



RESEARCH ARTICLE

Fuzzy Quadripartitioned Neutrosophic Soft Matrix Theory and Its Decision-Making Approach

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Abstract: Earlier several fuzzy oriented matrix theories have been proposed by the researchers to model uncertainty for real-world decision-making (DM). Among them, fuzzy neutrosophic soft matrix (FNSM) theory has been developed recently by combining the fuzzy neutrosophic set and soft set to handle the indeterminate information parametrically. The main purpose to represent this paper is to introduce the notion of fuzzy quadripartitioned neutrosophic soft matrix (FQNSM) theory to generalize the FNSM concept. Moreover, several properties and matrix operations on fuzzy quadripartitioned neutrosophic soft sets are investigated with appropriate examples. Finally, a DM model based on FQNSMs has been developed and thus validated by showing an application to real problems.

Keywords: soft set; quadripartitioned neutrosophic soft set; fuzzy quadripartitioned neutrosophic soft set; fuzzy quadripartitioned neutrosophic soft matrix; decision-making

1. Introduction

Nowadays, most real-world problems contain uncertainty and it arises due to the unknown, incomplete, imprecise, inconsistent, or vague information, driven by human knowledge. Therefore, it is difficult for the decision-makers to make a precise decision about a particular problem. The introduction of fuzzy set (FS) theory, proposed by Zadeh (1965), has been applied successfully to overcome such issues quite significantly over the decades. We use FS to measure the degree of belongingness of an object to a universe. It is not useful to measure the hesitation degree or non-belongingness of an object, which is important in scientific computation. Later on, the FS has been extended by introducing the intuitionistic fuzzy set (Atanassov, 1983), hesitant fuzzy set (Torra, 2010), Pythagorean fuzzy set (Yager, 2013), q-rung orthopair fuzzy set (Yager, 2016), linear Diophantine fuzzy set (Riaz & Hashmi, 2019), and fermatean fuzzy set (Senapati & Yager, 2019). But the main obstacle in using FS is to set up the membership function to address various types of uncertainty. Such obstacle has been removed quite significantly by introducing the SS by Molodtsov (1999). The introduction of SS theory provides a new dimension for the researchers and encourages them to enrich their research area with more variation while handling uncertain information. Using SS, we give an approximate description of a set of alternatives along with parameters. It becomes popular among the researchers which yield new theories such as fuzzy soft set (Cagman et al., 2011) and its application, intuitionistic fuzzy soft set (Çağman & Karataş, 2013) theory in decision-making (DM), introduction to interval-valued fuzzy soft set (Yang et al., 2009), interval-valued intuitionistic fuzzy

soft sets (Jiang et al., 2010) and their properties, neutrosophic soft set (NSS) (Maji, 2013), quadripartitioned neutrosophic soft set (Kumar & Mary, 2021), Pythagorean fuzzy soft set (Peng et al., 2015) and its application, etc.

The role of matrix theory is unparallel in scientific computing because of its concise representation of big data and it is helpful for computer storage for future applications. Based on the above discussion, the classical matrix theory has been generalized in the following manner: Çağman and Enginoğlu (2010) introduced the soft matrix (SM) theory in DM. Borah et al. (2012) proposed the fuzzy soft matrix (FSM) theory in DM. Rajarajeswari and Dhanalakshmi (2013) presented the intuitionistic fuzzy soft matrix (IFSM) theory and applied it in medical diagnosis. They also defined the interval-valued fuzzy soft matrix theory and interval-valued intuitionistic fuzzy soft matrix theory (2014(a); 2014(b)), respectively. Deli and Broumi (2015) initiated the neutrosophic soft matrices (NSMs) and utilize them for DM. More works are based on NSMs given in Basu and Mondal (2015), Das et al. (2019), Bera and Mahapatra (2016), and Jafar et al. (2020). Furthermore, Arockiarani (2014) introduced the fuzzy neutrosophic soft matrix (FNSM) approach in DM. New operations on FNSMs are proposed in Sumathi and Arockiarani (2014). Uma et al. (2021) propounded the Type-I and Type-II FNSMs. Kavitha et al. (2017) described the λ -robustness of FNSM. Petchimuthu et al. (2020) introduced the mean operators of fuzzy soft matrices and applied them in multicriteria group decision making (MCGDM). Murugadas and Kavitha (2021) introduced the convergence of fuzzy neutrosophic soft circulant matrices. Gulistan et al. (2019) presented a new approach to DM using the neutrosophic cubic soft matrices. Das et al. (2019) have shown an algorithmic approach using NSM in GDM. Khan et al. (2021) defined the complex NSMs in the application of signal processing. Guleria and Bajaj (2019) proposed different operations on Pythagorean fuzzy soft matrices.

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Smarandache (2005) invented a new mathematical tool known as a neutrosophic set (NS) that helps to extend the fuzzy logic concept. In NS, the truth membership, indeterminate membership, and falsity membership degrees are unrestricted by assigning the non-standard interval. So, it is worthy to represent any quantitative concept without any restriction, but it seems to be difficult for any real application. To overcome such difficulties, Wang et al. (2010) introduced the single-valued neutrosophic set (SVNS). Aiming to achieve the same goal, Arockiarani et al. defined the fuzzy neutrosophic set (FNS). To realize the complexity present in human knowledge while handling indeterminate information, Chaterjee et al. (2016), in their paper, defined the notion of quadripartitioned single-valued neutrosophic set (QSVNS). Also, Kamici et al. (2021) presented the bipolar trapezoidal neutrosophic sets and their Dombi operators in MCDM. Rough approximations of complex QSVNSs were studied in Kamaci (2021), and correlation coefficients of simplified neutrosophic multiplicative refined sets and their application in pattern recognition were described in Kamaci (2021). The QSVNS provides more knowledge for the decision-makers as the indeterminacy component in the NS is divided into two parts, namely contradiction and unknown. So, the use of QSVNS is more logical than the NS.

Combining the FNS and QNS, in this work, we have defined the fuzzy quadripartitioned neutrosophic set (FQNS) that is analogous to QSVNS. After that, embedding FQNS with SS, a fuzzy quadripartitioned neutrosophic soft set (FQNSS) is formed. Our study is mainly based on FQNSS and its matrix representation. Using the matrix representation of FQNSS, a new matrix theory called a fuzzy quadripartitioned neutrosophic soft matrix (FQNSM) theory and its associated properties are established.

Soft matrices are defined on soft sets (SSs) where each entry assigns only two values (0 and 1). To handle a more complex situation where we are not certain about the acceptance or rejection of the parametric attribute, the soft matrix theory failed to address such a situation. This led to the introduction of a FSM, IFSM, and Pythagorean fuzzy soft matrix, whereas to handle indeterminate parametric attributes in matrix form, NSM and FNSM are studied. Due to more complexity involved in indeterminate parametric data, we need to split the indeterminacy into two parts, namely the contradiction and the unknown; they analyze indeterminacy with more precision. So, we need another powerful tool to take care of such a situation. This is the main motivation for introducing the FQNSM theory in our study. We also believe that the matrix representation based on FQNSS has not been found in any research paper. Also, by introducing the FQNSM we extend the FSM, IFSM, NSM, FNSM, etc.

The objectives of the proposed study are furnished below:

- Redefine the QNS and then formulate the FQNS and FQNSS.
- Obtain the matrix representation of FQNSS called the FQNSM.
- Extend the FNSM to FQNSM.
- To obtain the score function, accuracy function, average product, etc. using the FQNSM in the DM.
- To construct an algorithm.
- Apply the proposed model in medical DM.

2. Preliminaries

In this section, we recollect some basic definitions that are essential for the rest of the paper.

Definition 2.1. (Zadeh, 1965)

Suppose X be a set of the universe. A fuzzy set F over X is defined as $F = \{ \langle x, \mu_F(x) \rangle : x \in X \}$, where the membership function μ_F is defined as $\mu_F : X \rightarrow [0, 1]$.

Definition 2.2. (Smarandache, 2005)

A NS N over X is defined as follows:

$$N = \{ \langle x, T_N(x), I_N(x), F_N(x) \rangle : x \in X \},$$

where $T_N(x), I_N(x)$, and $F_N(x)$ denote the truth membership degree, indeterminate membership degree, and the false membership degree such that $T_N, I_N, F_N : X \rightarrow]-0, 1^+[$ and $-0 \leq T_N(x) + I_N(x) + F_N(x) \leq 3^+$.

For quantitative measures, we use the SVNS (Wang et al., 2010) instead of the NS.

Definition 2.3. (Molodtsov, 1999)

Let X be a set of universe and E be a set of parameters. Also, let $A \subseteq E$ and I^X denote the power set of X . Then a SS is denoted by an ordered pair (H, I^X) , where H is an operator so that $H : A \rightarrow I^X$.

Definition 2.4. (Arockiarani, 2014)

Let X be the universe of discourse. Then, a FNS A over X is defined as $A = \{ \langle x, T_A(x), I_A(x), F_A(x) \rangle : x \in X \}$ with the restriction $T_A, I_A, F_A : X \rightarrow [0, 1]$ and $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$.

Definition 2.5. A quadripartitioned neutrosophic set (QNS) S over the set of the universe X is defined as

$S = \{ \langle x, T_S(x), C_S(x), U_S(x), F_S(x) \rangle : x \in X \}$ under the condition $T_S, C_S, U_S, F_S : X \rightarrow]-0, 1^+[$ such that $-0 \leq T_S(x) + C_S(x) + U_S(x) + F_S(x) \leq 4^+$. Kumar and Mary (2021) similarly defined the QNS but they consider the restriction as $0 \leq T_S(x) + C_S(x) + U_S(x) + F_S(x) \leq 4$.

As we know in NS, the sum of the membership triplet is not restricted; thus, we modify the restricted condition in this definition.

Here $T_S(x), C_S(x), U_S(x)$ and $F_S(x)$ denote the truth membership, contradiction membership, ignorance membership, and falsity membership, respectively. Clearly, in QNS there is no limitation of considering the membership degrees. Though it has an enormous advantage of making any qualitative DM, the scientific computation demands quantitative DM. For which QSVNS is introduced (Chaterjee et al., 2016).

Definition 2.6. (Sumathi & Arockiarani, 2014)

Suppose X be a set of the universe and E is a set of parameters. Again, let $B \subseteq E$ and $I^{FNS(X)}$ denote the collection of all FNSs of X . Then, a fuzzy neutrosophic soft set (FNSS) is denoted by the pair $(G, I^{FNS(X)})$, where G is a mapping denoted by $G : B \rightarrow I^{FNS(X)}$.

Definition 2.7. (Sumathi & Arockiarani, 2014)

Let $X = \{u_1, u_2, \dots, u_m\}$ be the set of the universe and $E = \{\varepsilon_1, \varepsilon_2, \dots, \varepsilon_n\}$ be the set of parameters. For $B \subseteq E$, $(G, I^{FNS(X)})$ be a FNSS over X . Then the relation set \mathfrak{R}_B can be considered as a subset of the Cartesian product $X \times E$ and it is defined by $\mathfrak{R}_B = \{ \langle u, \varepsilon \rangle : \varepsilon \in B, u \in h_B(\varepsilon) \}$ which is the relation form of (h_B, E) . Based on the relation set \mathfrak{R}_B , we define $T_{\mathfrak{R}_B}, I_{\mathfrak{R}_B}, F_{\mathfrak{R}_B} : X \times E \rightarrow [0, 1]$ as the truth, indeterminacy, and falsity membership functions, respectively, and they are used to obtain the respective membership values of $u \in X$ for each $\varepsilon \in E$.

The matrix representation of the FNSS $(G, I^{FNS(X)})$ over X is given by

$$[\langle T_{ij}, I_{ij}, F_{ij} \rangle]_{m \times n} = \begin{bmatrix} \langle T_{11}, I_{11}, F_{11} \rangle & \langle T_{12}, I_{12}, F_{12} \rangle & \cdots & \langle T_{1n}, I_{1n}, F_{1n} \rangle \\ \langle T_{21}, I_{21}, F_{21} \rangle & \langle T_{22}, I_{22}, F_{22} \rangle & \cdots & \langle T_{2n}, I_{2n}, F_{2n} \rangle \\ \langle T_{31}, I_{31}, F_{31} \rangle & \langle T_{32}, I_{32}, F_{32} \rangle & \cdots & \langle T_{3n}, I_{3n}, F_{3n} \rangle \\ \vdots & \vdots & \vdots & \vdots \\ \langle T_{m1}, I_{m1}, F_{m1} \rangle & \langle T_{m2}, I_{m2}, F_{m2} \rangle & \cdots & \langle T_{mn}, I_{mn}, F_{mn} \rangle \end{bmatrix}_{m \times n}$$

Definition 2.8. (Sumathi & Arockiarani, 2014) Let $(G, I^{FNS(X)})$ be a FNSS over X where $G : B \rightarrow I^{FNS(X)}$. Then the matrix representation of $(G, I^{FNS(X)})$ is $B_{m \times n} = [b_{ij}]_{m \times n}$ and it is defined as

$$b_{ij} = \begin{cases} \langle T_j(u_i), I_j(u_i), F_j(u_i) \rangle & \text{if } \varepsilon_j \in B \\ (0, 0, 1) & \text{if } \varepsilon_j \notin B \end{cases}$$

Definition 2.9. The FQNS K over the universe of discourse X is defined as $K = \{ \langle u, T_K(u), C_K(u), U_K(u), F_K(u) \rangle : u \in X \}$, where $T_K, C_K, U_K, F_K : X \rightarrow [0, 1]$ such that $0 \leq T_K(u) + C_K(u) + U_K(u) + F_K(u) \leq 4$.

It is to be noted that the FQNS is defined in a similar way to QSVNS (Chatterjee et al., 2016).

Definition 2.10. Let $I^{FQNS(X)}$ denote the collection of all FQNSs over X and E denote the set of parameters, where $Z \subseteq E$. Then the pair $(D, I^{FQNS(X)})$ is called a FQNSS over X where $D : Z \rightarrow I^{FQNS(X)}$.

3. Matrix Representation of Fuzzy Quadripartitioned Neutrosophic Soft Set

Definition 3.1. Let $\vec{X} = \{ \vec{u}_1, \vec{u}_2, \dots, \vec{u}_m \}$ be the set of the universe and $\vec{E} = \{ \vec{\varepsilon}_1, \vec{\varepsilon}_2, \dots, \vec{\varepsilon}_n \}$ be the set of parameters. For $\vec{B} \subseteq \vec{E}$, $(\vec{G}, I^{FQNS(\vec{X})})$ be a FQNSS over \vec{X} . Then the relation set $\mathfrak{R}_{\vec{B}}$ can be considered as a subset of the Cartesian product $\vec{X} \times \vec{E}$ and it is defined by $\mathfrak{R}_{\vec{B}} = \{ (\vec{u}, \vec{\varepsilon}) : \vec{\varepsilon} \in \vec{B}, \vec{u} \in \vec{X} \}$ which is the relation form of (\vec{X}, \vec{E}) . Based on the relation set $\mathfrak{R}_{\vec{B}}$, we define $T_{\mathfrak{R}_{\vec{B}}}, C_{\mathfrak{R}_{\vec{B}}}, U_{\mathfrak{R}_{\vec{B}}}, F_{\mathfrak{R}_{\vec{B}}} : \vec{X} \times \vec{E} \rightarrow [0, 1]$ as the truth, contradiction, ignorance, and falsity membership functions, respectively, and they are used to obtain the respective membership values of $\vec{u} \in \vec{X}$ for each $\vec{\varepsilon} \in \vec{E}$.

The matrix representation of the FQNSS $(\vec{G}, I^{FQNS(\vec{X})})$ over X is given by

$$[\langle T_{ij}, C_{ij}, U_{ij}, F_{ij} \rangle]_{m \times n} = \begin{bmatrix} \langle T_{11}, C_{11}, U_{11}, F_{11} \rangle & \langle T_{12}, C_{12}, U_{12}, F_{12} \rangle & \cdots & \langle T_{1n}, C_{1n}, U_{1n}, F_{1n} \rangle \\ \langle T_{21}, C_{21}, U_{21}, F_{21} \rangle & \langle T_{22}, C_{22}, U_{22}, F_{22} \rangle & \cdots & \langle T_{2n}, C_{2n}, U_{2n}, F_{2n} \rangle \\ \langle T_{31}, C_{31}, U_{31}, F_{31} \rangle & \langle T_{32}, C_{32}, U_{32}, F_{32} \rangle & \cdots & \langle T_{3n}, C_{3n}, U_{3n}, F_{3n} \rangle \\ \vdots & \vdots & \vdots & \vdots \\ \langle T_{m1}, C_{m1}, U_{m1}, F_{m1} \rangle & \langle T_{m2}, C_{m2}, U_{m2}, F_{m2} \rangle & \cdots & \langle T_{mn}, C_{mn}, U_{mn}, F_{mn} \rangle \end{bmatrix}_{m \times n}$$

Definition 3.2. Let $(\vec{G}, I^{FQNS(\vec{X})})$ be a FQNSS over \vec{X} , where $\vec{G} : \vec{B} \rightarrow I^{FQNS(\vec{X})}$. Then the matrix representation of $(\vec{G}, I^{FQNS(\vec{X})})$ is $\vec{B}_{m \times n} = [\vec{b}_{ij}]_{m \times n}$ and it is defined as

$$\vec{b}_{ij} = \begin{cases} \langle \langle T_j(\vec{u}_i), C_j(\vec{u}_i), U_j(\vec{u}_i), F_j(\vec{u}_i) \rangle \rangle & \text{if } \vec{\varepsilon}_j \in \vec{B} \\ ((0, 0, 1, 1)) & \text{if } \vec{\varepsilon}_j \notin \vec{B} \end{cases}$$

Note. The fuzzy quadripartitioned neutrosophic soft matrix is denoted by FQNSM.

We now discuss the different types of FQNSMs.

Definition 3.3. Let $\vec{A} = [\langle T_{ij}^{\vec{A}}, C_{ij}^{\vec{A}}, U_{ij}^{\vec{A}}, F_{ij}^{\vec{A}} \rangle]_{m \times n} = [\vec{a}_{ij}]_{m \times n}$, $\vec{B} = [\langle T_{ij}^{\vec{B}}, C_{ij}^{\vec{B}}, U_{ij}^{\vec{B}}, F_{ij}^{\vec{B}} \rangle]_{m \times n} = [\vec{b}_{ij}]_{m \times n} \in FQNSM_{m \times n}$. Then \vec{A} is called a fuzzy quadripartitioned neutrosophic soft sub-matrix of \vec{B} , denoted by $\vec{A} \subseteq \vec{B}$ if $T_{ij}^{\vec{A}} \leq T_{ij}^{\vec{B}}, C_{ij}^{\vec{A}} \leq C_{ij}^{\vec{B}}, U_{ij}^{\vec{A}} \geq U_{ij}^{\vec{B}}$, and $F_{ij}^{\vec{A}} \geq F_{ij}^{\vec{B}}, \forall i, j$. If $\vec{A} \subseteq \vec{B}$ and $\vec{B} \subseteq \vec{A}, \forall i, j$, then $\vec{A} = \vec{B}$. In other way, if $T_{ij}^{\vec{A}} = T_{ij}^{\vec{B}}, C_{ij}^{\vec{A}} = C_{ij}^{\vec{B}}, U_{ij}^{\vec{A}} = U_{ij}^{\vec{B}}$, and $F_{ij}^{\vec{A}} = F_{ij}^{\vec{B}}, \forall i, j$ then $\vec{A} = \vec{B}$.

Definition 3.4. A FQNSM of any order is said to be a fuzzy quadripartitioned neutrosophic soft null matrix if all its elements are $((0, 0, 1, 1))$ and it is denoted by $\vec{0}$.

Definition 3.5. A FQNSM of any order is said to be a fuzzy quadripartitioned neutrosophic soft universal matrix if all its elements are $((1, 1, 0, 0))$ and it is denoted by $\vec{1}$.

Definition 3.6. If $\vec{A} = [\langle T_{ij}^{\vec{A}}, C_{ij}^{\vec{A}}, U_{ij}^{\vec{A}}, F_{ij}^{\vec{A}} \rangle]_{m \times n} = [\vec{a}_{ij}]_{m \times n} \in FQNSM$. Then its complement is denoted by $\vec{A}^c = [a_{ij}^c]_{m \times n} \in FQNSM$ where $[a_{ij}^c] = [(1 - T_{ij}^{\vec{A}}, 1 - C_{ij}^{\vec{A}}, 1 - U_{ij}^{\vec{A}}, 1 - F_{ij}^{\vec{A}})]$.

Definition 3.7. Let $\vec{A} = [\vec{a}_{ij}]_{m \times n}, \vec{B} = [\vec{b}_{ij}]_{m \times n} \in FQNSM$. Then their sum is denoted and defined by

$$\vec{A} + \vec{B} = [\vec{c}_{ij}]_{m \times n} = \left\langle \begin{matrix} \max(T_{ij}^{\vec{A}}, T_{ij}^{\vec{B}}), & \max(C_{ij}^{\vec{A}}, C_{ij}^{\vec{B}}), \\ \min(U_{ij}^{\vec{A}}, U_{ij}^{\vec{B}}), & \min(F_{ij}^{\vec{A}}, F_{ij}^{\vec{B}}) \end{matrix} \right\rangle, \forall i, j$$

Definition 3.8. Let $\vec{A} = [\vec{a}_{ij}]_{m \times n}, \vec{B} = [\vec{b}_{ij}]_{m \times n} \in FQNSM$. Then their difference is denoted and defined by

$$\vec{A} - \vec{B} = [\vec{d}_{ij}]_{m \times n} = \left\langle \begin{matrix} \min(T_{ij}^{\vec{A}}, T_{ij}^{\vec{B}}), & \min(C_{ij}^{\vec{A}}, C_{ij}^{\vec{B}}), \\ \max(U_{ij}^{\vec{A}}, U_{ij}^{\vec{B}}), & \max(F_{ij}^{\vec{A}}, F_{ij}^{\vec{B}}) \end{matrix} \right\rangle, \forall i, j$$

Definition 3.9. If $\vec{A} = [\vec{a}_{ij}]_{m \times n}, \vec{B} = [\vec{b}_{jk}]_{n \times p} \in FQNSM$, then their product is denoted and defined as

$$\vec{A} * \vec{B} = [\vec{e}_{ik}]_{m \times p} = \left\langle \begin{matrix} \max(\min_j(T_{ij}^{\vec{A}}, T_{jk}^{\vec{B}})), & \max(\min_j(C_{ij}^{\vec{A}}, C_{jk}^{\vec{B}})), \\ \min(\max_j(U_{ij}^{\vec{A}}, U_{jk}^{\vec{B}})), & \min(\max_j(F_{ij}^{\vec{A}}, F_{jk}^{\vec{B}})) \end{matrix} \right\rangle, \forall i, j, k$$

Remark.. $\vec{A} * \vec{B} \neq \vec{B} * \vec{A}$ unless $\vec{A} = \vec{B}$.

Definition 3.10. Let $\vec{A} = [\vec{a}_{ij}]_{m \times n}$, $\vec{B} = [\vec{b}_{jk}]_{n \times p} \in FQNSM$. Then, their average product is denoted and defined by

$$\vec{A} \nabla^* \vec{B} = \left[\vec{f}_{ik} \right]_{m \times p} = \left[\left\langle \begin{matrix} \max_j \left\{ \frac{T_{ij}^A + T_{jk}^B}{2} \right\}, & \max_j \left\{ \frac{C_{ij}^A + C_{jk}^B}{2} \right\}, \\ \min_j \left\{ \frac{U_{ij}^A + U_{jk}^B}{2} \right\}, & \min_j \left\{ \frac{F_{ij}^A + F_{jk}^B}{2} \right\} \end{matrix} \right\rangle \right]$$

Example 3.11. Let $\vec{A} = \begin{bmatrix} \langle 0.3, 0.6, 0.4, 0.7 \rangle & \langle 0.6, 0.8, 0.3, 0.6 \rangle \\ \langle 0.3, 0.6, 0.2, 0.8 \rangle & \langle 0.4, 0.6, 0.5, 0.8 \rangle \end{bmatrix}$ and $\vec{B} = \begin{bmatrix} \langle 0.5, 0.7, 0.5, 0.3 \rangle & \langle 0.4, 0.6, 0.7, 0.2 \rangle \\ \langle 0.6, 0.5, 0.4, 0.7 \rangle & \langle 0.2, 0.4, 0.3, 0.6 \rangle \end{bmatrix}$ be two FQNSMs of same order. Then using the above definitions, we compute the following:

$$\begin{aligned} \vec{A}^c &= \begin{bmatrix} \langle 0.7, 0.4, 0.6, 0.3 \rangle & \langle 0.4, 0.2, 0.7, 0.4 \rangle \\ \langle 0.7, 0.4, 0.8, 0.2 \rangle & \langle 0.6, 0.4, 0.5, 0.2 \rangle \end{bmatrix} \\ \vec{A} + \vec{B} &= \begin{bmatrix} \langle 0.5, 0.7, 0.4, 0.3 \rangle & \langle 0.6, 0.8, 0.3, 0.2 \rangle \\ \langle 0.6, 0.6, 0.2, 0.7 \rangle & \langle 0.4, 0.6, 0.3, 0.6 \rangle \end{bmatrix} \\ \vec{A} - \vec{B} &= \begin{bmatrix} \langle 0.3, 0.6, 0.5, 0.7 \rangle & \langle 0.4, 0.6, 0.7, 0.6 \rangle \\ \langle 0.3, 0.5, 0.4, 0.8 \rangle & \langle 0.2, 0.4, 0.5, 0.8 \rangle \end{bmatrix} \\ \vec{A} * \vec{B} &= \begin{bmatrix} \langle 0.6, 0.6, 0.4, 0.7 \rangle & \langle 0.3, 0.6, 0.3, 0.6 \rangle \\ \langle 0.4, 0.6, 0.5, 0.8 \rangle & \langle 0.3, 0.6, 0.5, 0.8 \rangle \end{bmatrix} \\ \vec{A} \nabla^* \vec{B} &= \begin{bmatrix} \langle 0.6, 0.7, 0.35, 0.5 \rangle & \langle 0.4, 0.6, 0.3, 0.45 \rangle \\ \langle 0.5, 0.55, 0.35, 0.55 \rangle & \langle 0.35, 0.6, 0.4, 0.5 \rangle \end{bmatrix} \end{aligned}$$

Definition 3.12. Let $\vec{A} = \langle T_{ij}^A, C_{ij}^A, U_{ij}^A, F_{ij}^A \rangle_{m \times n} \in FQNSM$. Then, a score function defined on \vec{A} is a mapping $S: \vec{A} \rightarrow [-1, 1]$ such that $S(\vec{A}) = \frac{T_{ij}^A + C_{ij}^A - U_{ij}^A - F_{ij}^A}{2}, \forall i, j$.

Definition 3.13. Let $\vec{A} = \langle T_{ij}^A, C_{ij}^A, U_{ij}^A, F_{ij}^A \rangle_{m \times n} \in FQNSM$. Then, an accuracy function defined on \vec{A} is a mapping $H: \vec{A} \rightarrow [0, 1]$ such that $H(\vec{A}) = \frac{T_{ij}^A + C_{ij}^A + U_{ij}^A + F_{ij}^A}{4}, \forall i, j$.

Definition 3.14. Let $\vec{A} = \langle T_{ij}^A, C_{ij}^A, U_{ij}^A, F_{ij}^A \rangle_{m \times n} = [\vec{a}_{ij}]_{m \times n}$, $\vec{B} = \langle T_{ij}^B, C_{ij}^B, U_{ij}^B, F_{ij}^B \rangle_{m \times n} = [\vec{b}_{ij}]_{m \times n} \in FQNSM_{m \times n}$. Then, their linear sum is denoted by $\vec{D} = \vec{A} \oplus \vec{B} = [\vec{d}_{ij}] = \langle T_{ij}^D, C_{ij}^D, U_{ij}^D, F_{ij}^D \rangle$, where $T_{ij}^D = T_{ij}^A + T_{ij}^B - T_{ij}^A \cdot T_{ij}^B$, $C_{ij}^D = C_{ij}^A + C_{ij}^B - C_{ij}^A \cdot C_{ij}^B$, $U_{ij}^D = U_{ij}^A \cdot U_{ij}^B$, and $F_{ij}^D = F_{ij}^A \cdot F_{ij}^B, \forall i, j$.

Definition 3.15. Let $\vec{A} = \langle T_{ij}^A, C_{ij}^A, U_{ij}^A, F_{ij}^A \rangle_{m \times n} = [\vec{a}_{ij}]_{m \times n}$, $\vec{B} = \langle T_{ij}^B, C_{ij}^B, U_{ij}^B, F_{ij}^B \rangle_{m \times n} = [\vec{b}_{ij}]_{m \times n} \in FQNSM_{m \times n}$. Then, their linear product is denoted by $\vec{E} = \vec{A} \odot \vec{B} = [\vec{e}_{ij}] = \langle T_{ij}^E, C_{ij}^E, U_{ij}^E, F_{ij}^E \rangle$, where $T_{ij}^E = T_{ij}^A \cdot T_{ij}^B$, $C_{ij}^E = C_{ij}^A \cdot C_{ij}^B$, $U_{ij}^E = U_{ij}^A + U_{ij}^B - U_{ij}^A \cdot U_{ij}^B$, and $F_{ij}^E = F_{ij}^A + F_{ij}^B - F_{ij}^A \cdot F_{ij}^B, \forall i, j$.

4. Application of Fuzzy Quadripartitioned Neutrosophic Soft Matrix in Medical DM

The medical DM process is considered the most complicated DM as the medical diagnosis required proper knowledge about the history of the patient suffering from a certain disease with several symptoms and they need to undergo various medical treatments under the medical expert. Sometimes the information provided by the patient is incomplete, unknown, or vague and thus it is difficult for the expert to identify the disease. This may cause the wrong treatment of the patient. In such a case, the expert should list out all possible symptoms of the patient and keep monitoring the patient's health condition right from the beginning. For complicated symptoms, there may be a panel of experts required for proper treatment. We know that biopsy is the sure way for a proper diagnosis to identify the disease. But it is a risky process. We need a proper methodology that ensures us to avoid biopsy to the patient who is out of danger. As everything in the medical treatment involved uncertainty, we implement FQNSSs in the present scenario with the belief that it provides high precision DM that is required for the proper medical investigation.

We now construct an algorithm for the application of FQNSM in medical diagnosis.

Algorithm:

Step 1- Input the FQNSSs (M, S) and (N, D) called the patient-symptom and symptom-disease sets, respectively, and write their corresponding matrices Q, T .

Step 2- Using the definitions 3.9 and 3.10, compute $\vec{Q} * \vec{T}$ and $\vec{Q} \nabla^* \vec{T}$.

Step 3- Using the definition 3.12, compute $S(\vec{Q} * \vec{T})$ and $S(\vec{Q} \nabla^* \vec{T})$.

Step 4- Find the total score $V_S = S(\vec{Q} * \vec{T}) + S(\vec{Q} \nabla^* \vec{T})$.

Step 5- Identify the maximum total score $V_{S_{ij}}$ for each patient P_i and conclude that the patient P_i is surely suffering from the disease D_j .

4.1 Example

Suppose there are three patients denoted by the set $P = \{P_1, P_2, P_3\}$ with symptoms denoted by the set $S = \{e_1 = \text{headache}, e_2 = \text{chest pain}, e_3 = \text{cough}\}$. Let the possible diseases denoted by $D = \{d_1 = \text{Pneumonia}, d_2 = \text{Malaria}, d_3 = \text{Typhoid}\}$.

Let the FQNSS (M, S) over P is given by

$$(M, S) = \begin{cases} M(e_1) = \{ \langle p_1, 0.4, 0.3, 0.5, 0.6 \rangle, \langle p_2, 0.2, 0.4, 0.7, 0.3 \rangle, \langle p_3, 0.5, 0.4, 0.1, 0.5 \rangle \}, \\ M(e_2) = \{ \langle p_1, 0.2, 0.4, 0.3, 0.4 \rangle, \langle p_2, 0.4, 0.6, 0.5, 0.4 \rangle, \langle p_3, 0.6, 0.7, 0.3, 0.8 \rangle \}, \\ M(e_3) = \{ \langle p_1, 0.3, 0.6, 0.4, 0.5 \rangle, \langle p_2, 0.7, 0.2, 0.1, 0.2 \rangle, \langle p_3, 0.5, 0.3, 0.6, 0.7 \rangle \} \end{cases}$$

Again, the FQNSS (N, D) over S is given by

$$(N, D) = \begin{cases} M(d_1) = \{ \langle e_1, 0.5, 0.4, 0.7, 0.2 \rangle, \langle e_2, 0.6, 0.2, 0.4, 0.4 \rangle, \langle e_3, 0.2, 0.6, 0.4, 0.6 \rangle \}, \\ M(d_2) = \{ \langle e_1, 0.2, 0.5, 0.6, 0.5 \rangle, \langle e_2, 0.6, 0.4, 0.7, 0.2 \rangle, \langle e_3, 0.5, 0.6, 0.4, 0.5 \rangle \}, \\ M(d_3) = \{ \langle e_1, 0.4, 0.5, 0.6, 0.7 \rangle, \langle e_2, 0.6, 0.3, 0.4, 0.5 \rangle, \langle e_3, 0.4, 0.2, 0.4, 0.6 \rangle \} \end{cases}$$

The matrix representation of FQNSSs (M, S) and (N, D) is given by

$$Q = \begin{bmatrix} \langle 0.4, 0.3, 0.5, 0.6 \rangle & \langle 0.2, 0.4, 0.3, 0.4 \rangle & \langle 0.3, 0.6, 0.4, 0.5 \rangle \\ \langle 0.2, 0.4, 0.7, 0.3 \rangle & \langle 0.4, 0.6, 0.5, 0.4 \rangle & \langle 0.7, 0.2, 0.1, 0.2 \rangle \\ \langle 0.5, 0.4, 0.1, 0.5 \rangle & \langle 0.6, 0.7, 0.3, 0.8 \rangle & \langle 0.5, 0.3, 0.6, 0.7 \rangle \end{bmatrix}$$

And

$$T = \begin{bmatrix} \langle 0.5, 0.4, 0.7, 0.2 \rangle & \langle 0.2, 0.5, 0.6, 0.5 \rangle & \langle 0.4, 0.5, 0.6, 0.7 \rangle \\ \langle 0.6, 0.2, 0.4, 0.4 \rangle & \langle 0.6, 0.4, 0.7, 0.2 \rangle & \langle 0.6, 0.3, 0.4, 0.5 \rangle \\ \langle 0.2, 0.6, 0.4, 0.6 \rangle & \langle 0.5, 0.6, 0.4, 0.5 \rangle & \langle 0.4, 0.2, 0.4, 0.6 \rangle \end{bmatrix}$$

respectively.

$$\begin{aligned} \vec{Q} * \vec{T} &= \begin{bmatrix} \langle 0.4, 0.3, 0.5, 0.6 \rangle & \langle 0.2, 0.4, 0.3, 0.4 \rangle & \langle 0.3, 0.6, 0.4, 0.5 \rangle \\ \langle 0.2, 0.4, 0.7, 0.3 \rangle & \langle 0.4, 0.6, 0.5, 0.4 \rangle & \langle 0.7, 0.2, 0.1, 0.2 \rangle \\ \langle 0.5, 0.4, 0.1, 0.5 \rangle & \langle 0.6, 0.7, 0.3, 0.8 \rangle & \langle 0.5, 0.3, 0.6, 0.7 \rangle \end{bmatrix} * \\ &= \begin{bmatrix} \langle 0.5, 0.4, 0.7, 0.2 \rangle & \langle 0.2, 0.5, 0.6, 0.5 \rangle & \langle 0.4, 0.5, 0.6, 0.7 \rangle \\ \langle 0.6, 0.2, 0.4, 0.4 \rangle & \langle 0.6, 0.4, 0.7, 0.2 \rangle & \langle 0.6, 0.3, 0.4, 0.5 \rangle \\ \langle 0.2, 0.6, 0.4, 0.6 \rangle & \langle 0.5, 0.6, 0.4, 0.5 \rangle & \langle 0.4, 0.2, 0.4, 0.6 \rangle \end{bmatrix} \\ &= \begin{bmatrix} \langle 0.4, 0.6, 0.4, 0.4 \rangle & \langle 0.3, 0.6, 0.4, 0.4 \rangle & \langle 0.4, 0.3, 0.4, 0.5 \rangle \\ \langle 0.2, 0.4, 0.4, 0.3 \rangle & \langle 0.5, 0.4, 0.4, 0.4 \rangle & \langle 0.4, 0.4, 0.4, 0.5 \rangle \\ \langle 0.6, 0.4, 0.4, 0.5 \rangle & \langle 0.6, 0.5, 0.6, 0.5 \rangle & \langle 0.6, 0.4, 0.4, 0.7 \rangle \end{bmatrix} \end{aligned}$$

$$\begin{aligned} \vec{Q} \nabla^* \vec{T} &= \begin{bmatrix} \langle 0.4, 0.3, 0.5, 0.6 \rangle & \langle 0.2, 0.4, 0.3, 0.4 \rangle & \langle 0.3, 0.6, 0.4, 0.5 \rangle \\ \langle 0.2, 0.4, 0.7, 0.3 \rangle & \langle 0.4, 0.6, 0.5, 0.4 \rangle & \langle 0.7, 0.2, 0.1, 0.2 \rangle \\ \langle 0.5, 0.4, 0.1, 0.5 \rangle & \langle 0.6, 0.7, 0.3, 0.8 \rangle & \langle 0.5, 0.3, 0.6, 0.7 \rangle \end{bmatrix} \\ \nabla^* &= \begin{bmatrix} \langle 0.5, 0.4, 0.7, 0.2 \rangle & \langle 0.2, 0.5, 0.6, 0.5 \rangle & \langle 0.4, 0.5, 0.6, 0.7 \rangle \\ \langle 0.6, 0.2, 0.4, 0.4 \rangle & \langle 0.6, 0.4, 0.7, 0.2 \rangle & \langle 0.6, 0.3, 0.4, 0.5 \rangle \\ \langle 0.2, 0.6, 0.4, 0.6 \rangle & \langle 0.5, 0.6, 0.4, 0.5 \rangle & \langle 0.4, 0.2, 0.4, 0.6 \rangle \end{bmatrix} \\ &= \begin{bmatrix} \langle 0.45, 0.6, 0.35, 0.4 \rangle & \langle 0.4, 0.6, 0.4, 0.3 \rangle & \langle 0.4, 0.4, 0.35, 0.45 \rangle \\ \langle 0.5, 0.4, 0.25, 0.25 \rangle & \langle 0.6, 0.5, 0.25, 0.3 \rangle & \langle 0.55, 0.45, 0.25, 0.4 \rangle \\ \langle 0.6, 0.45, 0.35, 0.35 \rangle & \langle 0.6, 0.55, 0.35, 0.5 \rangle & \langle 0.6, 0.5, 0.35, 0.6 \rangle \end{bmatrix} \end{aligned}$$

Now,

$$S(\vec{Q} * \vec{T}) = \begin{bmatrix} 0.1 & 0.05 & -0.1 \\ -0.05 & 0.05 & -0.05 \\ 0.05 & 0.00 & -0.05 \end{bmatrix} \text{ and}$$

$$S(\vec{Q} \nabla^* \vec{T}) = \begin{bmatrix} 0.15 & 0.15 & 0.00 \\ 0.20 & 0.27 & 0.17 \\ 0.17 & 0.15 & 0.07 \end{bmatrix}$$

Finally,

$$V_S = S(\vec{Q} * \vec{T}) + S(\vec{Q} \nabla^* \vec{T}) = \begin{matrix} p_1 & d_1 & d_2 & d_3 \\ p_2 & \begin{bmatrix} 0.25 & 0.2 & -0.1 \\ 0.15 & 0.32 & 0.12 \\ 0.22 & 0.15 & 0.02 \end{bmatrix} \\ p_3 \end{matrix}$$

From the above matrix V_S , it is clear that the patients p_1 , p_2 , and p_3 are suffering from the disease d_1 , d_2 , and d_2 respectively. Also, no patient is suffering from the disease d_3 .

5. Conclusion and Scope

The present topic concentrates on a new type of matrix theory known as FQNSM theory and it is based on FQNSS. This new theory perceived significant attention to the decision-makers to make a precise decision while handling uncertainty that involved indeterminacy, where indeterminacy can be divided into two parts, namely contradiction and ignorance that we encounter more often in our real world. We also discuss some types of FQNSMs and

algebraic operations on them. The score and accuracy functions are defined on FQNSM. An algorithm has been introduced to make real decision under the FQNSM environment. Finally, for practical application of the algorithm, a medical diagnosis-based problem has been solved successfully. In the future, there is a scope for the researcher to extend the proposed topic by introducing the interval FQNSM theory where the truth, contradiction, ignorance, and the false membership degrees are not crisp. We also apply the proposed study in the field of game theory, similarity measures, risk management, group DM problem, etc.

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