

## RESEARCH ARTICLE

# Assessment of Some Proposed Replacement Models Involving Moderate Fix-Up



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**Abstract:** Among all reasonable dynamic replacement strategies, the age replacement strategy is the best. Furthermore, when compared to optimal replacement time of the other alternative formations, the optimal replacement time of system with series formation is shorter. For such reason, this paper compared the optimal replacement times obtained under the standard age replacement policy (SARP) with optimal replacement times obtained under some proposed two replacement policies (policy A and policy B) for some four multicomponent systems. Two numerical examples are provided for simple illustrations of the replacement policies under SARP, policy A and policy B, so as to select and apply the best replacement policy with respect to a particular system in making a projected precautionary replacement maintenance action of the system.

**Keywords:** replacement policy, reliability, optimal strategies, maintenance action

## 1. Introduction

Organizational management is constantly interested in establishing an effective precautionary replacement policy for regular system operation, so as to reduce the incidences of system failures. Modern technology has enabled us to form many multicomponent systems, whose availability and reliability depend on the components that form a system. Furthermore, in describing the reliability of a multicomponent system, it is necessary to specify how the components of the system are connected to provide the rule of the operation. The series and parallel configurations are the most basic types of system configurations.

There are many information about age replacement models, involving minor repair. Al-Chalabi (2022) developed a cost minimization model to optimize the lifetime of a drill rig used in Tara underground mine in Ireland, such that the model can be used to estimate the economic replacement time of fixable instrument applied in the mining and other production industries. Cha and Finkelstein (2019) proposed a novel sort of fix-up known as conditional statistical minimal repair, and their methodology extends to the corresponding minimal repair procedures for systems operating in a random environment. Chang and Chen (2018) discussed that effective replacement policies should be collaborative once data are gathered from time of operation, mission lengths, fix-up, and maintenance triggering approaches. Under bivariate order of statistics, Gheisary and Goli (2018) proposed an efficient method of computing the exact reliability of a system with  $n$  components. Huang and Wang (2019) proposed a time replacement policy for

multistate systems (MSSs) with aging multistate components (MSCs) to determine when the entire system should be replaced. Enogwe et al. (2018) suggested a replacement model unit that fails suddenly under the probability distribution of failure times. Lim et al. (2016) suggested different age replacement policies in which a system is replaced by new one at intended age and when failure happens before the scheduled replacement age, it can be either perfectly fixed with random probability  $p$  or minimally repaired with random probability  $1-p$ . Liu et al. (2018) developed mathematical models for simple repairable multiunit systems. Maihula et al. (2021) study some reliability measures such as reliability, mean time to failure (MTTF) availability, and profit function for a solar serial system with four subsystems, so as to seek for ways to improve the whole reliability of the system. Malki et al. (2015) suggested some age replacement strategies for a parallel system with stochastic dependence. Mirjalili and Kazempoor (2020) explored three replacement plans for a system consisting of independent components with rising failure rate, including cold standby and fix-up policies. Nakagawa et al. (2018) presented the benefits of some proposed replacement policies. To examine the behavior of an industrial system under the cost-free warranty policy, Niwas and Garg (2018) built a mathematical model for a system based on the Markov process. Rebaiaia and Ait-kadi (2020) presented some three preventive maintenance actions. In terms of several parameters, Safaei et al. (2018) evaluated the ideal interval for preventative maintenance and the best decision for repair or replacement. Safaei et al. (2020) used the copula framework to provide two optimal age replacement policies based on the expected cost function and maximum availability function. Sanoubar et al. (2020) considered an age replacement policy, in which the system is replaced at failure

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(reactive replacement) or at a predetermined replacement time whichever comes first, and replacement costs are assumed to be nondecreasing as the system ages. Sheu et al. (2019) proposed preventative replacement models for a system that is prone to shocks, in which the system is either replaced with a new one (type 2 failure) or minimally fixed when a shock occurs (type 1 failure). Sudheesh et al. (2019) looked at the discrete age replacement model before looking at the features of a system's MTTF. Wang et al. (2019) obtained the charge function  $C(T, N)$  for a fixable system with one repair worker, such that, as the system meets up a specific time  $T$ , the repairman will fix up the unit precautionary, and it will return back to operation as soon as the fixing is completed. Waziri et al. (2019) developed some discounted age replacement model involving minor fixed-up for a series system, such that the systems are subjected to two types of failures. Waziri (2021) offered a discrete planned replacement model for a unit subjected to three levels of breakdown. Wu et al. (2021) established corresponding replacement models for a deteriorating repairable system with multiple vacations of one repairman. Xie et al. (2020) investigated the implications of cascading failures of a particular system, as well as the effects of safety barriers on preventing failures from occurring. Zhang et al. (2022) studied optimizing aliment policy with fix-up limit time for a latest class of airliner unit with an estimated lifespan, and the outcomes of this study provides a helping guidance for airliner program and plays vital usage in engineering practice. Zhao et al. (2017) developed some analytical replacement cost rates under two proposed policies considering random mission durations time, so as to avoid preventive replacement during mission period.

To the best ability of the authors of this paper, they did not come across any existing paper that discussed the chances or possibilities of extending the optimal replacement time of any multicomponent systems obtained using the standard age replacement policy (SARP). So, this reason influenced the authors of this paper to come up with some proposed replacement policies, so as to see the possibility or chances of extending the optimal replacement time of some multicomponent systems, particularly the series system. Therefore, the purpose of this study is to provide some proposed age replacement policies, so as to see the possibility of extending the optimal replacement time of some multicomponent systems. Almost all the subsystems of an industrial plant and communication facilities are in series, parallel, series-parallel, and parallel-series formations, so as to maintain their reliability and avoid the breakdown. As it is already known that the series system is having the shortest optimal replacement time that balances the rising maintenance cost and avoid sudden failure. This paper will construct some replacement charge function under the existing SARP and some two proposed replacement policies (policy A and policy B) for some four different multicomponent systems, such that each of the four systems is formed by six components having nonuniformed failure rate. The two proposed replacement policies (policy A and policy B) are formed from the order of the failure rate of the six components. Policy A is based on the components with high failure rate, while policy B is based on the components with low failure rate. Noting that authors did not consider  $n$  component for the configuration of the

four systems, because the series and parallel formations have a unique representation with  $n$  components, while the series-parallel and parallel-series formations did not have a unique representation with  $n$  components.

## 2. Materials and Methods

Six components will be arranged in four different formations (series, parallel, series-parallel, and parallel-series), so as to form four different systems. The mathematical expressions of reliability functions and failure rates for the six components will be obtained using the mathematical expressions of replacement charge functions for the four systems based on the assumptions. The aim of this research is to construct some mathematical replacement models based on two proposed replacement policies (policy A and policy B), so as to see the possibility of extending the optimal replacement time obtained using SARP. This research will be accomplished through the following steps:

1. By developing replacement charge rates for some four systems (series, parallel, series-parallel, and parallel-series systems) under the SARP.
2. By developing replacement charge rates for the four systems according to two proposed replacement policies (policy A and policy B).
3. By providing numerical examples for simple illustration of the constructed replacement charge rate under SARP, policy A, and policy B.

## 3. Description of the Systems, Notations, and their Assumptions

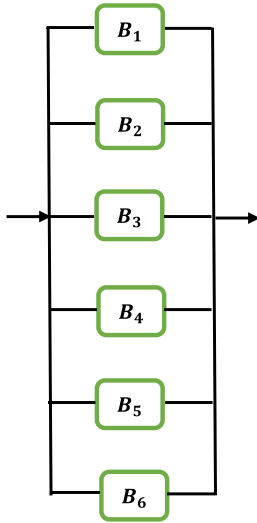
### 3.1. Description of systems

Consider six components  $B_1, B_2, B_3, B_4, B_5,$  and  $B_6$ , arranged in four different formations, so as to form series system –  $S_1$ , parallel system –  $S_2$ , series-parallel system –  $S_3$  and parallel-series system –  $S_4$ . All six components are subjected to category I and category II failures, with category I failure being fixable and category II failure being non-fixable. Because all six components are vulnerable to category I and category II failures, it follows that all the four systems are similarly vulnerable to category I and category II failures. See Figures 1, 2, 3, and 4 for the diagram of the four systems ( $S_1, S_2, S_3,$  and  $S_4$ ). For the series system  $S_1$ , the system fails due to category I failure, if at least one of the six components fails due to category I failure, while the system fails due to category II failure, if at least one of the six components fails due to category II failure. For the parallel system  $S_2$ , the system fails due to category I failure, if all the components fail due to category I failure, while the system fails due to category II failure, if all the component fail due to category II failure. For the series-parallel system  $S_3$ , the system fails due to category I failure, if at least one of the three subsystems fails due to category I failure, while the system fails due to category II failure, if at least one of the three subsystems fails due to category II failure. For the parallel-series system  $S_4$ , the system fails due to category I failure, if all the two subsystems fail due to category I failure, while the

Figure 1  
Block diagram of system  $S_1$



**Figure 2**  
Block diagram of system  $S_2$



system fails due to category II failure, if all the two subsystems fail due to category II failure.

**3.2. Notations used**

1.  $r_i(t)$ : Category I failure rate for component  $B_i$ , for  $i = 1, 2, 3, 4, 5, 6$ .
2.  $r_i^*(t)$ : Category II failure rate for component  $B_i$ , for  $i = 1, 2, 3, 4, 5, 6$ .
3.  $R_i^*(t)$ : Reliability function for component  $B_i$  due to category II failure for  $i = 1, 2, 3, 4, 5, 6$ .
4. *SARP*: Standard age replacement policy.
5.  $R_{S_i}^*(t)$ : Reliability function for system  $S_i$  due to category II failure, for  $i = 1, 2, 3, 4$ .
6.  $CS_i(T)$ : Charge rate for system  $S_i$  under SARP, for  $i = 1, 2, 3, 4$ .
7.  $CYS_i(T)$ : Charge rate for system  $S_i$  under policy A, for  $i = 1, 2, 3, 4$ .
8.  $CZS_i(T)$ : Charge rate for system  $S_i$  under policy B, for  $i = 1, 2, 3, 4$ .
9.  $T_{S_i}^*$ : Optimal replacement time for system  $S_i$  under SARP, for  $i = 1, 2, 3, 4$ .
10.  $T_{S_i}^{a*}$ : Optimal replacement time for system  $S_i$  under policy A, for  $i = 1, 2, 3, 4$ .
11.  $T_{S_i}^{b*}$ : Optimal replacement time for system  $S_i$  under policy B, for  $i = 1, 2, 3, 4$ .
12.  $C_{ir}$ : Charge of unspecified replacement for failed  $B_i$  due to category II failure, for  $i = 1, 2, 3, 4, 5, 6$ .

13.  $C_{im}$ : Charge for moderate fix-up of failed  $B_i$  due to category II failure, for  $i = 1, 2, 3, 4, 5, 6$ .
14.  $C_{sp}$ : Charge for specified replacement of system  $S_i$  at planned replacement time  $T$ , for  $i = 1, 2, 3, 4$ .
15.  $C_{ur}$ : Charge for unspecified replacement of system  $S_i$  due to category II failure, for  $i = 1, 2, 3, 4$ .

**3.3. Assumptions**

1. If a system fails due to category I failure, then the system undergoes moderate fix-up.
2. If a system fails due to category II failure, then the whole system is replaced completely with new one.
3. Both the two categories (category I and category II) of failures for the six components are generated by nonhomogeneous Poisson process.
4. Rate of category II failure follows the order:  $r_1^*(t) \geq r_3^*(t) \geq r_5^*(t) \geq r_2^*(t) \geq r_4^*(t) \geq r_6^*(t)$ .
5. Rate of category I failure follows the order:  $r_1(t) \geq r_3(t) \geq r_5(t) \geq r_2(t) \geq r_4(t) \geq r_6(t)$ .
6. A system is replaced at a specified replacement time  $T(T > 0)$  after its installation or when it fails due to category II, whichever comes first.
7. The charge of specified replacement is lower than the charge of unspecified replacement.
8. The charge of fixing a broken component is lower than the charge of replacing it.
9. All the charge of fix-up and replacement are positive numbers.

**4. Formulation of Mathematical Models**

The proposed mathematical models for the four systems  $S_1, S_2, S_3$ , and  $S_4$  under SARP, policy A, and policy B will be presented in this section.

**4.1. Mathematical model under SARP**

The reliability function for system  $S_1$  with respect to category II failure under SARP is

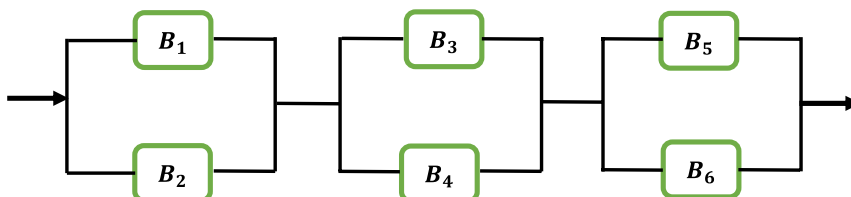
$$R_{S1}(T) = R_1^*(T)R_2^*(T)R_3^*(T)R_4^*(T)R_5^*(T)R_6^*(T). \tag{1}$$

The reliability function for system  $S_2$  with respect to category II failure under SARP is

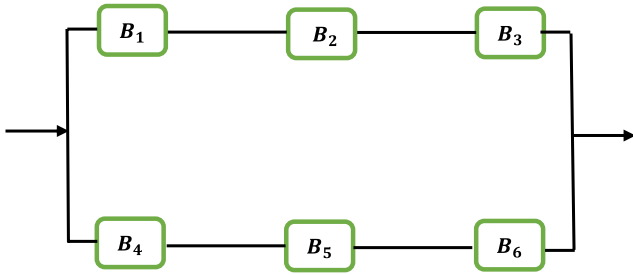
$$R_{S2}(T) = 1 - \prod_{i=1}^6 (1 - R_i^*(T)). \tag{2}$$

The reliability function for system  $S_3$  with respect to category II failure under SARP is

**Figure 3**  
Block diagram of system  $S_3$



**Figure 4**  
**Block diagram of system  $S_4$**



$$R_{S3}(T) = (1 - (1 - R_2^*(T))(1 - R_3^*(T))) \times (1 - (1 - R_4^*(T))(1 - R_5^*(T))) \times (1 - (1 - R_6^*(T))(1 - R_6^*(T))). \quad (3)$$

The reliability function for system  $S_4$  with respect to category II failure under SARP is

$$R_{S4}(T) = 1 - (1 - R_1^*(T)R_2^*(T)R_3^*(T)) \times (1 - R_4^*(T)R_5^*(T)R_6^*(T)). \quad (4)$$

The mean time for systems  $S_1, S_2, S_3,$  and  $S_4$  in one replacement cycle under SARP is

$$\int_0^T R_{Si}^*(t) dt, \text{ for } i = 1, 2, 3, 4. \quad (5)$$

The charge for unspecified replacement (failure due to category II failure) of systems  $S_1, S_2, S_3,$  and  $S_4$  in one replacement cycle under SARP is

$$C_{sr}(1 - R_{Si}^*(T)), \text{ for } i = 1, 2, 3, 4. \quad (6)$$

The charge for specified replacement at time T of systems  $S_1, S_2, S_3,$  and  $S_4$  in one replacement cycle under SARP is

$$C_{sp}R_{Si}^*(T), \text{ for } i = 1, 2, 3, 4. \quad (7)$$

The charge for moderate fix-up of components  $B_1, B_2, B_3, B_4, B_5,$  and  $B_6$  due to category I failure in one replacement cycle is

$$\int_0^T C_{1m}r_1(t)R_{Si}^*(t)dt + \int_0^T C_{2m}r_2(t)R_{Si}^*(t)dt + \int_0^T C_{3m}r_3(t)R_{Si}^*(t)dt + \int_0^T C_{4m}r_4(t)R_{Si}^*(t)dt + \int_0^T C_{4m}r_5(t)R_{Si}^*(t)dt + \int_0^T C_{5m}r_6(t)R_{Si}^*(t)dt. \quad (8)$$

The charge rate for systems  $S_1, S_2, S_3,$  and  $S_4$  under SARP is

$$CS_i(T) = \frac{C_{sr}(1 - R_{Si}(T)) + C_{sp}R_{Si}(T) + \int_0^T J(t)R_{Si}(t)dt}{\int_0^T R_{Si}(t)dt}, i = 1, 2, 3, 4, \quad (9)$$

where

$$J(t) = C_{1m}r_1(t) + C_{2m}r_2(t) + C_{3m}r_3(t) + C_{4m}r_4(t) + C_{5m}r_5(t) + C_{6m}r_6(t). \quad (10)$$

## 4.2. Mathematical model under Policy A

From assumptions 4 and 5, observe that category II failure of components  $B_1, B_3,$  and  $B_5$  is higher than that of components  $B_2, B_4,$  or  $B_6$ . Based on this reason, replacement under policy A is proposed. Policy A is a preventive maintenance policy, in which the unspecified replacement of a whole system depends on the failure of components  $B_1, B_3,$  and  $B_5$  in a system. But when any of the components  $B_2, B_4,$  or  $B_6$  fails due to category II failure, the failed component is replaced completely with new one and allowed the system to continue operating from where it stopped.

Under policy A, we have the following descriptions:

1. System  $S_1$ : the system is replaced completely with new one, when at least one of the components  $B_1, B_3,$  or  $B_5$  fails due to category II failure. Now, the reliability function of system  $S_1$  with respect to category II failure under policy A is

$$R_{S1}^{a*}(T) = R_1^*(T)R_3^*(T)R_5^*(T). \quad (11)$$

2. System  $S_2$ : the system is replaced completely with new one when all the three components  $B_1, B_3,$  and  $B_5$  fail due to category II failure. Now, the reliability function of system  $S_2$  with respect to category II failure under policy A is

$$R_{S2}^{a*}(T) = 1 - (1 - R_1^*(T))(1 - R_3^*(T))(1 - R_5^*(T)). \quad (12)$$

3. System  $S_3$ : the system is replaced completely with new one when at least one of the components  $B_1, B_3,$  or  $B_5$  fails due to category II failure. Now, the reliability function of system  $S_3$  with respect to category II failure under policy A is

$$R_{S3}^{a*}(T) = R_1^*(T)R_3^*(T)R_5^*(T). \quad (13)$$

4. System  $S_4$ : the system is replaced completely with new one when any of the combination fails:  $B_1$  and  $B_5,$  or  $B_3$  and  $B_5$  fails. Now, the reliability function of system  $S_4$  with respect to category II failure under policy A is

$$R_{S4}^{a*}(T) = 1 - (1 - R_1^*(T)R_3^*(T))(1 - R_2^*(T)). \quad (14)$$

The mean time for systems  $S_1, S_2, S_3,$  and  $S_4$  in one replacement cycle under policy A is

$$\int_0^T R_{Si}^{a*}(t) dt, \text{ for } i = 1, 2, 3, 4. \quad (15)$$

The charge for unspecified replacement (failure due to category II failure) of  $S_1, S_2, S_3,$  and  $S_4$  in one replacement cycle under policy A is

$$C_{sr}(1 - R_{Si}^{a*}(T)), \text{ for } i = 1, 2, 3, 4. \quad (16)$$

The charge for specified replacement at time T of systems  $S_1, S_2, S_3,$  and  $S_4$  in one replacement cycle under policy A is

$$C_{sp}R_{Si}^{a*}(T), \text{ for } i = 1, 2, 3, 4. \quad (17)$$

The charge for moderate fix-up of components  $B_1, B_2, B_3, B_4, B_5,$  and  $B_6$  due to category I failure in one replacement cycle is

$$\begin{aligned} & \int_0^T C_{1m}r_1(t)R_{Si}^{a*}(t)dt + \int_0^T C_{2m}r_2(t)R_{Si}^{a*}(t)dt \\ & + \int_0^T C_{3m}r_3(t)R_{Si}^{a*}(t)dt + \int_0^T C_{4m}r_4(t)R_{Si}^{a*}(t)dt \\ & + \int_0^T C_{4m}r_5(t)R_{Si}^{a*}(t)dt + \int_0^T C_{5m}r_6(t)R_{Si}^{a*}(t)dt. \end{aligned} \quad (18)$$

The charge for replacing components  $B_2, B_4,$  and  $B_6$  due to category II failure in one replacement cycle is

$$\begin{aligned} & \int_0^T C_{2r}r_2^*(t)R_{Si}^{a*}(t)dt + \int_0^T C_{4r}r_4^*(t)R_{Si}^{a*}(t)dt \\ & + \int_0^T C_{6r}r_6^*(t)R_{Si}^{a*}(t)dt. \end{aligned} \quad (19)$$

The charge rate for systems  $S_1, S_2, S_3,$  and  $S_4$  under policy A is

$$CYS_i(T) = \frac{C_{sr}(1 - R_{Si}^{a*}(T)) + C_{sp}R_{Si}^{a*}(T) + \int_0^T K(t)R_{Si}^{a*}(t)dt + \int_0^T L(t)R_{Si}^{a*}(t)dt}{\int_0^T R_{Si}^{a*}(t)dt}, \quad (20)$$

for  $i = 1, 2, 3, 4.$

where

$$K(t) = C_{1m}r_1(t) + C_{2m}r_2(t) + C_{3m}r_3(t) + C_{4m}r_4(t) + C_{5m}r_5(t) + C_{6m}r_6(t). \quad (21)$$

and

$$L(t) = C_{2r}r_2^*(t) + C_{4r}r_4^*(t) + C_{6r}r_6^*(t). \quad (22)$$

### 4.3. Mathematical model under Policy B

By the observation of assumptions 4 and 5, category II failure of components  $B_2, B_4,$  and  $B_6$  is lower than that of components  $B_1, B_3,$  or  $B_5.$  Based on this reason, replacement under policy B is proposed. Policy B is a preventive maintenance policy in which the unspecified replacement of a whole system depends on the failure of components  $B_2, B_4,$  and  $B_6$  due to category II. Noting that the reliability function of a system due to policy B depends on the location of components  $B_2, B_4,$  and  $B_6$  in a system. But when any of the components  $B_1, B_3,$  or  $B_5$  fails due to category II failure, the failed component is replaced completely with new one and allowed the system to continue operating from where it stopped.

Under policy B, we have the following descriptions:

1. System  $S_1$ : the system is replaced completely with new one when at least one of the components  $B_2, B_4,$  or  $B_6$  fails due to category

II failure. Now, the reliability function of system  $S_1$  with respect to category II failure under policy B is

$$R_{S1}^{b*}(T) = R_2^*(T)R_4^*(T)R_6^*(T). \quad (23)$$

2. System  $S_2$ : the system is replaced completely with new one when all the three components  $B_2, B_4,$  or  $B_6$  fail due to category II failure. Now, the reliability function of system  $S_2$  with respect to category II failure under policy B is

$$R_{S2}^{b*}(T) = 1 - (1 - R_2^*(T))(1 - R_4^*(T))(1 - R_6^*(T)). \quad (24)$$

3. System  $S_3$ : the system is replaced completely with new one when at least one of the components  $B_2, B_4,$  or  $B_6$  fails due to category II failure. Now, the reliability function of system  $S_3$  with respect to category II failure under policy B is

$$R_{S3}^{b*}(T) = R_2^*(T)R_4^*(T)R_6^*(T). \quad (25)$$

4. System  $S_4$ : the system is replaced completely with new one when any of the combination fails:  $B_4$  and  $B_2,$  or  $B_6$  and  $B_2$  fails. Now, the reliability function of system  $S_4$  with respect to category II failure under policy B is

$$R_{S4}^{b*}(T) = 1 - (1 - R_4^*(T)R_6^*(T))(1 - R_2^*(T)). \quad (26)$$

The mean time for systems  $S_1, S_2, S_3,$  and  $S_4$  in one replacement cycle under policy B is

$$\int_0^T R_{Si}^{b*}(t)dt, \text{ for } i = 1, 2, 3, 4. \quad (27)$$

The charge for unspecified replacement (failure due to category II failure) of systems  $S_1, S_2, S_3,$  and  $S_4$  in one replacement cycle under policy B is

$$C_{sr}(1 - R_{Si}^{b*}(T)), \text{ for } i = 1, 2, 3, 4. \quad (28)$$

The charge for specified replacement at time T of systems  $S_1, S_2, S_3,$  and  $S_4$  in one replacement cycle under policy B is

$$C_{sp}R_{Si}^{b*}(T), \text{ for } i = 1, 2, 3, 4. \quad (29)$$

The charge for moderate fix-up components  $B_1, B_2, B_3, B_4, B_5,$  and  $B_6$  due to category I failure in one replacement cycle is

$$\begin{aligned} & \int_0^T C_{1m}r_1(t)R_{Si}^{b*}(t)dt + \int_0^T C_{2m}r_2(t)R_{Si}^{b*}(t)dt \\ & + \int_0^T C_{3m}r_3(t)R_{Si}^{b*}(t)dt + \int_0^T C_{4m}r_4(t)R_{Si}^{b*}(t)dt \\ & + \int_0^T C_{4m}r_5(t)R_{Si}^{b*}(t)dt + \int_0^T C_{5m}r_6(t)R_{Si}^{b*}(t)dt. \end{aligned} \quad (30)$$

The charge for replacing components  $B_1, B_3,$  and  $B_5$  due to category II failure in one replacement cycle is

$$\int_0^T C_{1r}r_1^*(t)R_{S_1}^{b*}(t)dt + \int_0^T C_{3r}r_3^*(t)R_{S_3}^{b*}(t)dt + \int_0^T C_{5r}r_5^*(t)R_{S_5}^{b*}(t)dt. \tag{31}$$

The charge rate for systems  $S_1, S_2, S_3,$  and  $S_4$  under policy B is

$$CZ_{S_i}(T) = \frac{C_{sr}(1 - R_{S_i}^{b*}(T)) + C_{sp}R_{S_i}^{b*}(T) + \int_0^T M(t)R_{S_i}^{b*}(t)dt + \int_0^T N(t)R_{S_i}^{b*}(t)dt}{\int_0^T R_{S_i}^{b*}(t)dt}, \text{ for } i = 1, 2, 3, 4. \tag{32}$$

where

$$M(t) = C_{1m}r_1(t) + C_{2m}r_2(t) + C_{3m}r_3(t) + C_{4m}r_4(t) + C_{5m}r_5(t) + C_{6m}r_6(t), \tag{33}$$

and

$$N(t) = C_{1r}r_1^*(t) + C_{3r}r_3^*(t) + C_{5r}r_5^*(t). \tag{34}$$

$$r_3(t) = 0.09t^2; \tag{39}$$

$$r_4(t) = 0.003t^2; \tag{40}$$

$$r_5(t) = 0.004t^3; \tag{41}$$

$$r_6(t) = 0.002t; \tag{42}$$

$$r_1^*(t) = 0.00132t^3 \tag{43}$$

$$r_2^*(t) = 0.000875t^{2.5}; \tag{44}$$

$$r_3^*(t) = 0.00012t^3; \tag{45}$$

$$r_4^*(t) = 0.000805t^{2.5}; \tag{46}$$

$$r_5^*(t) = 0.001t^3; \tag{47}$$

$$r_6^*(t) = 0.0007t^{2.5}. \tag{48}$$

### 5. Numerical Examples

This section presents two numerical examples, so as to illustrate the characteristics and compare the constructed replacement charge functions under SARP, policy A, and policy B. In example 1, it is assumed that the rate of arrival of both category I and category II failures obeys Weibull distribution, while in example 2, it is assumed that the rate of arrival of both category I and category II failures obeys Power law distribution.

#### 5.1. Numerical example 1

Let the failure time of category I failure for the six components obeys Weibull distribution:

$$r_i(t) = \lambda_i \alpha_i t^{\alpha_i - 1}, \text{ for } i = 1, 2, 3, 4, 5, 6, \tag{35}$$

where  $\alpha_i > 1$  and  $t \geq 0$ .

Also, let the failure time of category II failure for the six components obeys Weibull distribution:

$$r_i^*(t) = \lambda_i^* \alpha_i^* t^{\alpha_i^* - 1}, \text{ for } i = 1, 2, 3, 4, 5, 6, \tag{36}$$

where  $\alpha_i^* > 1$  and  $t \geq 0$ .

The values of the parameters and charge of fix-up/replacement are assumed based on the conditions of 4 and 5 (orders of category I and category II failures) of the assumptions. The set of parameters and charge of fix-up/replacement are used throughout this particular example:

1.  $\alpha_1 = 4, \alpha_2 = 2, \alpha_3 = 3, \alpha_4 = 3, \alpha_5 = 4$  and  $\alpha_6 = 2$ .
2.  $\lambda_1 = 0.03, \lambda_2 = 0.002, \lambda_3 = 0.03, \lambda_4 = 0.001, \lambda_5 = 0.001$  and  $\lambda_6 = 0.001$ .
3.  $\alpha_1^* = 4, \alpha_2^* = 3.5, \alpha_3^* = 4, \alpha_4^* = 3.5, \alpha_5^* = 4,$  and  $\alpha_6^* = 3.5$ .
4.  $\lambda_1^* = 0.00033, \lambda_2^* = 0.00025, \lambda_3^* = 0.00030, \lambda_4^* = 0.00023,$   
 $\lambda_5^* = 0.00025$  and  $\lambda_6^* = 0.0002$ .
5.  $C_{sr} = 72, C_{sp} = 48$  and  $C_{im} = 0.3,$  for  $i = 1, 2, 3, 4, 5, 6$ .

By substituting the parameters of category I and category II failures in equations (35) and (36), the following equations are obtained:

$$r_1(t) = 0.12t^3; \tag{37}$$

$$r_2(t) = 0.06t; \tag{38}$$

Table 1 results are obtained by substituting the cost of replacement/repair and rates of category I and category II failures (equations (37)–(48)) in equation (9), so as to evaluate the system’s optimal replacement time of all the four systems under SARP. Table 2 results are obtained by substituting the cost of replacement/repair and rates of category I and category II failures

**Table 1**  
Results obtained from evaluating the charge rates of systems  $S_1, S_2, S_3,$  and  $S_4$  under SARP

T	$CS_1(T)$	$CS_2(T)$	$CS_3(T)$	$CS_4(T)$
1	240.42	240.04	240.04	240.03
2	122.77	120.16	120.16	120.11
3	88.58	80.36	80.42	80.41
4	79.17	60.69	61.06	61.61
5	82.73	49.16	50.68	53.17
6	91.47	41.80	46.28	52.86
7	94.78	36.93	46.64	59.29
8	95.87	33.87	53.74	67.98
9	97.93	32.50	61.01	72.17
10	99.00	32.92	63.97	74.11
11	99.52	34.91	70.03	76.16
12	100.22	37.87	73.92	78.97

**Table 2**  
Results obtained from evaluating the charge rates of systems  $S_1, S_2, S_3,$  and  $S_4$  under policy A

T	$CAS_1(T)$	$CAS_2(T)$	$CAS_3(T)$	$CAS_4(T)$
1	240.78	240.57	240.80	240.57
2	122.87	121.18	122.89	121.19
3	87.58	81.96	87.60	82.03
4	76.01	62.89	76.20	63.41
5	76.62	52.13	76.65	54.13
6	83.71	46.27	83.77	51.36
7	89.44	44.88	89.48	53.95
8	90.24	48.34	90.26	59.77
9	92.73	55.49	92.75	64.92
10	93.99	60.79	94.99	66.28
11	95.45	62.77	95.47	67.90
12	98.00	64.57	98.10	69.72



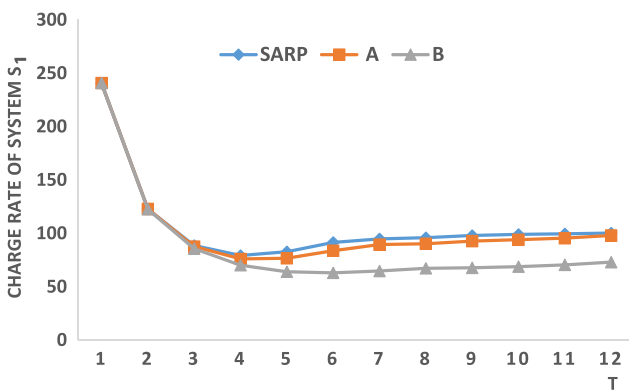
**Table 3**  
Results obtained from evaluating the charge rates of systems  $S_1, S_2, S_3,$  and  $S_4$  under policy B

T	$CBS_1(T)$	$CBS_2(T)$	$CBS_3(T)$	$CBS_4(T)$
1	240.80	240.64	240.80	240.64
2	122.54	121.62	122.54	121.63
3	85.61	83.11	85.61	83.13
4	70.10	65.11	70.10	65.20
5	63.97	55.61	63.97	55.94
6	62.95	50.64	62.95	51.48
7	64.67	49.55	64.67	50.22
8	67.21	48.55	67.21	51.27
9	67.67	50.20	68.67	53.79
10	68.74	53.05	70.74	56.80
11	70.46	56.43	74.46	59.10
12	72.97	59.34	76.97	59.61

**Table 4**  
The optimal replacement times of systems  $S_1, S_2, S_3,$  and  $S_4$  under SARP, policy A, and policy B

System	SARP	A	B
$S_1$	$T_{S_1}^* = 4.00$	$T_{S_1}^{a*} = 4.00$	$T_{S_1}^{b*} = 6.00$
$S_2$	$T_{S_2}^* = 9.00$	$T_{S_2}^{a*} = 7.00$	$T_{S_2}^{b*} = 8.00$
$S_3$	$T_{S_3}^* = 6.00$	$T_{S_3}^{a*} = 4.00$	$T_{S_3}^{b*} = 6.00$
$S_4$	$T_{S_4}^* = 6.00$	$T_{S_4}^{a*} = 6.00$	$T_{S_4}^{b*} = 7.00$

**Figure 5**  
The plot of charge rates of system  $S_1$  against specified replacement time T

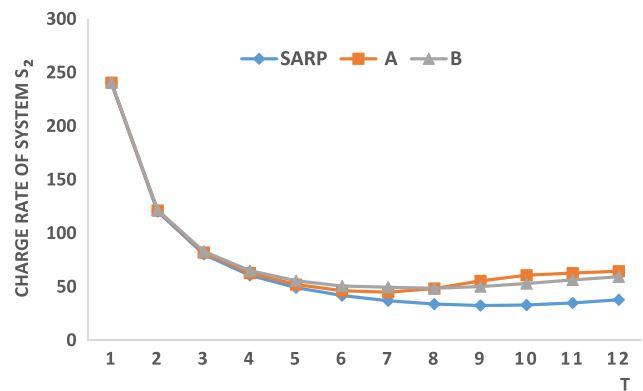


(equations (37)–(48)) in equation (20), so as to evaluate the system’s optimal replacement time of all the four systems under policy A. Table 3 results are obtained by substituting the cost of replacement/repair and rates of category I and category II failures (equations (37)–(48)) in equation (32), so as to evaluate the system’s optimal replacement time of all the four systems under policy B.

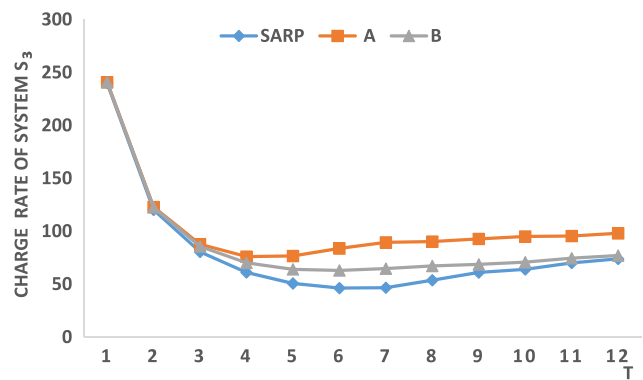
Some observations from example 1:

1. From Tables 1, 2, and 3, the Table 4 is formed:
2. Figure 5 showed that  $CBS_1(T) \leq CAS_1(T) \leq CS_1(T)$ .

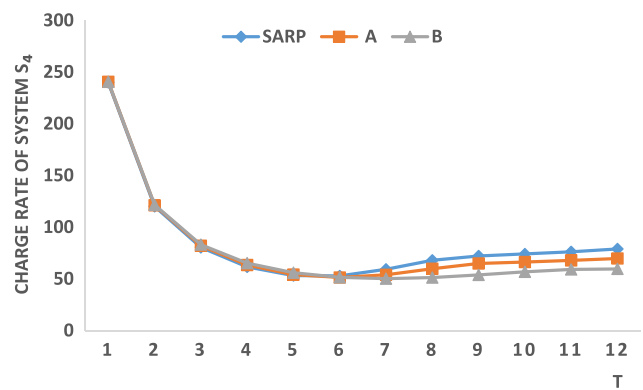
**Figure 6**  
The plot of charge rates of system  $S_2$  against specified replacement time T



**Figure 7**  
The plot of charge rates of system  $S_3$  against specified replacement time T



**Figure 8**  
The plot of charge rates of system  $S_4$  against specified replacement time T



3. Figure 6 showed that  $CS_2(T) \leq CBS_2(T) \leq CAS_2(T)$ .
4. Figure 7 showed that  $CS_3(T) \leq CBS_3(T) \leq CAS_3(T)$ .
5. Figure 8 showed that  $CBS_4(T) \leq CAS_4(T) \leq CS_4(T)$ .

5.2. Numerical example 2

Let the failure time of category I failure for the six components obeys Power law distribution:

$$r_i(t) = \lambda_i \alpha_i (\lambda_i t)^{\alpha_i - 1}, \quad \text{for } i = 1, 2, 3, 4, 5, 6, \quad (49)$$

where  $\alpha_i > 1$  and  $t \geq 0$ .

Also, let the failure time of category II failure for the six components obeys Power law distribution:

$$r_i^*(t) = \lambda_i^* \alpha_i^* (\lambda_i^* t)^{\alpha_i^* - 1}, \quad \text{for } i = 1, 2, 3, 4, 5, 6, \quad (50)$$

where  $\alpha_i^* > 1$  and  $t \geq 0$ .

The set of parameters and charge of fix-up/replacement are used throughout this particular numerical example 1 that is adopted for this numerical example 2. By substituting the parameters of category I and category II failures in equations (49) and (50), the following equations are obtained:

$$r_1(t) = 3.24 \times 10^{-6} t^3; \quad (51)$$

$$r_2(t) = 2.4 \times 10^{-8} t; \quad (52)$$

$$r_3(t) = 8.1 \times 10^{-5} t^2; \quad (53)$$

$$r_4(t) = 3 \times 10^{-9} t^2; \quad (54)$$

$$r_5(t) = 4 \times 10^{-12} t^3; \quad (55)$$

$$r_6(t) = 2 \times 10^{-6} t; \quad (56)$$

$$r_1^*(t) = 4.74 \times 10^{-14} t^3; \quad (57)$$

$$r_2^*(t) = 8.64 \times 10^{-13} t^{2.5}; \quad (58)$$

$$r_3^*(t) = 3.24 \times 10^{-14} t^3; \quad (59)$$

$$r_4^*(t) = 1.10 \times 10^{-10} t^{2.5}; \quad (60)$$

$$r_5^*(t) = 1.56 \times 10^{-4} t^3; \quad (61)$$

$$r_6^*(t) = 3.95 \times 10^{-3} t^{2.5}. \quad (62)$$

Table 5 results are obtained by substituting the cost of replacement/repair and rates of category I and category II failures (equations (1)–(62)) in equation (9), so as to evaluate the system’s optimal replacement time of all the four systems under SARP. Table 6 results are obtained by substituting the cost of replacement/repair and rates of category I and category II failures (equations (51)–(62)) in equation (20), so as to evaluate the system’s optimal replacement time of all the four systems under policy A. Table 7 results are obtained by substituting the cost of replacement/repair and rates of category I and category II failures (equations (51)–(62)) in equation (32), so as to evaluate the system’s optimal replacement time of the four systems under policy B.

Some observations from example 2:

1. From Tables 5, 6, and 7, the Table 8 is formed:
2. Figure 9 showed that  $CS_1(T) \leq CAS_1(T) \leq CBS_1(T)$ .
3. Figure 10 showed that  $CS_2(T) \leq CAS_2(T) \leq CBS_2(T)$ .
4. Figure 11 showed that  $CS_3(T) \leq CBS_3(T) \leq CAS_3(T)$ .
5. Figure 12 showed that  $CS_4(T) \leq CBS_4(T) \leq CAS_4(T)$ .

**Table 5**  
Results obtained from evaluating the charge rates of systems  $S_1, S_2, S_3,$  and  $S_4$  under SARP

T	$CS_1(T)$	$CS_2(T)$	$CS_3(T)$	$CS_4(T)$
10	224.67	224.29	224.29	224.29
20	107.02	104.41	104.41	104.41
30	72.83	64.61	64.67	64.62
40	63.42	44.94	45.31	45
50	66.98	33.41	34.93	33.9
60	75.72	26.05	30.53	28.48
70	79.03	21.18	30.89	28.95
80	80.12	18.12	37.99	35.56
90	82.18	16.75	45.26	44.99
100	83.25	17.17	48.22	49.14
110	83.77	19.16	54.28	49.56
120	84.47	22.12	58.17	52.12

**Table 6**  
Results obtained from evaluating the charge rates of systems  $S_1, S_2, S_3,$  and  $S_4$  under policy A

T	$CAS_1(T)$	$CAS_2(T)$	$CAS_3(T)$	$CAS_4(T)$
10	225.03	224.82	225.03	224.82
20	107.12	105.43	107.12	105.43
30	71.83	66.21	71.83	66.21
40	60.26	47.14	60.26	47.14
50	60.87	36.38	60.87	36.38
60	67.96	30.52	67.96	30.52
70	73.69	29.13	73.69	29.13
80	74.49	32.59	74.49	32.59
90	76.98	39.74	76.98	39.74
100	78.24	46.04	79.24	46.04
110	79.70	47.02	79.7	47.02
120	82.25	48.82	82.25	49.82

**Table 7**  
Results obtained from evaluating the charge rates of systems  $S_1, S_2, S_3,$  and  $S_4$  under policy B

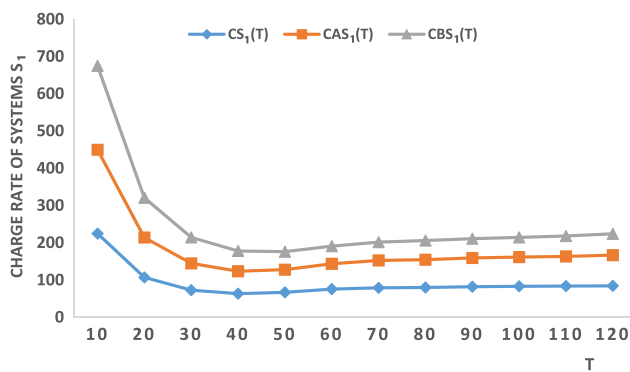
T	$CBS_1(T)$	$CBS_2(T)$	$CBS_3(T)$	$CBS_4(T)$
10	225.05	240.64	224.89	224.89
20	106.79	121.62	105.87	105.87
30	69.86	83.11	67.36	67.36
40	54.35	65.11	49.36	49.36
50	48.22	55.61	39.86	39.86
60	47.20	50.64	34.89	34.89
70	48.92	48.55	32.8	32.8
80	51.46	48.55	32.8	32.8
90	51.92	50.20	34.45	34.45
100	52.99	53.05	37.3	37.3
110	54.71	56.43	40.68	40.68
120	57.22	59.34	43.59	43.59



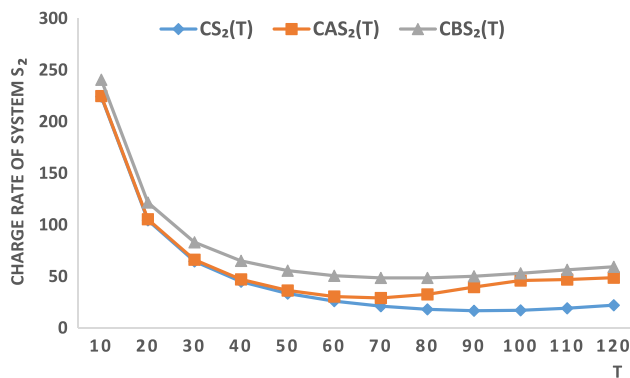
**Table 8**  
The optimal replacement times of systems  $S_1, S_2, S_3,$  and  $S_4$  under SARP, policy A, and policy B

System	SARP	A	B
$S_1$	$T_{S_1}^* = 40.00$	$T_{S_1}^{a*} = 40.00$	$T_{S_1}^{b*} = 60.00$
$S_2$	$T_{S_2}^* = 90.00$	$T_{S_2}^{a*} = 70.00$	$T_{S_2}^{b*} = 80.00$
$S_3$	$T_{S_3}^* = 60.00$	$T_{S_3}^{a*} = 40.00$	$T_{S_3}^{b*} = 60.00$
$S_4$	$T_{S_4}^* = 60.00$	$T_{S_4}^{a*} = 60.00$	$T_{S_4}^{b*} = 70.00$

**Figure 9**  
The plot of charge rates of system  $S_1$  against specified replacement time T



**Figure 10**  
The plot of charge rates of system  $S_2$  against specified replacement time T

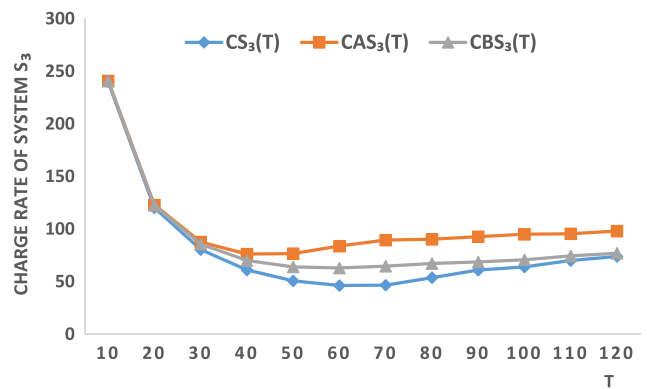


**6. Discussion of Results from Examples 1 and 2**

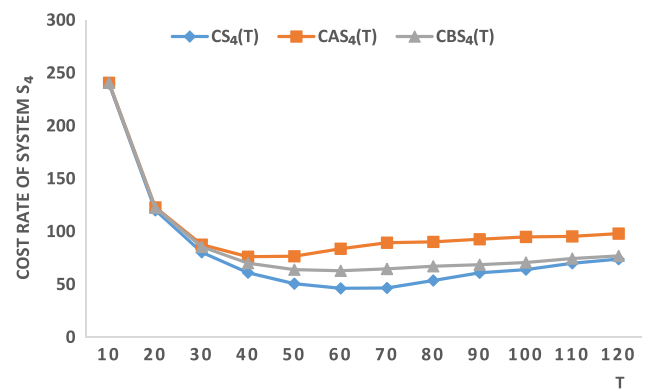
From examples 1 and 2, we have the observations as follows:

1. From both Tables 4 and 8, it can be seen that the optimal replacement time for series system ( $S_1$ ) and parallel-series system ( $S_4$ ) under policy B is higher than that of SARP and policy A.
2. From both Tables 4 and 8, it can be seen that the optimal replacement time for parallel system ( $S_2$ ) under SARP is higher than that of policy A and policy B.
3. From both Tables 4 and 8, it can be seen that the optimal replacement time for system  $S_3$  obtained under SARP and policy B are the same.
4. The optimal replacement times for all the four systems under SARP, policy A, and policy B obtained using Weibull law are less than that of the one obtained using Power law.

**Figure 11**  
The plot of charge rates of system  $S_3$  against specified replacement time T



**Figure 12**  
The plot of charge rates of system  $S_4$  against specified replacement time T



5. The values of charge rate for all the four systems obtained using the Weibull law vary with the one obtained using the Power law.

**7. Summary and Conclusion**

The main goal of writing this research paper is to come up with some proposed replacement policies, so as to see or explore the possibility of extending the optimal replacement time of a system obtained using SARP, because the optimal replacement time of a system obtained under SARP may be lower than that of the other systems. Six components were arranged in four different formations, so as to form series, parallel, series-parallel, and parallel-series systems. It is assumed that all the six components are subjected to two types of failures (category I and category II failures), which implied that all the four systems are also subjected to category I and category II failures. This paper presented some proposed replacement charge costs under SARP, policy A, and policy B for the four multicomponent systems. Two numerical examples were provided for simple presentation of the mathematical models constructed. The results obtained from both examples 1 and 2 showed that the optimal replacement time of series and parallel-series systems obtained under policy B is higher than that of the one obtained using SARP, that is to say, policy B extended that optimal replacement time of series and parallel-series systems obtained under SARP. Thus, the advantage of preventive replacement of series and parallel-series systems under policy B will reduce the chances of unplanned replacement of series and

parallel systems at an early stage. From the outcomes of this research, maintenance managers and plant management are advised to adopt SARP as a good preventive policy of maintaining their industrial operating machines or components which have parallel and series-parallel subsystems such as:

1. Series and parallel configurations of a combined heat and power (CHP) plant coupled with thermal networks.
2. Subsystems of industrial plants.
3. Subsystems of air crafts.

Also, the results showed that the values of charge costs of the four systems under three replacement policies (SARP, policy A, and policy B) vary.

### Conflicts of Interest

The authors declare that they have no conflicts of interest to this work.

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