_2 := \{m_1+1, \ldots, m\} \\ & \widetilde{x}_j \in F(\mathbb{R}), \text{ for } j \in J := \{1, 2, \ldots, n\} \end{aligned} \tag{1}$$

where $\widetilde{b}_i \in F(\mathbb{R})$, $c_j \in \mathbb{R}^+$ (or $c_j \in \mathbb{R}$ if $L = R$), $a_{ij} \in \mathbb{R}^+$ (or $a_{ij} \in \mathbb{R}$ if $L = R$), and $\preceq$ is an order relation defined for LR fuzzy numbers. Since only the decision variables and the right-hand side values of the constraints are represented by LR fuzzy numbers, FLP problem (1) is referred to as FVLP problem. In addition, we consider the following FLP problem:

$$\begin{aligned} \max & \sum_{j=1}^{n} \widetilde{c}_j x_j \\ \text{s.t.} & \sum_{j=1}^{n} \widetilde{a}_{ij}x_j = \widetilde{b}_i, \text{ for } i \in I_1 := \{1, 2, \ldots, m_1\} \\ & \sum_{j=1}^{n} \widetilde{a}_{ij}x_j \preceq \widetilde{b}_i, \text{ for } i \in I_2 := \{m_1+1, \ldots, m\} \\ & x_j \ge 0, \text{ for } j \in J := \{1, 2, \ldots, n\} \end{aligned} \tag{2}$$

where each $\widetilde{c}_j, \widetilde{a}_{ij}, \widetilde{b}_i \in F(\mathbb{R})$, and $\preceq$ is an order relation on $F(\mathbb{R})$. Since all objective function coefficients, technological coefficients, and right-hand side values of the constraints are represented by LR fuzzy numbers, and only the decision variables take on nonnegative real values, FLP problem (2) is referred to as FNLP problem.

Definition 6. A vector $\widetilde{x} = (\widetilde{x}_1, \widetilde{x}_2, \ldots, \widetilde{x}_n) \in F(\mathbb{R})^n$ is a feasible solution to FVLP problem (1) if all equality and inequality constraints are satisfied.

Definition 7. Denote by $\widetilde{X}$ the set of feasible solutions to problem (1). A vector $\widetilde{x}^* = (\widetilde{x}_1^*, \widetilde{x}_2^*, \ldots, \widetilde{x}_n^*) \in \widetilde{X}$ is an optimal fuzzy solution to FVLP problem (1) if $\sum_{j=1}^{n} c_j \widetilde{x}_j \preceq \sum_{j=1}^{n} c_j \widetilde{x}_j^*$ for all $\widetilde{x} \in \widetilde{X}$.

Definition 8. A vector $x = (x_1, x_2, \ldots, x_n) \in \mathbb{R}^n$ is a feasible solution to FNLP problem (2) if all equality and inequality constraints are satisfied.

Definition 9. Denote by X the set of feasible solutions to problem (2). A vector $x^* = (x_1^*, x_2^*, \ldots, x_n^*) \in X$ is an optimal solution to FNLP problem (2) if $\sum_{j=1}^{n} \widetilde{c}_j x_j \preceq \sum_{j=1}^{n} \widetilde{c}_j x_j^*$ for all $x \in X$.

Remark 1. Ebrahimnejad (2015) used a ranking function to solve instances of FVLP problem (1). The ranking function, which in practical applications approximates a decision-maker's ranking criterion, transforms fuzzy numbers into real numbers. Consequently, comparing two fuzzy numbers reduces to comparing the corresponding real numbers. Ebrahimnejad (2015) pointed out that two fuzzy numbers with the same ranking value are not necessarily equal. A major implication of this fact, as we will show, is that the fuzzy objective value of FVLP problem (1) obtained by using Ebrahimnejad's (2015) method is not unique in general, that is, there may be other solutions whose corresponding fuzzy values may seem better to the decision-maker. This observation also applies in FNLP and FFLP. We refer the reader to Kaur and Kumar (2012), Ezzati et al. (2015), and Kaur and Kumar (2016) for further discussions on this issue.

4. Lexicographic Method for FVLP and FNLP

Definition 10. Denote by $\prec_{lex}$ the lexicographic order on $\mathbb{R}^4$ and let f_1, f_2, f_3, and f_4 be linear functions with non-singular coefficient matrix. For $\widetilde{a}_1, \widetilde{a}_2 \in F(\mathbb{R})$, the strict inequality $\widetilde{a}_1 \prec \widetilde{a}_2$ holds, if and only if, $(f_1(\widetilde{a}_1), f_2(\widetilde{a}_1), f_3(\widetilde{a}_1), f_4(\widetilde{a}_1)) \prec_{lex} (f_1(\widetilde{a}_2), f_2(\widetilde{a}_2), f_3(\widetilde{a}_2), f_4(\widetilde{a}_2))$. The weak inequality $\widetilde{a}_1 \preceq \widetilde{a}_2$ holds, if and only if, $(f_1(\widetilde{a}_1), f_2(\widetilde{a}_1), f_3(\widetilde{a}_1), f_4(\widetilde{a}_1)) \prec_{lex} (f_1(\widetilde{a}_2), f_2(\widetilde{a}_2), f_3(\widetilde{a}_2), f_4(\widetilde{a}_2))$ or $(f_1(\widetilde{a}_1), f_2(\widetilde{a}_1), f_3(\widetilde{a}_1), f_4(\widetilde{a}_1)) = (f_1(\widetilde{a}_2), f_2(\widetilde{a}_2), f_3(\widetilde{a}_2), f_4(\widetilde{a}_2))$.

Remark 2. The order relation $\preceq$ is a total order, that is, it has the following properties (provided that $\widetilde{a}_1$, $\widetilde{a}_2$ and $\widetilde{a}_3$ are LR fuzzy numbers of the same type):

1. $\forall \widetilde{a}_1 \in F(\mathbb{R})\ \widetilde{a}_1 \preceq \widetilde{a}_1$ (reflexivity).
2. $\forall \widetilde{a}_1, \widetilde{a}_2 \in F(\mathbb{R})\ \ \widetilde{a}_1 \preceq \widetilde{a}_2$ and $\widetilde{a}_2 \preceq \widetilde{a}_1$ implies $\widetilde{a}_1 = \widetilde{a}_2$ (anti-symmetry).
3. $\forall \widetilde{a}_1, \widetilde{a}_2, \widetilde{a}_3 \in F(\mathbb{R})\ \ \widetilde{a}_1 \preceq \widetilde{a}_2$ and $\widetilde{a}_2 \preceq \widetilde{a}_3$ implies $\widetilde{a}_1 \preceq \widetilde{a}_3$ (transitivity).
4. $\forall \widetilde{a}_1, \widetilde{a}_2 \in F(\mathbb{R})\ \widetilde{a}_1 \preceq \widetilde{a}_2$ or $\widetilde{a}_2 \preceq \widetilde{a}_1$ (comparability).

The steps of the lexicographic method are as follows:

Step 1. If FVLP problem (1) is to be solved, then assume $\widetilde{x}_j = (x_j^1, x_j^2, \alpha_j^x, \beta_j^x)_{LR}$. By using Definition 5, write $c_j\widetilde{x}_j = (c_j^1, c_j^2, \alpha_j^c, \beta_j^c)_{LR}$, $a_{ij}\widetilde{x}_j = (a_{ij}^1, a_{ij}^2, \alpha_{ij}^a, \beta_{ij}^a)_{LR}$, and $\widetilde{b}_i = (b_i^1, b_i^2, \alpha_i^b, \beta_i^b)_{LR}$. On the other hand, if FNLP problem (2) is to be solved, then write $\widetilde{c}_j x_j = (c_j^1, c_j^2, \alpha_j^c, \beta_j^c)_{LR}$ and $\widetilde{a}_{ij}x_j = (a_{ij}^1, a_{ij}^2, \alpha_{ij}^a, \beta_{ij}^a)_{LR}$. By using the linearity property of each f_k, write the constraint set of FVLP problem (1) (or FNLP problem (2)) as:

$$\mathbb{X}_{lex} := \begin{cases} \sum_{j=1}^{n} a_{ij}^1 = b_i^1, \sum_{j=1}^{n} a_{ij}^2 = b_i^2, \sum_{j=1}^{n} \alpha_{ij}^a = \alpha_i^b, \sum_{j=1}^{n} \beta_{ij}^a = \beta_i^b, \text{ for } i \in I_1 \\ \Big(\sum_{j=1}^{n} f_1\big((a_{ij}^1, a_{ij}^2, \alpha_{ij}^a, \beta_{ij}^a)_{LR}\big), \sum_{j=1}^{n} f_2\big((a_{ij}^1, a_{ij}^2, \alpha_{ij}^a, \beta_{ij}^a)_{LR}\big), \\ \sum_{j=1}^{n} f_3\big((a_{ij}^1, a_{ij}^2, \alpha_{ij}^a, \beta_{ij}^a)_{LR}\big), \sum_{j=1}^{n} f_4\big((a_{ij}^1, a_{ij}^2, \alpha_{ij}^a, \beta_{ij}^a)_{LR}\big) \Big) \preceq_{lex} \\ \big(f_1\big((b_i^1, b_i^2, \alpha_i^b, \beta_i^b)_{LR}\big), f_2\big((b_i^1, b_i^2, \alpha_i^b, \beta_i^b)_{LR}\big), \\ f_3\big((b_i^1, b_i^2, \alpha_i^b, \beta_i^b)_{LR}\big), f_4\big((b_i^1, b_i^2, \alpha_i^b, \beta_i^b)_{LR}\big)\big), \text{ for } i \in I_2 \end{cases}$$

Step 2. Transform FVLP problem (1) (resp. FNLP problem (2)) into the following lexicographic optimization problem:

$$\begin{aligned} &\text{lex max}\Bigg(\sum_{j=1}^{n} f_1\Big(\big(c_j^1, c_j^2, \alpha_j^c, \beta_j^c\big)_{LR}\Big), \sum_{j=1}^{n} f_2\Big(\big(c_j^1, c_j^2, \alpha_j^c, \beta_j^c\big)_{LR}\Big), \\ &\qquad \sum_{j=1}^{n} f_3\Big(\big(c_j^1, c_j^2, \alpha_j^c, \beta_j^c\big)_{LR}\Big), \sum_{j=1}^{n} f_4\Big(\big(c_j^1, c_j^2, \alpha_j^c, \beta_j^c\big)_{LR}\Big) \Bigg) \\ &\text{s.t.} \begin{cases} \mathbb{X}_{lex} \\ \alpha_j^x, \beta_j^x \geq 0,\ x_j^1 \leq x_j^2, \text{ for } j \in J \end{cases} \left(\text{resp. } \begin{cases} \mathbb{X}_{lex} \\ x_j \geq 0, \text{ for } j \in J \end{cases} \right) \end{aligned} \tag{3}$$

Step 3. By introducing auxiliary variables $s_i^1, s_i^2, s_i^3, s_i^4$ and binary variables $y_i^1, y_i^2, y_i^3, y_i^4$, write $\mathbb{X}_{lex}$ as:

$$\mathbb{X} := \begin{cases} \sum_{j=1}^{n} a_{ij}^1 = b_i^1, \sum_{j=1}^{n} a_{ij}^2 = b_i^2, \sum_{j=1}^{n} \alpha_{ij}^a = \alpha_i^b, \sum_{j=1}^{n} \beta_{ij}^a = \beta_i^b, \text{ for } i \in I_1 \\ \sum_{j=1}^{n} f_1\Big(\big(a_{ij}^1, a_{ij}^2, \alpha_{ij}^a, \beta_{ij}^a\big)_{LR}\Big) + s_i^1 = f_1\big((b_i^1, b_i^2, \alpha_i^b, \beta_i^b)_{LR}\big), \text{ for } i \in I_2 \\ \sum_{j=1}^{n} f_2\Big(\big(a_{ij}^1, a_{ij}^2, \alpha_{ij}^a, \beta_{ij}^a\big)_{LR}\Big) + s_i^2 = f_2\big((b_i^1, b_i^2, \alpha_i^b, \beta_i^b)_{LR}\big), \text{ for } i \in I_2 \\ \sum_{j=1}^{n} f_3\Big(\big(a_{ij}^1, a_{ij}^2, \alpha_{ij}^a, \beta_{ij}^a\big)_{LR}\Big) + s_i^3 = f_3\big((b_i^1, b_i^2, \alpha_i^b, \beta_i^b)_{LR}\big), \text{ for } i \in I_2 \\ \sum_{j=1}^{n} f_4\Big(\big(a_{ij}^1, a_{ij}^2, \alpha_{ij}^a, \beta_{ij}^a\big)_{LR}\Big) + s_i^4 = f_4\big((b_i^1, b_i^2, \alpha_i^b, \beta_i^b)_{LR}\big), \text{ for } i \in I_2 \\ \epsilon y_i^1 \leq s_i^1 \leq L y_i^1, -L y_i^1 + \epsilon y_i^2 \leq s_i^2 \leq L y_i^2, \text{ for } i \in I_2 \\ -L(y_i^1 + y_i^2) + \epsilon y_i^3 \leq s_i^3 \leq L y_i^3, \text{ for } i \in I_2 \\ -L(y_i^1 + y_i^2 + y_i^3) + \epsilon y_i^4 \leq s_i^4 \leq L y_i^4; y_i^1, y_i^2, y_i^3, y_i^4 \in \{0,1\}, \text{ for } i \in I_2 \end{cases}$$

for positive ϵ and L sufficiently small and large, respectively.

Step 4. Transform problem (3) into the following mixed-integer lexicographic linear programming (MILLP) problem:

$$\begin{aligned}&\text{lex max}\left(\sum_{j=1}^{n} f_1\left(\left(c_j^1, c_j^2, \alpha_j^c, \beta_j^c\right)_{LR}\right), \sum_{j=1}^{n} f_2\left(\left(c_j^1, c_j^2, \alpha_j^c, \beta_j^c\right)_{LR}\right),\right.\\&\qquad\left.\sum_{j=1}^{n} f_3\left(\left(c_j^1, c_j^2, \alpha_j^c, \beta_j^c\right)_{LR}\right), \sum_{j=1}^{n} f_4\left(\left(c_j^1, c_j^2, \alpha_j^c, \beta_j^c\right)_{LR}\right)\right)\\&\text{s.t.}\begin{cases}\mathbb{X}\\ \alpha_j^x, \beta_j^x \geq 0, x_j^1 \leq x_j^2, \text{ for } j \in J\end{cases} \left(\text{resp. } \begin{cases}\mathbb{X}\\ x_j \geq 0, \text{ for } j \in J\end{cases}\right)\end{aligned} \tag{4}$$

Step 5. Solve MILLP problem (4) with the lexicographic method (Ehrgott, 2005).

Step 6. If FVLP problem (1) was solved, then put the values of x_j^1, x_j^2, α_j^x, and β_j^x into $\widetilde{x}_j = (x_j^1, x_j^2, \alpha_j^x, \beta_j^x)_{LR}$ and evaluate $\sum_{j=1}^{n} c_j \widetilde{x}_j$ to get an optimal fuzzy solution and its corresponding unique fuzzy value. On the other hand, if FNLP problem (2) was solved, then the solution is already given by the variables $x_1, x_2, \ldots, x_n$. Evaluate $\sum_{j=1}^{n} \widetilde{c}_j x_j$ to get the unique optimal fuzzy objective value of FNLP problem (2).

Theorem 1. Problems (3) and (4) have the same feasible set (Pérez-Cañedo & Concepción-Morales, 2019).

Proof. It is sufficient to prove the equivalence between the lexicographic constraints of problem (3) and the following set of constraints of problem (4):

$$\begin{aligned}&\sum_{j=1}^{n} f_k\left(\left(a_{ij}^1, a_{ij}^2, \alpha_{ij}^a, \beta_{ij}^a\right)_{LR}\right) + s_i^k = f_k\left((b_i^1, b_i^2, \alpha_i^b, \beta_i^b)_{LR}\right),\\&k \in \{1,2,3,4\}, i \in I_2 - L\sum_{p=1}^{k-1} y_i^p + \epsilon y_i^k \leq s_i^k \leq L y_i^k, y_i^k \in \{0,1\},\\&k \in \{1,2,3,4\}, i \in I_2\end{aligned} \tag{5}$$

To simplify the notation, let us define $l_{ik} := \sum_{j=1}^{n} f_k\left(\left(a_{ij}^1, a_{ij}^2, \alpha_{ij}^a, \beta_{ij}^a\right)_{LR}\right)$ and $r_{ik} := f_k\left((b_i^1, b_i^2, \alpha_i^b, \beta_i^b)_{LR}\right)$. We first show that any solution satisfying the lexicographic constraints of problem (3), necessarily satisfies constraint set (5). Thus, we have for every $i \in I_2$ $(l_{i1}, \ldots, l_{i4}) \prec_{lex} (r_{i1}, \ldots, r_{i4})$ or $(l_{i1}, \ldots, l_{i4}) = (r_{i1}, \ldots, r_{i4})$ in problem (3). If $(l_{i1}, \ldots, l_{i4}) = (r_{i1}, \ldots, r_{i4})$, then clearly $s_i^k = 0$ for all k which is only possible by setting $y_i^k = 0$ for all k. On the other hand, if $(l_{i1}, \ldots, l_{i4}) \prec_{lex} (r_{i1}, \ldots, r_{i4})$, then there is $1 \leq k_i \leq 4$ such that $l_{ik} = r_{ik}$ for $k < k_i$ and $l_{ik_i} < r_{ik_i}$. This means that $s_i^k = 0$ for $k < k_i$ and $s_i^{k_i} > 0$, which is obtained only by setting $y_i^k = 0$ for $k < k_i$, $y_i^{k_i} = 1$ and $y_i^k = 0$ (or 1) for $k > k_i$ with positive ϵ and L, respectively, small and large enough so that $s_i^{k_i} \in [\epsilon, L]$. Conversely, if $y_i^k = 0$ for all k, then $s_i^k = 0$ for all k implying $(l_{i1}, \ldots, l_{i4}) = (r_{i1}, \ldots, r_{i4})$ in problem (3). Assume now that there is $1 \leq k_i \leq 4$ such that $y_i^{k_i} = 1$ and $y_i^k = 0$ for $k < k_i$. In such a case, we obtain the inequality $\epsilon \leq s_i^{k_i} \leq L$, which implies that $s_i^{k_i} > 0$. Therefore, we have $s_i^k = 0$ for $k < k_i$ and $s_i^{k_i} > 0$ implying $(l_{i1}, \ldots, l_{i4}) \prec_{lex} (r_{i1}, \ldots, r_{i4})$.

Remark 3. By virtue of Theorem 1 and since problems (3) and (4) have the same objective function, both problems are equivalent.

5. Advantages and Limitations of the Lexicographic Method

The lexicographic method has the following advantages on solving FVLP and FNLP problems with inequality constraints:

1. All problems solved by the existing methods (Ebrahimnejad, 2015; Lai & Hwang, 1992; Mahdavi-Amiri & Nasseri, 2007; Maleki et al., 2000, Shaocheng, 1994) can be also solved by the lexicographic method. However, uniqueness of optimal fuzzy objective values is only guaranteed by the lexicographic method.
2. Most of the existing methods (Ebrahimnejad, 2015; Mahdavi-Amiri & Nasseri, 2007) use a linear ranking function to define fuzzy inequality relations in $F(\Re)$. Such an approach neglects the fuzziness in the objective function and inequality relations in the constraint set. On the other hand, by using several ranking criteria, the lexicographic method takes all information into account in the solving process.
3. The lexicographic method makes very few assumptions on the nature of the ranking functions, which allows for a great deal of flexibility in practical decision-making.

Despite its advantages, the lexicographic method transforms the FLP problem into an MILLP problem, and it is therefore more complex than the other methods.

6. Numerical Examples and Applications

This section presents four FLP problems. Examples 1 and 2 are an FVLP problem and an FNLP problem, respectively. These examples are used to illustrate how to carry out the steps of the lexicographic method. We provide the optimal solution to both examples by using three different lexicographic ranking criteria. In Subsection 6.1, a diet problem presented in Ebrahimnejad (2015) is solved by using the lexicographic method, and the results are compared with those obtained via Ebrahimnejad's (2015) method. Lastly, an application of the lexicographic method to a time–cost trade-off problem with fuzzy variables is discussed in subsection 6.2. Numerical optimization was carried out by using PuLP linear programming modeller (Mitchell et al., 2011) version 1.6.0 and CBC MILP solver (Forrest & Lougee-Heimer, 2005) version 2.8.12. The computer codes of each example are available from the first author upon request.

Example 1. Let us consider the following FVLP problem:

$$\begin{aligned}&\max\ 2\widetilde{x}_1 \oplus \widetilde{x}_2 \oplus \widetilde{x}_3\\&\text{s.t. } \widetilde{x}_1 \oplus \widetilde{x}_2 \oplus \widetilde{x}_3 = (6, 8, 2, 1)_{LR}\\&\quad \widetilde{x}_1 = \frac{1}{2}\widetilde{x}_2 \oplus \left(0, 2, \frac{1}{2}, \frac{1}{2}\right)_{LR}\\&\quad \widetilde{x}_1 \preceq \widetilde{x}_3\\&\quad \widetilde{x}_1, \widetilde{x}_2, \widetilde{x}_3 \geq 0\\&\quad L(x) = R(x) = 1 - x\end{aligned} \tag{6}$$

Some authors (Ezzati et al., 2015; Giri et al., 2015) use fuzzy slack and surplus variables to transform fuzzy inequalities into equalities.

Thus, in FVLP problem (6), the inequality $\widetilde{x}_1 \preceq \widetilde{x}_3$ would be replaced by $\widetilde{x}_1 \oplus \widetilde{s} = \widetilde{x}_3$ (with $\widetilde{s} \geq 0$), and the solution to the resulting FVLP problem is assumed to be that of the original problem. If we carry out this sort of transformation, then the resulting FVLP problem is not feasible. However, we next show, by using the lexicographic method, that FVLP problem (6) is feasible.
The RMDS ranking criterion (Kaur & Kumar, 2016) is used to solve FVLP problem (6). Note that, in this example, $\int_0^1 L^{-1}(\lambda)d\lambda = \int_0^1 R^{-1}(\lambda)d\lambda = 1/2$.

Step 1. Assume $\widetilde{x}_j = (x_j, y_j, \alpha_j, \beta_j)_{LR}$ and write the constraint set as:

$$\mathbb{X}_{lex} := \begin{cases} x_1 + x_2 + x_3 = 6, y_1 + y_2 + y_3 = 8, \\ \alpha_1 + \alpha_2 + \alpha_3 = 2, \beta_1 + \beta_2 + \beta_3 = 1, \\ x_1 = \frac{1}{2}x_2, y_1 = \frac{1}{2}y_2 + 2, \\ \alpha_1 = \frac{1}{2}\alpha_2 + \frac{1}{2}, \beta_1 = \frac{1}{2}\beta_2 + \frac{1}{2}, \\ \left(\frac{1}{2}\left(x_1 + y_1 + \frac{1}{2}\beta_1 - \frac{1}{2}\alpha_1\right), \frac{(x_1+y_1)}{2},\right. \\ \left. y_1 - x_1 + \frac{1}{2}\beta_1 + \frac{1}{2}\alpha_1, \frac{1}{2}\alpha_1\right) \preceq_{lex} \\ \left(\frac{1}{2}\left(x_3 + y_3 + \frac{1}{2}\beta_3 - \frac{1}{2}\alpha_3\right), \frac{(x_3+y_3)}{2},\right. \\ \left. y_3 - x_3 + \frac{1}{2}\beta_3 + \frac{1}{2}\alpha_3, \frac{1}{2}\alpha_3\right) \end{cases}$$

Step 2. FVLP problem (6) into the following lexicographic optimization problem:

$$\begin{aligned} \text{lex max} & \left(x_1 + y_1 + \frac{1}{2}\beta_1 - \frac{1}{2}\alpha_1 + \frac{1}{2}\left(x_2 + y_2 + \frac{1}{2}\beta_2 - \frac{1}{2}\alpha_2\right) + \right. \\ & \frac{1}{2}\left(x_3 + y_3 + \frac{1}{2}\beta_3 - \frac{1}{2}\alpha_3\right), (x_1 + y_1) + \frac{(x_2 + y_2)}{2} + \frac{(x_3 + y_3)}{2}, \\ & 2y_1 - 2x_1 + \beta_1 + \alpha_1 + y_2 - x_2 + \frac{1}{2}\beta_2 + \frac{1}{2}\alpha_2 + y_3 - x_3 + \\ & \left.\frac{1}{2}\beta_3 + \frac{1}{2}\alpha_3, \alpha_1 + \frac{1}{2}\alpha_2 + \frac{1}{2}\alpha_3\right) \\ \text{s.t.} & \begin{cases} \mathbb{X}_{lex} \\ \alpha_j, \beta_j \geq 0,\ x_j \leq y_j,\ x_j - \alpha_j \geq 0,\ \text{for } j = 1,2,3 \end{cases} \end{aligned} \tag{7}$$

Step 3. Write constraint set $\mathbb{X}_{lex}$ as:

$$\mathbb{X} := \begin{cases} x_1 + x_2 + x_3 = 6, y_1 + y_2 + y_3 = 8, \\ \alpha_1 + \alpha_2 + \alpha_3 = 2, \beta_1 + \beta_2 + \beta_3 = 1, \\ x_1 = \frac{1}{2}x_2, y_1 = \frac{1}{2}y_2 + 2, \alpha_1 = \frac{1}{2}\alpha_2 + \frac{1}{2}, \beta_1 = \frac{1}{2}\beta_2 + \frac{1}{2}, \\ \frac{1}{2}\left(x_1 + y_1 + \frac{1}{2}\beta_1 - \frac{1}{2}\alpha_1\right) + s_1^1 = \frac{1}{2}\left(x_3 + y_3 + \frac{1}{2}\beta_3 - \frac{1}{2}\alpha_3\right), \\ \frac{(x_1+y_1)}{2} + s_1^2 = \frac{(x_3+y_3)}{2}, \\ y_1 - x_1 + \frac{1}{2}\beta_1 + \frac{1}{2}\alpha_1 + s_1^3 = y_3 - x_3 + \frac{1}{2}\beta_3 + \frac{1}{2}\alpha_3, \frac{1}{2}\alpha_1 + s_1^4 = \frac{1}{2}\alpha_3 \\ \epsilon y_1^1 \leq s_1^1 \leq Ly_1^1, -Ly_1^1 + \epsilon y_1^2 \leq s_1^2 \leq Ly_1^2, -L(y_1^1 + y_1^2) + \epsilon y_1^3 \leq s_1^3 \leq Ly_1^3, \\ -L(y_1^1 + y_1^2 + y_1^3) + \epsilon y_1^4 \leq s_1^4 \leq Ly_1^4, \tilde{\ } y_1^k \in \{0,1\} \text{ for } k = 1, \ldots, 4 \end{cases}$$

where $\epsilon = 10^{-5}$ and $L = 10$.

Step 4. Transform problem (7) into the following MILLP problem:

$$\begin{aligned} \text{lex max} & \left(x_1 + y_1 + \frac{1}{2}\beta_1 - \frac{1}{2}\alpha_1 + \frac{1}{2}\left(x_2 + y_2 + \frac{1}{2}\beta_2 - \frac{1}{2}\alpha_2\right) + \right. \\ & \frac{1}{2}\left(x_3 + y_3 + \frac{1}{2}\beta_3 - \frac{1}{2}\alpha_3\right), (x_1 + y_1) + \frac{(x_2 + y_2)}{2} + \frac{(x_3 + y_3)}{2}, \\ & 2y_1 - 2x_1 + \beta_1 + \alpha_1 + y_2 - x_2 + \frac{1}{2}\beta_2 + \frac{1}{2}\alpha_2 + y_3 - x_3 + \\ & \left.\frac{1}{2}\beta_3 + \frac{1}{2}\alpha_3, \alpha_1 + \frac{1}{2}\alpha_2 + \frac{1}{2}\alpha_3\right) \\ \text{s.t.} & \begin{cases} \mathbb{X} \\ \alpha_j, \beta_j \geq 0,\ x_j \leq y_j,\ x_j - \alpha_j \geq 0,\ \text{for } j = 1,2,3 \end{cases} \end{aligned} \tag{8}$$

Step 5. Solve MILLP (8) problem with the lexicographic method (Ehrgott, 2005). This produces $x_1 = 1.24$, $y_1 = 3.24$, $\alpha_1 = 0.91$, $\beta_1 = 0.66$, $x_2 = y_2 = 2.49$, $\alpha_2 = 0.83$, $\beta_2 = 0.33$, $x_3 = y_3 = 2.25$, $\alpha_3 = 0.25$, and $\beta_3 = 0$.

Step 6. Put the values of x_j, y_j, α_j, and β_j into $\widetilde{x}_j = (x_j, y_j, \alpha_j, \beta_j)_{LR}$. Thus, we obtain $\widetilde{x}_1 = (1.24, 3.24, 0.91, 0.66)_{LR}$, $\widetilde{x}_2 = (2.49, 2.49, 0.83, 0.33)_{LR}$, and $\widetilde{x}_3 = (2.25, 2.25, 0.25, 0)_{LR}$, with unique optimal fuzzy objective value $\widetilde{u} = (7.24, 11.24, 2.91, 1.66)_{LR}$.

Now, we verify that the obtained solution satisfies the inequality constraint of FVLP problem (6) lexicographically. By using Definition 10, we have $(f_1(\widetilde{x}_1), f_2(\widetilde{x}_1), f_3(\widetilde{x}_1), f_4(\widetilde{x}_1)) = (2.18, 2.24, 2.79, 0.45)$ and $(f_1(\widetilde{x}_3), f_2(\widetilde{x}_3), f_3(\widetilde{x}_3), f_4(\widetilde{x}_3)) = (2.18, 2.25, 0.12, 0.12)$. Since $(2.18, 2.24, 2.79, 0.45)$ is lexicographically less than $(2.18, 2.25, 0.12, 0.12)$, we conclude that $\widetilde{x}_1 \prec \widetilde{x}_3$; therefore, the obtained solution satisfies the inequality constraint of FVLP problem (6). It should be noted that replacing the lexicographic constraints with mere inequalities in MILLP problem (8), that is, dropping all constraints involving binary variables y_1^k and setting $s_1^k \geq 0$ for $k = 1, 2, 3, 4$, makes the resulting optimization problem not feasible.

Additionally, Table 1 shows the solution to FVLP problem (6) using the lexicographic method with the ranking criteria of Wang et al. (2005) and Farhadinia (2009). We must stress that although the optimal fuzzy objective values obtained by using three different ranking criteria are not equal, they are unique with respect to the corresponding ranking criteria.

Example 2. Let us consider the following FNLP problem:

$$\begin{aligned} \max\ & (2, 2, 1, 1)_{LR}x_1 \oplus (4, 8, 2, 3)_{LR}x_2 \\ \text{s.t.}\ & (4, 6, 2, 1)_{LR}x_1 \oplus (5, 8, 3, 2)_{LR}x_2 \preceq (20, 24, 10, 11)_{LR} \\ & (1, 3, 1, 1)_{LR}x_1 \oplus (9, 13, 1, 1)_{LR}x_2 \preceq (22, 30, 11, 13)_{LR} \\ & x_1, x_2 \geq 0, L(x) = R(x) = 1 - x \end{aligned} \tag{9}$$

Table 1
Solution to FVLP problem (6) using the lexicographic method with two different ranking criteria

	Ranking criterion	
Solution	(Wang et al., 2005)	(Farhadinia, 2009)
$\widetilde{x}_1$	$(1.24, 3.24, 1, 0.66)_{LR}$	$(1.50, 3.50, 0.75, 0.66)_{LR}$
$\widetilde{x}_2$	$(2.49, 2.49, 1, 0.33)_{LR}$	$(3, 3, 0.50, 0.33)_{LR}$
$\widetilde{x}_3$	$(2.25, 2.25, 0, 0)_{LR}$	$(1.50, 1.50, 0.74, 0)_{LR}$
Fuzzy value	$(7.24, 11.24, 3, 1.66)_{LR}$	$(7.50, 11.50, 2.75, 1.66)_{LR}$

To solve this FNLP problem, we use Farhadinia's (2009) ranking criterion and follow the steps of the lexicographic method. Note that, in this example, $\int \mu_{\tilde{a}}(x)dx = n - m + \frac{1}{2}(\alpha + \beta)$. Thus, we obtain the following equivalent MILLP problem:

$$\begin{aligned}
&\text{lex max}\,(2x_1+4x_2, x_1+2x_2, 2x_1+9x_2, x_1+6.5x_2)\\
&\quad \text{s.t. } 4x_1+5x_2+s_1^1=20,\ 2x_1+2x_2+s_1^2=10,\\
&\qquad 5x_1+8x_2+s_1^3=25,\ 3.5x_1+5.5x_2+s_1^4=14.5,\\
&\qquad x_1+9x_2+s_2^1=22,\ 8x_2+s_2^2=11,\\
&\qquad 4x_1+6x_2+s_2^3=32,\ 3x_1+5x_2+s_2^4=20,\\
&\qquad \epsilon y_1^1 \le s_1^1 \le Ly_1^1, -Ly_1^1+\epsilon y_1^2 \le s_1^2 \le Ly_1^2,\\
&\qquad -L(y_1^1+y_1^2)+\epsilon y_1^3 \le s_1^3 \le Ly_1^3,\\
&\qquad -L(y_1^1+y_1^2+y_1^3)+\epsilon y_1^4 \le s_1^4 \le Ly_1^4,\\
&\qquad \epsilon y_2^1 \le s_2^1 \le Ly_2^1, -Ly_2^1+\epsilon y_2^2 \le s_2^2 \le Ly_2^2,\\
&\qquad -L(y_2^1+y_2^2)+\epsilon y_2^3 \le s_2^3 \le Ly_2^3,\\
&\qquad -L(y_2^1+y_2^2+y_2^3)+\epsilon y_2^4 \le s_2^4 \le Ly_2^4,\\
&\qquad x_1, x_2 \ge 0,\ y_1^k, y_2^k \in \{0,1\} \text{ for } k=1,\dots,4
\end{aligned} \tag{10}$$

where $\epsilon = 10^{-5}$ and $L = 10$. Solving MILLP problem (10) produces $x_1 = 2.25$ and $x_2 = 2.19$, with unique optimal fuzzy objective value $\tilde{u} = (13.29, 22.06, 6.64, 8.83)_{LR}$. Table 2 shows the solution to FNLP problem (9) using the lexicographic method with the ranking criteria of Wang et al. (2005) and Kaur and Kumar (2016). Figure 1 depicts the fuzzy objective values of FNLP problem (9) obtained with Farhadinia's (2009) criterion (Figure 1(a)), Wang et al.'s (2005) criterion (Figure 1(b)) and Kaur & Kumar's (2016) criterion (Figure 1(c)), and also the fuzzy objective values obtained when adding fuzzy slack variables and replacing the lexicographic constraints with linear inequalities. In Figure 1(b), the solution obtained by adding fuzzy slack variables (dashed line) and the solution derived from replacing the lexicographic constraints with linear inequality constraints have equal fuzzy objective values. From Figure 1, the reader may see that the lexicographic method produced better results in all cases.

Table 2
Solution to FNLP problem (9) using the lexicographic method with two different ranking criteria

Solution	Ranking criterion (Wang et al., 2005)	(Kaur & Kumar, 2016)
x_1	1.73	1.99
x_2	2.04	2.04
Fuzzy value	$(11.66, 19.85, 5.83, 7.88)_{LR}$	$(12.16, 20.35, 6.08, 8.13)_{LR}$

Examples 1 and 2 demonstrate that it is better to use the lexicographic method as compared to the methods that introduce fuzzy slack and surplus variables or those that transform each fuzzy inequality into a set of crisp inequalities. By using the lexicographic method, feasibility issues occasioned by the latter fuzzy inequality constraint handling approaches are avoided, better fuzzy objective values are obtained, and the decision-maker's ranking criterion is used properly in the solving process.

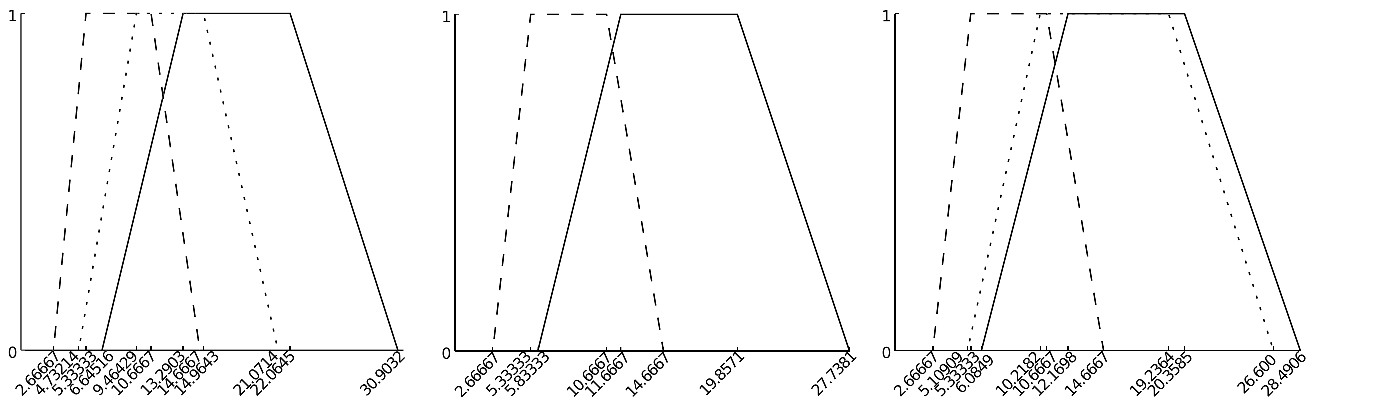
6.1. Application in a diet problem

Let us consider the diet problem solved by Ebrahimnejad (2015), whose formulation is given below.

$$\begin{aligned}
&\min\ 3\tilde{x}_1 \oplus 2\tilde{x}_2 \oplus 4\tilde{x}_3 \oplus 5\tilde{x}_4\\
&\text{s.t. } \tilde{x}_1 \oplus 2\tilde{x}_2 \oplus \tilde{x}_3 \oplus 4\tilde{x}_4 \succeq (196, 202, 1, 5)_{LR}\\
&\quad 3\tilde{x}_1 \oplus \tilde{x}_2 \oplus 2\tilde{x}_3 \oplus 2\tilde{x}_4 \succeq (118, 120, 2, 6)_{LR}\\
&\quad (8, 12, 1, 1)_{LR} \preceq \tilde{x}_1 \preceq (18, 22, 2, 2)_{LR}\\
&\quad (18, 22, 2, 2)_{LR} \preceq \tilde{x}_2 \preceq (36, 42, 2, 6)_{LR}\\
&\quad (8, 12, 1, 1)_{LR} \preceq \tilde{x}_3 \preceq (18, 22, 2, 2)_{LR}\\
&\quad (8, 12, 1, 1)_{LR} \preceq \tilde{x}_4 \preceq (26, 32, 1, 5)_{LR}\\
&\quad L(x) = R(x) = 1 - x
\end{aligned} \tag{11}$$

Solving FVLP problem (11) by using Ebrahimnejad's (2015) method produces: $\tilde{x}_1 = (8, 12, 1, 1)_{LR}$, $\tilde{x}_3 = (8, 12, 1, 1)_{LR}$, $\tilde{x}_2 = (36, 42, 2, 6)_{LR}$ and $\tilde{x}_4 = (16, 34.5, 7.75, 6.75)_{LR}$ with fuzzy value $\tilde{v}_E = (208, 340.5, 49.75, 52.75)_{LR}$. To solve FVLP problem (11) by using the lexicographic method, we define $f_1(\tilde{a}) := \frac{1}{2}\left(m + n + \frac{(\beta - \alpha)}{2}\right)$,

Figure 1
Unique optimal fuzzy objective value of FNLP problem (9) (solid line), obtained by using the lexicographic method with three different ranking criteria. Suboptimal fuzzy values were obtained by adding fuzzy slack variables (dashed line) and alternatively by replacing lexicographic constraints with linear inequalities (dotted line)



(a) Farhadinia's (2009) ranking criterion. (b) Wang et al.'s (2005) ranking criterion. (c) Kaur & Kumar's (2016) ranking criterion.

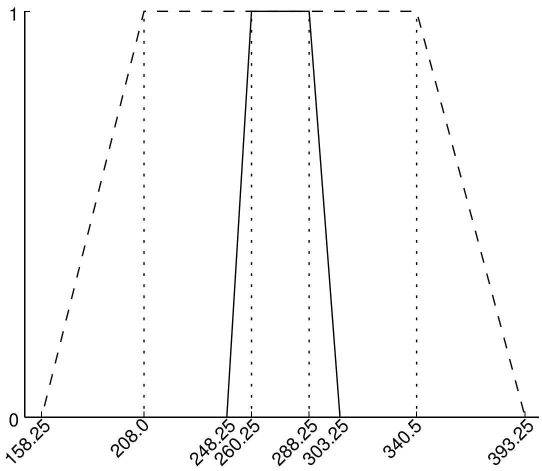
which is the same ranking function used by Ebrahimnejad (2015), and additionally three more criteria $f_2(\widetilde{a}) := \frac{1}{2}(m+n)$, $f_3(\widetilde{a}) := (n-m+\alpha+\beta)$ and $f_4(\widetilde{a}) := \alpha$. By following the steps of the lexicographic method, the solution is $\widetilde{x}_1 = \widetilde{x}_3 = (8, 12, 1, 1)_{LR}$, $\widetilde{x}_2 = (39, 39, 0, 4)_{LR}$ and $\widetilde{x}_4 = (25.25, 25.25, 1, 0)_{LR}$ with optimal fuzzy value $\widetilde{v}_{\text{Lex}} = (260.25, 288.25, 12, 15)_{LR}$. According to this solution, the special mix should include about 10 units of corn, about 39 units of lime, about 10 units of alfalfa, and about 25.25 units of soy. Total cost is about \$274.25. The total cost estimation is the same as the one obtained by using Ebrahimnejad's method because both fuzzy costs have the same modal value; however, the total fuzzy cost obtained by using the lexicographic method has less fuzziness and is therefore preferred. Figure 2 depicts the graphs of $\widetilde{v}_{\text{Lex}}$ (solid line) and $\widetilde{v}_{\text{E}}$ (dashed line). The reader may agree with us that intuitively $\widetilde{v}_{\text{Lex}} \prec \widetilde{v}_{\text{E}}$; hence, a decision-maker should prefer $\widetilde{v}_{\text{Lex}}$ rather than $\widetilde{v}_{\text{E}}$. By using Definition 10 to compare $\widetilde{v}_{\text{Lex}}$ and $\widetilde{v}_{\text{E}}$, we obtain $(f_1(\widetilde{v}_{\text{Lex}}), f_2(\widetilde{v}_{\text{Lex}}), f_3(\widetilde{v}_{\text{Lex}}), f_4(\widetilde{v}_{\text{Lex}})) = (275, 274.25, 55, 12)$ and $(f_1(\widetilde{v}_{\text{E}}), f_2(\widetilde{v}_{\text{E}}), f_3(\widetilde{v}_{\text{E}}), f_4(\widetilde{v}_{\text{E}})) = (275, 274.25, 235, 49.75)$; since $(275, 274.25, 55, 12) \prec_{lex} (275, 274.25, 235, 49.75)$, we conclude that $\widetilde{v}_{\text{Lex}} \prec \widetilde{v}_{\text{E}}$. It can also be easily checked that the obtained solution is feasible according to the given lexicographic criterion. Lastly, according to the total order properties of $\preceq$, any alternative fuzzy solution must have the same fuzzy objective value $\widetilde{v}_{\text{Lex}}$.

6.2. Application in a time–cost trade-off problem

Time–cost trade-off problems are a special type of project scheduling problems, in which the project activity durations are modified to achieve a balance between the project cost and its completion time. The following assumptions are made regarding the problem parameters and decision variables (Göçken & Baykasoğlu 2016):

1. Events are presented on the nodes and activities on the arcs.
2. Crashing cost value of activity (i, j), denoted by c_{ij}, is assumed crisp.
3. Normal and crash durations of activity (i,j), denoted by $\widetilde{T}^n_{ij}$ and $\widetilde{T}^c_{ij}$, respectively, are uncertain and defined as LR fuzzy numbers with $L(x) = R(x) = 1 - x$.
4. Problem variables, occurrence time of event i, and duration of activity (i,j) denoted by $\widetilde{t}_i$ and $\widetilde{t}_{ij}$, respectively, are defined as LR fuzzy numbers with $L(x) = R(x) = 1 - x$.

Figure 2
Membership functions of optimal fuzzy value $\widetilde{v}_{Lex}$ (solid line) and $\widetilde{v}_E$ (dashed line)



In the time–cost trade-off problem with n nodes (events), two objectives are considered: (1) minimization of crashing cost $\sum_{i=1}^{n-1} \sum_{j \in S_i} c_{ij}\left(\widetilde{T}^n_{ij} \ominus \widetilde{t}_{ij}\right)$, where S_i is the set of activities that begin with event i; (2) minimization of the makespan of project execution $\widetilde{t}_n$. Since $\widetilde{T}^n_{ij}$ is a constant LR fuzzy number, we choose to maximize $\sum_{i=1}^{n-1} \sum_{j \in S_i} c_{ij}\widetilde{t}_{ij}$. The problem is then formulated as the bi-objective FVLP (BOFVLP) problem (12):

$$\begin{aligned} &\max \sum_{i=1}^{n-1} \sum_{j \in S_i} c_{ij}\widetilde{t}_{ij} \\ &\min \widetilde{t}_n \\ &\text{s.t. } \widetilde{t}_i \oplus \widetilde{t}_{ij} \preceq \widetilde{t}_j, \text{ for } i = 1, \ldots, n-1;\ j \in S_i \\ &\qquad \widetilde{T}^c_{ij} \preceq \widetilde{t}_{ij} \preceq \widetilde{T}^n_{ij}, \text{ for} i = 1, \ldots, n-1;\ j \in S_i \end{aligned} \tag{12}$$

In order to solve BOFVLP problem (12) with the lexicographic method, we use the popular ϵ-constraint scalarization approach. In the ϵ-constraint scalarization approach, the objective function with the highest priority is optimized, while the other objective functions are bounded from above (in case of minimization) or below (in case of maximization) by means of additional constraints.

If we assume that a decision-maker wishes to complete the project in at most $\widetilde{\epsilon}$ days, then BOFVLP problem (12) is transformed into single-objective FVLP problem (13) and solved by using the lexicographic method with different fuzzy values of parameter $\widetilde{\epsilon}$. To illustrate this approach, let us consider the project network depicted in Figure 3. Table 3 shows the crash duration, normal duration, and crashing cost of each activity. Tables 4 and 5 show the solution to FVLP problem (13) with different values of $\widetilde{\epsilon}$. A decision-maker can now choose the most preferable solution

Figure 3
Network of problem (12)

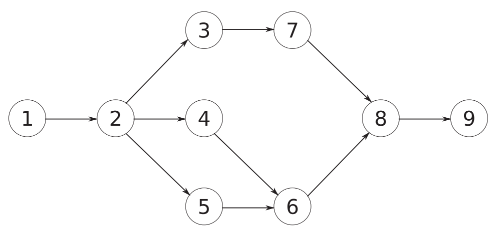


Table 3
Data of project network depicted in Figure 3

Activity index	Crash duration	Normal duration	Crashing cost
(1, 2)	$(1, 0, 1)_{LR}$	$(8, 3, 1)_{LR}$	15
(2, 3)	$(2, 0, 0)_{LR}$	$(6, 1, 0)_{LR}$	25
(2, 4)	$(2, 1, 1)_{LR}$	$(7, 2, 2)_{LR}$	23
(2, 5)	$(4, 1, 1)_{LR}$	$(10, 2, 2)_{LR}$	35
(5, 6)	$(4, 1, 0)_{LR}$	$(7, 0, 2)_{LR}$	18
(4, 6)	$(4, 1, 1)_{LR}$	$(7, 2, 0)_{LR}$	32
(3, 7)	$(3, 1, 1)_{LR}$	$(8, 2, 2)_{LR}$	20
(7, 8)	$(3, 1, 1)_{LR}$	$(7, 0, 0)_{LR}$	17
(6, 8)	$(4, 2, 2)_{LR}$	$(7, 1, 1)_{LR}$	30
(8, 9)	$(4, 1, 2)_{LR}$	$(10, 1, 3)_{LR}$	27

Table 4
Activity durations obtained with the lexicographic method and Farhadinia's criterion

	Values of $\widetilde{\epsilon}$				
Activity index	$(42, 10, 10)_{LR}$	$(39, 8, 8)_{LR}$	$(36, 8, 10)_{LR}$	$(31, 4, 4)_{LR}$	$(18, 7, 7)_{LR}$
(1, 2)	$(8, 6, 2)_{LR}$	$(5, 4, 0)_{LR}$	$(2, 2, 0)_{LR}$	$(1, 0, 1)_{LR}$	$(1, 0, 1)_{LR}$
(2, 3)	$(6, 1, 0)_{LR}$	$(6, 1, 0)_{LR}$	$(6, 1, 0)_{LR}$	$(6, 1, 0)_{LR}$	$(6, 1, 0)_{LR}$
(2, 4)	$(7, 2, 2)_{LR}$	$(7, 2, 2)_{LR}$	$(7, 2, 2)_{LR}$	$(7, 2, 2)_{LR}$	$(2, 1, 1)_{LR}$
(2, 5)	$(10, 2, 2)_{LR}$	$(10, 2, 2)_{LR}$	$(10, 2, 2)_{LR}$	$(10, 2, 2)_{LR}$	$(5, 3, 2)_{LR}$
(5, 6)	$(7, 0, 2)_{LR}$	$(7, 0, 2)_{LR}$	$(7, 2, 4)_{LR}$	$(4, 0, 0)_{LR}$	$(4, 1, 0)_{LR}$
(4, 6)	$(7, 2, 0)_{LR}$	$(7, 2, 0)_{LR}$	$(7, 2, 0)_{LR}$	$(7, 2, 0)_{LR}$	$(7, 3, 1)_{LR}$
(3, 7)	$(8, 2, 2)_{LR}$	$(8, 2, 2)_{LR}$	$(8, 2, 2)_{LR}$	$(8, 2, 2)_{LR}$	$(4, 4, 3)_{LR}$
(7, 8)	$(7, 0, 0)_{LR}$	$(7, 0, 0)_{LR}$	$(7, 0, 0)_{LR}$	$(7, 0, 0)_{LR}$	$(3, 1, 1)_{LR}$
(6, 8)	$(7, 1, 1)_{LR}$	$(7, 1, 1)_{LR}$	$(7, 1, 1)_{LR}$	$(7, 1, 1)_{LR}$	$(4, 2, 2)_{LR}$
(8, 9)	$(10, 1, 3)_{LR}$	$(10, 1, 3)_{LR}$	$(10, 1, 3)_{LR}$	$(9, 1, 0)_{LR}$	$(4, 1, 2)_{LR}$
$\sum_{i=1}^{n-1}\sum_{j\in S_i} c_{ij}\widetilde{t}_{ij}$	$(1890, 392, 333)_{LR}$	$(1845, 362, 303)_{LR}$	$(1800, 368, 339)_{LR}$	$(1704, 302, 201)_{LR}$	$(1041, 451, 331)_{LR}$
$\widetilde{t}_9$	$(42, 10, 10)_{LR}$	$(39, 8, 8)_{LR}$	$(36, 8, 10)_{LR}$	$(31, 4, 4)_{LR}$	$(18, 7, 7)_{LR}$

Table 5
Event times obtained with the lexicographic method and Farhadinia's criterion

	Values of $\widetilde{\epsilon}$				
Node index	$(42, 10, 10)_{LR}$	$(39, 8, 8)_{LR}$	$(36, 8, 10)_{LR}$	$(31, 4, 4)_{LR}$	$(18, 7, 7)_{LR}$
1	$(0, 0, 0)_{LR}$	$(0, 0, 0)_{LR}$	$(0, 0, 0)_{LR}$	$(0, 0, 0)_{LR}$	$(0, 0, 0)_{LR}$
2	$(8, 6, 2)_{LR}$	$(5, 4, 0)_{LR}$	$(2, 2, 0)_{LR}$	$(1, 0, 1)_{LR}$	$(1, 0, 1)_{LR}$
3	$(14, 7, 5)_{LR}$	$(11, 5, 0)_{LR}$	$(8, 0, 0)_{LR}$	$(7, 1, 2)_{LR}$	$(7, 1, 1)_{LR}$
4	$(15, 0, 12)_{LR}$	$(12, 6, 2)_{LR}$	$(9, 0, 0)_{LR}$	$(8, 0, 3)_{LR}$	$(3, 1, 2)_{LR}$
5	$(18, 8, 4)_{LR}$	$(15, 6, 2)_{LR}$	$(12, 4, 2)_{LR}$	$(11, 2, 3)_{LR}$	$(6, 3, 3)_{LR}$
6	$(25, 8, 6)_{LR}$	$(22, 6, 4)_{LR}$	$(19, 6, 6)_{LR}$	$(15, 2, 3)_{LR}$	$(10, 4, 3)_{LR}$
7	$(22, 9, 7)_{LR}$	$(19, 0, 0)_{LR}$	$(16, 2, 12)_{LR}$	$(15, 3, 4)_{LR}$	$(11, 5, 4)_{LR}$
8	$(32, 9, 7)_{LR}$	$(29, 7, 5)_{LR}$	$(26, 7, 7)_{LR}$	$(22, 3, 4)_{LR}$	$(14, 6, 5)_{LR}$
9	$(42, 10, 10)_{LR}$	$(39, 8, 8)_{LR}$	$(36, 8, 10)_{LR}$	$(31, 4, 4)_{LR}$	$(18, 7, 7)_{LR}$

among the available ones or provide more values for $\widetilde{\epsilon}$ and analyze the corresponding solutions.

$$\begin{aligned} \max\ & \sum_{i=1}^{n-1}\sum_{j\in S_i} c_{ij}\widetilde{t}_{ij} \\ \text{s.t. } & \widetilde{t}_n \preceq \widetilde{\epsilon} \\ & \widetilde{t}_i \oplus \widetilde{t}_{ij} \preceq \widetilde{t}_j, \ \text{for } i = 1,\dots,n-1;\ j \in S_i \\ & \widetilde{T}^c_{ij} \preceq \widetilde{t}_{ij} \preceq \widetilde{T}^n_{ij}, \ \text{for } i = 1,\dots,n-1;\ j \in S_i \end{aligned} \tag{13}$$

7. Conclusions and Future Research Lines

In this paper, we used the lexicographic method proposed by Pérez-Cañedo and Concepción-Morales (2019) to solve FVLP problems and FNLP problems. All fuzzy parameters and fuzzy decision variables of the FLP problems were considered LR fuzzy numbers of the same type. By using the lexicographic method, the FLP problems were transformed into equivalent MILLP problems. Numerical examples, including applications in diet and time–cost trade-off problems, illustrated the lexicographic method. By following the steps of the lexicographic method, we found optimal solutions with unique fuzzy objective values. However, the lexicographic method increases the complexity of the solution process since solving an FLP problem requires to solve four mixed-integer linear programming problems. Future research may therefore be devoted to assessing the lexicographic method in high-dimensional real-world FLP problems and using metaheuristic algorithms as substitutes of exact approaches. In the future, emphasis could be done on the multiobjective case, since by nature those models provide more realistic and richer solutions to real-world problems. In this case, it seems promising the application of the Grossone methodology for lexicographic optimization described in Lai et al. (2020); Lai et al. (2021). To extend the lexicographic method for solving FLP problems with different types of fuzzy numbers is also an interesting research line to address in the future. From an application point of view, given the vagueness of the available data around all topics covered by the sustainable development goals (UN, 2015), FLP models, and solution methods are ideal for them (e.g., the case of fish harvesting (FAO, 1995) or sustainable power generation (Khan et al., 2021)). This is a topic that we will address in the immediate future.

Acknowledgment

The authors are grateful to the editor and the anonymous reviewers for their constructive comments and suggestions. Research supported by projects PID2020-112754GB-I0, MCIN/AEI, and B-TIC-640-UGR20 FEDER/Junta de Andalucía.

Conflicts of Interest

The authors declare that they have no conflicts of interest to this work.

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How to Cite: Pérez-Cañedo, B. & Verdegay, J. L. (2023). On the Application of a Lexicographic Method to Fuzzy Linear Programming Problems. *Journal of Computational and Cognitive Engineering* 2(1), 47–56, https://doi.org/10.47852/bonviewJCCE20235142025