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Development of *q*-Rung Orthopair Trapezoidal Fuzzy Einstein Aggregation Operators and Their Application in MCGDM Problems

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Abstract: Compared to previous extensions, the q-rung orthopair fuzzy sets are superior to intuitionistic ones and Pythagorean ones because they allow decision-makers to use a more extensive domain to present judgment arguments. The purpose of this study is to explore the multicriteria group decision-making (MCGDM) problem with the q-rung orthopair trapezoidal fuzzy (q-ROTrF) context by employing Einstein t-conorms and t-norms. Firstly, some arithmetical operations for q-ROTrF numbers, such as Einstein-based sum, product, scalar multiplication, and exponentiation, are introduced based on Einstein t-conorms and t-norms. Then, Einstein operations-based averaging and geometric aggregation operators (AOs), viz., q-ROTrF Einstein weighted averaging and weighted geometric operators, are developed. Further, some prominent characteristics of the suggested operators are investigated. Then, based on defined AOs, a MCGDM model with q-ROTrF numbers is developed. In accordance with the proposed operators and the developed model, two numerical examples are illustrated. The impacts of the rung parameter on decision results are also analyzed in detail to reflect the suitability and supremacy of the developed approach.

Keywords: Multicriteria group decision-making, q-rung orthopair trapezoidal fuzzy number, Einstein operations, weighted averaging and weighted geometric aggregating operators

1. Introduction

Multicriteria group decision-making (MCGDM) is a technique for choosing the most desirable alternatives from a collection of finite alternatives based on a group of decision-makers' (DMs) aggregate assessment values. However, because it incorporates the complexity of human cognitive thinking, the MCGDM process tends to be vague and imprecise, making it difficult for DMs to provide precise evaluations or preference information during the evaluation process. To cope with such issues, Atanassov's intuitionistic fuzzy set (IFS) (Atanassov, 1986) might be considered an appealing method for dealing with data fuzziness and inaccuracy. IFS is characterized by membership and nonmembership degrees in which their sum is not beyond one. Despite numerous IFS's advantages, there may be situations in which the sum of membership and nonmembership degrees is greater than 1. Yager (2013a) and Yager (2013b) introduced the Pythagorean fuzzy set (PFS) to address these issues, ensuring that the squared sum of its degree of membership and degree of nonmembership is ≤1. As a result, PFS have a more extensive region to model real-life situations than IFSs. Wang and Garg (2021) introduced Archimedean t-conorm and t-norm-based Pythagorean fuzzy interactive weighted averaging (WA) and weighted geometric (WG) operators as novel interaction Pythagorean operators. After the inception of PFS, it has been broadly studied and employed by scholars (Fei & Deng, 2020; Zeng et al., 2016; Sarkar & Biswas, 2019).

However, in real-world situations, the square sum of the degrees of membership and degree of nonmembership is more than 1. In such situations, PFS and IFS are inadequate for describing DMs' evaluation information. To address this flaw, Yager (2016) redefined the notion of q-rung orthopair fuzzy (q-ROF) set (q-ROFS) as a generalization of PFS and IFS, wherein the sum of q^{th} power of membership and nonmembership degrees is less than or equal to unity. It is important to keep in mind that the space of admissible orthopairs expands as the rung q increases, making q-ROFs better suited to unpredictable environments. Based on q-ROF environment, Peng et al. (2021) defined entropy measure, distance measure, and similarity measure and solved decision-making problems utilizing those measures. Under q-ROF context, Riaz et al. (2021a) established numerous WA and WG aggregation operators (AOs),

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viz., q-ROF fuzzy interaction-ordered and hybrid averaging AOs as well as geometric versions of these AOs. Zeng et al. (2021) defined induced weighted logarithmic-based two distance measures of q-ROFSs. Recently, Alkan and Kahraman (2021) developed two different TOPSIS methods under the q-ROF context and applied to determine the most appropriate strategy. Ever since q-ROFSs' appearance, many studies (Liu et al., 2018; Liu & Wang, 2020; Sarkar & Biswas, 2021) have been conducted on decision-making methods under q-ROF environment.

The use of trapezoidal fuzzy numbers (TrFNs) (Abbasbandy & Hajjari, 2009) has also become increasingly widespread as a starting point for developing fuzzy sets. TrFN is the best fit for conveying the uncertainty of the alternative. If the alternative's uncertainty is expressed as an interval, TrFN is the best choice for representing it. Gupta et al. (2021) presented the notion of q-rung orthopair TrFNs (q-ROTrFNs), which was inspired by the ideas of q-ROFS (Yager, 2016) and TrFN (Wang & Zhang, 2009). For q-ROTrFNs, Wan et al. (2021a) established a novel ranking algorithm and Hamming distance measure. They also recommended using q-ROTrFNs for developing a new TODIM group decision-making approach.

1.1 Motivations

It is worth noting that operational regulations play a crucial role in data integration. Gupta et al. (2021) proposed the basic operations laws and defined WA and WG AOs for q-ROTrFNs and moreover developed a TOPSIS approach for solving the MAGDM problem. As an alternative to algebraic sum and product, Einstein-based t-norm and t-conorm provide the best approximation for sum and product of q-ROTrFNs. The AOs are most typically employed to aggregate each individual preference into the overall preference information and generate a collective preference value for each alternative. There appear to be limited studies into aggregation approaches for aggregating a collection of q-ROTrF data in the literature. From the above motivation, the aim of this research is to design some information AOs using Einstein operations on q-ROTrFNs.

1.2 Contributions

In the present paper, we will research some Einstein-based operational laws of the q-ROTrNs. Moreover, as the applications, we give two novel AOs. As can be summarized from the motivations above, the contributions are shown in the following:

- Using Einstein t-conorm and t-norms, the current study prolonged the concept of aggregating distinct q-ROTrFNs. For this purpose, firstly Einstein operating laws for q-ROTrFNs have been devised.
- Using defined operational rules, a set of q-ROTrF Einstein WA (q-ROTrFEWA) and q-ROTrF Einstein WG (q-ROTrFEWG) operators have been proposed for integrating q-ROTrF information. Some desirable properties of these developed operators are also investigated in detail.
- A novel MCGDM method based on the proposed operators has been described under q-ROTrF context.
- By comparing the proposed approach to the existing method, it is determined that the method proposed in this study has proven to be useful in q-ROTrFNs research.

The following is the outline of the paper: Section 2 briefly recalls fundamental conceptions related to q-ROFS, q-ROTrFN, and Einstein operations. Based on Einstein operations, some basic operational rules for q-ROTrFN are defined in Section 3. To aggregate q-ROTrFNs, Section 4 introduces some operators based on Einstein operations, viz., q-ROTrFEWA and q-ROTrFEWG operators. Further, some characteristics of these developed operators are also exhibited in this section. Section 5 illustrates a MCGDM approach utilizing the developed AOs. Utilizing the proposed approach, two numerical examples have been solved in Section 6, and comparative and sensitivity analyses are also presented here. Finally, in Section 7, an overall summary of the current study is depicted.

2. Preliminaries

Several basic principles that will be used throughout the article are briefly reviewed in this section. In order to better understand this paper, we will introduce some basic and useful concepts of q-ROFSs (Yager, 2016), q-ROTrFN (Gupta et al., 2021), and Einstein operations (Klement et al., 2004) in this section.

2.1 q-ROFS

The notion of q-ROFS is introduced by Yager (2016). In the following, some basic notions pertaining to q-ROF sets are presented from Yager (2016).

Definition 2.1. (Yager, 2016) On a universal set X, a q-ROFS, $\mathcal{P}$ is presented by:

$$\mathcal{P} = \{(x, \mu_{\mathcal{P}}(x), \nu_{\mathcal{P}}(x)) | x \in X\},$$

where the values of $\mu_{\mathcal{P}}$ and $\nu_{\mathcal{P}}$ that lie in the closed unit interval designate membership and nonmembership values, respectively, following the requirement that

$$((\mu_{\mathcal{P}}(x))^q + (\nu_{\mathcal{P}}(x))^q) \in [0,1], \text{ where rung parameter } q \geq 1.$$

For convenience, Yager (2016) named the pair $(\mu_{\mathcal{P}}(x), \nu_{\mathcal{P}}(x))$ as a q-ROF number (q-ROFN) and symbolized it by $\tilde{\wp} = (\mu, \nu)$.

2.2 q-ROTrFN

The concept of q-ROTrFN suggested by Gupta et al. (2021) as a generalization of intuitionistic TrFN and Pythagorean fuzzy number is as follows:

Definition 2.2. (Gupta et al., 2021) Suppose X be a fixed set. A q-ROFN $\tilde{R}$ is said to be q-ROTrFN explained on [0,1], denoted by $\tilde{R} = \langle([a, b, c, d]; \gamma_{\tilde{R}}), ([a_1, b, c, d_1]; \delta_{\tilde{R}})\rangle$ if

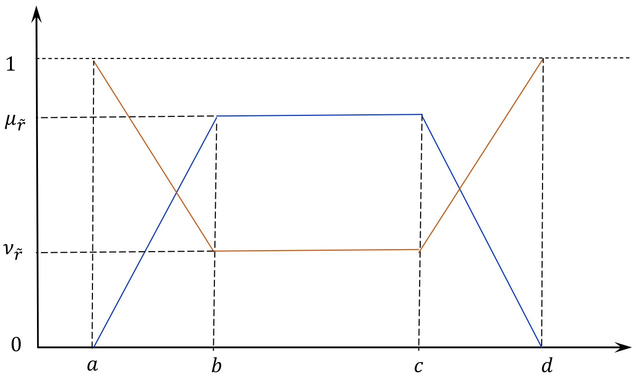
$$\gamma_{\tilde{R}}(x) = \begin{cases} \frac{(x-a)\mu_{\tilde{R}}}{(b-a)}, & a \leq x \leq b \\ \mu_{\tilde{R}}, & b \leq x \leq c \\ \frac{(d-x)\mu_{\tilde{R}}}{(d-c)}, & c \leq x \leq d \\ 0 & \mathit{Otherwise} \end{cases} \tag{1}$$

$$\delta_{\tilde{R}}(x) = \begin{cases} \frac{(b-x)+(x-a_1)\nu_{\tilde{R}}}{(b-a_1)}, & a_1 \leq x \leq b \\ \nu_{\tilde{R}}, & b \leq x \leq c \\ \frac{(x-c)+(d_1-x)\nu_{\tilde{R}}}{(d_1-c)}, & c \leq x \leq d_1 \\ 1 & \mathit{Otherwise} \end{cases} \tag{2}$$

where a, a_1, b, c, d, and d_1 are given numbers, and $\gamma_{\tilde{R}}(x) \in [0,1]$ denotes the degree of membership and $\delta_{\tilde{R}}(x) \in [0,1]$ denotes the degree of nonmembership with the condition that $0 \leq (\gamma_{\tilde{R}}(x))^q + (\delta_{\tilde{R}}(x))^q \leq 1$ where $x \in X$ and rung parameter $q \geq 1$.

For convenience, consider $a = a_1$ and $d = d_1$; therefore, the real numbers a, b, c, and d and $\mu_{\tilde{r}}$, $\nu_{\tilde{r}}$ define the q-ROTrFN $\tilde{r}$ which is denoted by $\langle[a, b, c, d]; \mu_{\tilde{r}}, \nu_{\tilde{r}}\rangle$. The membership function $\gamma_{\tilde{R}}(x)$ and nonmembership function $\delta_{\tilde{R}}(x)$ of a q-ROTrFN have a graphical representation, as shown in Figure 1, of a trapezoidal with $[a, d]$ being the base of the trapezoidal.

Figure 1
Graphical representation of *q*-ROTrFN $\tilde{r} = \langle [a, b, c, d]; \mu_{\tilde{r}}, \nu_{\tilde{r}} \rangle$
Note. Several fuzzy numbers can be generated from *q*-ROTrFN based on changing the rung parameter *q*



- When $q = 1$ is considered, q-ROTrFN reduces to an intuitionistic trapezoidal fuzzy number (Ye, 2011).
- For $q = 2$, q-ROTrFN reduces to the Pythagorean trapezoidal fuzzy number (Shakeel et al., 2018; Shakeel et al., 2019).
- If $q = 1$ and $b = c$ are considered, q-ROTrFN is converted to an intuitionistic triangular fuzzy number (Riaz et al., 2021a).
- The q-ROTrFN is converted to Pythagorean triangular fuzzy number (Zhang & Liu, 2010) for considering $q = 2$ and $b = c$.
- When $b = c$, q-ROTrFN changes in q-rung orthopair triangular fuzzy number (Fahmi & Aslam, 2021; Wan et al., 2021a).

There are many different t-conorms and t-norms families to choose from when modeling intersections and unions, and Einstein product and Einstein sum are good choices because they typically yield the same smooth approximation as algebraic product and algebraic sum, respectively.

2.3 Einstein operations

Klement et al. (2004) introduced one of generalized t-norm and t-conorm, which is known as Einstein t-norms and t-conorms and expressed as:

- Einstein t-norm: $T^E(x, y) = \frac{xy}{1+(1-x)(1-y)}$,
- Einstein t-conorm: $S^E(x, y) = \frac{x+y}{1+xy}$.

2.4 Score and accuracy functions

Wan et al. (2021b) proposed the definition of a score and accuracy functions for q-ROTrFNs in order to compare them.

Definition 2.3. (Wan et al., 2021b) Let $\tilde{r} = \langle [a, b, c, d]; \mu, \nu \rangle$ be a q-ROTrFN, then score function $S(\tilde{r})$ and accuracy function $A(\tilde{r})$ are presented as:

$$S(\tilde{r}) = \frac{a+b+c+d}{4}(\mu^q - \nu^q); \tag{3}$$

$$A(\tilde{r}) = \frac{a+b+c+d}{4}(\mu^q - \nu^q). \tag{4}$$

To effectively compare the two q-ROTrFNs, using the score $S(\tilde{r})$ and accuracy $A(\tilde{r})$ functions, Wan et al. (2021b) defined a comparison law presented as follows:

Definition 2.4. (Wan et al., 2021b) Let $\tilde{r}_1 = \langle [a_1, b_1, c_1, d_1]; \mu_1, \nu_1 \rangle$ and $\tilde{r}_2 = \langle [a_2, b_2, c_2, d_2]; \mu_2, \nu_2 \rangle$ are any two q-ROTrFNs, then comparison rule between $\tilde{r}_1$ and $\tilde{r}_2$ are presented in the following way:

(i) If $S(\tilde{r}_1) > S(\tilde{r}_2)$, then $\tilde{r}_1 \succ \tilde{r}_2$;
(ii) If $S(\tilde{r}_1) = S(\tilde{r}_2)$, then
- If $A(\tilde{r}_1) < A(\tilde{r}_2)$, then $\tilde{r}_1 \prec \tilde{r}_2$;
- If $A(\tilde{r}_1) = A(\tilde{r}_2)$, then $\tilde{r}_1 \approx \tilde{r}_2$.

3. Einstein Operations-Based *q*-ROTrF AOs

This section first introduces some basic operational laws for q-ROTrFNs based on Einstein t-norm and t-conorm, and then using defined operational rules, two new AOs were constructed.

3.1. Einstein operations for *q*-ROTrFNs

In this part, the Einstein t-conorm, S^E, and t-norm, T^E, are used to propose several q-ROTrF Einstein AOs.

The Einstein sum and product on two q-ROTrFNs $\tilde{r}_1$ and $\tilde{r}_2$ are also be a q-ROTrFN denoted by $\tilde{r}_1 \oplus_E \tilde{r}_2$ and $\tilde{r}_1 \otimes_E \tilde{r}_2$, respectively, as follows.

Definition 3.1. Let $\tilde{r}_i = \langle [a_i, b_i, c_i, d_i]; \mu_i, \nu_i \rangle$, $(i = 1, 2)$ and $\tilde{r} = \langle [a, b, c, d]; \mu, \nu \rangle$ be any three q-ROTrFNs, then their addition, $\tilde{r}_1 \oplus_E \tilde{r}_2$, multiplication, $\tilde{r}_1 \otimes_E \tilde{r}_2$, $(\lambda > 0)$

(i) $\tilde{r}_1 \oplus_E \tilde{r}_2 = \left\langle [a_1 + a_2, b_1 + b_2, c_1 + c_2, d_1 + d_2]; \left(\frac{\mu_1{}^q + \mu_2{}^q}{1+\mu_1{}^q \mu_2{}^q}\right)^{\frac{1}{q}}, \left(\frac{\nu_1{}^q \nu_2{}^q}{1+(1-\nu_1{}^q)(1-\nu_2{}^q)}\right)^{\frac{1}{q}} \right\rangle$;

(ii) $\tilde{r}_1 \otimes_E \tilde{r}_2 = \left\langle [a_1 a_2, b_1 b_2, c_1 c_2, d_1 d_2]; \left(\frac{\mu_1{}^q \mu_2{}^q}{1+(1-\mu_1{}^q)(1-\mu_2{}^q)}\right)^{\frac{1}{q}}, \left(\frac{\nu_1{}^q + \nu_2{}^q}{1+\nu_1{}^q \nu_2{}^q}\right)^{\frac{1}{q}} \right\rangle$;

(iii) $\lambda \odot_E \tilde{r} = \left\langle [\lambda a, \lambda b, \lambda c, \lambda d]; \left(\frac{(1+\mu^q)^\lambda - (1-\mu^q)^\lambda}{(1+\mu^q)^\lambda + (1-\mu^q)^\lambda}\right)^{\frac{1}{q}}, \left(\frac{2\nu^{q\lambda}}{(2-\nu^q)^\lambda + \nu^{q\lambda}}\right)^{\frac{1}{q}} \right\rangle$;

(iv) $\tilde{r}^\lambda = \left\langle [a^\lambda, b^\lambda, c^\lambda, d^\lambda]; \left(\frac{2\mu^{q\lambda}}{(2-\mu^q)^\lambda + \mu^{q\lambda}}\right)^{\frac{1}{q}}, \left(\frac{(1+\nu^q)^\lambda - (1-\nu^q)^\lambda}{(1+\nu^q)^\lambda + (1-\nu^q)^\lambda}\right)^{\frac{1}{q}} \right\rangle$.

Proof **(i).** Since $a_i, b_i, c_i, d_i \in \mathbb{R}$, then it is evident that $a_1 + a_2, b_1 + b_2, c_1 + c_2, d_1 + d_2 \in \mathbb{R}$.

We have to show that $\left(\left(\frac{\mu_1{}^q + \mu_2{}^q}{1+\mu_1{}^q \mu_2{}^q}\right)^{\frac{1}{q}}\right)^q + \left(\left(\frac{\nu_1{}^q + \nu_2{}^q}{1+(1-\nu_1{}^q)(1-\nu_2{}^q)}\right)^{\frac{1}{q}}\right)^q \le 1$, i.e., $\frac{\mu_1{}^q + \mu_2{}^q}{1+\mu_1{}^q \mu_2{}^q} + \frac{\nu_1{}^q + \nu_2{}^q}{1+(1-\nu_1{}^q)(1-\nu_2{}^q)} \le 1$.

From the definition of q-ROTrFNs, $\tilde{r}_i$, the membership and nonmembership degrees satisfy the condition that

$$\mu_1{}^q + \nu_1{}^q \le 1; \tag{5}$$

and

$$\mu_2{}^q + \nu_2{}^q \le 1; \tag{6}$$

i.e., $\mu_1{}^q \le 1 - \nu_1{}^q$, $\mu_2{}^q \le 1 - \nu_2{}^q$,

$$\begin{aligned} &\Rightarrow \mu_2{}^q \mu_2{}^q \le (1 - \nu_1{}^q)(1 - \nu_2{}^q)(\because \mu_i, \nu_i \in [0, 1] \\ &\Rightarrow 1 + \mu_2{}^q \mu_2{}^q \le 1 + (1 - \nu_1{}^q)(1 - \nu_2{}^q). \end{aligned} \tag{7}$$

Adding (5) and (6),

$$\mu_1{}^q + \nu_1{}^q + \mu_2{}^q + \nu_2{}^q \le 2, \tag{8}$$

and since $\mu_i \in [0, 1]$

$$\mu_1{}^q\mu_2{}^q \leq 1, i.e., 1+\mu_1{}^q\mu_2{}^q \leq 2. \tag{9}$$

From (8) and (9),

$$\frac{\mu_1{}^q+\mu_2{}^q+\nu_1{}^q+\nu_2{}^q}{1+\mu_1{}^q\mu_2{}^q} \leq 1, \text{or}, \frac{\mu_1{}^q+\mu_2{}^q}{1+\mu_1{}^q\mu_2{}^q}+\frac{\nu_1{}^q+\nu_2{}^q}{1+\mu_1{}^q\mu_2{}^q} \leq 1,$$

Now using (3.3), $\frac{\mu_1{}^q+\mu_2{}^q}{1+\mu_1{}^q\mu_2{}^q}+\frac{\nu_1{}^q+\nu_2{}^q}{1+(1-\nu_1{}^q)(1-\nu_2{}^q)} \leq 1$.

So $\tilde{r}_1 \oplus_E \tilde{r}_2$ is an q-ROTrFN.

In a parallel way, it can be proven that each of $\tilde{r}_1 \otimes_E \tilde{r}_2$, $\lambda \odot_E \tilde{r}$, and $\tilde{r}^\lambda$ is a q-ROTrFN.

Example 1: Let $\tilde{r}_1 = \langle[0.4, 0.5, 0.6, 0.7]; 0.7, 0.3\rangle$ and $\tilde{r}_2 = \langle[0.3, 0.4, 0.6, 0.8]; 0.8, 0.5\rangle$ be any two q-ROTrFNs. Then for taking $q = 3$, some Einstein operations of $\tilde{r}_1$ and $\tilde{r}_2$ can be defined as follows:

$$\tilde{r}_1 \oplus_E \tilde{r}_2 = \left\langle [0.4+0.3, 0.5+0.4, 0.6+0.6, 0.7+0.8]; \left(\frac{0.7^3+0.8^3}{1+0.7^3 0.8^3}\right)^{\frac{1}{3}}, \left(\frac{0.3^3 0.5^3}{1+(1-0.3^3)(1-0.5^3)}\right)^{\frac{1}{3}}\right\rangle; = \langle[0.7, 0.9, 1.2, 1.5]; 0.8993, 0.1222\rangle,$$

$$\tilde{r}_1 \otimes_E \tilde{r}_2 = \left\langle [[0.4\times0.3, 0.5\times0.4, 0.6\times0.6, 0.7\times0.8]]; \left(\frac{0.7^3 0.8^3}{1+(1-0.7^3)(1-0.8^3)}\right)^{\frac{1}{3}}, \left(\frac{0.3^3+0.5^3}{1+0.3^3 0.5^3}\right)^{\frac{1}{3}}\right\rangle; = \langle[0.12, 0.2, 0.36, 0.56]; 0.5104, 0.5331\rangle,$$

$$2 \odot_E \tilde{r}_1 = \left\langle [2\times0.4, 2\times0.5, 2\times0.6, 2\times0.7]; \left(\frac{(1+0.7^3)^2-(1-0.7^3)^2}{(1+0.7^3)^2+(1-0.7^3)^2}\right)^{\frac{1}{3}}, \left(\frac{2\times0.3^{3\times2}}{(2-0.3^3)^2+0.3^{3\times2}}\right)^{\frac{1}{q}}\right\rangle; = \langle[0.8, 1.0, 1.2, 1.4]; 0.8498, 0.0721\rangle,$$

$$\tilde{r}_1^2 = \left\langle [0.4^2, 0.5^2, 0.6^2, 0.7^2]; \left(\frac{2\times0.7^{3\times2}}{(2-0.7^3)^2+0.7^{3\times2}}\right)^{\frac{1}{3}}, \left(\frac{(1+0.3^3)^2-(1-0.3^3)^2}{(1+0.3^3)^2+(1-0.3^3)^2}\right)^{\frac{1}{3}}\right\rangle; = \langle[0.16, 0.25, 0.36, 0.49]; 0.4348, 0.3779\rangle.$$

3.2 Einstein operations-based q-ROTrF AOs

With the help of Einstein operations, the q-ROTrF averaging and geometric AOs are introduced in the section.

- ***q*-ROTrFEWA operator**

Definition 3.2. Let $\{\tilde{r}_j = \langle [a_j, b_j, c_j, d_j]; \mu_j, \nu_j\rangle | j = 1, 2, \ldots, n\}$ be a collection of q-ROTrFNs. The q-ROTrFEWA operator is defined as follows:

$$q-ROTrFEWA(\tilde{r}_1, \tilde{r}_2, \ldots, \tilde{r}_n) = \oplus_{Ej=1}^{n}(\omega_j \odot_E \tilde{r}_j), \tag{10}$$

In which addition $\oplus_E$ and scalar multiplication $\odot_E$, laws are presented in Definition 3.1, where $\omega = (\omega_1, \omega_2, \ldots, \omega_n)^T$ is a weight vector of q-ROTrFNs $\tilde{r}_j$ with $\omega_j \in [0, 1]$ and $\sum_{j=1}^{n}\omega_j = 1$.

Theorem 3.1. Let $\{\tilde{r}_j = \langle [a_j, b_j, c_j, d_j]; \mu_j, \nu_j\rangle | j = 1, 2, \ldots, n\}$ be a group of q-ROTrFNs and $\omega = (\omega_1, \omega_2, \ldots, \omega_n)^T$ be a weight vector of $\tilde{r}_j$ where $\omega_j \in [0, 1]$, $\sum_{j=1}^{n}\omega_j = 1$. Then aggregated value of $\{\tilde{r}_1, \tilde{r}_2, \ldots, \tilde{r}_n\}$ by the q-ROTrFEWA operator is still a q-ROTrFN and

$$q-ROTrFEWA(\tilde{r}_1, \tilde{r}_2, \ldots, \tilde{r}_n) = \left\langle \left[\sum_{j=1}^{n}\omega_j a_j, \sum_{j=1}^{n}\omega_j b_j, \sum_{j=1}^{n}\omega_j c_j, \sum_{j=1}^{n}\omega_j d_j\right]; \left(\frac{\prod_{j=1}^{n}(1+\mu_j{}^q)^{\omega_j}-\prod_{j=1}^{n}(1-\mu_j{}^q)^{\omega_j}}{\prod_{j=1}^{n}(1+\mu_j{}^q)^{\omega_j}+\prod_{j=1}^{n}(1-\mu_j{}^q)^{\omega_j}}\right)^{\frac{1}{q}}, \left(\frac{2\prod_{j=1}^{n}\nu_j{}^{q\omega_j}}{\prod_{j=1}^{n}(2-\nu_j{}^q)^{\omega_j}+\prod_{j=1}^{n}\nu_j{}^{q\omega_j}}\right)^{\frac{1}{q}}\right\rangle. \tag{11}$$

Proof. Based on Definition 3.1,

$$\omega_j \odot_E \tilde{r}_j = \left\langle [\omega_j a_j, \omega_j b_j, \omega_j c_j, \omega_j d_j]; \left(\frac{(1+\mu_j{}^q)^{\omega_j}-(1-\mu_j{}^q)^{\omega_j}}{(1+\mu_j{}^q)^{\omega_j}+(1-\mu_j{}^q)^{\omega_j}}\right)^{\frac{1}{q}}, \left(\frac{2\nu_j{}^{q\omega_j}}{(2-\nu_j{}^q)^{\omega_j}+\nu_j{}^{q\omega_j}}\right)^{\frac{1}{q}}\right\rangle; \text{now, } \omega_1\tilde{r}_1 \oplus_E \omega_2\tilde{r}_2$$

$$= \langle[\omega_1 a_1+\omega_2 a_2, \omega_1 b_1+\omega_2 b_2, \omega_1 c_1+\omega_2 c_2, \omega_1 d_1+\omega_2 d_2]; \left(\frac{\prod_{j=1}^{2}(1+\mu_j{}^q)^{\omega_j}-\prod_{j=1}^{2}(1-\mu_j{}^q)^{\omega_j}}{\prod_{j=1}^{2}(1+\mu_j{}^q)^{\omega_j}+\prod_{j=1}^{2}(1-\mu_j{}^q)^{\omega_j}}\right)^{\frac{1}{q}}, \left(\frac{2\prod_{j=1}^{2}\nu_j{}^{q\omega_j}}{\prod_{j=1}^{2}(2-\nu_j{}^q)^{\omega_j}+\prod_{j=1}^{2}\nu_j{}^{q\omega_j}}\right)^{\frac{1}{q}}\rangle$$

i.e., the theorem holds for $n = 2$. Now, assume that the theorem is valid for $n = k$.

Hence, $q-ROTrFEWA(\tilde{r}_1, \tilde{r}_2, \ldots, \tilde{r}_k) = \left\langle \left[\sum_{j=1}^{k}\omega_j a_j, \sum_{j=1}^{k}\omega_j b_j, \sum_{j=1}^{k}\omega_j c_j, \sum_{j=1}^{k}\omega_j d_j\right];\right.$

$$\left.\left(\frac{\prod_{j=1}^{k}(1+\mu_j{}^q)^{\omega_j}-\prod_{j=1}^{k}(1-\mu_j{}^q)^{\omega_j}}{\prod_{j=1}^{k}(1+\mu_j{}^q)^{\omega_j}+\prod_{j=1}^{k}(1-\mu_j{}^q)^{\omega_j}}\right)^{\frac{1}{q}}, \left(\frac{2\prod_{j=1}^{k}\nu_j{}^{q\omega_j}}{\prod_{j=1}^{k}(2-\nu_j{}^q)^{\omega_j}+\prod_{j=1}^{k}\nu_j{}^{q\omega_j}}\right)^{\frac{1}{q}}\right\rangle.$$

Then for $n = k+1$,

$$\begin{aligned}
&q-ROTrFEWA(\tilde{r}_1, \tilde{r}_2, \ldots, \tilde{r}_k, \tilde{r}_{k+1}) \\
&= q-ROTrFEWA(\tilde{r}_1, \tilde{r}_2, \ldots, \tilde{r}_k) \oplus_E (\omega_{k+1}\tilde{r}_{k+1}) \\
&= \left\langle \left[\sum_{j=1}^{k}\omega_j a_j, \sum_{j=1}^{k}\omega_j b_j, \sum_{j=1}^{k}\omega_j c_j, \sum_{j=1}^{k}\omega_j d_j\right]; \left(\frac{\prod_{j=1}^{k}(1+\mu_j{}^q)^{\omega_j}-\prod_{j=1}^{k}(1-\mu_j{}^q)^{\omega_j}}{\prod_{j=1}^{k}(1+\mu_j{}^q)^{\omega_j}+\prod_{j=1}^{k}(1-\mu_j{}^q)^{\omega_j}}\right)^{\frac{1}{q}}, \left(\frac{2\prod_{j=1}^{k}\nu_j{}^{q\omega_j}}{\prod_{j=1}^{k}(2-\nu_j{}^q)^{\omega_j}+\prod_{j=1}^{k}\nu_j{}^{q\omega_j}}\right)^{\frac{1}{q}}\right\rangle \\
&\oplus_E \left\langle [\omega_{k+1}a_{k+1}, \omega_{k+1}b_{k+1}, \omega_{k+1}c_{k+1}, \omega_{k+1}d_{k+1}]; \left(\frac{(1+\mu_{k+1}{}^q)^{\omega_{k+1}}-(1-\mu_{k+1}{}^q)^{\omega_j}}{(1+\mu_{k+1}{}^q)^{\omega_j}+(1-\mu_{k+1}{}^q)^{\omega_j}}\right)^{\frac{1}{q}}, \left(\frac{2\nu_{k+1}{}^{q\omega_{k+1}}}{(2-\nu_j{}^q)^{\omega_{k+1}}+\nu_{k+1}{}^{q\omega_{k+1}}}\right)^{\frac{1}{q}}\right\rangle, \\
&= \left\langle \left[\sum_{j=1}^{k+1}\omega_j a_j, \sum_{j=1}^{k+1}\omega_j b_j, \sum_{j=1}^{k+1}\omega_j c_j, \sum_{j=1}^{k+1}\omega_j d_j\right]; \left(\frac{\prod_{j=1}^{k+1}(1+\mu_j{}^q)^{\omega_j}-\prod_{j=1}^{k+1}(1-\mu_j{}^q)^{\omega_j}}{\prod_{j=1}^{k+1}(1+\mu_j{}^q)^{\omega_j}+\prod_{j=1}^{k+1}(1-\mu_j{}^q)^{\omega_j}}\right)^{\frac{1}{q}}, \left(\frac{2\prod_{j=1}^{k+1}\nu_j{}^{q\omega_j}}{\prod_{j=1}^{k+1}(2-\nu_j{}^q)^{\omega_j}+\prod_{j=1}^{k+1}\nu_j{}^{q\omega_j}}\right)^{\frac{1}{q}}\right\rangle.
\end{aligned}$$

Therefore, the theorem is true for $n = k+1$ also and is valid $\forall n$.

Example 2: Let $\tilde{r}_1 = \langle[0.4, 0.5, 0.6, 0.7]; 0.4, 0.3\rangle$, $\tilde{r}_2 = \langle[0.1, 0.2, 0.3, 0.4]; 0.5, 0.2\rangle$, and $\tilde{r}_3 = \langle[0.4, 0.5, 0.7, 0.8]; 0.2, 0.5\rangle$

be a collection of q-ROTrFNs. If the weights of three q-ROTrFNs are taken, respectively, such as 0.3, 0.25, and 0.35, then their aggregated value by using the q-ROTrFEWA operator is also a q-ROTrF and obtained as:

$$q-ROTrFEWA(\tilde{r}_1,\tilde{r}_2,\tilde{r}_3) = \left\langle \left[\sum_{j=1}^{3}\omega_j a_j, \sum_{j=1}^{3}\omega_j b_j, \sum_{j=1}^{3}\omega_j c_j, \sum_{j=1}^{3}\omega_j d_j\right]; \left(\frac{\prod_{j=1}^{3}(1+\mu_j{}^3)^{\omega_j}-\prod_{j=1}^{3}(1-\mu_j{}^3)^{\omega_j}}{\prod_{j=1}^{3}(1+\mu_j{}^3)^{\omega_j}+\prod_{j=1}^{3}(1-\mu_j{}^3)^{\omega_j}}\right)^{\frac{1}{3}}, \left(\frac{2\prod_{j=1}^{3}\nu_j{}^{3\omega_j}}{\prod_{j=1}^{3}(2-\nu_j{}^3)^{\omega_j}+\prod_{j=1}^{3}\nu_j{}^{3\omega_j}}\right)^{\frac{1}{3}}\right\rangle; = \langle[0.2850, 0.3750, 0.5000, 0.5900]; 0.3765, 0.5734\rangle.$$

Now, some fundamental characteristics of the proposed q-ROTrFEWA operator are stated in the following section.

Theorem 3.2. (*Idempotency*) Suppose $\{\tilde{r}_j = \langle[a_j, b_j, c_j, d_j]; \mu_j, \nu_j\rangle \mid j = 1, 2, \ldots, n\}$ be a group of q-ROTrFNs. If $\tilde{r}_j = \tilde{r} = \langle[a, b, c, d]; \mu, \nu\rangle$ $\forall j$, then

$$q-ROTrFEWA(\tilde{r}_1,\tilde{r}_2,\ldots,\tilde{r}_n) = \tilde{r}.$$

.

Proof. $q-ROTrFEWA(\tilde{r}_1,\tilde{r}_2,\ldots,\tilde{r}_n) =$

$$\left\langle\left[\sum_{j=1}^{n}\omega_j a_j, \sum_{j=1}^{n}\omega_j b_j, \sum_{j=1}^{n}\omega_j c_j, \sum_{j=1}^{n}\omega_j d_j\right]; \left(1-\prod_{j=1}^{n}(1-\mu_j{}^q)^{\omega_j}\right)^{\frac{1}{q}}, \prod_{j=1}^{n}\nu_j^{\omega_j}\right\rangle.$$

Since $\tilde{r}_j = \tilde{r}$ $\forall j$,

$$q-ROTrFEWA(\tilde{r},\tilde{r},\ldots,\tilde{r}) = \left\langle\left[\left(\sum_{j=1}^{n}\omega_j\right)a, \left(\sum_{j=1}^{n}\omega_j\right)b, \left(\sum_{j=1}^{n}\omega_j\right)c, \left(\sum_{j=1}^{n}\omega_j\right)d\right]; \left(1-(1-\mu^q)^{\sum_{j=1}^{n}\omega_j}\right)^{\frac{1}{q}}, \nu^{\sum_{j=1}^{n}\omega_j}\right\rangle = \langle[a, b, c, d]; \mu, \nu\rangle = \tilde{r}.$$

Theorem 3.3. (*Monotonicity*) Let $\{\tilde{r}_j = \langle[a_j, b_j, c_j, d_j]; \mu_j, \nu_j\rangle\}$ and $\left\{\tilde{r}_j{}' = \langle\left[a'_j, b'_j, c'_j, d'_j\right]; \mu'_j, \nu'_j\rangle\right\}$ be two collections of n q-ROTrFNs. If $a_j \le a'_j$, $b_j \le b'_j$, $c_j \le c'_j$, $d_j \le d'_j$, $\mu_j \le \mu'_j$ and $\nu_j \ge \nu'_j$ $\forall j$, then,

$$q-ROTrFEWA(\tilde{r}_1,\tilde{r}_2,\ldots,\tilde{r}_n) \preccurlyeq q-ROTrFEWA(\tilde{r}_1{}',\tilde{r}_2{}',\ldots,\tilde{r}_n{}'). \quad (12)$$

Proof. Let $g(t) = \frac{1+t}{1-t}$, $t \in [0, 1)$, then $g'(t) = \frac{2}{(1-t)^2} > 0$, thus g is an increasing function. Since for every $\tilde{r}_j$ and $\tilde{r}_j{}'$, $\mu_j \le \mu'_j$,

$$\frac{(1+\mu_j{}^q)}{(1-\mu_j{}^q)} \le \frac{\left(1+\mu_j'^q\right)}{\left(1-\mu_j'^q\right)}. \ So, \ \left(\frac{1+\mu_j{}^q}{1-\mu_j{}^q}\right)^{\omega_j} \le \left(\frac{1+\mu_j'^q}{1-\mu_j'^q}\right)^{\omega_j},$$

$$\Leftrightarrow \prod_{j=1}^{n}\left(\frac{1+\mu_j{}^q}{1-\mu_j{}^q}\right)^{\omega_j} \le \prod_{j=1}^{n}\left(\frac{1+\mu_j'^q}{1-\mu_j'^q}\right)^{\omega_j},$$

$$\Leftrightarrow \prod_{j=1}^{n}\left(\frac{1+\mu_j{}^q}{1-\mu_j{}^q}\right)^{\omega_j}+1 \le \prod_{j=1}^{n}\left(\frac{1+\mu_j'^q}{1-\mu_j'^q}\right)^{\omega_j}+1,$$

$$\Leftrightarrow \frac{1}{\prod_{j=1}^{n}\left(\frac{1+\mu_j{}^q}{1-\mu_j{}^q}\right)^{\omega_j}+1} \ge \frac{1}{\prod_{j=1}^{n}\left(\frac{1+\mu_j'^q}{1-\mu_j'^q}\right)^{\omega_j}+1},$$

$$\Leftrightarrow \frac{2\prod_{j=1}^{n}(1-\mu_j{}^q)^{\omega_j}}{\prod_{j=1}^{n}(1+\mu_j{}^q)^{\omega_j}+\prod_{j=1}^{n}(1-\mu_j{}^q)^{\omega_j}} \ge \frac{2\prod_{j=1}^{n}\left(1-\mu_j'^q\right)^{\omega_j}}{\prod_{j=1}^{n}\left(1+\mu_j'^q\right)^{\omega_j}+\prod_{j=1}^{n}\left(1-\mu_j'^q\right)^{\omega_j}},$$

$$\Leftrightarrow 1-\frac{2\prod_{j=1}^{n}(1-\mu_j{}^q)^{\omega_j}}{\prod_{j=1}^{n}(1+\mu_j{}^q)^{\omega_j}+\prod_{j=1}^{n}(1-\mu_j{}^q)^{\omega_j}} \le 1-\frac{2\prod_{j=1}^{n}\left(1-\mu_j'^q\right)^{\omega_j}}{\prod_{j=1}^{n}\left(1+\mu_j'^q\right)^{\omega_j}+\prod_{j=1}^{n}\left(1-\mu_j'^q\right)^{\omega_j}},$$

$$\Leftrightarrow \frac{\prod_{j=1}^{n}(1+\mu_j{}^q)^{\omega_i}-\prod_{j=1}^{n}(1-\mu_j{}^q)^{\omega_j}}{\prod_{j=1}^{n}(1+\mu_j{}^q)^{\omega_j}+\prod_{j=1}^{n}(1-\mu_j{}^q)^{\omega_j}} \le \frac{\prod_{j=1}^{n}\left(1+\mu_j'^q\right)^{\omega_j}-\prod_{j=1}^{n}\left(1-\mu_j'^q\right)^{\omega_j}}{\prod_{j=1}^{n}\left(1+\mu_j'^q\right)^{\omega_j}+\prod_{j=1}^{n}\left(1-\mu_j'^q\right)^{\omega_j}}, \quad (13)$$

Again let $f(u) = \frac{2-u}{u}$, $u \in (0, 1]$, then $f'(u) = -\frac{2}{u^2} < 0$, thus $f(u)$ is a decreasing function.

Since, $\nu_j{}^q \ge \nu_j'^q \forall j$, then

$$\frac{2-\nu_j{}^q}{\nu_j{}^q} \le \frac{2-\nu_j'^q}{\nu_j'^q}, \ \text{thus}, \ \left(\frac{2-\nu_j{}^q}{\nu_j{}^q}\right)_j^{\omega} \le \left(\frac{2-\nu_j'^q}{\nu_j'^q}\right)^{\omega_j},$$

$$\Leftrightarrow \prod_{j=1}^{n}\left(\frac{2-\nu_j^q}{\nu_j^q}\right)^{\omega_j} \le \prod_{j=1}^{n}\left(\frac{2-\nu_j'^q}{\nu_j'^q}\right)^{\omega_j}, \Leftrightarrow \prod_{j=1}^{n}\left(\frac{2-\nu_j^q}{\nu_j^q}\right)^{\omega_j}+1 \le \prod_{j=1}^{n}\left(\frac{2-\nu_j'^q}{\nu_j'^q}\right)^{\omega_j}+1, \Leftrightarrow \frac{1}{\prod_{j=1}^{n}\left(\frac{2-\nu_j{}^q}{\nu_j{}^q}\right)^{\omega_j}+1}$$

$$\ge \frac{1}{\prod_{j=1}^{n}\left(\frac{2-\nu_j'^q}{\nu_j'^q}\right)^{\omega_j}+1}, \Leftrightarrow \frac{2\prod_{j=1}^{n}\nu_j{}^{q\omega_j}}{\prod_{j=1}^{n}(2-\nu_j{}^q)^{\omega_j}+\prod_{j=1}^{n}\nu_j{}^{q\omega_j}} \ge \frac{2\prod_{j=1}^{n}\nu_j'^{q\omega_j}}{\prod_{j=1}^{n}\left(2-\nu_j'^q\right)^{\omega_j}+\prod_{j=1}^{n}\nu_j'^{q\omega_j}}. \quad (14)$$

From (13) and (14) and using the relations $\sum_{j=1}^{n}\omega_j a_j \le \sum_{j=1}^{n}\omega_j a'_j$, $\sum_{j=1}^{n}\omega_j b_j \le \sum_{j=1}^{n}\omega_j b'_j$, $\sum_{j=1}^{n}\omega_j c_j \le \sum_{j=1}^{n}\omega_j c'_j$ and $\sum_{j=1}^{n}\omega_j d_j \le \sum_{j=1}^{n}\omega_j d'_j$, it is clear that

$$S(q-ROTrFEWA(\tilde{r}_1,\tilde{r}_2,\ldots,\tilde{r}_n)) \le S(q-ROTrFEWA(\tilde{r}_1{}',\tilde{r}_2{}',\ldots,\tilde{r}_n{}')).$$

Therefore, $q-ROTrFEWA(\tilde{r}_1,\tilde{r}_2,\ldots,\tilde{r}_n) \preccurlyeq q-ROTrFEWA(\tilde{r}_1{}',\tilde{r}_2{}',\ldots,\tilde{r}_n{}')$.

Hence, inequality (12) follows.

Theorem 3.4. (Boundedness) Let $\{\tilde{r}_j = \langle [a_j, b_j, c_j, d_j]; \mu_j, \nu_j\rangle \,|\, j = 1, 2, \ldots, n\}$ be a group of q-ROTrFNs and assume

$\tilde{r}_j^- = \langle [min_j\{a_j\}, min_j\{b_j\}, min_j\{c_j\}, min_j\{d_j\}]; min_j\{\mu_j\}, max_j\{\nu_j\}\rangle$, and
$\tilde{r}_j^+ = \langle [max_j\{a_j\}, max_j\{b_j\}, max_j\{c_j\}, max_j\{d_j\}]; max_j\{\mu_j\}, min_j\{\nu_j\}\rangle$,

then, $\tilde{r}_j^- \leq q - ROTrFEWA(\tilde{r}_1, \tilde{r}_2, \ldots, \tilde{r}_n) \leq \tilde{r}_j^+$.

Proof. Since $min\{a_j\} \leq a_j \leq max\{a_j\}$, $min\{b_j\} \leq b_j \leq max\{b_j\}$, $min\{c_j\} \leq c_j \leq max\{c_j\}$, $min\{d_j\} \leq d_j \leq max\{d_j\}$, $min\{\mu_j\} \leq \mu_j \leq max\{\mu_j\}$ and $min\{\nu_j\} \leq \nu_j \leq max\{\nu_j\} \forall j$, then $\tilde{r}_j^- \leq \tilde{r}_j \forall j$.

Thus, from monotonicity

$$q - ROTrFEWA(\tilde{r}_j^-, \tilde{r}_j^-, \ldots, \tilde{r}_j^-) \leq q - ROTrFEWA(\tilde{r}_1, \tilde{r}_2, \ldots, \tilde{r}_n).$$

Now applying the idempotency theorem, the above inequality takes the form as:

$$\tilde{r}_j^- \leq q - ROTrFEWA(\tilde{r}_1, \tilde{r}_2, \ldots, \tilde{r}_n). \tag{15}$$

Similarly, it can be shown that

$$q - ROTrFEWA(\tilde{r}_1, \tilde{r}_2, \ldots, \tilde{r}_n) \leq \tilde{r}_j^+. \tag{16}$$

So, by combining (15) and (16), it follows that

$$\tilde{r}_j^- \leq q - ROTrFEWA(\tilde{r}_1, \tilde{r}_2, \ldots, \tilde{r}_n) \leq \tilde{r}_j^+.$$

- ***q*-ROTrFEWG operator**

In this subsection, q-ROTrFEWG operator is developed based on Einstein operational rules.

Definition 3.3. Let $\{\tilde{r}_j = \langle [a_j, b_j, c_j, d_j]; \mu_j, \nu_j\rangle | j = 1, 2, \ldots, n\}$ be a collection of q-ROTrFNs. The q-ROTrFEWG operator is defined as follows:

$$q - ROTrFEWG(\tilde{r}_1, \tilde{r}_2, \ldots, \tilde{r}_n) = \tilde{r}_1^{\omega_1} \otimes_E \tilde{r}_2^{\omega_2} \otimes_E \ldots \otimes_E \tilde{r}_n^{\omega_n}, \tag{17}$$

In which multiplication $\otimes_E$ and exponential laws are presented in Definition 3.1, where $\omega = (\omega_1, \omega_2, \ldots, \omega_n)^T$ is a vector of q-ROTrFNs $\tilde{r}_j$ with $\omega_j \in [0, 1]$ and $\sum_{j=1}^n \omega_j = 1$.

Theorem 3.5. Let $\{\tilde{r}_j = \langle [a_j, b_j, c_j, d_j]; \mu_j, \nu_j\rangle | j = 1, 2, \ldots, n\}$ be a set of q-ROTrFNs and $\omega = (\omega_1, \omega_2, \ldots, \omega_n)^T$ represent the weight vector of $\tilde{r}_j$ where $\omega_j \in [0, 1]$, $\sum_{j=1}^n \omega_j = 1$. Then their aggregated value using q-ROTrFEWG operator is furthermore a q-ROTrFN and

$$\begin{aligned} &q - ROTrFEWG(\tilde{r}_1, \tilde{r}_2, \ldots, \tilde{r}_n) \\ &= \left\langle \left[\prod_{j=1}^n a_j^{\omega_j}, \prod_{j=1}^n b_j^{\omega_j}, \prod_{j=1}^n c_j^{\omega_j}, \prod_{j=1}^n d_j^{\omega_j}\right]; \left(\frac{2\prod_{j=1}^n \mu_j^{q\omega_j}}{\prod_{j=1}^n (2 - \mu_j^q)^{\omega_j} + \prod_{j=1}^n \mu_j^{q\omega_j}}\right)^{\frac{1}{q}}, \right. \\ &\left. \left(\frac{\prod_{j=1}^n (1 + \nu_j^q)^{\omega_j} - \prod_{j=1}^n (1 - \nu_j^q)^{\omega_j}}{\prod_{j=1}^n (1 + \nu_j^q)^{\omega_j} + \prod_{j=1}^n (1 - \nu_j^q)^{\omega_j}}\right)^{\frac{1}{q}} \right\rangle. \end{aligned} \tag{18}$$

Proof. The proof is same as Theorem 3.1.

Example 3:. In Example 2, if the geometric aggregation operator is used q-ROTrFEWG, then the aggregating values of the three q-ROTrFNs, $\tilde{r}_1$, $\tilde{r}_2$, and $\tilde{r}_3$, are computed as:

$$\begin{aligned} &q - ROTrFEWG(\tilde{r}_1, \tilde{r}_2, \tilde{r}_3) = \left\langle \left[\prod_{j=1}^3 a_j^{\omega_j}, \prod_{j=1}^3 b_j^{\omega_j}, \prod_{j=1}^3 c_j^{\omega_j}, \prod_{j=1}^3 d_j^{\omega_j}\right]; \right. \\ &\left. \left(\frac{2\prod_{j=1}^3 \mu_j^{3\omega_j}}{\prod_{j=1}^3 (2 - \mu_j^3)^{\omega_j} + \prod_{j=1}^3 \mu_j^{3\omega_j}}\right)^{\frac{1}{3}}, \left(\frac{\prod_{j=1}^3 (1 + \nu_j^3)^{\omega_j} - \prod_{j=1}^3 (1 - \nu_j^3)^{\omega_j}}{\prod_{j=1}^3 (1 + \nu_j^3)^{\omega_j} + \prod_{j=1}^3 (1 - \nu_j^3)^{\omega_j}}\right)^{\frac{1}{3}} \right\rangle; \\ &= \langle [0.3100, 0.4262, 0.5604, 0.6609]; 0.7112, 0.3780\rangle. \end{aligned}$$

Next, the characteristics of the defined q-ROTrFEWG operator are presented.

Theorem 3.6. (Idempotency) Let $\tilde{r}_j = \langle [a_j, b_j, c_j, d_j]; \mu_j, \nu_j\rangle$ $(j = 1, 2, \ldots, n)$ be a set of n Lq-ROFNs. If $\tilde{r}_j = \tilde{r} = \langle [a, b, c, d]; \mu, \nu\rangle$ $\forall j$, then

$$q - ROTrFEWG(\tilde{r}_1, \tilde{r}_2, \ldots, \tilde{r}_n) = \tilde{r}$$

.

Proof. The proof is same as Theorem 3.2.

Theorem 3.7. (*Monotonicity*) Suppose $\tilde{r}_j = \langle [a_j, b_j, c_j, d_j]; \mu_j, \nu_j\rangle$ and $\tilde{r}_j' = \langle [a_j', b_j', c_j', d_j']; \mu_j', \nu_j'\rangle$ be two set of n q-ROTrFNs. If $a_j \leq a_j', b_j \leq b_j', c_j \leq c_j', d_j \leq d_j'$, $\mu_j \leq \mu_j'$ and $\nu_j \geq \nu_j'$ $\forall j$, then

$$q - ROTrFEWG(\tilde{r}_1, \tilde{r}_2, \ldots, \tilde{r}_n) \leq q - ROTrFEWG(\tilde{r}_1', \tilde{r}_2', \ldots, \tilde{r}_n')$$

.

Proof. The proof is similar as Theorem 3.3.

Theorem 3.8. (*Boundedness*) If $\{\tilde{r}_j = \langle [a_j, b_j, c_j, d_j]; \mu_j, \nu_j\rangle\}$ represents a set of n q-ROTrFNs, and

$\tilde{r}_j^- = \langle [min_j\{a_j\}, min_j\{b_j\}, min_j\{c_j\}, min_j\{d_j\}]; min\{\mu_j\}, max_j\{\nu_j\}\rangle$ and
$\tilde{r}_j^+ = \langle [max_j\{a_j\}, max_j\{b_j\}, max_j\{c_j\}, max_j\{d_j\}]; max_j\{\mu_j\}, min_j\{\nu_j\}\rangle$, then
$\tilde{r}_j^- \leq q - ROTrFEWG(\tilde{r}_1, \tilde{r}_2, \ldots, \tilde{r}_n) \leq \tilde{r}_j^+$.

Proof. The proof is same as Theorem 3.4.

4. MCGDM Approach Based on the Proposed AOs under *q*-ROTrF Environment

In this part, a novel MCGDM method has been propounded in which the evaluation information is in the form of q-ROTrFNs.

For a MCGDM problem, let $E = \{e^{(1)}, e^{(2)}, \ldots, e^{(k)}\}$ be the group of the DMs with their associated weight vector $\Omega = (\Omega_1, \Omega_2, \ldots, \Omega_k)^T$. Suppose $A = \{A_i | i = 1, 2, \ldots, m\}$ be a set of m discrete alternatives and $\mathcal{C} = \{\mathcal{C}_j | j = 1, 2, \ldots, n\}$ represents the set of n criteria along with their weight vector $\omega = (\omega_1, \omega_2, \ldots, \omega_n)^T$, satisfying $\omega_i \in [0, 1]$ and $\sum_{i=1}^n \omega_i = 1$. DMs give their assessment values in terms of q-ROTrFNs. The DMs use q-ROTrFNs to express their judgment values, and q-ROTrF decision matrix (q-ROTrFDM) is provided as $\mathcal{D}^{(l)} = [\tilde{r}_{ij}^{(l)}]_{m\times n} = [\langle [a_{ij}^{(l)}, b_{ij}^{(l)}, c_{ij}^{(l)}, d_{ij}^{(l)}]; \mu_{ij}^{(l)}, \nu_{ij}^{(l)}\rangle]_{m\times n}$ $(l = 1, 2, .., k)$, where $\tilde{r}_{ij}^{(l)} = \langle [a_{ij}^{(l)}, b_{ij}^{(l)}, c_{ij}^{(l)}, d_{ij}^{(l)}]; \mu_{ij}^{(l)}, \nu_{ij}^{(l)}\rangle$ denotes a q-ROTrFN given by the DM $e^{(l)}$ for the alternative A_i under the criteria $\mathcal{C}_j$.

The purpose is to find the best suitable alternative(s) in light of the presented approach. The following is a step-by-step breakdown of the computing procedure.

Step 1. Normalize $\mathcal{D}^{(l)}$, if required, into $\mathcal{N}^{(l)} = \left[\tilde{\wp}_{ij}{}^{(l)}\right]_{m\times n}$ as follows:

$$\tilde{\wp}_{ij}{}^{(l)} = \begin{cases} \left\langle \left[a_{ij}^{(l)}, b_{ij}^{(l)}, c_{ij}^{(l)}, d_{ij}^{(l)}\right]; \mu_{ij}^{(l)}, \nu_{ij}^{(l)} \right\rangle & \textit{if } C_j \textit{ is type of benefit criteria} \\ \left\langle \left[a_{ij}^{(l)}, b_{ij}^{(l)}, c_{ij}^{(l)}, d_{ij}^{(l)}\right]; \nu_{ij}^{(l)}, \mu_{ij}^{(l)} \right\rangle & \textit{if } C_j \textit{ is type of cost criteria,} \end{cases} \tag{19}$$

$i = 1, 2, \ldots, m, j = 1, 2, \ldots, n.$

Step 2. Utilize q-ROTrFEWA (or q-ROTrFEWG) operator to aggregate all the individual normalized q-ROTrFDMs, $\mathcal{N}^{(l)} = \left[\tilde{\wp}_{ij}{}^{(l)}\right]_{m\times n}$ $(l = 1, 2, \ldots, k)$ into a single q-ROTrFDM, $\mathcal{N} = \left[\tilde{\wp}_{ij}\right]_{m\times n}$ $(i = 1, 2, \ldots, m; j = 1, 2, \ldots, n)$ as:

$$\tilde{\wp}_{ij} = \left\langle \left[\sum_{l=1}^{3} \Omega^{(l)} a_{ij}^{(l)}, \sum_{l=1}^{3} \Omega^{(l)} b_{ij}^{(l)}, \sum_{l=1}^{3} \Omega^{(l)} c_{ij}^{(l)}, \sum_{l=1}^{3} \Omega^{(l)} d_{ij}^{(l)}\right]; \right.$$

$$\left(\frac{\prod_{l=1}^{3}\left(1+\left(\mu_{ij}^{(l)}\right)^q\right)^{\Omega^{(l)}} - \prod_{l=1}^{3}\left(1-\left(\mu_{ij}^{(l)}\right)^q\right)^{\Omega^{(l)}}}{\prod_{l=1}^{3}\left(1+\left(\mu_{ij}^{(l)}\right)^q\right)^{\Omega^{(l)}} + \prod_{l=1}^{3}\left(1-\left(\mu_{ij}^{(l)}\right)^q\right)^{\Omega^{(l)}}}\right)^{\frac{1}{q}},$$

$$\left.\left(\frac{2\prod_{l=1}^{3}\left(\nu_{ij}^{(l)}\right)^{q\Omega^{(l)}}}{\prod_{l=1}^{3}\left(2-\left(\nu_{ij}^{(l)}\right)^q\right)^{\Omega^{(l)}} + \prod_{l=1}^{3}\left(\nu_{ij}^{(l)}\right)^{q\Omega^{(l)}}}\right)^{\frac{1}{q}}\right\rangle; \tag{20}$$

$$\text{or } \tilde{\wp}_{ij}{}' = \left\langle \left[\prod_{l=1}^{3}\left(a_{ij}^{(l)}\right)^{\Omega^{(l)}}, \prod_{l=1}^{3}\left(a_{ij}^{(l)}\right)^{\Omega^{(l)}}, \prod_{l=1}^{3}\left(a_{ij}^{(l)}\right)^{\Omega^{(l)}}, \prod_{l=1}^{3}\left(a_{ij}^{(l)}\right)^{\Omega^{(l)}}\right]; \right.$$

$$\left(\frac{2\prod_{l=1}^{3}\left(\mu_{ij}^{(l)}\right)^{q\Omega^{(l)}}}{\prod_{l=1}^{3}\left(2-\left(\mu_{ij}^{(l)}\right)^q\right)^{\Omega^{(l)}} + \prod_{l=1}^{3}\left(\mu_{ij}^{(l)}\right)^{q\Omega^{(l)}}}\right)^{\frac{1}{q}},$$

$$\left.\left(\frac{\prod_{l=1}^{3}\left(1+\left(\nu_{ij}^{(l)}\right)^q\right)^{\Omega^{(l)}} - \prod_{l=1}^{3}\left(1-\left(\nu_{ij}^{(l)}\right)^q\right)^{\Omega^{(l)}}}{\prod_{l=1}^{3}\left(1+\left(\nu_{ij}^{(l)}\right)^q\right)^{\Omega^{(l)}} + \prod_{l=1}^{3}\left(1-\left(\nu_{ij}^{(l)}\right)^q\right)^{\Omega^{(l)}}}\right)^{\frac{1}{q}}\right\rangle. \tag{21}$$

Step 3. Aggregate the q-ROTrFN $\tilde{\wp}_{ij}$ (or $\tilde{\wp}_{ij}{}'$) for each A_i $(i = 1, 2, \ldots, m)$ applying q-ROTrFEWA (or q-ROTrFEWG) operator as follows:

$$\tilde{\wp}_i = q - ROTrFEWA(\tilde{\wp}_{i1}, \tilde{\wp}_{i2}, \ldots, \tilde{\wp}_{in}); \tag{22}$$

or,

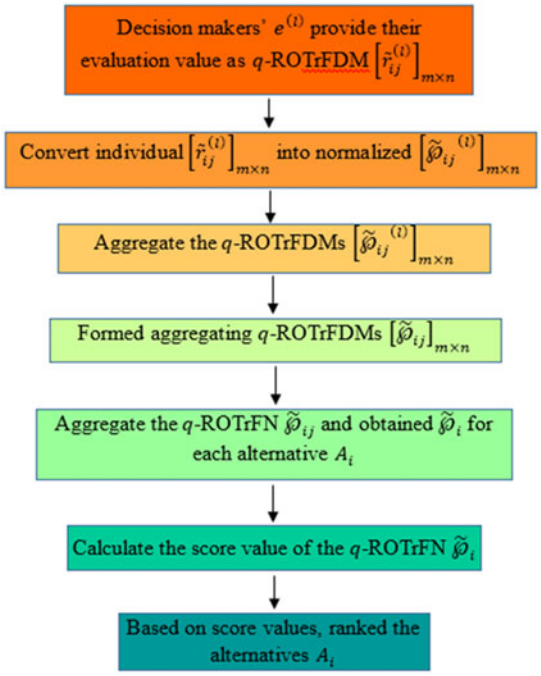
$$\tilde{\wp}_i{}' = q - ROTrFEWG(\tilde{\wp}_{i1}, \tilde{\wp}_{i2}, \ldots, \tilde{\wp}_{in}). \tag{23}$$

Step 4. Compute the score values $S(\tilde{\wp}_i)$ (or $S(\tilde{\wp}_i'))$ of the $\tilde{\wp}_i$ (or $\tilde{\wp}_i'$) for obtaining ordering among the alternatives, A_i.

Step 5. Sort the scores of all the alternatives in descending order, then choose the one with the highest score function.

The flowchart of the above methodology is presented through the Figure 2.

Figure 2
Flowchart of the proposed methodology



5. Illustrative Examples

In this part, two numerical examples, previously studied by Aydin et al. (2020) and Zhao et al. (2017), are given to illustrate the application of the proposed q-ROTrFEWA and q-ROTrFEWG operators.

5.1 Example 4

The human resources department of a corporation is looking to hire a sales consultant. Three human resource specialists will assess the four candidates based on the following criteria:

C_1: experience;
C_2: competencies;
C_3: foreign language skills;
C_4: human relationship management.

where C_1, C_2, and C_3, are benefit type, and last one is cost type. DMs evaluate four candidates $\{A_1, A_2, A_3, A_4\}$ with q-ROTrFNs presented in Tables 1, 2 and 3. Let $\omega = (0.15, 0.25, 0.25, 0.35)^T$ and $\Omega = (0.45, 0.25, 0.30)^T$ represent the weight vector of criteria and DMs, respectively.

Now q-ROTrFEWA and q-ROTrFEWG operators are implemented to choose the ideal candidate.

Step 1: The criteria are classified into two groups: criteria $C_1 - C_3$ are classified as benefit criteria. The cost criterion is C_4. So, by using Eq. (19), the normalized q-ROTrFDMs is obtained, which is shown in Tables 4, 5 and 6, respectively.

Step 2: Apply the q-ROTrFEWA operator, presented in Eq. (20), to aggregate all the normalized q-ROTrFDM $\mathcal{N}^{(l)} = \left[\tilde{\wp}_{ij}{}^{(l)}\right]_{m\times n}$ $(l = 1, 2, 3, 4)$. The integrated q-ROTrFDM, $\mathcal{N} = \left[\tilde{\wp}_{ij}\right]_{m\times n}$ is shown in Table 7.

Step 3: Again, by Eq. (22) and Table 7, the final aggregated values $\tilde{\wp}_i$ of A_i are found as:

Table 1
q-ROTrFDM $e^{(1)}$

	C_1	C_2	C_3	C_4
A_1	$\langle[0.5, 0.6, 0.8, 0.9]; 0.5, 0.6\rangle$	$\langle[0.6, 0.7, 0.8, 0.9]; 0.5, 0.2\rangle$	$\langle[0.4, 0.7, 0.8, 0.9]; 0.7, 0.4\rangle$	$\langle[0.2, 0.3, 0.4, 0.5]; 0.6, 0.9\rangle$
A_2	$\langle[0.2, 0.3, 0.5, 0.6]; 0.3, 0.6\rangle$	$\langle[0.1, 0.3, 0.6, 0.9]; 0.7, 0.2\rangle$	$\langle[0.4, 0.6, 0.7, 0.9]; 0.3, 0.3\rangle$	$\langle[0.5, 0.6, 0.7, 0.8]; 0.4, 0.8\rangle$
A_3	$\langle[0.3, 0.4, 0.5, 0.9]; 0.4, 0.8\rangle$	$\langle[0.2, 0.3, 0.5, 0.7]; 0.6, 0.1\rangle$	$\langle[0.3, 0.4, 0.5, 0.6]; 0.4, 0.7\rangle$	$\langle[0.4, 0.5, 0.7, 0.8]; 0.3, 0.6\rangle$
A_4	$\langle[0.5, 0.7, 0.8, 0.9]; 0.8, 0.4\rangle$	$\langle[0.2, 0.4, 0.6, 0.8]; 0.3, 0.8\rangle$	$\langle[0.4, 0.5, 0.8, 0.9]; 0.8, 0.5\rangle$	$\langle[0.3, 0.5, 0.6, 0.8]; 0.6, 0.4\rangle$

Table 2
q-ROTrFDM $e^{(2)}$

	C_1	C_2	C_3	C_4
A_1	$\langle[0.4, 0.6, 0.7, 0.8]; 0.6, 0.7\rangle$	$\langle[0.6, 0.7, 0.8, 0.9]; 0.1, 006\rangle$	$\langle[0.5, 0.6, 0.7, 0.8]; 0.3, 0.6\rangle$	$\langle[0.4, 0.5, 0.8, 0.9]; 0.4, 0.4\rangle$
A_2	$\langle[0.5, 0.6, 0.7, 0.8]; 0.6, 0.7\rangle$	$\langle[0.5, 0.7, 0.8, 0.9]; 0.3, 0.4\rangle$	$\langle[0.1, 0.3, 0.5, 0.6]; 0.9, 0.5\rangle$	$\langle[0.3, 0.6, 0.7, 0.8]; 0.5, 0.6\rangle$
A_3	$\langle[0.6, 0.7, 0.8, 0.9]; 0.6, 0.9\rangle$	$\langle[0.2, 0.4, 0.5, 0.7]; 0.4, 0.7\rangle$	$\langle[0.6, 0.7, 0.8, 0.9]; 0.2, 0.6\rangle$	$\langle[0.4, 0.5, 0.7, 0.8]; 0.2, 0.3\rangle$
A_4	$\langle[0.5, 0.6, 0.8, 0.9]; 0.5, 0.6\rangle$	$\langle[0.4, 0.5, 0.6, 0.7]; 0.3, 0.8\rangle$	$\langle[0.4, 0.6, 0.8, 0.9]; 0.5, 0.6\rangle$	$\langle[0.3, 0.4, 0.7, 0.9]; 0.6, 0.4\rangle$

Table 3
q-ROTrFDM $e^{(3)}$

	C_1	C_2	C_3	C_4
A_1	$\langle[0.5, 0.6, 0.8, 0.9]; 0.3, 0.7\rangle$	$\langle[0.5, 0.7, 0.8, 0.9]; 0.2, 0.4\rangle$	$\langle[0.4, 0.7, 0.8, 0.9]; 0.2, 0.4\rangle$	$\langle[0.4, 0.7, 0.8, 0.9]; 0.4, 0.4\rangle$
A_2	$\langle[0.6, 0.7, 0.8, 0.9]; 0.4, 0.6\rangle$	$\langle[0.3, 0.5, 0.7, 0.9]; 0.9, 0.5\rangle$	$\langle[0.4, 0.5, 0.7, 0.9]; 0.4, 0.3\rangle$	$\langle[0.3, 0.5, 0.8, 0.9]; 0.6, 0.3\rangle$
A_3	$\langle[0.5, 0.6, 0.7, 0.8]; 0.1, 0.3\rangle$	$\langle[0.2, 0.3, 0.5, 0.7]; 0.5, 0.8\rangle$	$\langle[0.3, 0.4, 0.5, 0.6]; 0.6, 00.7\rangle$	$\langle[0.4, 0.5, 0.7, 0.8]; 0.3, 0.4\rangle$
A_4	$\langle[0.1, 0.3, 0.5, 0.7]; 0.2, 0.7\rangle$	$\langle[0.2, 0.3, 0.4, 0.5]; 0.3, 0.2\rangle$	$\langle[0.1, 0.2, 0.4, 0.5]; 0.3, 0.7\rangle$	$\langle[0.3, 0.5, 0.6, 0.8]; 0.7, 0.2\rangle$

Table 4
Normalized q-ROTrFDM $\mathcal{N}^{(1)}$

	C_1	C_2	C_3	C_4
A_1	$\langle[0.5, 0.6, 0.8, 0.9]; 0.5, 0.6\rangle$	$\langle[0.6, 0.7, 0.8, 0.9]; 0.5, 0.2\rangle$	$\langle[0.4, 0.7, 0.8, 0.9]; 0.7, 0.4\rangle$	$\langle[0.2, 0.3, 0.4, 0.5]; 0.9, 0.6\rangle$
A_2	$\langle[0.2, 0.3, 0.5, 0.6]; 0.3, 0.6\rangle$	$\langle[0.1, 0.3, 0.6, 0.9]; 0.7, 0.2\rangle$	$\langle[0.4, 0.6, 0.7, 0.9]; 0.3, 0.3\rangle$	$\langle[0.5, 0.6, 0.7, 0.8]; 0.8, 0.4\rangle$
A_3	$\langle[0.3, 0.4, 0.5, 0.9]; 0.4, 0.8\rangle$	$\langle[0.2, 0.3, 0.5, 0.7]; 0.6, 0.1\rangle$	$\langle[0.3, 0.4, 0.5, 0.6]; 0.4, 0.7\rangle$	$\langle[0.4, 0.5, 0.7, 0.8]; 0.6, 0.3\rangle$
A_4	$\langle[0.5, 0.7, 0.8, 0.9]; 0.8, 0.4\rangle$	$\langle[0.2, 0.4, 0.6, 0.8]; 0.3, 0.8\rangle$	$\langle[0.4, 0.5, 0.8, 0.9]; 0.8, 0.5\rangle$	$\langle[0.3, 0.5, 0.6, 0.8]; 0.4, 0.6\rangle$

Table 5
Normalized q-ROTrFDM $\mathcal{N}^{(2)}$

	C_1	C_2	C_3	C_4
A_1	$\langle[0.4, 0.6, 0.7, 0.8]; 0.6, 0.7\rangle$	$\langle[0.6, 0.7, 0.8, 0.9]; 0.1, 0.6\rangle$	$\langle[0.5, 0.6, 0.7, 0.8]; 0.3, 0.6\rangle$	$\langle[0.4, 0.5, 0.8, 0.9]; 0.4, 0.4\rangle$
A_2	$\langle[0.5, 0.6, 0.7, 0.8]; 0.6, 0.7\rangle$	$\langle[0.5, 0.7, 0.8, 0.9]; 0.3, 0.4\rangle$	$\langle[0.1, 0.3, 0.5, 0.6]; 0.9, 0.5\rangle$	$\langle[0.3, 0.6, 0.7, 0.8]; 0.6, 0.5\rangle$
A_3	$\langle[0.6, 0.7, 0.8, 0.9]; 0.6, 0.9\rangle$	$\langle[0.2, 0.4, 0.5, 0.7]; 0.4, 0.7\rangle$	$\langle[0.6, 0.7, 0.8, 0.9]; 0.2, 0.6\rangle$	$\langle[0.4, 0.5, 0.7, 0.8]; 0.3, 0.2\rangle$
A_4	$\langle[0.5, 0.6, 0.8, 0.9]; 0.5, 0.6\rangle$	$\langle[0.4, 0.5, 0.6, 0.7]; 0.3, 0.8\rangle$	$\langle[0.4, 0.6, 0.8, 0.9]; 0.5, 0.6\rangle$	$\langle[0.3, 0.4, 0.7, 0.9]; 0.4, 0.6\rangle$

Table 6
Normalized q-ROTrFDM $\mathcal{N}^{(3)}$

	C_1	C_2	C_3	C_4
A_1	$\langle[0.5, 0.6, 0.8, 0.9]; 0.3, 0.7\rangle$	$\langle[0.5, 0.7, 0.8, 0.9]; 0.2, 0.4\rangle$	$\langle[0.4, 0.7, 0.8, 0.9]; 0.2, 0.4\rangle$	$\langle[0.4, 0.7, 0.8, 0.9]; 0.4, 0.4\rangle$
A_2	$\langle[0.6, 0.7, 0.8, 0.9]; 0.4, 0.6\rangle$	$\langle[0.3, 0.5, 0.7, 0.9]; 0.9, 0.5\rangle$	$\langle[0.4, 0.5, 0.7, 0.9]; 0.4, 0.3\rangle$	$\langle[0.3, 0.5, 0.8, 0.9]; 0.3, 0.6\rangle$
A_3	$\langle[0.5, 0.6, 0.7, 0.8]; 0.1, 0.3\rangle$	$\langle[0.2, 0.3, 0.5, 0.7]; 0.5, 0.8\rangle$	$\langle[0.3, 0.4, 0.5, 0.6]; 0.6, 0.7\rangle$	$\langle[0.4, 0.5, 0.7, 0.8]; 0.4, 0.3\rangle$
A_4	$\langle[0.1, 0.3, 0.5, 0.7]; 0.2, 0.7\rangle$	$\langle[0.2, 0.3, 0.4, 0.5]; 0.3, 0.2\rangle$	$\langle[0.1, 0.2, 0.4, 0.5]; 0.3, 0.7\rangle$	$\langle[0.3, 0.5, 0.6, 0.8]; 0.2, 0.7\rangle$

Table 7
Collective q-ROTrFDM using q-ROTrFEWA operator

	$\mathcal{C}_1$	$\mathcal{C}_2$	$\mathcal{C}_3$	$\mathcal{C}_4$
A_1	$\langle[0.4750, 0.6000, 0.7750, 0.8750]; 0.4918, 0.6540\rangle$	$\langle[0.5700, 0.7000, 0.8000, 0.9000]; 0.3896, 0.3265\rangle$	$\langle[0.4250, 0.6750, 0.7750, 0.8750]; 0.5522, 0.4439\rangle$	$\langle[0.3100, 0.4700, 0.6200, 0.7200]; 0.7511, 0.4819\rangle$
A_2	$\langle[0.3950, 0.4950, 0.6400, 0.7400]; 0.4415, 0.6242\rangle$	$\langle[0.2600, 0.4600, 0.6800, 0.9000]; 0.7479, 0.3145\rangle$	$\langle[0.3250, 0.4950, 0.6500, 0.8250]; 0.6359, 0.3416\rangle$	$\langle[0.3900, 0.5700, 0.7300, 0.8300]; 0.6747, 0.4790\rangle$
A_3	$\langle[0.4350, 0.5350, 0.6350, 0.8700]; 0.4376, 0.6372\rangle$	$\langle[0.2000, 0.3250, 0.5000, 0.7000]; 0.5323, 0.3159\rangle$	$\langle[0.3750, 0.4750, 0.5750, 0.6750]; 0.4585, 0.6743\rangle$	$\langle[0.4000, 0.5000, 0.7000, 0.8000]; 0.4988, 0.2712\rangle$
A_4	$\langle[0.3800, 0.5550, 0.7100, 0.8400]; 0.6546, 0.5277\rangle$	$\langle[0.4350, 0.2500, 0.5400, 0.6850]; 0.3000, 0.5496\rangle$	$\langle[0.3100, 0.4350, 0.6800, 0.7800]; 0.6587, 0.5810\rangle$	$\langle[0.3000, 0.4750, 0.6250, 0.8250]; 0.3615, 0.6291\rangle$

$$\tilde{\wp}_1 = \langle[0.4285, 0.5982, 0.7270, 0.8270]; 0.6121, 0.4502\rangle,$$
$$\tilde{\wp}_2 = \langle[0.3420, 0.5125, 0.7270, 0.6840]; 0.6637, 0.4140\rangle,$$
$$\tilde{\wp}_3 = \langle[0.3490, 0.4552, 0.6090, 0.7542]; 0.4905, 0.4070\rangle,$$
$$\tilde{\wp}_4 = \langle[0.3483, 0.4207, 0.6302, 0.7810]; 0.5182, 0.5811\rangle.$$

Step 4: Utilizing Eq. (3), calculate the scores of $\tilde{\wp}_1$, $\tilde{\wp}_2$, $\tilde{\wp}_3$, and $\tilde{\wp}_4$ as $S(\tilde{\wp}_1) = 0.0891$, $S(\tilde{\wp}_2) = 0.1254$, $S(\tilde{\wp}_3) = 0.0274$, and $S(\tilde{\wp}_4) = -0.0312$.

Step 5: Conferring to the score function, using Definition 2.4, alternatives' ranking is achieved as follows:
$A_2 \succ A_1 \succ A_3 \succ A_4$. Thus, the best alternative is A_2.

Now, developed geometric operator, i.e., q-ROTrFEWG is used to aggregate the separable q-ROTrF data into a communal one.

Step 3: Apply the geometric operator, q-ROTrFEWG, to aggregate all the individual q-ROTrFDMs into a collective q-ROTrFDM $\mathcal{N}' = \left[\tilde{\wp}_{ij}'\right]_{m\times n}$, as shown in Table 8.

Step 4: For collecting overall values $\tilde{\wp}_i'$, aggregate all the preference values $\tilde{\wp}_{ij}'$ $(i = 1, 2, \ldots, 4; j = 1, 2, \ldots, 4)$.

$$\tilde{\wp}_1' = \langle[0.4071, 0.5125, 0.5756, 0.8108]; 0.4258, 0.5093\rangle,$$
$$\tilde{\wp}_2' = \langle[0.2991, 0.4922, 0.6775, 0.8258]; 0.5255, 0.4738\rangle,$$
$$\tilde{\wp}_3' = \langle[0.3288, 0.4414, 0.5985, 0.7477]; 0.4212, 0.5941\rangle,$$
$$\tilde{\wp}_4' = \langle[0.2753, 0.4374, 0.6152, 0.7674]; 0.3882, 0.6393\rangle.$$

Step 5: Use the score function, as displayed in Eq. (3), for finding the score value of $\tilde{\wp}_1'$, $\tilde{\wp}_2'$, $\tilde{\wp}_3'$, and $\tilde{\wp}_4'$. The score values are found as $S(\tilde{\wp}_1') = -0.0352$, $S(\tilde{\wp}_2') = 0.0178$, $S(\tilde{\wp}_3') = -0.0792$, and $S(\tilde{\wp}_4') = -0.1100$.

Step 6: Rank the alternatives based on the above score values, $S(\tilde{\wp}_i')$, using Definition 2.4. Alternatives' ordering is obtained as $A_2 \succ A_1 \succ A_3 \succ A_4$. So, the best alternative is identified as A_2.

We can see that the rankings are the same in two cases, viz., using q-ROTrFWA and q-ROTrFWG operators. Hence, the candidate A_2 is the most potential sales consultant over the other three candidates. As q is assigned different values, the developed approach provides more general and versatile properties when combined with Einstein operations. The proposed approach is superior to other recent research works in real practical decision-making situations.

5.2 Example 5

Another MCGDM problem is previously studied by Zhao et al. (2017) which is looking for the best green supplier for one of the essential components in the automobile production process. Suppose a company sets up a panel with three DMs, viz., e_1, e_2 and e_3, whose weighting vector is $\Omega = (0.35, 0.4, 0.25)^T$. Let there be five supplier A_i $(i = 1, 2, 3, 4, 5)$. We have to evaluate the most suitable alternative through the evaluation process on the basis of four criteria: product quality $\mathcal{C}_1$, technology capability $\mathcal{C}_2$, pollution control $\mathcal{C}_3$, and environment management $\mathcal{C}_4$, whose weighting vector is $\omega = (0.2, 0.1, 0.3, 0.4)^T$), and construct the following three normalized intuitionistic trapezoidal fuzzy decision matrices, $\mathcal{N}^{(l)} = \left(\tilde{\wp}_{ij}^{(l)}\right)_{5\times 4}$ $(l = 1, 2, 3)$ as shown in Tables 9, 10 and 11.

Table 8
Collective q-ROTrFDM using q-ROTrFEWG operator

	$\mathcal{C}_1$	$\mathcal{C}_2$	$\mathcal{C}_3$	$\mathcal{C}_4$
A_1	$\langle[0.4729, 0.6000, 0.7737, 0.8739]; 0.4512, 0.6597\rangle$	$\langle[0.5681, 0.7000, 0.8000, 0.9000]; 0.2559, 0.4264\rangle$	$\langle[0.4229, 0.6735, 0.7737, 0.8739]; 0.3965, 0.4681\rangle$	$\langle[0.2928, 0.4395, 0.5856, 0.6908]; 0.5975, 0.5107\rangle$
A_2	$\langle[0.3497, 0.4600, 0.6262, 0.7281]; 0.3908, 0.6288\rangle$	$\langle[0.2079, 0.4322, 0.6753, 0.9000]; 0.6295, 0.3854\rangle$	$\langle[0.2828, 0.4777, 0.6435, 0.8132]; 0.4428, 0.3723\rangle$	$\langle[0.3775, 0.5681, 0.7286, 0.8288]; 0.5678, 0.5006\rangle$
A_3	$\langle[0.4158, 0.5196, 0.6221, 0.8688]; 0.2950, 0.7706\rangle$	$\langle[0.2000, 0.3224, 0.5000, 0.7000]; 0.5147, 0.6331\rangle$	$\langle[0.3568, 0.4601, 0.5623, 0.6640]; 0.3826, 0.6784\rangle$	$\langle[0.4000, 0.5000, 0.7000, 0.8000]; 0.4495, 0.2813\rangle$
A_4	$\langle[0.3085, 0.5224, 0.6948, 0.8346]; 0.4839, 0.5735\rangle$	$\langle[0.2378, 0.3880, 0.5313, 0.6720]; 0.3000, 0.7233\rangle$	$\langle[0.2639, 0.3975, 0.6498, 0.7545]; 0.5425, 0.5993\rangle$	$\langle[0.3000, 0.4729, 0.6236, 0.8239]; 0.3256, 0.6342\rangle$

After evaluation, the final score values of alternatives are achieved by the proposed methodology as shown in Table 12.

5.2.1 Result and discussion

Using q-ROTrFEWA and q-ROTrFEWG operators, the achieved results are discussed by varying rung parameter, q, continuously in a specified interval as shown in Figures 3 and 4.

Using the q-ROTrFEWA operator and adjusting the rung parameter, q, between 1 and 10, Figure 2 provides the graphical clarification of the score values of the alternatives.

As the value of q changes from 1 to 10, it is noticed in Figure 3 that several ranking results are obtained.

When $q \in [1, 2.1061]$, the alternative's rank is achieved as $A_2 > A_5 > A_3 > A_4 > A_1$.

When $q \in [2.377, 3.3901]$, the alternative's rank is achieved as $A_2 > A_5 > A_4 > A_3 > A_1$. And when $q \in [3.3901, 10]$, the alternative's rank is achieved as $A_2 > A_4 > A_5 > A_3 > A_1$.

Further, Figure 4 signifies the graphical interpretation of score values of the alternatives by varying the rung parameter, q, between 1 and 10, using q-ROTrFEWG operator.

From Figure 4, it is experimental that many ordering results are obtained, as q changes from 1 to 10.

When $q \in (1, 4.1443)$, the ranking of alternatives is achieved as $A_2 > A_5 > A_3 > A_4 > A_1$.

And when $q \in (4.1443, 10)$, the ranking of alternative is achieved as $A_2 > A_3 > A_5 > A_4 > A_1$. So in all cases, we obtained that the A_2 is the best alternative and A_1 is the worst alternative.

5.3 Comparative analysis

The new method is compared to various existing methods in this section.

First, we have compared the results of Example 4 with Aydin's (Aydin et al., 2020) method. The rankings of the Aydin's method (Aydin et al., 2020) and our method are presented in Table 13. The rankings of both Aydin's (Aydin et al., 2020) and proposed methods are the same. However, the score value difference of two consecutive alternatives (rank-wise) in the proposed method is higher than existing Aydin's method (Aydin et al., 2020) almost everywhere.

Next, Example 5 is compared with some existing operators such as ITFWAA (Wang & Zhang, 2009), ITFWG (Wu & Cao, 2013), ITFEWA and ITFEWG (Zhao et al., 2017), PTFWA (Shakeel et al., 2019), and PTFEWG (Shakeel et al., 2019) operators. The score values and rankings of alternatives are described in Table 14.

Table 14 shows that the rankings of the alternatives acquired by different operators are almost identical to the proposed operators, indicating that the proposed ranking technique is effective.

The suggested MCDM strategy based on q-ROTrFN AOs is found to have two key advantages. On the one hand, the q-ROTrFNs used in this work can be used to represent assessment data in a variety of ways. They can also manage a variety of specific situations where a variety of alternative values can cause confusion about the best option while maintaining the accuracy of the original data. The proposed operators, on the other hand, are based on Einstein t-norms and t-conorms, which makes them more beneficial than regular algebraic operations. Furthermore, the proposed approach can provide a variety of options for implementing decision-making with q-ROTrF data. This circumstance can prevent the preferred information from being lost or distorted. As a result, the final outcomes are more closely related to real-world decision-making issues.

Table 9
Normalized decision matrix given by DM e_1

	C_1	C_2	C_3	C_4
A_1	⟨[0.5, 0.6, 0.7, 0.8]; 0.5, 0.4⟩	⟨[0.1,0.2,0.3,0.4]; 0.6,0.3⟩	⟨[0.5,0.6,0.8,0.9]; 0.3,0.6⟩	⟨[0.4,0.5,0.6,0.7]; 0.2,0.7⟩
A_2	⟨[0.6, 0.7, 0.8, 0.9]; 0.7, 0.3⟩	⟨[0.5,0.6,0.7,0.8]; 0.7,0.2⟩	⟨[0.4,0.5,0.7,0.8]; 0.7,0.2⟩	⟨[0.5,0.6,0.7,0.9]; 0.4,0.5⟩
A_3	⟨[0.1, 0.2, 0.4, 0.5]; 0.6, 0.4⟩	⟨[0.2,0.3,0.5,0.6]; 0.5,0.4⟩	⟨[0.5,0.6,0.7,0.8]; 0.5,0.3⟩	⟨[0.3,0.5,0.7,0.9]; 0.2,0.3⟩
A_4	⟨[0.3, 0.4, 0.5, 0.6]; 0.8, 0.1⟩	⟨[0.1,0.3,0.4,0.5]; 0.6,0.3⟩	⟨[0.1,0.3,0.5,0.7]; 0.3,0.4⟩	⟨[0.6,0.7,0.8,0.9]; 0.2,0.6⟩
A_5	⟨[0.2, 0.3, 0.4, 0.5]; 0.6, 0.2⟩	⟨[0.3,0.4,0.5,0.6]; 0.4,0.3⟩	⟨[0.2,0.3,0.4,0.5]; 0.7,0.1⟩	⟨[0.5,0.6,0.7,0.8]; 0.1,0.3⟩

Table 10
Normalized decision given by DM e_2

	C_1	C_2	C_3	C_4
A_1	⟨[0.4,0.5,0.6,0.7]; 0.4,0.3⟩	⟨[0.1,0.2,0.3,0.4]; 0.5,0.2⟩	⟨[0.4,0.5,0.7,0.8]; 0.2,0.5⟩	⟨[0.3,0.4,0.5,0.6]; 0.1,0.6⟩
A_2	⟨[0.5,0.6,0.7,0.8]; 0.6,0.2⟩	⟨[0.4,0.5,0.6,0.7]; 0.6,0.1⟩	⟨[0.3,0.4,0.6,0.7]; 0.6,0.1⟩	⟨[0.3,0.4,0.6,0.8]; 0.3,0.4⟩
A_3	⟨[0.1,0.2,0.3,0.4]; 0.5,0.3⟩	⟨[0.1,0.2,0.4,0.5]; 0.4,0.3⟩	⟨[0.4,0.5,0.6,0.7]; 0.4,0.2⟩	⟨[0.2,0.4,0.6,0.8]; 0.5,0.2⟩
A_4	⟨[0.2,0.3,0.4,0.5]; 0.7,0.1⟩	⟨[0.1,0.2,0.3,0.5]; 0.5,0.2⟩	⟨[0.1,0.2,0.4,0.6]; 0.2,0.3⟩	⟨[0.5,0.6,0.7,0.8]; 0.1,0.5⟩
A_5	⟨[0.1,0.2,0.3,0.4]; 0.5,0.1⟩	⟨[0.2,0.3,0.4,0.5]; 0.3,0.2⟩	⟨[0.1,0.2,0.3,0.4]; 0.6,0.2⟩	⟨[0.4,0.5,0.6,0.7]; 0.4,0.2⟩

Table 11
Normalized decision by DM e_3

	C_1	C_2	C_3	C_4
A_1	⟨[0.6,0.7,0.8,0.9]; 0.4,0.5⟩	⟨[0.2,0.3,0.4,0.5]; 0.5,0.4⟩	⟨[0.6,0.7,0.9,1.0]; 0.2,0.7⟩	⟨[O.5,0.6,0.7,0.8]; 0.1,0.8⟩
A_2	⟨[0.7,0.8,0.9,1.0]; 0.6,0.4⟩	⟨[0.6,0.7,0.8,0.9]; 0.6,0.3⟩	⟨[0.5,0.6,0.8,0.9]; 0.6,0.3⟩	⟨[0.6,0.7,0.8,1.0]; 0.3,0.6⟩
A_3	⟨[0.2,0.3,0.5,0.6]; 0.5,0.5⟩	⟨[0.3,0.4,0.6,0.7]; 0.4 0.5⟩	⟨[0.6,0.7,0.8,0.9]; 0.4,0.4⟩	⟨0.4,0.6,0.8,1.0]; 0.5,0.4⟩
A_4	⟨[0.4,0.5,0.6,0.7]; 0.7,0.2⟩	⟨[0.2,0.4,0.5,0.6]; 0.5,0.4⟩	⟨[0.2,0.4,0.6,0.8]; 0.2,0.5⟩	⟨[0.7,0.8,0.9,1.0]; 0.6,0.3⟩
A_5	⟨[0.3,0.4,0.5,0.6]; 0.5,0.3⟩	⟨[0.4,0.5,0.6,0.7]; 0.3,0.4⟩	⟨[0.3,0.4,0.5,0.6]; 0.6,0.2⟩	⟨[0.6,0.7,0.8,0.9]; 0.4,0.4⟩

Table 12
Score values obtained through the proposed method

Proposed method	Score values					Ranking
	$S(A_1)$	$S(A_2)$	$S(A_3)$	$S(A_4)$	$S(A_5)$	
q-ROTrFEWA	−0.0646	0.0989	0.0377	0.0513	0.0571	$A2 > A_5 > A_4 > A3 > A_1$
q-ROTrFEWG	−0.1143	0.0392	0.0177	−0.0237	0.0195	$A2 > A_5 > A3 > A_4 > A_1$

Figure 3
Effect of rung parameter (*q*) on *q*-ROTrFEWA operator

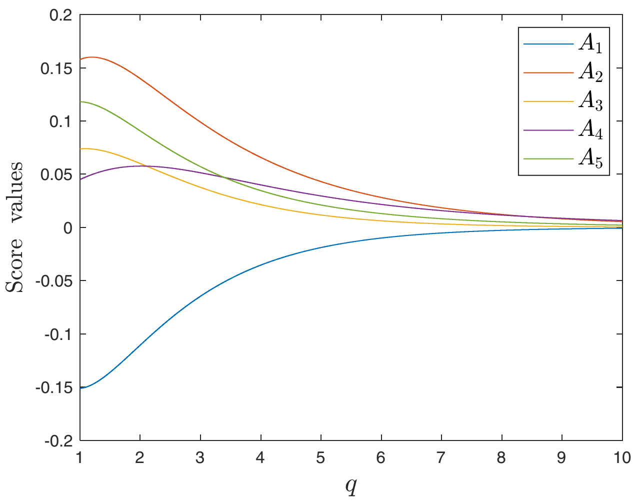


Figure 4
Effect of rung parameter (*q*) on *q*-ROTrFEWG operator

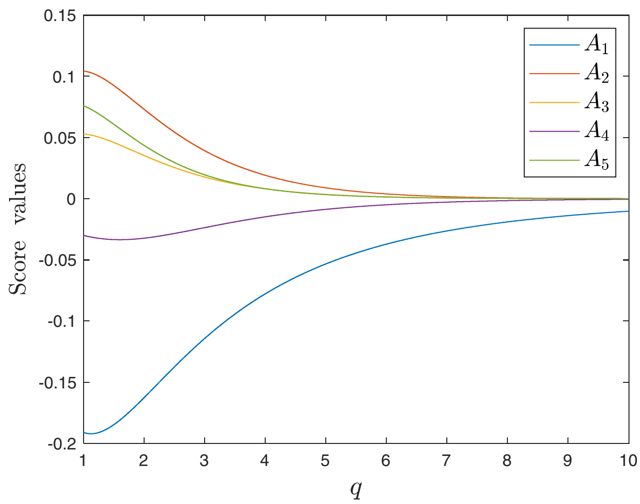


Table 13
Comparison of score value and ranking for Example 4

Method	Score values				Ranking
	$S(A_1)$	$S(A_2)$	$S(A_3)$	$S(A_4)$	
Aydin et al. (2020) method	0.100	0.143	0.093	0.048	$A_2 > A_1 > A_3 > A_4$
q-ROTrFEWA	0.0891	0.1254	0.0274	−0.0312	$A_2 > A_1 > A_3 > A_4$
q-ROTrFEWG	−0.0352	0.0178	−0.0792	−0.1100	$A_2 > A_1 > A_3 > A_4$

Table 14
Comparison based on Example 5

Operators	Score values				Ranking
	$S(A_1)$	$S(A_2)$	$S(A_3)$	$S(A_4)$	
ITFEWA (Shakeel et al., 2019)	−0.1512	0.1577	0.0738	0.0450	$A2 > A_5 > A_3 > A_4 > A_1$
PTFEWG (Shakeel et al., 2019)	−0.1627	0.0730	0.0353	−0.0323	$A2 > A_5 > A_3 > A_4 > A_1$
q-ROTrFEWA	−0.0665	0.0958	0.0370	0.0466	$A2 > A_5 > A_4 > A_3 > A_1$
q-ROTrFEWG	−0.1108	0.0412	0.0181	−0.0224	$A2 > A_5 > A_3 > A_4 > A_1$

6. Conclusion

This research looks into the MCGDM problem using assessment values in the form of q-ROTrFN and proposes a novel MCGDM approach. On the basis of Einstein t-conorm and t-norm, some basic operation laws for q-ROTrFNs are defined. Two AOs based on Einstein operations, q-ROTrFEWA and q-ROTrFEWG, are introduced in this study. Their appropriate characteristics, viz., idempotency, monotonicity, and boundedness, are also defined. Two motives for these expansions are as follows: (1) q-ROTrFN comprise more information than other kinds of fuzzy numbers and (2) Einstein averaging and Einstein geometric operators have the capability to catch the value if there are outliers of data. So, merging Einstein averaging and geometric operators and q-ROTrFN provides advantages in the MCGDM problem. This article tackles a personnel selection problem to demonstrate the applicability of the presented methodology. It proves that the proposed methodology can handle the MCGDM problem efficiently.

However, our study still has some limitations. Our methodology will be unable to determine the best alternative when DMs' and criteria weight are totally unknown. The developed AOs are insufficient to evaluate information when DMs hesitate to make the decision. Our proposed method neglects the preference information of DMs.

In the future, we will develop the concept of hesitant q-ROTrFN. Moreover, various decision-making methods will be extended to handle hesitant q-ROTrFNs. The proposed operators could be used in a variety of domains, viz., bipolar fuzzy (Poulik & Ghorai, 2021), cubic fuzzy (Riaz et al., 2021b), T-spherical fuzzy (Chen, 2021), and other environments. We will continue to work on expanding and applying the proposed operators to other disciplines, such as medical diagnostics (Šušteršič et al., 2021) and pattern recognition (Sánchez-Salgado et al., 2021) and, in the future.

Conflicts of Interest

The authors declare that they have no conflicts of interest to this work.

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