

RESEARCH ARTICLE



Development of a Statistical Reliability-Based Model for the Estimation and Optimization of a Spur Gear System

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Abstract: In this paper, a statistical model, which is based on the stress–strength theorem, has been proposed for the estimation and optimization of spur gear systems embedded (running) in a mechanical device. First, the model is used for the estimation of the variation of the bending stress and the contact pressure distribution in the spur gear systems, which several studies have confirmed as the main source of failure (failure modes) in the spur gear system. The optimal variation values of the stress and strength distribution as well as the reliability of the spur gear systems are then determined via a statistical reliability analysis method embedded in model, which is otherwise referred to as the proactive approach. The proactive approach is mainly applied to evaluate and predict the system failures. The computational results from the analysis show that the higher the stress distribution function, the lower the reliability of the spur gear system. Also, with a distributed strength function of 696.3 MPa and a mean value of 100 MPa of the spur gear system, the optimal stress value is between the range 0–369.5 MPa.

Keywords: proactive model, statistical reliability analysis method, spur gear system, reliability

1. Introduction

Spur gears, which are classified under the cylindrical gear type, are designed to transmit power and motion between parallel shafts which rotate in opposite directions (Mobley, 2001); they are regarded as the most visualized common gears. Spur gears like any other type of gear are generally manufactured from materials with the best physical, mechanical, and chemical properties depending on the industry and the application they are to be deployed. Some of the most common materials that have found application in spur gear design and manufacturing include steel, aluminum, brass, and plastic and are selected using a decision-based method (Aikhuele & Turan, 2017; Drago, 2009).

In the past years, the operation and application of spur gear systems in industries and plant design management have increased significantly, where they have been mostly pushed to their design limits. They are made to operate under higher and variable load and speeds condition and sometimes affected by vibration. These operating conditions, however, have resulted in an increase in the number of failures that is observed in today’s modern gear systems applied in so many industries (Endeshaw et al., 2017). The fatigue strength and wear conditions of the gears, which are one of the most important criteria considered while designing, can be regarded as a key contributory factor to the high failure rate recorded in the

system, where this is due to the billions of load cycles each of the gear tooth need to experience during operation. Thus, in designing a spur gear system a large safety margin which is termed “over-conservative” is usually incorporated into the design (Stoker et al., 2010). The design of spur gear strength is based on the Lewis bending stress and Hertzian contact stress models, and they are related to the stress at the gear base and the wear at the contact surface, respectively (Balaji et al., 2017; Tiwari & Joshi, 2012).

Spur gear systems are subjected to design parameters, speed, and loading uncertainties which emanate from the inherent randomness, manufacturing, and assembly errors, which in some cases are propagated across the whole gear system (Kumar et al., 2017; Riche et al., 2009). For gears that are properly designed and manufactured, abnormal distress or failure can result from misapplication, poor installation, or poor maintenance. Failure in the spur gears can be classified into two categories, that is (Kumar et al. (2017),

1. Tooth breakage (damage) – from excessive bending stress and
2. Surface pitting/wear – from excessive contact stress.

Several design variables can be optimized and analyzed to reduce both the contact pressure and bending stresses, some of which include the center distance, face width, and pressure angle (Arulmozhi et al., 2017; Gopal et al., 2016). The accurate calculation of the stress in spur gears during operation is critically important to the proper analysis and optimization of the gear system.

In the convectional stress models/approaches used in the spur gear system analysis, the main parameter that is assumed to vary

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is the axial separation which is between the gear and pinion (Stoker et al., 2010). However, in practice, both the bending stress and the contact pressure models all vary according to the variation of the axial separation. Some other parameters, which affect the stress and pressure depending on the condition of operation, may include the elastic modulus, applied torque, contact pressure, surface temperature, and the tooth shape (Markovic & Franulovic, 2011).

To predict, estimate, and optimize the variation of the bending stress which usually results in gear tooth breakages, and the contact pressure which is due to the surface pitting/wear, the effect of the parameters needs to be considered more intelligently. These parameters which by nature are uncertain due to variations in their material properties, tolerances in manufacturing, and the random nature of the applied torques (Aziz & Chassapis, 2011; Stoker et al., 2010) and are estimated and sometimes optimized by performing uncertainty analysis, reliability analysis of gear assembly, or using robustness analysis method.

This study, however, will be concerned with the reliability analysis of gear assembly. Several authors and researchers alike have conducted and contributed to the reliability analysis of the spur gears system by enhancing the state of the art, among include. Raja et al. (2014) investigate the root causes of failure in a fractured gear tooth using several metallurgical-based techniques like visual examination technique, micro-hardness technique, tensile strength analysis, chemical composition analysis, and microstructure analysis. Furthermore, they used a genetic algorithm method to optimize the design of the gear train. Alemayehu and Ekwaro-Osire (2014) performed a probabilistic multi-body dynamic analysis in a high-speed-parallel-helical-stage of a wind turbine gearbox by considering and measuring the uncertainties in the following gear parameters: generator-side, torque-loading, input-shaft speed, and the different assembly and design parameters which causes failure due to variations.

Endeshaw et al. (2017) presented a framework that consists of a dynamic model of a one-stage gearbox, a finite element method, as well as a degradation model for the estimation of fatigue crack propagation in the gear system. A torque time historical data for a wind turbine rotor are used to simulate the stochastic characteristic of the loading and uncertainties in the material constants of the degradation model. Sahu et al. (2017) presented a parametric analysis to improve the transmission performance of spur and helical gear by considering the bending and surface strength of the gear tooth which is one of the main causes of failure in gear. The determination of the stresses in the gear system which is aimed at minimizing the failures in the gear system also helps in the optimization of the gear design. Riche et al. (2009) applied a Monte Carlo simulation and kriging method for the optimization of the following helical involute gear parameters: the gear stresses, geometrical feasibility, teeth sliding velocity, contact ratio, and the static transmission error (STE).

Balaji et al. (2017), upon applying the ANSYS software model, estimated the bending and contact stress of a gear system using the following geometrical parameters: the face width and module which are related to the variation of stresses. Barbieri et al. (2008) applied two optimization strategies – the static finite element calculations and the numerical ODE integration method – to model the spur gear noise reduction, where the STE and dynamic transmission error are optimized via genetic algorithms. Marjanović et al. (2017) applied a real-coded genetic algorithm method to study and optimize the gear train volume of a gear system. The objective function is used to minimize the volume with changes in the position of the shaft axes, at the same time comply with all

physical constraints. Mohan and Seshiah (2012) applied a genetic algorithm technique for the optimization of the spur gear set, where the center distance, weight, and the tooth deflections, face width, module, and the number of teeth on the pinion were used as the objective functions, while bending and contact stresses are used as the constraints. Sun et al. (2018) presented a new method based on the genetic algorithm for parameter optimization, optimization which is aimed at minimizing the contact stress in the gear system and uses equal bending fatigue life at the tooth root of pinion and that of the gear. The equation of the contact stress is solved by a genetic algorithm to obtain reasonable parameter estimates for the design.

Pei et al. (2021) proposed an approach for lubrication reliability analysis of spur gear systems that takes into account the randomness of external load and surface roughness. A gear stochastic dynamics model and an elasto-hydrodynamic lubrication reliability model which describe the instantaneous failure probability of gear lubrication are presented and used for determining the numerical features of the meshing force in the gear. The results from the analysis show that the chance of lubrication failure is higher in the dedendum and single meshing regions than in the addendum. A lumped mass model of the spur gear drive system for a railway locomotive is constructed by taking into account internal and external excitations such as time-varying mesh stiffness (TVMS), backlash, transmission error, traction motor torque, and load torque of the wheel/rail. The TVMS is calculated by simplification of a gear tooth as a cantilever beam on the root circle, with the effects of prolonged tooth contact and updated foundation stiffness taken into consideration. The study's findings give theoretical support for the locomotive transmission system's model parameter design (Wang et al. 2018).

Yanzhong et al. (2022) develop a prediction model for stable spiral bevel gear transmission time during a loss-of-lubrication event in a helicopter transmission system. A spiral bevel gear test rig was designed according to the specifications of the tests, and verification experiments were carried out to monitor the temperature change of the spiral bevel gear during operation. The accuracy of the prediction is confirmed by a test for determining the temperature of an oil pool. Tooth surface burn is the most common type of damage to helicopter spiral bevel gears when they are deprived of lubrication. The degree of tooth surface burn control determines the steady running time under oil-free lubrication. The spiral bevel gear oil-free lubrication process experimental data are largely consistent with the computer forecast results. The results provide a foundation for spiral bevel gear working life design in helicopter transmission systems with limited lubrication.

From the foregoing, it is obvious that several attempts have been made to improve spur gear analysis and management. However, there is still a need to extend the current state of the art. First, there is sparse information on a holistic approach that exhaustively quantifies and estimates the physical and operational failure of the spur gear system. Also, to the best of my understanding, there is no model or fully reported tool that could probe or allow for a proactive failure determination and maintenances when the spur gear is in operation or applied in plant design. These knowledge gaps, however, have served as the motivations for the present work; hence, the current study is aimed at presenting a statistical reliability model that could determine the probability of failure and the reliability of the system based on the stresses and strength of the spur gear system. As well as a proactive failure determination and maintenances model for the spur gear system when in operation.

In this paper, the optimal variation values of the stress and strength distribution in spur gears are determined, as well as the reliability of the spur gear system, via a statistical reliability analysis model otherwise referred to a proactive model. Reliability of the system is evaluated when the stresses in the spur gear system are exponentially distributed, and the strength of the system, normally distributed. The statistical reliability analysis method which is based on a stress–strength theorem is used to address the variability in the stress and strength distribution, as well as the reliability of the system. The proactive model is a robust decision-making technique used to find and provide reliability for the spur gear system. The models contribute to the study of spur gear systems by providing a reliability solution that are focus on not only on a specific failure mode type but on several modes that allows for the understanding of the system before it fails and to identify and predict the variations in the stresses and strength that the gear could withstands before it fails, as well as the probability of failure which to the best of my knowledge have not been fully research.

The other parts of the paper are organized as follows: in Section 2, the methodology which consists of the statistical reliability model is introduced. In Section 3, a numerical illustration of the implementation of the model for spur gear systems is presented, while a concluding remark and recommendation are presented in Section 4.

2. Methodology

2.1. Statistical reliability model

Spur gears whose functions are mainly to transmit power and motion, depending on their application in a mechanical system, increase or decrease the torque (power) and speed of the system. The continuous operation and application of the spur gear system most especially under a higher and variable load condition, higher speed, and sometimes under or with vibration can lead to variation in the stresses distributed across the spur gear system. If these stresses exceed the strength of the spur gear system, research has shown that it could result in several failures in the spur gear system (Endeshaw et al., 2017).

To determine the failure in the spur gear system, a statistical reliability analysis model which is based on the stress–strength theorem is applied. The model which used in finding the optimal variation values of the stress and strength distribution as well as the reliability of the spur gear system addresses the stresses in the system when they are exponentially distributed, and when the strength of the system is normally distributed. A definition of the formation of the statistical reliability analysis model is given below.

Definition 1. Let the respective exponentially distributed stresses and normally distributed strength according to Kapur and Lamberson (1941), used for the determination of the probability of failure (reliability function) for the spur gear system, be denoted as,

$$f_s(\delta) = \frac{1}{\sigma_s \sqrt{2\pi}} \exp\left[-\frac{1}{2} \left(\frac{\delta - \mu_s}{\sigma_s}\right)^2\right], \quad -\infty < \delta < \infty \quad (1)$$

and

$$f_s(s) = \lambda e^{-\lambda s}, \quad s \geq 0, \quad (2)$$

From the above, the reliability function of the spur gear system can therefore be derived as follows: when $\mu_s = 1/\lambda$ and $\sigma_s = 1/\lambda$, the

reliability is given as

$$R = \int_0^\infty f_s(\delta) \left[\int_0^\delta f_s(s) ds \right] d\delta \quad (3)$$

Hence,

$$\int_0^\delta f_s(s) ds = \int_0^\delta \lambda e^{-\lambda s} ds = 1 - e^{-\lambda \delta}$$

Upon substituting, we have

$$\begin{aligned} R &= \int_0^\infty \frac{1}{\sigma_s \sqrt{2\pi}} \exp\left[-\frac{1}{2} \left(\frac{\delta - \mu_s}{\sigma_s}\right)^2\right] (1 - e^{-\lambda \delta}) d\delta \\ &= \frac{1}{\sigma_s \sqrt{2\pi}} \int_0^\infty \exp\left[-\frac{1}{2} \left(\frac{\delta - \mu_s}{\sigma_s}\right)^2\right] d\delta \\ &\quad - \frac{1}{\sigma_s \sqrt{2\pi}} \int_0^\infty \exp\left[-\frac{1}{2} \left(\frac{\delta - \mu_s}{\sigma_s}\right)^2\right] e^{-\lambda \delta} d\delta \\ &= 1 - \Phi\left(-\frac{\mu_s}{\sigma_s}\right) - \frac{1}{\sigma_s \sqrt{2\pi}} * T \end{aligned} \quad (4)$$

where $T = \int_0^\infty \exp\left[-\frac{1}{2\sigma_s^2} ((\delta - \mu_s + \lambda\sigma_s^2)^2 + 2\mu_s\sigma_s^2 - \lambda^2\sigma_s^4)\right] d\delta$
If $t = (\delta - \mu_s + \lambda\sigma_s^2)^2/d\delta$, then $\sigma_s dt = d\delta$; the expression for the reliability R can then be rewritten as

$$\begin{aligned} R &= 1 - \Phi\left(-\frac{\mu_s}{\sigma_s}\right) - \frac{1}{\sqrt{2\pi}} \int_{-\frac{\mu_s - \lambda\sigma_s^2}{\sigma_s}}^\infty \exp\left[-\frac{t^2}{2}\right] \\ &\quad * \exp\left[-\frac{1}{2}(2\mu_s\lambda - \lambda^2\sigma_s^2)\right] dt \end{aligned}$$

If $X = -\frac{\mu_s - \lambda\sigma_s^2}{\sigma_s}$, the above can be rewritten as,

$$\begin{aligned} R &= 1 - \Phi\left(-\frac{\mu_s}{\sigma_s}\right) - \frac{1}{\sqrt{2\pi}} \int_X^\infty \exp\left[-\frac{t^2}{2}\right] \\ &\quad * \exp\left[-\frac{1}{2}(2\mu_s\lambda - \lambda^2\sigma_s^2)\right] dt \end{aligned} \quad (5)$$

Hence, the reliability function R is given as

$$\begin{aligned} R &= 1 - \Phi\left(-\frac{\mu_s}{\sigma_s}\right) - \exp\left[-\frac{1}{2}(2\mu_s\lambda - \lambda^2\sigma_s^2)\right] \\ &\quad * \left[1 - \Phi\left(-\frac{\mu_s - \lambda\sigma_s^2}{\sigma_s}\right) \right] \end{aligned} \quad (6)$$

where μ_s is the strength, σ_s is the stress, and λ is the distribution mean value.

3. Numerical Illustration of the Model for the Spur Gear System

In this section, the model presented in the above section is applied for the evaluation of the reliability of 10 spur gear systems used in running a mechanical device and for the determination of the optimal variation values of the stress and strength distribution of a spur gear system designed with a carbon steel material. A pictorial diagram of the mechanical device which contains the spur gear systems is presented in Figure 1.

Figure 1
A pictorial diagram of the spur gear system



3.1. Application of the statistical reliability model

Using the statistical reliability model, it is assumed that each of the spur gear systems has a varying deteriorating strength, such that the spur gear systems are modeled using a normally distributed process with a strength function μ_δ and a common distribution mean value λ . The reliability of each of the spur gear systems can be evaluated if the spur gear systems are subjected to an exponentially distributed stress function σ_δ using equation (6). Using the above information, the 10 spur gear systems in the mechanical device are evaluated, first by varying the numerical distributed stress–strength values with a common distribution mean value. The computation and their corresponding reliability values and results for the spur gear system as calculated with equation (6) are presented in Table 1.

From the computational result presented in Table 1, it is not hard to see that the reliability of each of the 10 gear systems is greatly affected by the variability of the stress and strength distribution functions when distribution mean value is constant. Also, the results agree with an existing result originally presented in Hassan et al. (2015), who concluded that the higher the strength distribution function of a system, the higher the reliability of such system, and the lower the stress values, the higher the reliability.

To determine the optimal variation values of the stress and strength distribution functions as well as the distribution mean value and to validate the above results, a common stress value of 409.5 MPa was used for the spur gear systems evaluation with a varying strength distribution function. The results of the computation and their corresponding graphical representation have been shown and depicted in Table 2 and Figure 2, respectively. The results show that the higher the strength distribution functions in the spur gear system, the higher their reliability. The study can therefore confirm that the system reliability fails at a lower strength distribution function of the spur gear system and becomes better at a higher strength distribution function. Hence, the

Table 1
Reliability of the spur gear systems with variation in strength and stress function

<i>n</i> spur gear systems	Strength function (μ_δ) MPa	Distribution means value (λ) MPa	Stress function (σ_δ) MPa	Reliability value (<i>R</i>)
1	696.3	100	369.5	0.1276
2	753.7	100	353.3	0.7264
3	755.1	100	369.4	0.5172
4	794.3	100	390.6	0.2699
5	853.2	100	378.5	0.7456
6	863.5	100	409.5	0.2216
7	882.6	100	360.8	0.9014
8	980.7	100	360.8	0.9630
9	1010.1	100	400.5	0.8752
10	1108.2	100	369.5	0.9858

optimal variation values of the strength distribution at a constant stress value of 409.5 MPa for high reliability of a spur gear system are anything between and above 853.2 MPa.

To further verify the rationality of the model for finding the optimal variation values of the stress and strength distribution, as well as the reliability of the system, a common strength distribution function of 696.3 MPa and a common distribution mean value of 100 MPa are used for the spur gear system’s evaluation with several varying stress distribution functions which were obtained via mathematical calculation.

The results of the computation are presented in Table 3, showing that the higher the stress distributions function, the lower the reliability of the spur gear system. The result also shows that for a spur gear system with a distributed strength function of 696.3 MPa and a mean distribution value of 100 MPa, the optimal

Table 2
Reliability of the spur gear systems with variation in the strength distribution function

<i>n</i> spur gear systems	Strength function (μ_δ) MPa	Distribution means value (λ) MPa	Stress function (σ_δ) MPa	Reliability value (<i>R</i>)	Remarks
1	696.3	100	409.5	-3.1434	Fail
2	753.7	100	409.5	-1.3338	Fail
3	755.1	100	409.5	-1.3014	Fail
4	794.3	100	409.5	-0.5551	Fail
5	853.2	100	409.5	0.2181	Good
6	863.5	100	409.5	0.2216	Good
7	882.6	100	409.5	0.3569	Good
8	980.7	100	409.5	0.7589	Good
9	1010.1	100	409.5	0.8203	Good
10	1108.2	100	409.5	0.9326	Good

Figure 2
Increased reliability of the spur gear due to increase in strength distribution function

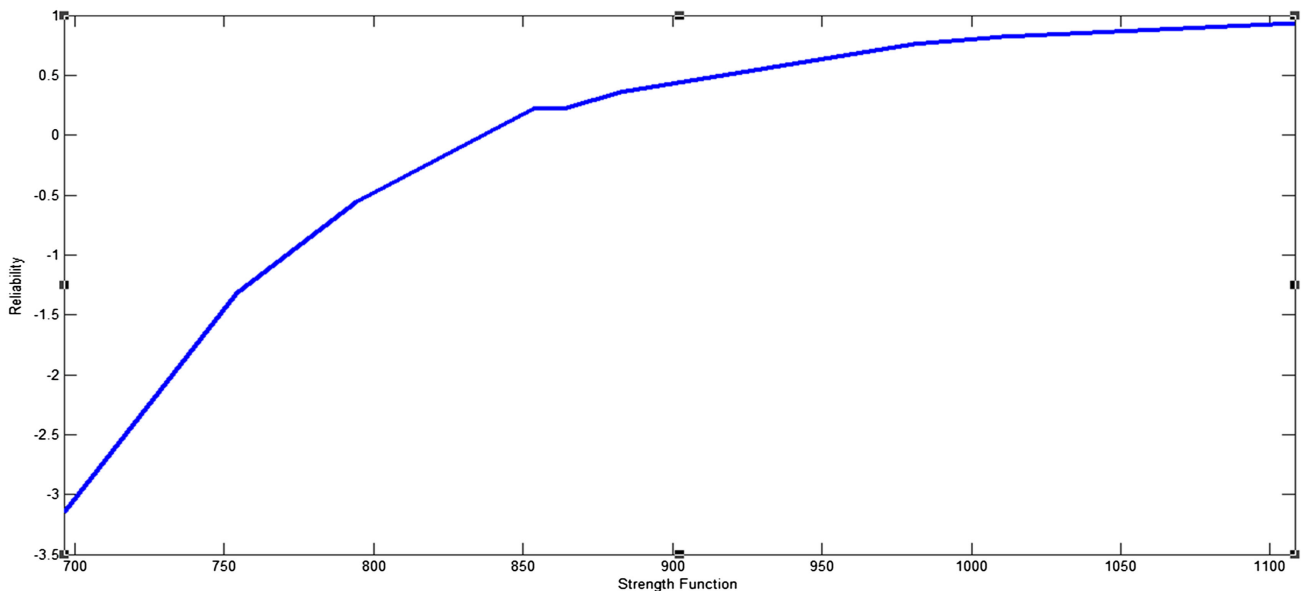


Table 3
Reliability of the spur gear systems with variation in the stress distribution function

<i>n</i> spur gear systems	Strength function (μ_δ) MPa	Distribution means value (λ) MPa	Stress function (σ_δ) MPa	Reliability value (<i>R</i>)	Remarks
1	696.3	100	353.3E-3	0.514	Good
2	696.3	100	360.8 E-3	0.365	Good
3	696.3	100	360.8 E-3	0.365	Good
4	696.3	100	369.4 E-3	0.131	Good
5	696.3	100	369.5 E-3	0.128	Good
6	696.3	100	369.5 E-3	0.128	Good
7	696.3	100	400.5 E-3	-0.188	Fail
8	696.3	100	409.5 E-3	-3.143	Fail

stress values the system can take are between the ranges 0–369.5 MPa. This implies that, at a stress level above this range (stress limit), the system is bound to fail; hence, the material used for the design of the spur gear system needs to be improved or checked.

4. Conclusions

Spur gears, which are designed to transmit power and motion between parallel shafts which rotate in opposite directions, are

regarded as the most visualized common gear in the market. To predict, estimate, and optimize the variation of the bending stress and the contact pressure distribution which is the broad classification of failure modes in the spur gear system, an intelligent and proactive model is needed.

In this paper, the optimal variability values of the stress and strength distribution of spur gear system are determined, as well as the reliability of the spur gear system, via a statistical reliability analysis method otherwise referred to as proactive model of the spur gear system. When the stresses in the spur gear system are exponentially distributed and the strength normally distributed. The statistical reliability analysis model, which is based on a stress-strength reliability theorem, addresses the variability in the stress and strength distribution, as well as the probability of failure.

The computational results via the statistical reliability analysis show that the higher the stress distribution function, the lower the reliability of the spur gear system, and it is in agreement with an existing result originally presented in Hassan et al. (2015). The result also shows that for a spur gear system with a distributed strength functions of 696.3 MPa and a mean value of 100 MPa, the optimal stress values the spur gear system can withstand are between 0 and 369.5 MPa. Which implies that, at a stress level above this range (stress limit), the system is bound to fail; hence, the material used for the design of the spur gear system needs to be improved or checked. In the future, the study will be focused on investigating failure modes associated with the spur gear system, by prioritizing the modes to obtain the highest and least risk failure modes as well as the failure modes with the reliability concerns.

Conflicts of Interest

The author declares that he has no conflicts of interest to this work.

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