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Performance Measure Using a Multi-Attribute Decision-Making Approach Based on Complex T-Spherical Fuzzy Power Aggregation Operators

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Abstract: Performance measurement has a vital role in almost every field of life especially when uncertainty is involved in processing information. The purpose of this research is to use the concept of power aggregation operators (PAOs) of complex T-spherical fuzzy sets (CTSFSs) to analyze the performance measures of software packages. In comparison to other studies, the main advantage of the PAOs of CTSFSs is its ability to feature four possible aspects of the information under uncertainty. In CTSFSs, each component has further two aspects denoted by their amplitude and phase terms and hence provides us a better ground to deal with practical problems. Another advantage of using PAOs for performance evaluation is the involvement of the relationship of the input arguments that play an essential role in aggregation. Other traditional aggregation operators (AOs) do not have such capabilities. We aim to develop complex T-spherical fuzzy (TSF) power weighted averaging and complex TSF power weighted geometric (CTSFPWG) operators and evaluated their validity using some tests. Ultimately, using the recommended operators, multi-attribute decision-making (MADM) algorithm is established for the performance evaluation of the software packages. With the help of a numerical example, we demonstrated the proposed MADM algorithm using complex uncertain information. The results show a positive impact by analyzing them after a comparative analysis where the results obtained using complex TSF PAOs seem to be more reliable than the results obtained using previously developed AOs. To see the effectiveness of the algorithm, the results are numerically compared using some existing approaches, and conclusions are drawn.

Keywords: multi-attribute decision-making, complex T-spherical fuzzy set, power aggregation operators, performance measure

1. Introduction

Multi-attribute decision-making (MADM) procedure entails examining a small number of options and arranging them in such a way that they are trusted by the decision-makers. To manage it, Zadeh (1965) introduced the idea of a membership grade (MG) defined by a membership function on the interval [0, 1]. Zadeh's fuzzy set (FS) can be used to describe the uncertainty that lies in human opinion and hence provides the solution to many real-world complications such as decision-making (Nasir et al., 2021) and pattern recognition (Chau et al., 2021).

Human opinion about a certain thing is not always unidirectional, while the framework of FS only describes the MG of an uncertain event hence not providing any information about the non-membership grade (NMG) of the event. Furthermore, Atanassov (1986) contributed by connecting the NMG and the MG with elements and objects and proposed the idea of an intuitionistic fuzzy set (IFS) and intuitionistic fuzzy number. This concept of the IFS is likely to model any uncertain events with the help of the MG and NMG with some certain limitations.

The IFS solely addressed two components of human opinion regarding an uncertain occurrence, namely yes or no aspects symbolized by the MG and NMG. The recognition that human perception may include abstinence and denial to some extent which is not covered by the frame of IFS. To cover this situation due to the reason, Cuong (2014) developed the concept of picture FS (PFS). The human perception is related with an extra grade, an

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abstention grade (AG), in addition to the NMG and MG, and any such triplets with a summation on the scale of 0 and 1 are allowed by the frame of PFS. However, PFS also faced considerable resistance while assigning the degree of the MG, AG, and the NMG due to its strict condition. By expanding the range of PFS, Mahmood et al. (2019) established the structure of spherical fuzzy set (SFS) by giving some larger space for assigning the degree of MG, AG, and the NMG. Consequently, with the addition of the parameter t the concept of SFS is further expanded to T-spherical fuzzy sets (TSFS). A TSFS is the most widespread fuzzy framework that may be used to describe the human perception of an uncertain event in a flexible and unrestricted manner.

Recently, the notion of complex FS has been introduced by Ramot et al. (2002), where the description of the MG is reshaped using complex numbers with magnitude less than or equal to 1. This notion was extended by Alkouri and Salleh (2012), to propose the frame complex IFS (CIFS). Shanthi et al. (2021) introduced the layout of complex PFS (CPFS) as the enhanced form of CIFS by covering the AG along with the MG and the NMG. This frame of CPFS was further extended by the Ali et al. (2020) by introducing the notion of complex SFS (CSFS) and complex T-spherical fuzzy set (CTSFS). A CTFS is one the advanced frame that can cope with four kinds of uncertain information.

MADM is a popular topic in fuzzy mathematics. This is one of the most discussed topics in virtually and hazy context. AOs, distance measures, similarity measures, entropy measures, and divergence measures are some well-known tools for the MADM process in specific cases. So far, many AOs have been developed in various fuzzy environments to deal with MADM problems. Muhammad et al., (2021) used the ranking approach to determine the superiority and inferiority in MADM issues using new operations on PyFSs such as division and subtraction. Umar and Saraswat (2021) proposed some divergence measures for MADM in intuitionistic fuzzy settings. Ali and Smarandache (2017) investigated the idea of complex single-valued neutrosophic sets and their use in MADM. Wei (2017) defined WA and GA in picture fuzzy (PF) context; Wang et al. (2017) developed several geometric operators to deal with the problem of decision-making. Some other work on AOs and their applications in MADM can be found in Riaz et al. (2020), Riaz and Hashmi (2019) and Pamucar (2020).

To overcome the MADM difficulties, all of the above-mentioned measures and operators were extensively used. The investigations, however, show that all of these algorithms are based on the assumption that the traits being assessed are unrelated to one another. However, it is self-evident that in our everyday lives, all of the factors or attributes are dependent on each other and their relation is of importance. The AOs discussed in Muhammad et al. (2021), Umar and Saraswat (2021), Ali and Smarandache (2017), Wei (2017), Wang et al. (2017), Riaz et al. (2020), Riaz and Hashmi (2019), Pamucar (2020), Yager (2001), Xu and Yager (2009), Wei and Lu (2018), Xu (2011), Mu et al. (2021), Zhou et al. (2012), and Meng and Chen (2015) do not consider the relationship of the input information. For example, consider the purchasing of a home as a decision-making problem. The parameters that correlate, such as location, features, and price, are all dependent on one another. Such characteristics are ignored in the research design discussed in Muhammad et al. (2021), Umar and Saraswat (2021), Ali and Smarandache (2017), Wei (2017), Wang et al. (2017), Riaz et al. (2020), Riaz and Hashmi (2019), Pamucar (2020), Yager (2001), Xu and Yager (2009), Wei and Lu (2018), Xu (2011), Mu et al. (2021), Zhou et al. (2012), and Meng and Chen (2015). To address this problem, the idea of a power aggregation operator (PAO) was developed by Yager (2001) and Xu and Yager (2009).

Until now, extensive work has been performed in several fuzzy framework on the topic of PAOs and the results found were compatible. To address the MADM problem, the power WA and power WG operators for PyFSs are expressed by Wei and Lu (2018), using the features of IFSs. Xu (2011) investigated these PAOs. To describe the applications of MADM approach in power plant selection, Mu et al. (2021) used the power Maclaurin symmetric mean operators. To tackle problems with group decision-making, the extended PAOs were defined by Zhou et al. (2012). To address the issues, Meng and Chen (2015) developed interactive power Hamacher operators based on PyFS data. To find a solution of MADM problem in the PyFS environment, Schweizer and Sklar PAOs are investigated by Biswas and Deb (2021). Rani and Garg (2018) introduced the notion of IFPWA operators and their applications in the MADM process. Jana and Pal (2021) addressed the MADM problem using Dombi PAOs. Some Dombi PAOs in the frame of neutrosophic sets are discussed by Jana and Pal (2021). Rong et al. (2020) investigate the neutrosophic PAOs based on the Archimedean co-copula and copula used in MADM. The selection process on the bases of power Bonferroni AOs is discussed by Qin et al. (2020). Luo et al. (2022) solved the MADM problem by Maclaurin PAOs. Some other recent work can be seen in Jiang et al. (2018), Đorđević et al. (2019), Stević et al. (2018), and Garg et al. (2021).

In real-life scenarios, whenever the uncertain information has several aspects, dealing with such situations becomes a challenge. In this paper, our aim is to use the frame of CTSFSs where the uncertain information is described by four degrees, each of them has further two aspects expressed by the amplitude and phase terms of the four degrees. Due to the quality of expressing the relationship of the aggregated information of PAOs and the significance of the frame of CTSFSs, the goal of this paper is to develop the notion of PAOs in the frame of CTSFSs and utilized it in MADM problems. We observed the effectiveness of the PAOs of CTSFSs numerically.

The rest of the paper is designed as follows: In Section 2, we discussed brief history of several fuzzy frameworks and their particulars. In Section 3, we construct the PAOs in the framework of CTSFSs and investigate their properties. We investigated the specific circumstances and significance of the suggested operators in Section 4. In Section 5, we developed an algorithm based on the proposed operators to solve MADM issues. In Section 6, a numerical example to exemplify the stated algorithm is discussed, while Section 7 summarizes a comparative study of the current work. Lastly, a solid conclusion is given in Section 8.

2. Preliminaries

In this section, we present some fundamental definitions that will help us to understand the proposed work.

Definition 1. The TSFS is specified by Mahmood et al. (2019), three functions called abstinence (u) and non-member (d) and membership (s) with a limitation that $0 \leq s^t + u^t + d^t \leq 1 \forall t \in \mathbb{Z}^+$ and $t \geq 1$. In addition, r represents the refusal grade which is given by $= \sqrt[t]{1 - (s^t + u^t + d^t)}$. For simplicity, the (s, u, d) is named as T-spherical fuzzy number (TSFN).

Definition 2. A CTSFS will be specified in three functions called abstinence $(r_i.e^{2\pi i\theta i})$, non-member $(r_n.e^{2\pi i\theta n})$, and membership $(r_m.e^{2\pi i\theta m})$ with a limitation that $0 \leq r_m.e^{2\pi i\theta m} + r_i.e^{2\pi i\theta i} + r_n.e^{2\pi i\theta n} \leq 1 \forall q \in \mathbb{Z}^+$ and $q \geq 1$. In addition, r indicates and gets the name refusal grade which is given by

$r = \sqrt[q]{1 - (r_m.e^{2\pi i\theta m} + r_i.e^{2\pi i\theta i} + r_n.e^{2\pi i\theta n})}$. For simplicity, the CTSFN is named $(r_m.e^{2\pi i\theta m}, r_i.e^{2\pi i\theta}, r_n.e^{2\pi i\theta n})$.

Remark 1. A CTSFS is turned into:

i. SFS; if q is considered as 2.
ii. PFs; if q is considered as 1.
iii. IFS; if r_i is taken as 0 and q is considered as 1.

In remark 1, the advantage of CTSFS over the previous notions is clearly defined. The four membership features that denote membership, abstinence, non-member, and refusal status describe a CTSFS. Therefore, it can better describe any human opinion as compared to the CSFS, CPFS, Cq-ROPFS, CPyFS, and CIFS.

Now we will discuss the importance of PAOs which are firstly introduced by Yager (2001). Yager (2001) introduced the concept of power weighted AOs in a fuzzy environment. The development of PAOs was motivated by the fact that all current aggregation tools ignore the relationship between the data or information being used (aggregated).

The relationship of the information being used (aggregated) is taken into account by T-spherical fuzzy PAO (TSFPAOs) proposed by Garg et al. (2021), whereas the preceding AOs do not. The abstinence and refusal degrees are taken into account by TSFPAOs, whereas IFSs, PyFSs, and q-ROFSs PAOs only examined the MG and NMG. Due to the parameter t, decision-makers have a greater range for assigning MG when applying T-spherical fuzzy PAOs than when using PF PAOs or spherical fuzzy PAOs.

3. Complex TSF PAOs

In this section, we aim to develop the PAOs in the frame of CTSFSs. First, we proposed the complex T-spherical fuzzy power weighted averaging (CTSFPWA) operator. Then we discuss the CTSFPOWA operator and the complex TSF power hybrid averaging (CTSFPHA) operator.

Definition 1: Let $T_i = (r_m.e^{2\pi i\theta m}, r_i.e^{2\pi i\theta i}, r_n.e^{2\pi i\theta n})$ represent the collection of complex TSF numbers (CTSFNs). Then CTSFPWA operator is a map $CTSPWA : \Gamma^n \rightarrow \Gamma$ and defined as:

$$CTSFPWA\ (T_1,\ T_2,\ T_3,\ \ldots, T_n) = \frac{\overset{n}{\underset{j=1}{\oplus}} \left(w_i(1 + r(T_j)T_j\right)}{\sum_{j=1}^{n} w_i\left(1 + r\left(T_j\right)\right)}$$

$$= \begin{pmatrix} \sqrt[q]{1 - \prod_{j=1}^{n}\left(1 - \left(r_m^q\right)_j\right)^{\delta_j}}.e^{2\pi i \sqrt[q]{1 - \prod_{j=1}^{n}\left(1 - \left(\theta_m^q\right)_j\right)^{\delta_j}}}, \\ \prod_{j=1}^{n}\left(r_i^{\delta_j}\right)_j.e^{2\pi i \prod_{j=1}^{n}\left(\theta_i^{\delta_j}\right)_j}, \prod_{j=1}^{n}\left(r_n^{\delta_j}\right)_j.e^{2\pi i \prod_{j=1}^{n}\left(\theta_n^{\delta_j}\right)_j} \end{pmatrix} \quad (1)$$

Here $\delta_{ij} = \frac{w_i\left(1 - S\left(T_{ij}\right)\right)}{\sum_{j=1}^{n} w_i\left(1 - S\left(T_{ij}\right)\right)}$ $\delta_{ij} > 0$ such that $\sum_{j=1} \delta_{ij} = 1$. Where $r(T_j) = \sum_{i=1, i\neq j}^{n} w_i\ \mathrm{Sup}\left(T_i, T_j\right)$ is the support function. The above PAO satisfies the following axioms of aggregation.

Property 1. (Monotonicity) Let $T_j = \left(r_{m_{T_j}}.e^{2\pi i\theta m_{T_j}}, r_{i_{T_j}}.e^{2\pi i\theta i_{T_j}}, r_{n_{T_j}}.e^{2\pi i\theta n_{T_j}}\right)$ and $P_j = \left(r_{m_{P_j}}.e^{2\pi i\theta m_{P_j}}, r_{i_{P_j}}.e^{2\pi i\theta i_{P_j}}, r_{n_{P_j}}.e^{2\pi i\theta n_{P_j}}\right)$ be two CTSFNs such that $T_j \leq P_j\ \forall\, j$. Then

$$CTSFPWA(T_1,\ T_2,\ T_3,\ \ldots, T_n) \leq CTSFPWA(P_1,\ P_2,\ P_3,\ \ldots, P_n)$$

Property 2. (Boundedness) If $T^- = (\min_j r_{m_j}.e^{2\pi i\theta m_j}, \max_j r_{i_j}.e^{2\pi i\theta i_j}, \max_j r_{n_j}.e^{2\pi i\theta n_j})$ and $T^+ \left(\max_j r_{m_j}.e^{2\pi i\theta m_j},\ \min_j r_{i_j}.e^{2\pi i\theta i_j}, \min_j r_{n_j}.e^{2\pi i\theta n_j}\right)$. Then

$$T^- \leq CTSFPWA(T_1,\ T_2,\ T_3,\ \ldots, T_n) \leq T^+$$

Property 3. (Idempotency) If $\forall\, j = 1,\ 2,\ 3, \ldots, n$ $\left(r_{m_j}.e^{2\pi i\theta m_j}, r_{i_j}.e^{2\pi i\theta i_j}, r_{n_j}.e^{2\pi i\theta n_j}\right) = T$. Then

$$CTSFPWA(T_1,\ T_2,\ T_3,\ \ldots, T_n) = T$$

The ordered position of CTSFNs is ignored by the CTSFPWA operator in Definition 1 and it only weighs the argument of the CTSFNs. In challenges involving decision-making, the data are sometimes presented in ascending or descending order, that is, the input data's ordered position is critical. To deal with such a predicament, we were inspired by Yeager's proposal of ordered weighted averaging operators (Yager, 2001). The following definition explains the concept of the CTSFPOWA operator.

Definition 2: Let $T_j = \left(r_m.e^{2\pi i\theta m_{T_j}}, r_i.e^{2\pi i\theta i_{T_j}}, r_n.e^{2\pi i\theta n_{T_j}}\right)$ represent some CTSFNs. Then CTSFPOWA operator is defined as

$$\mathrm{CTSFPOWA}\ (T_1,\ T_2,\ T_3,\ \ldots, T_n) = \frac{\overset{n}{\underset{j=1}{\oplus}} \left(w_i\left(1 + r\left(T_j\right)\right)T_{\sigma_j}\right)}{\sum_{j=1}^{n}\left(w_i\left(1 + r\left(T_{\sigma_j}\right)\right)\right)}$$

$$= \begin{pmatrix} \sqrt[q]{1 - \prod_{1}^{n}\left(1 - r_{m_{\sigma_j}}^q\right)^{\delta_j}}.\ e^{2\pi i \sqrt[q]{1 - \prod_{i}^{n}\left(1 - \theta_{m_{\sigma_j}}^q\right)^{\delta_j}}}, \\ \prod_{1}^{n} r_{i_{\sigma_j}}^{\delta_j}\ e^{2\pi i\theta_{i_{\sigma_j}}^{\delta_j}},\ \prod_{1}^{n} r_{n_{\sigma_j}}^{\delta_j}\ e^{2\pi i\theta_{n_{\sigma_j}}^{\delta_j}} \end{pmatrix} \quad (2)$$

where $\sigma(j)$ is the permutation such that $T_{\sigma_{(j-1)}} \geq T_{\sigma_j} \forall\, j$ and w_i denotes the collection of weights such that $w_i = g\left(\frac{R_j}{rv}\right) - g\left(\frac{R_{j-1}}{rv}\right)$, $R_j = \sum_{i=1}^{j} V_{\sigma_i}SV$, $SV = \sum_{i=1}^{j} V_{\sigma_i}SV$, $SV = \sum_{i=1}^{n} V_{\sigma_i} = 1 + r\left(T_{\sigma_i}\right)$ and $r\left(T_{\sigma_j}\right) = \sum_{i=1, i\neq j}^{n} Sup\left(T_{\sigma_j}, T_{\sigma_i}\right)$. Where $r\left(T_{\sigma_j}\right)$ denotes the support of j^{th} largest CTSFN T_{σ_j} for i^{th} greatest CTSFNs T_{σ_j} and $g : [0,\ 1] \rightarrow [0,\ 1]$ be a monotonic function with the axioms $g(0) = 0$, $g(1) = 1$ and $g(x) \geq g(y)\ if\ x > y$. Equation (5) specifies an AO that is sufficient to convince the following fundamental features of AOs labeled as Property 1, Property 2, and Property 3, respectively.

The CTSFPWA operator simply considers the argument, whereas the CTSFPOWA operator considers the argument's ordered position, that is, two distinct aspects exist for both operators. Therefore, in the following terminology, we suggest the idea of a hybrid operator, also abbreviated as the CTSPHA operator, which weighs both the argument and the ordered position and hence possesses the taste of both CTSFPWA and complex TSF power weighted geometric (CTSFPWG) operators.

Definition 3: Let $T_i = (r_m.e^{2\pi i\theta m}, r_i.e^{2\pi i\theta i}, r_n.e^{2\pi i\theta n})$ represent some CTSPNs. Then CTSFPHA operator is termed as complex TSFPWA.

$$CTSFPHA\ (\mathrm{T}_1,\ \mathrm{T}_2,\ \mathrm{T}_3,\ \ldots,\mathrm{T}_\mathrm{n}) = \frac{\underset{j=1}{\overset{n}{\oplus}} \left(w_i\left(1+r\left(\dot{T}_j\right)\right)c\right)}{\sum_{j=1}^{n} w_i\left(1+r\left(\dot{T}_{\sigma_j}\right)c\right)}$$

$$= \begin{pmatrix} \sqrt[q]{1-\prod_{1}^{n}\left(1-\dot{r}_{\sigma_m}^{q}\right)^{\delta_j}}\ .\ e^{2\pi i\sqrt[q]{1-\prod_{i}^{n}\left(1-\theta_{\sigma_m}^{q}\right)^{\delta_j}}}, \\ \prod_{1}^{n}\dot{r}_{\sigma_i}^{\delta_j}\ e^{2\pi i\theta_{\sigma_i}^{\delta_j}}\ ,\ \prod_{1}^{n}\dot{r}_{\sigma_n}^{\delta_j}\ e^{2\pi i\theta_{\sigma_n}^{\delta_j}} \end{pmatrix} \quad (3)$$

where $r\left(\dot{T}_{\sigma_j}\right)$ is the j^{th} largest of the CTSFN $\dot{T}_j = mw_j\dot{T}_j$ with w_j as the weight vector of complex TSF arguments $\dot{T}_j$ such that $w_j \in [0,1]$ and $\sum_1^n w_j = 1$ and the coefficient of balancing is m. Additionally, w_j as a result $w_i = g\left(\frac{R_j}{rv}\right) - g\left(\frac{R_{j-1}}{rv}\right)$, $R_j = \sum_{i=1}^{j} V_{\sigma_i}SV$, $SV = \sum_{i=1}^{j} V_{\sigma_i}SV$, $SV = \sum_{i=1}^{n} V_{\sigma_i}$ $V_{\sigma_i} = 1 + r\left(\dot{T}_{\sigma_j}\right)$ and $r\left(\dot{T}_{\sigma_j}\right) = \sum_{\substack{i=1\\ i\neq j}}^{n} Sup\left(\dot{T}_{\sigma_j}, \dot{T}_{\sigma_i}\right)$ Where $r\left(\dot{T}_{\sigma_j}\right)$ denotes the support of j^{th} the largest CTSFN $\dot{T}_{\sigma_j}$ for i^{th} greatest *CTSFNs* $\dot{T}_{\sigma_j}$ and $g:[0,\ 1]\rightarrow[0,\ 1]$ is a monotonic function and their axioms $g(0)=0,\ g(1)=1$ and $g(x)\geq g(y)$ if $x>y$. In the subject of information fusion, the weighted geometric (WG) and ordered weighted geometric (OWG) operators are two common aggregating operators. When the given arguments are expressed as crisp numbers or linguistic values, then we use these two AOs.

Definition 4: Let $\mathrm{T}_i = (r_m.e^{2\pi i\theta m}, r_i.e^{2\pi i\theta i}, r_n.e^{2\pi i\theta n})$ represent the CTSFNs. Then CTSFPWG operators are defined as:

$$\mathrm{CTSFPWG}\ (T_1,\ T_2,\ T_3,\ \ldots,T_n) = \underset{j=1}{\overset{n}{\oplus}} T_j^{\frac{(w_i(1+r(T_j)T_j)}{\sum_{j=1}^{n} w_i(1+r(T_j))}}$$

$$= \begin{pmatrix} \prod_{j=1}^{n}\left(r_m^{\delta_j}\right)_j.\ e^{2\pi i\prod_{j=1}^{n}\left(\theta_m^{\delta_j}\right)_j},\ \sqrt[q]{1-\prod_{j=1}^{n}\left(1-\left(r_i^q\right)_j\right)^{\delta_j}}.e^{2\pi i\sqrt[q]{1-\prod_{j=1}^{n}\left(1-\left(\theta_i^q\right)_j\right)^{\delta_j}}}, \\ \sqrt[q]{1-\prod_{j=1}^{n}\left(1-\left(r_n^q\right)_j\right)^{\delta_j}}.e^{2\pi i\sqrt[q]{1-\prod_{j=1}^{n}\left(1-\left(\theta_n^q\right)_j\right)^{\delta_j}}} \end{pmatrix} \quad (4)$$

where $r\left(\dot{T}_j\right) = \sum_{\substack{i=1\\ i\neq j}}^{n} w_j\,Sup\left(T_j, T_i\right)$ is the aggregated operator. Definition 1 satisfies the following aggregation criteria.

Property 4. (Monotonicity) Consider $\mathrm{T_j}\left(T_j^{r_{m_j}.e^{2\pi i\theta m_j}},\ T_j^{r_{ij}.e^{2\pi i\theta i_j}},\ T_j^{r_{n_j}.e^{2\pi i\theta n_j}}\right)$ and $P_j = \left(r_{m_j}.e^{2\pi i\theta m_j}, r_{ij}.e^{2\pi i\theta i_j}, r_{n_j}.e^{2\pi i\theta n_j}\right)$ be two CTSFN such that $\mathrm{T_j} \leq \mathrm{P_j}\ \forall$ j. Then $CTSFWG\ \left(T_1, T_2, T_3,\ \ldots, T_n\right) \leq \left(P_1,\ P_2,\ P_3,\ \ldots, P_n\right)$.

Property 5. (Boundedness) If $T^- = \left(min_j r_{m_j}.e^{2\pi i\theta m_j},\ max_j r_{i_j}.e^{2\pi i\theta i_j},\ max_j r_{n_j}.e^{2\pi i\theta n_j}\right)$ and $T^+ = \left(max_j r_{m_j}.e^{2\pi i\theta m_j},\ min_j r_{i_j}.e^{2\pi i\theta i_j},\ min_j r_{n_j}.e^{2\pi i\theta n_j}\right)$. Then $T^- \leq CTSFPWG\left(T1, T_2, T_3,\ \ldots, T_n\right) \leq T^+$

Property 6. (Idempotency) If $\forall\, j = 1,\ 2,\ 3,\ \ldots,\ n\ \ T_j = T = \left(r_m.e^{2\pi i\theta m}, r_i.e^{2\pi i\theta i}, r_n.e^{2\pi i\theta n}\right)$. Then $CTSFPWG\left(T_1, T_2, T_3,\ \ldots, T_n\right) = T$. The CTSFPWG operator only considers the CTSF argument, which is not in any particular order.

Definition 5: Let $T_j = (r_m.e^{2\pi i\theta m}, r_i.e^{2\pi i\theta i}, r_n.e^{2\pi i\theta n})$ represent some CTSFNs. Then CTSFPOWG operator is defined as

$$CTSFPOWG\left(T_1, T_2, T_3, \ldots, T_n\right) = \underset{j=1}{\overset{n}{\oplus}} T_{\sigma(j)}^{\frac{(w_i(1+r(T_{\sigma_j}))}{\sum_{j=1}^{n}(1+r(T_{\sigma_j}))}}$$

$$= \begin{pmatrix} \prod_{j=1}^{n}\left(r_m^{\delta_j}\right)_{\sigma_j}.e^{2\pi i\prod_{j=1}^{n}\left(\theta_m^{\delta_j}\right)_{\sigma_j}},\ \sqrt[q]{1-\prod_{j=1}^{n}\left(1-\left(r_i^q\right)_{\sigma_j}\right)^{\delta_j}}.e^{2\pi i\sqrt[q]{1-\prod_{j=1}^{n}\left(1-\left(\theta_i^q\right)\sigma_j\right)^{\delta_j}}}, \\ \sqrt[q]{1-\prod_{j=1}^{n}\left(1-\left(r_n^q\right)_{\sigma_j}\right)^{\delta_j}}.e^{2\pi i\sqrt[q]{1-\prod_{j=1}^{n}\left(1-\left(\theta_n^q\right)\sigma_j\right)^{\delta_j}}} \end{pmatrix} \quad (5)$$

where σ_j is a permutation such that $T_{\sigma_{(j-1)}} \geq T_j\ \forall\, j$ and w_j represents the collection of weights such that $w_i = g\left(\frac{R_j}{rv}\right) - g\left(\frac{R_{j-1}}{rv}\right)$, $R_j = \sum_{i=1}^{j} V_{\sigma_j}SV$, $SV = \sum_{i=1}^{j} V_{\sigma_j}SV$, $SV = \sum_{i=1}^{n} V_{\sigma_j}$ $V_{\sigma_j} = 1 + r\left(T_{\sigma_j}\right)$ and $r\left(T_{\sigma_j}\right) = \sum_{\substack{i=1\\ i\neq j}}^{n} Sup\left(T_{\sigma_j}, T_{\sigma_i}\right)$. Where $r\left(T_{\sigma_j}\right)$ denotes the support of j^{th} the largest CTSFN T_{σ_j} for ith greatest CTSFNs T_{σ_j} and $g:[0,1]\rightarrow[0,1]$ is a monotonic function and their axioms $g(0)=0, g(1)=1$ and $g(x)\geq g(y)$ if $x>y$. The fundamental axioms (4–6) of AOs are fulfilled by definition. The CTSFPWG operator simply considers the CTSF argument, but the CTSFPOWG operator considers the argument's ordered position. As a result, we invented the CTSFHG operators, which consider both the CTSF ordered position and argument.

Definition 6: Consider $T_j = (r_m.e^{2\pi i\theta m}, r_i.e^{2\pi i\theta i}, r_n.e^{2\pi i\theta n})$ represent the CTSFNs. Then CTSFPHG operator is defined as

$$\mathrm{CTSFPOWG}\left(T_1, T_2, T_3, \ldots, T_n\right) = \underset{j=1}{\overset{n}{\oplus}} \mathrm{T}_{\sigma_j}^{\frac{(w_i(1+r(\dot{T}_{\sigma_j}))}{\sum_{j=1}^{n}(1+r(T_{\sigma_j}))}}$$

$$= \begin{pmatrix} \prod_{j=1}^{n}\left(\dot{r}_m^{\delta_j}\right)_{\sigma_j}.e^{2\pi i\prod_{j=1}^{n}\left(\theta_m^{\delta_j}\right)_{\sigma_j}},\ \sqrt[q]{1-\prod_{j=1}^{n}\left(1-\left(\dot{r}_i^q\right)_{\sigma_j}\right)^{\delta_j}}.e^{2\pi i\sqrt[q]{1-\prod_{j=1}^{n}\left(1-\left(\theta_i^q\right)\sigma_j\right)^{\delta_j}}}, \\ \sqrt[q]{1-\prod_{j=1}^{n}\left(1-\left(\dot{r}_n^q\right)_{\sigma_j}\right)^{\delta_j}}.e^{2\pi i\sqrt[q]{1-\prod_{j=1}^{n}\left(1-\left(\theta_n^q\right)\sigma_j\right)^{\delta_j}}} \end{pmatrix} \quad (6)$$

where $\dot{T}_{\sigma_j}$ is the j^{th} largest of the CTSFN $\dot{T}_{\sigma_j} = T_j^{mw_i}$ with weight vector w_i of CTSF arguments T_j such that $w_i \in [0,\ 1]$ and $\sum_1^n w_j = 1$ and m is the coefficient of the balancing, w_i such that $w_i = g\left(\frac{R_j}{rv}\right) - g\left(\frac{R_{j-1}}{rv}\right)$, $R_j = \sum_{i=1}^{j} V_{\sigma_i}SV$, $SV = \sum_{i=1}^{j} V_{\sigma_i}$ SV, $SV = \sum_{i=1}^{n} V_{\sigma_i}$ $V_{\sigma_i} = 1 + r\left(\dot{T}_{\sigma_j}\right)$ and $r\left(\dot{T}_{\sigma_j}\right) = \sum_{\substack{i=1\\ i\neq j}} Sup$ $\left(\dot{T}_{\sigma_j}, \dot{T}_{\sigma_t}\right)$. Where $r\left(\dot{T}_{\sigma_j}\right)$ denotes the support of j^{th} the largest CTSFN $\dot{T}_{\sigma(j)}$ for i^{th} largest *CTSFNs* $\dot{T}_{\sigma_j}$ and $g:[0,1]\rightarrow[0,1]$ are a monotonic function and their axioms $g(0)=0, g(1)=1$ and $g(x)\geq g(y)$ if $x>y$.

Remark 2. Placing $w_j = \left(\frac{1}{n},\ \frac{1}{n},\ \frac{1}{n}, \ldots,\ \frac{1}{n}\right)^T$ and $w_j = \left(\frac{1}{n},\ \frac{1}{n},\ \frac{1}{n}, \ldots,\ \frac{1}{n}\right)^T$ reduces the CTSFHG operator to TSFPWG operator and CTSFPOWG operator correspondingly.

4. The Significance of Proposed Work

The implications of the theory produced in Section 3 are examined in this segment, demonstrating the superiority of the recommended work over previously held beliefs. We show that the proposed CTSFWA and CTSFPWG operators generalize the previously defined PAOs. With the help of some restrictions, we show that the previously defined AOs can be obtained from the proposed CTSFWA and CTSFPWG operators.

Consider the following CTSFPWA and CTSFPWG operators labeled by Equations (7) and (8), respectively.

$$CTSFPWA(T_1, T_2, T_3, \ldots, T_n) = \begin{pmatrix} \sqrt[q]{1-\prod_{j=1}^{n}\left(1-\left(r_m^q\right)_j\right)^{\delta_j}}.e^{2\pi i \sqrt[q]{1-\prod_{j=1}^{n}\left(1-\left(\theta_m^q\right)_j\right)^{\delta_j}}}, \\ \prod_{j=1}^{n}\left(r_i^{\delta_j}\right)_j.e^{2\pi i \prod_{j=1}^{n}\left(\theta_i^{\delta_j}\right)_j}, \prod_{j=1}^{n}\left(r_n^{\delta_j}\right)_j.e^{2\pi i \prod_{j=1}^{n}\left(\theta_n^{\delta_j}\right)_j} \end{pmatrix} \tag{7}$$

$$CTSFPWG(T_1, T_2, T_3, \ldots, T_n) = \begin{pmatrix} \prod_{j=1}^{n}\left(r_m^{\delta_j}\right)_j.e^{2\pi i \prod_{j=1}^{n}\left(\theta_m^{\delta_j}\right)_j}, \prod_{j=1}^{\dot{n}}\left(r_i^{\delta_j}\right)_j.e^{2\pi i \prod_{j=1}^{n}\left(\theta_i^{\delta_j}\right)_j}, \\ \sqrt[q]{1-\prod_{j=1}^{n}\left(1-\left(r_n^q\right)_j\right)^{\delta_j}}.e^{2\pi i \sqrt[q]{1-\prod_{j=1}^{n}\left(1-\left(\theta_n^q\right)_j\right)^{\delta_j}}} \end{pmatrix} \tag{8}$$

1. Taking $q = 2$ reduces the PWA and PGA operators of CSFS in Equations (7) and (8) to the spherical fuzzy environment as shown below:

$$CSFPWA\ (T_1, T_2, T_3, \ldots, T_n) = \begin{pmatrix} \sqrt[2]{1-\prod_{j=1}^{n}\left(1-\left(r_m^2\right)_j\right)^{\delta_j}}.e^{2\pi i \sqrt[2]{1-\prod_{j=1}^{n}\left(1-\left(\theta_m^2\right)_j\right)^{\delta_j}}}, \\ \prod_{j=1}^{n}\left(r_i^{\delta_j}\right)_j.\ e^{2\pi i \prod_{j=1}^{n}\left(\theta_i^{\delta_j}\right)_j}, \prod_{j=1}^{n}\left(r_n^{\delta_j}\right)_j.\ e^{2\pi i \prod_{j=1}^{n}\left(\theta_n^{\delta_j}\right)_j} \end{pmatrix} \tag{9}$$

$$CSFPWG\ (T_1, T_2, T_3, \ldots, T_n) = \begin{pmatrix} \prod_{j=1}^{n}\left(r_m^{\delta_j}\right)_j.\ e^{2\pi i \prod_{j=1}^{n}\left(\theta_m^{\delta_j}\right)_j}, \prod_{j=1}^{\dot{n}}\left(r_i^{\delta_j}\right)_j.\ e^{2\pi i \prod_{j=1}^{n}\left(\theta_i^{\delta_j}\right)_j}, \\ \sqrt[2]{1-\prod_{j=1}^{n}\left(1-\left(r_n^2\right)_j\right)^{\delta_j}}.e^{2\pi i \sqrt[2]{1-\prod_{j=1}^{n}\left(1-\left(\theta_n^2\right)_j\right)^{\delta_j}}} \end{pmatrix} \tag{10}$$

2. Taking $q = 1$ reduces the PWA and PGA operators of CSFS in Equations (7) and (8) to PF environment as shown below:

$$CPFPWA\ (T_1, T_2, T_3, \ldots, T_n) = \begin{pmatrix} 1-\prod_{j=1}^{n}\left(1-\left(r_m\right)_j\right)^{\delta_j}.e^{2\pi i\left(1-\prod_{j=1}^{n}\left(1-\left(\theta_m\right)_j\right)^{\delta_j}\right)}, \\ \prod_{j=1}^{n}\left(r_i^{\delta_j}\right)_j.\ e^{2\pi i \prod_{j=1}^{n}\left(\theta_i^{\delta_j}\right)_j}, \prod_{j=1}^{n}\left(r_n^{\delta_j}\right)_j.\ e^{2\pi i \prod_{j=1}^{n}\left(\theta_n^{\delta_j}\right)_j} \end{pmatrix} \tag{11}$$

$$\mathrm{CPFPWG}\ (T_1, T_2, T_3, \ldots, T_n) = \begin{pmatrix} \prod_{j=1}^{n}\left(r_m^{\delta_j}\right)_j.\ e^{2\pi i \prod_{j=1}^{n}\left(\theta_m^{\delta_j}\right)_j}, \prod_{j=1}^{\dot{n}}\left(r_i^{\delta_j}\right)_j.\ e^{2\pi i \prod_{j=1}^{n}\left(\theta_i^{\delta_j}\right)_j}, \\ 1-\prod_{j=1}^{n}\left(1-\left(r_n\right)_j\right)^{\delta_j}.\ e^{2\pi i\left(1-\prod_{j=1}^{n}\left(1-\left(\theta_n\right)_j\right)^{\delta_j}\right)} \end{pmatrix} \tag{12}$$

3. Taking $r_i = 0$ and $\theta_i = 0$ reduces the PWA and PGA operators of Cq-ROPFPWG in Equations (7) and (8) to the spherical fuzzy environment as shown below:

$$Cq-ROPFPWA\ (T_1, T_2, T_3, \ldots, T_n) = \begin{pmatrix} \sqrt[q]{1-\prod_{j=1}^{n}\left(1-\left(r_m^q\right)_j\right)^{\delta}}.e^{2\pi i \sqrt[q]{1-\prod_{j=1}^{n}\left(1-\left(\theta_m^q\right)_j\right)^{\delta_j}}}, \\ \prod_{j=1}^{n}\left(r_n^{\delta_j}\right)_j.\ e^{2\pi i \prod_{j=1}^{n}\left(\theta_n^{\delta_j}\right)_j} \end{pmatrix} \tag{13}$$

$$Cq-ROPFPWG(T_1, T_2, T_3, \ldots, T_n) = \begin{pmatrix} \prod_{j=1}^{n}\left(r_m^{\delta_j}\right)_j.\ e^{2\pi i \prod_{j=1}^{n}\left(\theta_m^{\delta_j}\right)_j}, \\ \sqrt[q]{1-\prod_{j=1}^{n}\left(1-\left(r_n^q\right)_j\right)^{\delta_j}}.e^{2\pi i \sqrt[q]{1-\prod_{j=1}^{n}\left(1-\left(\theta_n^q\right)_j\right)^{\delta_j}}} \end{pmatrix} \tag{14}$$

4. Taking $q = 2$ $r_i = 0$ and $\theta_i = 0$ reduces the PWA and PGA operators of CPyFSs in Equations (7) and (8) to a spherical fuzzy environment, as illustrated below:

$$CPyFPWA\ (T_1, T_2, T_3, \ldots, T_n) = \begin{pmatrix} \sqrt[2]{1-\prod_{j=1}^{n}\left(1-\left(r_m^2\right)_j\right)^{\delta_j}}.e^{2\pi i \sqrt[2]{1-\prod_{j=1}^{n}\left(1-\left(\theta_m^2\right)_j\right)^{\delta_j}}}, \\ \prod_{j=1}^{n}\left(r_n^{\delta_j}\right)_j.\ e^{2\pi i \prod_{j=1}^{n}\left(\theta_n^{\delta_j}\right)_j} \end{pmatrix} \tag{15}$$

$$CPyFPWG\ (T_1, T_2, T_3, \ldots, T_n) = \begin{pmatrix} \prod_{j=1}^{n}\left(r_m^{\delta_j}\right)_j.\ e^{2\pi i \prod_{j=1}^{n}\left(\theta_m^{\delta_j}\right)_j}, \\ \sqrt[2]{1-\prod_{j=1}^{n}\left(1-\left(r_n^2\right)_j\right)^{\delta_j}}.e^{2\pi i \sqrt[2]{1-\prod_{j=1}^{n}\left(1-\left(\theta_n^2\right)_j\right)^{\delta_j}}} \end{pmatrix} \tag{16}$$

5. Taking $q = 1$ $r_i = 0$ and $\theta_i = 0$ reduces the PWA and PGA operators of CIFSs in Equations (7) and (8) to a spherical fuzzy environment, as illustrated below:

$$CPyFPWA\ (T_1, T_2, T_3, \ldots, T_n) = \begin{pmatrix} \sqrt[2]{1-\prod_{j=1}^{n}\left(1-\left(r_m^2\right)_j\right)^{\delta_j}}.e^{2\pi i \sqrt[2]{1-\prod_{j=1}^{n}\left(1-\left(\theta_m^2\right)_j\right)^{\delta_j}}}, \\ \prod_{j=1}^{n}\left(r_n^{\delta_j}\right)_j.\ e^{2\pi i \prod_{j=1}^{n}\left(\theta_n^{\delta_j}\right)_j} \end{pmatrix} \tag{17}$$

$$CPyFPWG\ (T_1, T_2, T_3, \ldots, T_n) = \begin{pmatrix} \prod_{j=1}^{n}\left(r_m^{\delta_j}\right)_j.\ e^{2\pi i \prod_{j=1}^{n}\left(\theta_m^{\delta_j}\right)_j}, \\ \sqrt[2]{1-\prod_{j=1}^{n}\left(1-\left(r_n^2\right)_j\right)^{\delta_j}}.e^{2\pi i \sqrt[2]{1-\prod_{j=1}^{n}\left(1-\left(\theta_n^2\right)_j\right)^{\delta_j}}} \end{pmatrix} \tag{18}$$

For CSFSs, CPFSs, and Cq-ROPFSs which are not currently covered by these types of AOs, Equations (9)–(18) gives the PWA and PWG operators. Equations (15, 16, 17 and 18) are distinct examples of CTSFPWA and CTSFPWG operators, which are reduced to the environment of CPyFSs and CIFSs examined in this paper. All of this demonstrates the importance and diversity of CTSPWA and CTSFPWG operators.

5. A MADM Algorithm Is Proposed

Purpose to create a CTSFPWA and CTSFPWG operator-based algorithm for solving MADM problems. Following a brief demonstration of the MADM process, an algorithm approach is presented. In the MADM process, the best option is chosen based on some weighted attribute and the decision-makers provided data. Suppose n alternatives are available $(\mathcal{A}_i)$ with m attributes (g_j) having weight w_i. Further, let $T_{m\times n} = (r_m.e^{2\pi i\theta m}, r_i.e^{2\pi i\theta i}, r_n.e^{2\pi i\theta n})$ represent the decision matrix. To solve MADM problems in the CTSFN environment depends on the indicated aggregate operators, the following steps are followed.

Step 1. The 1st phase involves the enlargement of a matrix of decisions and information analysis to determine the value of t at which all triplets become CTSFNs.

Step 2. This phase is constructed on the calculation $Sup(T_{ij}, T_{ik})$ given by

$$Sup(T_{ij}, T_{ik}) = 1 - \mathbb{D}(T_{ij}, T_{ik}) \tag{19}$$

where $\mathbb{D}(T_{ij}, T_{ik})$ represents the normalized Hamming distance suggested by Mahmood et al. (Shanthi et al., 2021), provided by

$$\mathbb{D}(T_{ij}, T_{ik}) = \frac{1}{n}\sum_{j,k=1}^{n}\left(\left|r^t_{m_{ij}} - r^t_{m_{ik}}\right| + \left|r^t_{i_{ij}} - r^t_{i_{ik}}\right| + \left|r^t_{n_{ij}} - r^t_{n_{ik}}\right|\right)$$

Step 3. Calculate the weighted support $S(T_{ij})$ of CTSFNs T_{ij} using the following formula in

$$S(T_{ij}) = \frac{1}{n}\sum_{j,k=1}^{n} w_j Sup(T_{ij}, T_{ik}) \tag{20}$$

where w_j be the weight vector of the attribute g_j.

Step 4. Compute the weight δ_{ij} associated with CTSFNs T_{ij} using the equation given below

$$\delta_{ij} = \frac{w_j(1 - S(T_{ij}))}{\sum_{j=1}^{n} w_j(1 - S(T_{ij}))} \tag{21}$$

where $\delta_{ij} > 0$ such that $\sum_{j=1}^{n}\delta_{ij} = 1$.

Step 5. The aggregation of the decision matrix is used in this stage, with the CTSFPWA and CTSFPWG operators, as stated below

$$CTSFPWA\ (T_1,\ T_2,\ T_3,\ \ldots, T_n) = \begin{pmatrix} \sqrt[q]{1 - \prod_{j=1}^{n}\left(1 - \left(r^q_m\right)_j\right)^{\delta_j}}.e^{2\pi i\sqrt[q]{1-\prod_{j=1}^{n}\left(1-\left(\theta^q_m\right)_j\right)^{\delta_j}}}, \\ \prod_{j=1}^{n}\left(r_i^{\delta_j}\right)_j.\ e^{2\pi i\prod_{j=1}^{n}\left(\theta_i^{\delta_j}\right)_j},\ \prod_{j=1}^{n}\left(r_n^{\delta_j}\right)_j.\ e^{2\pi i\prod_{j=1}^{n}\left(\theta_n^{\delta_j}\right)_j} \end{pmatrix} \tag{22}$$

$$CTSFPOWG\ (T_1,\ T_2,\ T_3,\ \ldots, T_n) = \begin{pmatrix} \sqrt[q]{1 - \prod_{j=1}^{n}\left(1 - \left(r^q_i\right)_j\right)^{\delta_j}}.e^{2\pi i\sqrt[q]{1-\prod_{j=1}^{n}\left(1-\left(\theta^q_i\right)_j\right)^{\delta_j}}}, \\ \sqrt[q]{1 - \prod_{j=1}^{n}\left(1 - \left(r^q_n\right)_j\right)^{\delta_j}}.e^{2\pi i\sqrt[q]{1-\prod_{j=1}^{n}\left(1-\left(\theta^q_n\right)_j\right)^{\delta_j}}} \end{pmatrix} \tag{23}$$

6. Numerical Example

In this section, we solve the real-life problem using the CTSFPWA and CTSFPWG operators of aggregation.

Example 1. In this example, we investigate the well-known software selection problem, in which an appropriate software package is chosen from a list of software packages based on a set of criteria. According to this issue, the university computer center must choose an information system that will allow for more efficient research work creation. The computer center, in collaboration with the university's HR department, came up with four options in this scenario $\mathcal{A}_i = (1, 2, 3, 4)$ as best option, "participation in university performance", "Information system trustworthiness," and "efforts to shift from the current system". These four attributes have a weight vector $w = (0.25, 0.35, 0.25, 0.15)^T$. In Table 1, following decision-making matrix summarizes the decision-makers analytically monitoring four alternatives based on four attributes. This tough problem is illuminated by the MADM algorithm and the procedure is explained in detail:

Step 1. The decision matrix contains data from anonymous decision-makers on the four alternatives. The data provided in Table 1 are noted in the investigation because its sum exceeds 1. So, it is not considered a PF number. Table 1 provides a summary of all membership data.

Step 2. Using the CTSFPWA and CTSFPWG operators to derive the overall CTSFNs. Table 2 shows the aggregation results.

Table 1
Matrix for choosing the best data system

	g_1	g_2	g_3	g_4
$\mathcal{A}_1$	$\begin{pmatrix}(0.77, 0.77),\\(0.7, 0.6),\\(0.2, 0.4)\end{pmatrix}$	$\begin{pmatrix}(0.5, 0.88),\\(0.3, 0.5),\\(0.3, 0.3)\end{pmatrix}$	$\begin{pmatrix}(0.6, 0.8),\\(0.3, 0.7),\\(0.7, 0.5)\end{pmatrix}$	$\begin{pmatrix}(0.2, 0.5),\\(0.7, 0.9),\\(0.7, 0.4)\end{pmatrix}$
$\mathcal{A}_2$	$\begin{pmatrix}(0.15, 0.12),\\(0.16, 0.55),\\(0.31, 0.51)\end{pmatrix}$	$\begin{pmatrix}(0.21, 0.22),\\(0.33, 0.61),\\(0.53, 0.65)\end{pmatrix}$	$\begin{pmatrix}(0.71, 0.11)\\(0.33, 0.5),\\(0.71, 0.67)\end{pmatrix}$	$\begin{pmatrix}(0.12, 0.11),\\(0.54, 0.99),\\(0.9, 0.91)\end{pmatrix}$
$\mathcal{A}_3$	$\begin{pmatrix}(0.11, 0.58),\\(0.43, 0.64),\\(0.82, 0.39)\end{pmatrix}$	$\begin{pmatrix}(0.55, 0.66),\\(0.38, 0.37),\\(0.64, 0.8)\end{pmatrix}$	$\begin{pmatrix}(0.19,\ 0.14),\\(0.33,\ 0.12),\\(0.71, 0.71)\end{pmatrix}$	$\begin{pmatrix}(0.67, 0.11),\\(0.19, 0.16),\\(0.16, 0.91)\end{pmatrix}$
$\mathcal{A}_4$	$\begin{pmatrix}(0.13, 0.18),\\(0.17, 0.11),\\(0.11, 0.41)\end{pmatrix}$	$\begin{pmatrix}(0.21, 0.31),\\(0.41, 0.61),\\(0.71, 0.91)\end{pmatrix}$	$\begin{pmatrix}(0.15,\ 0.18),\\(0.17,\ 0.18),\\(0.85,\ 0.52)\end{pmatrix}$	$\begin{pmatrix}(0.72, 0.66),\\(0.22, 0.61),\\(0.23, 0.82)\end{pmatrix}$

Table 2
The aggregation outcomes of the emerging technology enterprises by the CTSFPWA and CTSFPWG operators

	CTSFPWA	CTSFPWG
$\mathcal{A}_1$	$\begin{pmatrix}(0.63, 0.79), \\ (0.46, 0.66), \\ (0.41, 0.39)\end{pmatrix}$	$\begin{pmatrix}(0.46, 072), \\ (0.62, 0.76) \\ (0.62, 0.42)\end{pmatrix},$
$\mathcal{A}_2$	$\begin{pmatrix}(0.58, 0.18), \\ (0.20, 0.53), \\ (0.45, 0.57)\end{pmatrix}$	$\begin{pmatrix}(0.13, 0.06), \\ (0.45, 0.92), \\ (0.80, 0.82)\end{pmatrix}$
$\mathcal{A}_3$	$\begin{pmatrix}(0.54, 0.55), \\ (0.32, 0.26), \\ (0.49, 0.67)\end{pmatrix}$	$\begin{pmatrix}(0.30, 0, 28), \\ (0.37, 0.50), \\ (0.71, 0.80)\end{pmatrix}$
$\mathcal{A}_4$	$\begin{pmatrix}(0.50, 0.46), \\ (0.41, 0.48), \\ (0.53, 0.76)\end{pmatrix}$	$\begin{pmatrix}(0.42, 0.47), \\ (0.28, 0.48), \\ (0.64, 0.73)\end{pmatrix}$

Table 3
The score function of the technology emerging firms

	CTSFPWA	CTSFPWG
$\mathcal{A}_1$	0.1179	−0.0801
$\mathcal{A}_2$	−0.1177	−0.5937
$\mathcal{A}_3$	−0.0272	0.201
$\mathcal{A}_4$	−0.2725	−0.107

Step 3. By the aggregation outcomes, the score function of the technology emerging firms is given in Table 3.

Step 4. The ordering of technology emerging firms is given in Table 4 based on the score function as given in Table 3. Also, the comparison formula of the score functions. It's worth noting that "$>$" stands for "preferred to." As can be seen, Figure 1 the technology emerging firms are ordered similarly regardless of the AOs used, with $\mathcal{A}_1$being the best emerging technology enterprise.

7. Comparative Studies

To demonstrate the competence of the proposed method, we analyze the following current operators to make comparisons of our proposed technique. We consider the following existing operators and compare aggregated findings to Ali et al. (2020), Jiang et al. (2018), Wei (2017), Rani and Garg (2018), and Wei and Lu (2018). Table 5 and Figure 2 shows the comparison of the proposed work with these existing operators for Table 1.

Table 4
The order of technology emerging firms

	Ordering
CTSFPWA	$\mathcal{A}_1 > \mathcal{A}_3 > \mathcal{A}_2 > \mathcal{A}_4$
CTSFPWG	$\mathcal{A}_1 > \mathcal{A}_4 > \mathcal{A}_3 > \mathcal{A}_2$

8. Conclusion

The uncertainty in the data has been addressed using the CTSFS concept. Favor, abstention, rejection, and degree disfavor are four functions in use by CTSFSs to describe the imprecision of an event. In comparison to current IFS, PyFS, PFS, and Cq-ROFS, the geometry of CTSFS shows that it has a greater variety of information. The CTSFS concept, therefore, provides a different way of dealing with uncertainties. Due to the significance of the PAOs, we developed the CTSFPWA and CTSFPWG operators in the layout of CTSFSs. The t parameter for the decision-maker explains a wider and more adaptable MADM algorithm depending on the specified operators. A mathematical example is used to determine the usefulness of the algorithm. We have shown that in comparison with traditional AOs, that is, Ali et al. (2020), the results obtained using the proposed PAOs are more reliable and significant as these incorporate the interrelationship of the aggregated information. We also observed that the existing theories presented by Jiang et al. (2018), and Rani and Garg (2018) and Wei and Lu (2018) fails to deal with the information discussed in the example given in our proposed work. In near future, we aim to apply the idea of PAOs to introduce the idea of Dombi PAOs (Jana & Pal, 2021), Einstein PAOs (Riaz et al., 2020), for CTSFSs. We also aim to induce the notion of power Heronian means (Đorđević et al., 2019) and power Hamy means (Stević et al., 2018) for CTSFSs.

Figure 1
Geometrical interpretation of the score values

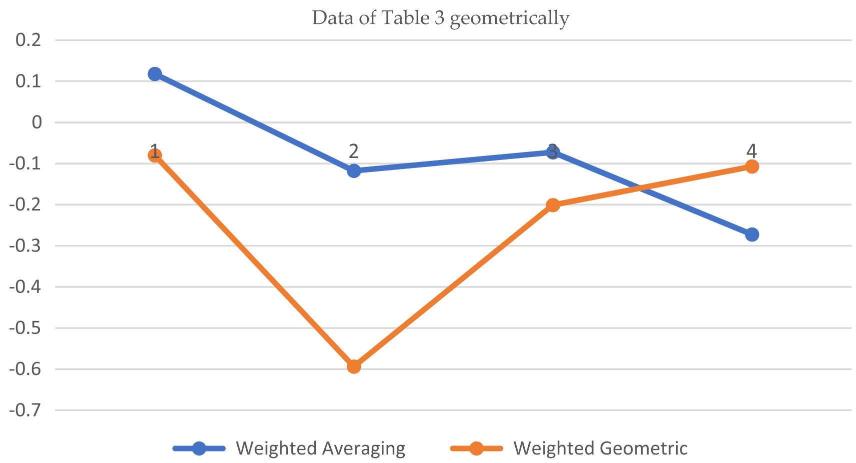
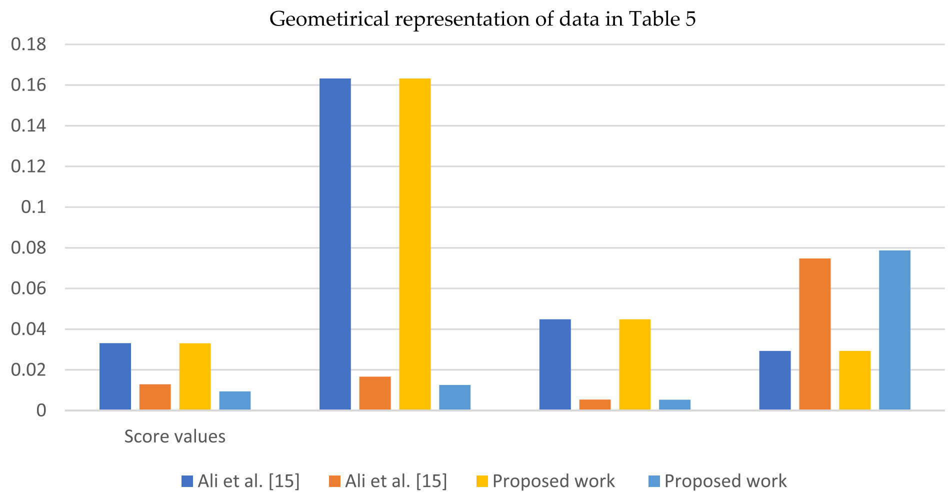


Table 5
A comparison of the proposed work with existing operators for Table 1

Methods	Operators	Score Values	Ranking Results
Proposed Operators	CTSFPWA	$S(c_1) = 0.1128,\ S(c_2) = 0.0221,\ S(c_3) = 0.0297,\ S(c_4) = 0.0149$	$\mathcal{A}_1 > \mathcal{A}_3 > \mathcal{A}_2 > \mathcal{A}_4$
	CTSFPWG	$S(c_1) = 0.05341,\ S(c_2) = 0.00001,\ S(c_3) = 0.00097,\ (c_4) = 0.00973$	$\mathcal{A}_1 > \mathcal{A}_4 > \mathcal{A}_3 > \mathcal{A}_2$
Ali et al. (2020)	CTSFWA	$S(c_1) = -0.126,\ S(c_2) = -0.482,\ S(c_3) = -0.656,\ S(c_4) = -0.052$	$\mathcal{A}_4 > \mathcal{A}_1 > \mathcal{A}_2 > \mathcal{A}_3$
	CTSFWG	$S(c_1) = 0.1085,\ S(c_2) = 0.6064,\ S(c_3) = -0.4333,\ S(c_4) = -0.3251$	$\mathcal{A}_2 > \mathcal{A}_1 > \mathcal{A}_3 > \mathcal{A}_2$
Jiang et al. (2018)	PAOs of IFSs	Unable to specify	Not applicable
Rani & Garg (2018)	PAOs of CIFSs	Unable to specify	Not applicable
Wei (2017)	Arithmetic AOs of PFSs	Unable to specify	Not applicable
Wei and Lu (2018)	PAOs of PyFSs	Unable to specify	Not applicable

Figure 2
Geometrical interpretation of comparative study



Conflicts of Interest

The authors declare that they have no conflicts of interest to this work.

References

Ali, M., & Smarandache, F. (2017). Complex neutrosophic set. *Neural Computing & Applications*, *28*, 1817–1834. https://doi.org/10.1007/s00521-015-2154-y.

Ali, Z., Mahmood, T., & Yang, M. S. (2020). Complex T-spherical fuzzy aggregation operators with application to multi-attribute decision making. *Symmetry*, *12*(8), 1311. https://doi.org/10.3390/sym12081311.

Alkouri, A. M. D. J. S., & Salleh, A. R. (2012). Complex intuitionistic fuzzy sets. *AIP Conference Proceedings*, *1482*(1), 464–470. https://doi.org/10.1063/1.4757515.

Atanassov, K. (1986). Intuitionistic fuzzy sets. *Fuzzy Sets and Systems*, *20*(1), 87–96. https://doi.org/10.1016/S0165-0114(86)80034-3.

Biswas, A., & Deb, N. (2021). Pythagorean fuzzy Schweizer and Sklar power aggregation operators for solving multi-attribute decision-making problems. *Granular Computing*, *6*, 991–1007. https://doi.org/10.1007/s41066-020-00243-1.

Cuong, B. C. (2014). Picture fuzzy sets. *Journal of Computer Science and Cybernetics*, *30*(4), 409–420. https://doi.org/10.15625/1813-9663/30/4/5032.

Chau, N. M., Lan, N. T., & Thao, N. X. (2021). A new similarity measure of picture fuzzy sets and application in pattern recognition. *American Journal of Business and Operations Research*, *1*(1), 5–18. https://americaspg.com/article/pdf/6

Đorđević, D., Stojić, G., Stević, Ž., Pamučar, D., Vulević, A., & Mišić, V. (2019). A new model for defining the criteria of service quality in rail transport: The full consistency method based on a rough power Heronian aggregator. *Symmetry*, *11*(8), 992. https://doi.org/10.3390/sym11080992.

Garg, H., Ullah, K., Mahmood, T., Hassan, N., & Jan, N. (2021). T-spherical fuzzy power aggregation operators and their applications in multi-attribute decision making. *Journal of Ambient Intelligence and Humanized Computing*, *12*, 9067–9080. https://doi.org/10.1007/s12652-020-02600-z.

Jana, C., & Pal, M. (2021). Multi-criteria decision making process based on some single-valued neutrosophic dombi power aggregation operators. *Soft Computing* , *25*, 207–213. https://doi.org/10.1007/s00500-020-05131-z.

Jiang, W., Wei, B., Liu, X., Li, X., & Zheng, H. (2018). Intuitionistic fuzzy power aggregation operator based on entropy and its application in decision making. *International Journal of Intelligent Systems*, *33*(1), 49–67. https://doi.org/10.1002/int.21939.

Luo, S., Xing, L., & Ren, T. (2022). Performance evaluation of human resources based on linguistic neutrosophic maclaurin symmetric mean operators. *Cognitive Computation*, *4*, 1–16. https://doi.org/10.1007/s12559-021-09963-1.

Mahmood, T., Ullah, K., Khan, Q., & Jan, N. (2019). An approach toward decision-making and medical diagnosis problems using the concept of spherical fuzzy sets. *Neural Computing and Applications*, *31*, 7041–7053. https://doi.org/10.1007/s00521-018-3521-2.

Meng, F., & Chen, X. (2015). Interval-valued intuitionistic fuzzy multi-criteria group decision making based on cross entropy and 2-additive measures. *Soft Computing*, *19*, 2071–2082. https://doi.org/10.1007/s00500-014-1393-7.

Mu, Z., Zeng, S., & Wang, P. (2021). Novel approach to multi-attribute group decision-making based on interval-valued pythagorean fuzzy power Maclaurin symmetric mean operator. *Computers & Industrial Engineering*, *155*, 107049. https://doi.org/10.1016/j.cie.2020.107049.

Muhammad, L. J., Badi, I., Haruna, A. A., & Mohammed, I. A. (2021). Selecting the best municipal solid waste management techniques in Nigeria using multi criteria decision making techniques. *Reports in Mechanical Engineering*, *2*(1), 180–189. https://doi.org/10.31181/rme2001021801b.

Nasir, A., Jan, N., Yang, M. S., & Khan, S. U. (2021). Complex T-spherical fuzzy relations with their applications in economic relationships and international trades. *IEEE Access*, *9*, 66115–66131. https://doi.org/10.1109/ACCESS.2021.3074557.

Pamucar, D. (2020). Normalized weighted geometric Dombi Bonferroni mean operator with interval grey numbers: Application in multicriteria decision making. *Reports in Mechanical Engineering*, *1*(1), 44–52. https://doi.org/10.31181/rme200101044p.

Qin, Y., Qi, Q., Scott, P. J., & Jiang, X. (2020). An additive manufacturing process selection approach based on fuzzy archimedean weighted power Bonferroni aggregation operators. *Robotics and Computer-Integrated Manufacturing*, *64*, 101926. https://doi.org/10.1016/j.rcim.2019.101926.

Ramot, D., Milo, R., Friedman, M., & Kandel, A. (2002). Complex fuzzy sets. *IEEE Transactions on Fuzzy Systems*, *10*(2), 171–186. https://doi.org/10.1109/91.995119.

Rani, D., & Garg, H. (2018). Complex intuitionistic fuzzy power aggregation operators and their applications in multicriteria decision-making. *Expert Systems*, *35*(6), 12325. https://doi.org/10.1111/exsy.12325.

Riaz, M., Athar Farid, H. M., Kalsoom, H., Pamučar, D., & Chu, Y. M. (2020).. A robust Q-rung orthopair fuzzy einstein prioritized aggregation operators with application towards MCGDM. *Symmetry*, *12*(6), 1058. https://doi.org/10.3390/sym12061058.

Riaz, M., & Hashmi, M. R. (2019). Linear diophantine fuzzy set and its applications towards multi-attribute decision-making problems. *Journal of Intelligent & Fuzzy Systems*, *37*(4), 5417–5439. https://doi.org/10.3233/JIFS-190550.

Rong, Y., Liu, Y., & Pei, Z. (2020). Generalized single-valued neutrosophic power aggregation operators based on archimedean copula and co-copula and their application to multi-attribute decision-making. *IEEE Access*, *8*, 35496–35519. https://doi.org/10.1109/ACCESS.2020.2974767.

Shanthi, S. A., Umamakeswari, T., & Saranya, M. (2021). MCDM method on complex picture fuzzy soft environment. *Materials Today: Proceedings*, *51*(8), 2375–2379. https://doi.org/10.1016/j.matpr.2021.11.583.

Stević, Ž., Pamučar, D., Subotić, M., Antuchevičiene, J., & Zavadskas, E. K. (2018). The location selection for roundabout construction using rough BWM-rough WASPAS approach based on a new rough hamy aggregator. *Sustainability*, *10*(8), 2817. https://doi.org/10.3390/su10082817.

Umar, A., & Saraswat, R. N. (2021). New generalized intuitionistic fuzzy divergence measure with applications to multi-attribute decision making and pattern recognition. *Recent Advances in Computer Science and Communications (Formerly: Recent Patents on Computer Science)*, *14*(7), 2247–2266. https://doi.org/10.2174/2666255813666200224093221.

Wang, C., Zhou, X., Tu, H., & Tao, S. (2017). Some geometric aggregation operators based on picture fuzzy sets and their application in multiple attribute decision making. *Italian Journal of Pure and Applied Mathematics*, *37*, 477–492.

Wei, G. (2017). Picture fuzzy aggregation operators and their application to multiple attribute decision making. *Journal of Intelligent & Fuzzy Systems*, *33*(2), 713–724. https://doi.org/10.3233/JIFS-161798.

Wei, G., & Lu, M. (2018). Pythagorean fuzzy power aggregation operators in multiple attribute decision making. *International Journal of Intelligent Systems*, *33*(2), 169–186. https://doi.org/10.1002/int.21946.

Xu, Z. (2011). Approaches to multiple attribute group decision making based on intuitionistic fuzzy power aggregation operators. *Knowledge-Based Systems*, *24*(6), 749–760. https://doi.org/10.1016/j.knosys.2011.01.011.

Xu, Z., & Yager, R. R. (2009). Power-geometric operators and their use in group decision making. *IEEE Transactions on Fuzzy Systems*, *18*(1), 94–105. https://doi.org/10.1109/TFUZZ.2009.2036907.

Yager, R. R. (2001). The power average operator. *IEEE Transactions on Systems, Man, and Cybernetics-Part A: Systems and Humans*, *31*(6), 724–731. https://doi.org/10.1109/3468.983429.

Zadeh, L. A. (1965). Fuzzy sets. *Information and Control*, *8*(3), 338–353. https://doi.org/10.1016/S0019-9958(65)90241-X.

Zhou, L., Chen, H., & Liu, J. (2012). Generalized power aggregation operators and their applications in group decision making. *Computers & Industrial Engineering*, *62*(4), 989–999. https://doi.org/10.1016/j.cie.2011.12.025.

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