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RESEARCH ARTICLE

Discrete Fix up Limit Model of a Device Unit

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Abstract: Sometimes, a failed device cannot be fixed up completely within the precise limit time due to some reasons. This paper addresses the problem of completing a fix up action of a failed device unit within a projected discrete precise limit time. A discrete fix up limit model is constructed for the device based on a fix up limit policy. A numerical example is provided for simple illustration of the fix up limit model constructed, so as to investigate the characteristics of the constructed model.

Keywords: discrete time, limit policy, reliability, repair policy, Weibull distribution

1. Introduction

In the reliability and maintenance field, all systems deteriorate and subsequently fail with time and usage, and after repair, the systems will be as good as new. These deficiencies can affect the production of items, which can lead to scarcity of products. Due to unavailability of a repair man needed to fix up a failed device, the failed device sometimes cannot be fixed up completely at the precise specified limit time. Bai and Pham (2005) presented a repair-limit risk-free warranty policy and provided the first and second moments of the warranty cost per unit sold through censored quasi-renewal processes. From the outcome of this study, the proposed repair-limit risk-free warranty may be a good candidate for marketing purposes, since it provides extra compensation to consumers suffering from low-quality products with a relatively low cost. Kapur et al. (2007) proposed some aliment cost function of a unit subjected to two types of breakdown under some proposed conditions. Aven and Castro (2008) constructed a minimal replacement policy with a discounting rate for a system subjected to two types of failures, which determined the discounted optimal replacement time for the system. Chang et al. (2010) presented a replacement model with minimal repair based on a cumulative repair-cost limit policy for a system subjected to two types of failures, such that the information of all repair costs is used to decide whether the system is repaired or replaced. Jain and Gupta (2013) presented an optimal replacement policy for a fixable system with multiple vacations and imperfect coverage. Beichelt (2014) developed a fix up charge function for a single unit such that the fixing or replacement is subjected to a single repairman. Zaharadeen and Bashir (2014) developed a replacement model for a unit exposed to two different forms of failures. Chen and Chang (2015) presented a charge function of a system involving two levels of alarms, such that the system undergoes a precautionary care at a projected time T or immediately after the nth level-I alarm, and a restorative care at the projected time T, when the entire damage exceeds a catastrophic limit or immediately after any level-II alarm, whichever comes first. Coria et al. (2015) proposed an analytical optimization method for preventive maintenance replacement cost rate. Briš et al. (2017) presented a latest mathematical program for system's aliment plan, which depends on a given reliability measures. Lai et al. (2017) studied a bivariate (n, k) replacement policy with a cumulative repair cost limit for a two-unit system, which is subjected to shock damage, where they constructed a long-term expected cost per unit time that incorporates costs related to replacement and repair. To examine the properties of an industrial plant, Niwas and Garg (2018) built a mathematical model of a system based on the Markov process, and they further derived various reliability parameters. Safaei et al. (2018) explored the optimal precautionary aliment actions of a system based on some stated terms. Sheu et al. (2019) proposed precautionary replacement charge functions for a system that is prone to a particular distress, in which the system is either replaced with a latest one or undergoes fixed up, when a distress occurs. Sudheesd et al. (2019) looked at the discontinuous replacement charge function before looking at the features of a system's mean time to failure. Wang et al. (2019) obtained the charge function $C(T, N)$ for a fixable system with a single repairman. The challenge of adopting the best aliment strategy among three charge-effective aliment planning approaches was investigated by Rebaiaia and Ait-kadi (2020). Mirjalili and Kazempoor (2020) investigated some three replacement policies, including cold standby and minimal repair policies for a system consisting of independent components with an increasing failure rate functions. Sanoubar et al. (2020) considered time replacement strategy for a system that is replaced at breakdown or at a specified replacement time, whichever comes first. Waziri

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(2021) offered a discontinuous projected replacement charge function for a unit subjected to three forms of breakdown. Some authors made some studies on some reliability measures, so as to look for ways to make improvement on the reliability of some multi-unit systems. For example, Gheisary and Goli (2018) investigated an efficient method to compute the exact reliability of a multi-state system consisting of some n components by using the distribution of bivariate order statistics. In trying to improve the reliability measures of a solar system, Maihula et al. (2021) studied some reliability measures such as reliability, mean time to failure availability, and profit function for a solar serial system with some subsystems. Danjuma et al. (2022) recently studied some reliability measures of a system consisting of four subsystems, where some of the subsystems are having two units in cold standby.

Nakagawa (2005) presents the continuous case of repair limit policy for a unit, so as to consider the duration of repair of a failed unit, because a longer repair time of a failed unit or system is very dangerous to industries and power plants. To the best ability of the authors of this paper, they did not come across any existing paper that addressed such a problem of completing a fix up at the precise fix up limit time T. This reason influenced the authors of this paper to come up with discrete fix up limit model for a single device unit, so as to provide the possibility or chance of getting the optimal discrete fix up limit time. Therefore, the purpose of this study is to provide some proposed discrete fix up limit time model for a single device unit. This study provides a discrete fix up limit model for a device that is exposed to a fixable failure. The subsequent sections of this paper are arranged in this order: Section 2 presents the methodology. Section 3 presents the proposed model. Section 4 presents the numerical example. Finally, Section 5 presents the conclusion.

2. Methodology

Some reliability measures, such as fix up distribution function and fix up rate, are used in coming up with the discrete fix up limit model for a device unit subjected to fixable failure.

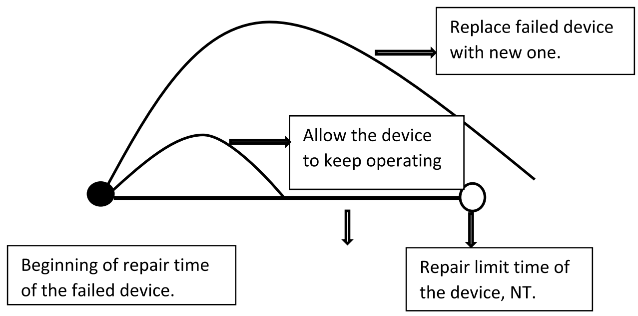
2.1. Notations

1. $C(N)$: Expected charge rate.
2. N^{*}: Optimal discrete fix up limit time of the device.
3. μ: Mean failure time of the device.
4. $r(t)$: Fix up rate of the device.
5. $H(t)$: Fix up distribution function of the failed device.
6. C_r: Charge of changing of the failed device when the fix up is not over within the specified discrete time NT, for a fixed T and $N = 1, 2, 3, \ldots$
7. $C_m(t)$: Charge of fix up during $(0, NT]$, for a fixed T and $N = 1, 2, 3, \ldots$

2.2. Description of the device

Consider a device exposed to a failure, such that the failure is rectified by fix up. When the device fails, its fix up is started immediately, and when the fix up is not over within the discrete limit time NT $(N = 1, 2, 3, \ldots)$ for a fixed T, the failed device is replaced with a new one. Let C_r be the charge of replacing the unfixable device that includes all charges caused by failure and replacement. Let $C_r(NT)$ be the expected fix up charge, which also includes all charges incurred due to fix up and downtime. Sometimes, issues such as insufficient resources or repairman(s) needed to complete fixing up of the failed device within a limit time, the failed device cannot be fix up completely within the exact fix up limit time; therefore, a discrete fix up limit time NT $(N = 1, 2, 3, \ldots)$ can be considered. Figure 1 shows the process of the repair limit time of the device.

Figure 1
Process of repair limit time of the device



3. Proposed Model

The proposed discrete fix up limit model for a single device unit will be presented in this section based on the following estimations below:

1. The device is subjected to a failure, which is rectified by fix up.
2. The fix up rate follows non-homogeneous Poisson process, such that fix up rate is an increasing function.
3. If the fix up of the failed device is not over within the specified discrete time $NT(N = 1, 2, 3, \ldots)$ for a fixed T, it is changed with a latest one.
4. The charge of fix up is proportional to time.

The probability that the device will be fix up within the discrete time $NT(N = 1, 2, 3, \ldots)$ for a fixed T in one cycle is

$$H(NT) = e^{-\int_0^{NT} r(t)dt}. \tag{1}$$

The probability that the device will not be fixed up within the discrete time NT $(N = 1, 2, 3, \ldots)$ for a fixed T in one cycle is

$$\overline{H}(NT) = 1 - H(NT). \tag{2}$$

The charge of changing of failed device that is not fixed up within the discrete time NT $(N = 1, 2, 3, \ldots)$ for a fixed T in one cycle is

$$\text{Charge of changing} = (C_r + C_m(NT))\overline{H}(NT). \tag{3}$$

The charge of fixing up of the failed device within the discrete time NT $(N = 1, 2, 3, \ldots)$ for a fixed T in one cycle is

$$\text{Charge of minimal repair} = \int_0^{NT} C_m(t)dH(t). \tag{4}$$

Using equations (3) and (4), the expected charge within the discrete time NT $(N = 1, 2, 3, \ldots)$ for a fixed T in one cycle is

$$(C_r + C_m(NT))\overline{H}(NT) + \int_0^{NT} C_m(t)dH(t) = C_r\overline{H}(NT) + \int_0^{NT} \overline{H}(t)dC_m(t). \tag{5}$$

The mean failure time of the device within the discrete time NT $(N = 1, 2, 3, \ldots)$ for a fixed T in one cycle is

$$Mean\ time = \mu + \int_0^{NT} \overline{H}(t)dt. \tag{6}$$

Using equations (5) and (6), the device's expected charge rate within the discrete time NT $(N = 1, 2, 3, \ldots)$ for a fixed T in one cycle is

$$C(N) = \frac{C_r\overline{H}(NT) + \int_0^{NT} \overline{H}(t)dC_m(t)}{\mu + \int_0^{NT} \overline{H}(t)dt}. \tag{7}$$

Noting following :

1. Observed that, as N approaches zero, we have

$$C(0) = \frac{C_r}{\mu}. \tag{8}$$

2. Observed that, as N approaches infinity, we have

$$C(\infty) = \frac{\int_0^{NT} \overline{H}(t)dC_m(t)}{\mu + \int_0^{\infty} \overline{H}(t)dt}. \tag{9}$$

4. Numerical Example

Let the fix-up rate of the device obeys Weibull distribution

$$r(t) = \lambda \propto t^{\propto -1}, \text{ for } \propto > 1 \text{ and } t \geq 0. \quad 10$$

Let the set of the parameters, charge of fix up and change be used in this specific example:

1. Following that the fix up is an increasing function from the assumption, let $\propto = 3$ and $\lambda = 0.02$.
2. From the assumption, the charge of fix up depends on time, let $C_r = 20$, $\mu = 2$ and $C_m = 2t^2$.

Now, by putting the parameters in equation (10), the fix up rate is

$$r(t) = 0.06t^2. \tag{11}$$

Table 1 is obtained by presenting the charges of change/fix up ($C_r = 20$, $\mu = 2$ and $C_3 = 2t^2$) and rate of the failure (equation (11)) in equation (7), so as to determine the device's optimal discrete projected fix up limit time. The value of T in equation (7) is fixed, while N is varied. So for the computation of the optimal fixed up limit time of the device, we will compute the optimal fixed up limit time N^*; for sensitivity analysis, we obtained N^* with different values of T. Below are the results obtained as follows:

1. Table 1 is obtained by computing the values of $C(N)$ by taking the index of T to be 1, 2, 3, 4, 5.
2. Figure 2 is obtained by sketching $C(N)$ against N as $T = 1$, so as see the behavior of $C(N)$.
3. Figure 3 is obtained by sketching $C(N)$ against N as $T = 2$, so as see the behavior of $C(N)$.
4. Figure 4 is obtained by sketching $C(N)$ against N as $T = 3$, so as see the behavior of $C(N)$.

Table 1
Values of $C(N)$ versus NT with different values of T

N	$C(N)$ for $T = 1$	$C(N)$ for $T = 2$	$C(N)$ for $T = 3$	$C(N)$ for $T = 4$	$C(N)$ for $T = 5$
1	0.00	−8.38	−15.62	−21.99	−27.49
2	−8.38	−21.99	−31.82	−34.58	−24.32
3	−15.62	−31.82	−31.46	4.55	91.21
4	−21.99	−34.58	4.55	133.46	384.77
5	−27.49	−24.32	91.21	384.77	934.99
6	−31.82	4.55	241.35	800.28	1828.00
7	−34.43	55.97	471.53	1424.14	3150.45
8	−34.58	133.46	800.28	2300.56	4990.00
9	−31.46	241.35	1246.23	3474.18	7435.00
10	−24.32	384.77	1828.00	4990.00	10574.29
11	−12.49	569.24	2564.35	6893.31	14497.00
12	4.55	800.28	3474.18	9229.56	19292.50
13	27.23	1083.41	4576.49	12044.36	25050.29
14	55.97	1424.14	5890.38	15383.41	31860.00
15	91.21	1828.00	7435.00	19292.50	39811.32

Figure 2
The plot of $C(N)$ versus N with T taking the index 1

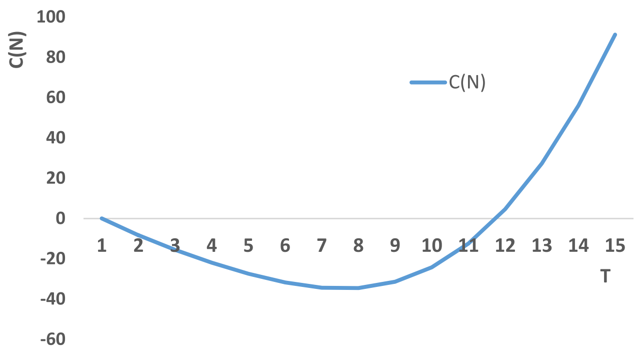


Figure 3
The plot of $C(N)$ versus N with T taking the index 2

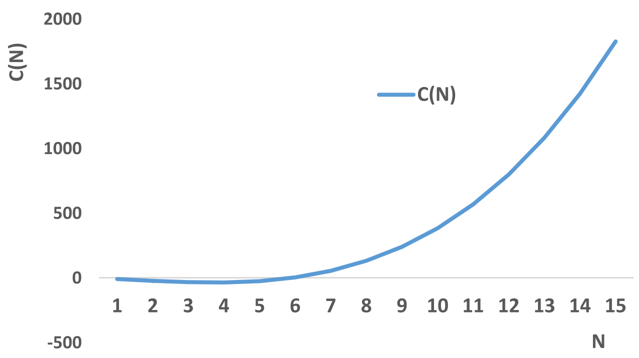


Figure 4
The plot of $C(N)$ versus N with T taking the index 3

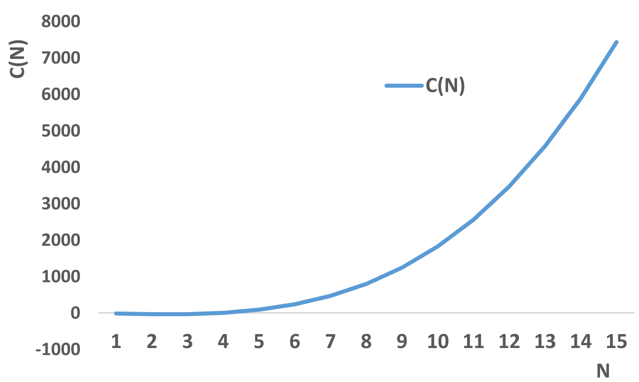


Figure 5
The plot of *C*(*N*) versus *N* with *T* taking the index 4

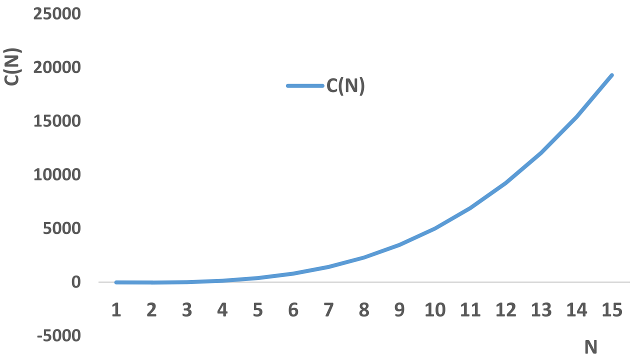


Figure 6
The plot of *C*(*N*) versus *N* with *T* taking the index 5

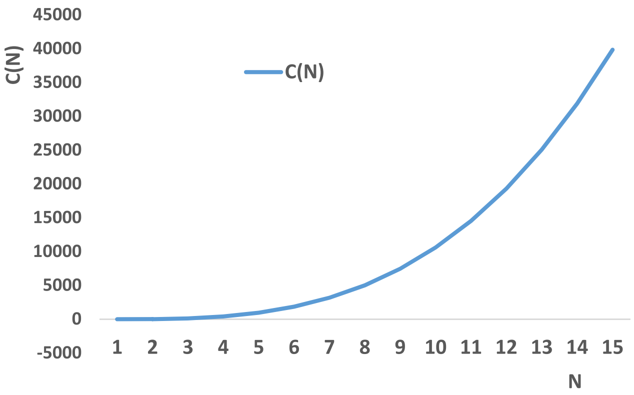
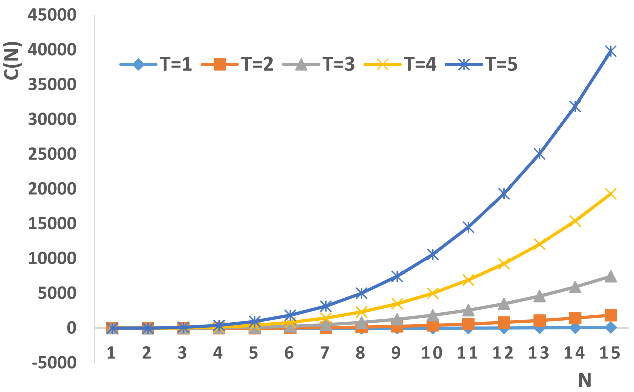


Figure 7
The plot analyzing *C*(*N*) with the different indices of *T*



5. Figure 5 is obtained by sketching $C(N)$ against N as $T = 4$, so as see the behavior of *C*(*N*).
6. Figure 6 is obtained by sketching $C(N)$ against N as $T = 5$, so as see the behavior of *C*(*N*).
7. Figure 7 is obtained by sketching $C(N)$ against N as T is 1, 2, 3, 4, 5, so as see the behavior of *C*(*N*) as T is 1, 2, 3, 4, 5.

From Table 1 and Figures 1, 2, 3, 4, 5, 6 and 7, we have the following observations :

1. Regarding Table 1, the optimal discrete fix up limit time is 8, when $T = 1$, that is, $N^* = 8$, with $C(N^* = 8) = -34.58$, when $T = 1$. See Figure 2, for the sketch of $C(N)$ versus N as $T = 1$.
2. Regarding Table 1, the optimal discrete fix up limit time is 4, when $T = 2$, that is, $N^* = 4$, with $C(N^* = 4) = -34.58$, when $T = 2$. See Figure 3, for the sketch of $C(N)$ versus N as $T = 2$.
3. Regarding Table 1, the optimal discrete fix up limit time is 2, when $T = 3$, that is, $N^* = 2$, with $C(N^* = 2) = -31.82$, when $T = 3$. See Figure 4, for the sketch of $C(N)$ versus N as $T = 3$.
4. Regarding Table 1, the optimal discrete fix up limit time is 2, when $T = 4$, that is, $N^* = 2$, with $C(N^* = 2) = -34.58$, when $T = 5$. See Figure 5, for the sketch of $C(N)$ versus N as $T = 4$.
5. Regarding Table 1, the optimal discrete fix up limit time is 1, when $T = 5$, that is, $N^* = 1$, with $C(N^* = 1) = -27.49$, when $T = 5$. See Figure 6, for the sketch of $C(N)$ versus N as $T = 5$.
6. Regarding Figure 7 : $(C(N),\ T = 1) < (C(N),\ T = 2) < (C(N),\ T = 3) < (C(N),\ T = 4) < (C(N),\ T = 5)$.
7. Regarding Table 1, as the value of T increases, the optimal discrete fix up limit time decreases.

5. Conclusion

In this paper, we constructed a discrete fix up limit model of a device unit subjected to a failure, such that the failure is rectified by fix up, so as to address the problem of completing a fix up action of a failed device unit within a projected discrete precise limit time. A numerical example was provided to determine the optimal discrete fix up limit time (N^*) of the device unit. From the results obtained, one can see that the index T really played a vital role in determining the optimal discrete fix up limit time (N^*) of the device unit. For future extension of this paper, one can involve discounting factor in the model, and one can also construct for multi-component system as a future research.

Conflicts of Interest

The authors declare that they have no conflicts of interest to this work.

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