Received: 18 November 2021 | Revised: 18 December 2021 | Accepted: 24 December 2021 | Published online: 31 January 2022

Journal of Computational and Cognitive Engineering
2022, Vol. 1(1) 21–31
DOI: 10.47852/bonviewJCCE2022010105

RESEARCH ARTICLE

Cosine and Cotangent Similarity Measures Based on Choquet Integral for Spherical Fuzzy Sets and Applications to Pattern Recognition

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Abstract: The notion of trigonometric similarity measure (SM) for spherical fuzzy sets (SFSs) has become very important in solving various problems in pattern recognition and medical diagnosis. This study proposes some trigonometric SMs with the help of Choquet integral for SFSs. The proposed trigonometric SMs clearly satisfy the axiomatic definition of classical SMs. We also perform these SMs in pattern recognition problems to examine a comparative analysis of the proposed trigonometric SMs with some existing SMs. Spearman's rank correlation coefficient is utilized to demonstrate the effectiveness of the proposed SMs.

Keywords: spherical fuzzy set, cosine and cotangent similarity measure, fuzzy measure, Choquet integral, pattern recognition, medical diagnosis

1. Introduction

Zadeh (1965) introduced the notion of fuzzy set (FS) with the help of a membership function and then it has been widely used to model incomplete information. Later, different FSs have been improved to better model vague information (see., Atanassov, 1986; Cuong, 2014; Yager, 2013). One of them is the concept of spherical fuzzy set (SFS) (Ashraf et al., 2019; Ashraf & Abdullah, 2019; Gündoğdu & Kahraman, 2019). It is a generalization of the concepts of Pythagorean fuzzy set (Yager, 2013) and picture fuzzy set (Cuong, 2014). A SFS A is characterized via a membership function μ_A, a neutral membership function η_A, and a non-membership function ν_A such that $\mu_A^2 + \eta_A^2 + \nu_A^2 \leq 1$. Clearly, the concept of SFS is more flexible than the concept of picture fuzzy set when dealing with the indeterminate and indefinite information in the practical real-life problems. Further information about SFSs and applications can be found in Gündoğdu and Kahraman (2019), Gündoğdu and Kahraman (2020) and Yuan et al. (2021).

A similarity measure (SM) is a powerful tool to determine how similar two items are. Recently, researchers have focused on various kinds of SMs under various types of fuzzy environment. Trigonometric SMs are examples of them, and there are many applications of trigonometric SMs in the literature (Rafiq et al., 2019; Rajarajeswari & Uma, 2013; Tian, 2013; Wei, 2017; Ye, 2011; Ye, 2016). The weighted arithmetic mean (WAM) is used in the majority of these SMs. A trigonometric SM aggregates trigonometric values of the angles among conjugate components of the vector representation of two SFSs by using the WAM. Since the WAM cannot consider the interaction between criteria, this concept is not always reasonable in some cases. In this paper, a Choquet integral (CI) similarity model which considers the interaction between criteria is proposed to overcome this deficiency. Similar idea was used for various FSs by several authors as well (Olgun et al., 2021; Olgun et al., 2021; Türkarslan et al., 2021; Yang & Ha, 2008).

Gustave Choquet proposed the concept of the CI (Choquet, 1953) in 1953, and it can be regarded as a non-linear aggregation function. As opposed to additive integrals like the Lebesgue integral, the CI has a more complex structure since there is no requirement to provide the additivity. The CI is substantially more powerful than the WAM at performing orders, as shown in Meyer and Pirlot (2012), and the difference grows as the number of members in the set grows. Furthermore, in comparison to the WAM, it has been shown in Lust (2015) that as the finite set's number of elements grows, the likelihood of finding a higher optimum ranking in the CI grows. In fact, fuzzy measures (FMs) and fuzzy integrals allow us to regard preferences that are not reflected in the weights in the WAM (Torra & Narukawa, 2007). The CI employs the FM notion (Sugeno, 1974), which can be used to simulate the interaction of criteria in a variety of circumstances. In this research, we propose some trigonometric SMs for SFSs using various types of FMs, 2-additive FMs and λ-FMs to define the interaction among criteria.

The remainder of the paper is illustrated in Table 1.

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Table 1
Outline of the paper

Section	Content
2	Some existing trigonometric SMs between SFSs are recalled
3	Some trigonometric SMs for SFSs based on CI are proposed
4	Some applications are given
5	Comparison analysis with some existing results is discussed by using Spearman's rank correlation
6	The paper is concluded and some future studies are suggested.

2. Some Existing Trigonometric SMs Between SFSs

We start with some previous weighted trigonometric SMs for SFSs in literature. Let $X = \{\iota_1, \ldots, \iota_n\}$ be a finite set and let

$$A = \{< \iota_i, \mu_A(\iota_i), \eta_A(\iota_i), \nu_A(\iota_i) > | i = 1, \ldots, n\}$$

and

$$B = \{< \iota_i, \mu_B(\iota_i), \eta_B(\iota_i), \nu_B(\iota_i) > | i = 1, \ldots, n\}$$

be two SFSs of X. Rafiq et al. (2019) and Wei et al. (2019) introduced the following weighted cosine SMs:

$$WSFC^1(A,B) := \sum_{i=1}^{n} w_i \frac{\mu_A^2(\iota_i)\mu_B^2(\iota_i) + \eta_A^2(\iota_i)\eta_B^2(\iota_i) + \nu_A^2(\iota_i)\nu_B^2(\iota_i)}{\sqrt{\mu_A^4(\iota_i) + \eta_A^4(\iota_i) + \nu_A^4(\iota_i)}\sqrt{\mu_B^4(\iota_i) + \eta_B^4(\iota_i) + \nu_B^4(\iota_i)}} \tag{1}$$

and

$$WSFC^2(A,B) := \sum_{i=1}^{n} w_i \frac{\begin{pmatrix} \mu_A^2(\iota_i)\mu_B^2(\iota_i) + \eta_A^2(\iota_i)\eta_B^2(\iota_i) \\ +\nu_A^2(\iota_i)\nu_B^2(\iota_i) + \pi_A^2(\iota_i)\pi_B^2(\iota_i) \end{pmatrix}}{\begin{pmatrix} \sqrt{\mu_A^4(\iota_i) + \eta_A^4(\iota_i) + \nu_A^4(\iota_i) + \pi_A^4(\iota_i)} \\ \sqrt{\mu_B^4(\iota_i) + \eta_B^4(\iota_i) + \nu_B^4(\iota_i) + \pi_B^4(\iota_i)} \end{pmatrix}}. \tag{2}$$

They also proposed the following weighted cosine and cotangent SMs:

$$WSFCS^1(A,B) := \sum_{i=1}^{n} w_i \cos\left[\frac{\pi}{2}\begin{pmatrix} |\mu_A^2(\iota_i) - \mu_B^2(\iota_i)| \vee |\eta_A^2(\iota_i) - \eta_B^2(\iota_i)| \vee \\ |\nu_A^2(\iota_i) - \nu_B^2(\iota_i)| \end{pmatrix}\right], \tag{3}$$

$$WSFCS^2(A,B) := \sum_{i=1}^{n} w_i \cos\left[\frac{\pi}{4}\begin{pmatrix} |\mu_A^2(\iota_i) - \mu_B^2(\iota_i)| + |\eta_A^2(\iota_i) - \eta_B^2(\iota_i)| + \\ |\nu_A^2(\iota_i) - \nu_B^2(\iota_i)| \end{pmatrix}\right], \tag{4}$$

$$WSFCS^3(A,B) := \sum_{i=1}^{n} w_i \cos\left[\frac{\pi}{2}\begin{pmatrix} |\mu_A^2(\iota_i) - \mu_B^2(\iota_i)| \vee |\eta_A^2(\iota_i) - \eta_B^2(\iota_i)| \vee \\ |\nu_A^2(\iota_i) - \nu_B^2(\iota_i)| \vee |\pi_A^2(\iota_i) - \pi_B^2(\iota_i)| \end{pmatrix}\right], \tag{5}$$

$$WSFCS^4(A,B) := \sum_{i=1}^{n} w_i \cos\left[\frac{\pi}{4}\begin{pmatrix} |\mu_A^2(\iota_i) - \mu_B^2(\iota_i)| + |\eta_A^2(\iota_i) - \eta_B^2(\iota_i)| + \\ |\nu_A^2(\iota_i) - \nu_B^2(\iota_i)| + |\pi_A^2(\iota_i) - \pi_B^2(\iota_i)| \end{pmatrix}\right], \tag{6}$$

and

$$WSFCT^1(A,B) := \sum_{i=1}^{n} w_i \cot\left[\frac{\pi}{4} + \frac{\pi}{4}\begin{pmatrix} |\mu_A^2(\iota_i) - \mu_B^2(\iota_i)| \vee |\eta_A^2(\iota_i) - \eta_B^2(\iota_i)| \vee \\ |\nu_A^2(\iota_i) - \nu_B^2(\iota_i)| \end{pmatrix}\right], \tag{7}$$

$$WSFCT^2(A,B) := \sum_{i=1}^{n} w_i \cot\left[\frac{\pi}{4} + \frac{\pi}{8}\begin{pmatrix} |\mu_A^2(\iota_i) - \mu_B^2(\iota_i)| + |\eta_A^2(\iota_i) - \eta_B^2(\iota_i)| + \\ |\nu_A^2(\iota_i) - \nu_B^2(\iota_i)| \end{pmatrix}\right], \tag{8}$$

$$WSFCT^3(A,B) := \sum_{i=1}^{n} w_i \cot\left[\frac{\pi}{4} + \frac{\pi}{4}\begin{pmatrix} |\mu_A^2(\iota_i) - \mu_B^2(\iota_i)| \vee |\eta_A^2(\iota_i) - \eta_B^2(\iota_i)| \vee \\ |\nu_A^2(\iota_i) - \nu_B^2(\iota_i)| \vee |\pi_A^2(\iota_i) - \pi_B^2(\iota_i)| \end{pmatrix}\right], \tag{9}$$

$$WSFCT^4(A,B) := \sum_{i=1}^{n} w_i \cot\left[\frac{\pi}{4} + \frac{\pi}{8}\begin{pmatrix} |\mu_A^2(\iota_i) - \mu_B^2(\iota_i)| + |\eta_A^2(\iota_i) - \eta_B^2(\iota_i)| + \\ |\nu_A^2(\iota_i) - \nu_B^2(\iota_i)| + |\pi_A^2(\iota_i) - \pi_B^2(\iota_i)| \end{pmatrix}\right], \tag{10}$$

where the symbol " $\vee$ " is the maximum operator, $w = (w_1, \ldots, w_n)$ is the weight vector with $w_i \in [0,1]$ and $\sum_{i=1}^{n} w_i = 1$.

3. Some Trigonometric SMs for SFSs Based on the CI

In this section, the CI is used to model the similarity between SFSs. The concept of FM (see., Chateauneuf & Jaffray, 1989; Sugeno, 1974) and its Möbius inverse are important to model interaction between criteria. Moreover, the Möbius inverse of a set function and relationship between a FM and its Möbius inverse are given in Chateauneuf and Jaffray (1989). According to Chateauneuf and Jaffray (1989), if we have a FM, then we obtain its Möbius inverse and if we have a Möbius inverse, we can determine the corresponding FM. Furthermore, the Möbius inverse of singletons equals the FM itself. There are some properties that the Möbius inverse of a FM must satisfy (see, item (ii) of Proposition 2 in Chateauneuf and Jaffray (1989)). One of these important properties is used to solve medical diagnosis problems in Section 4. The FM identification process is rather challenging, since it is defined on the power set. Exponential increase of the number of subsets of a given universal set complicates the FM identification in a set with excessive elements. Grabisch (1996) suggested the concept of k-additive FM to help with this complex situation. When a 2-additive FM is evaluated, it is sufficient to find the FMs of $(n(n-1))/2$ subsets rather than 2^n subsets in order to determine the entire FM. As a result, we concentrate on developing new trigonometric SMs that take into consideration the interaction of the criteria. 2-additive FMs are employed to lessen the computing cost in this case.

The concept of interaction index I_T for any $T \subset X$ is another key concept connected to FM theory. When FM is 2-additive, we have

$$I_T = \begin{cases} \Psi(\{T\}), & |T| = 2 \\ 0, & |T| > 2, \end{cases} \tag{11}$$

where Ψ is Möbius inverse of a 2-additive FM (see, e.g., Grabisch, 1996).

If $I_{ij} > 0$, $I_{ij} < 0$ and $I_{ij} = 0$, then there are positive interaction, redundancy, and non-interaction between the criteria i and j, respectively. Let σ is a FM and Ψ is a Möbius inverse of a FM. We simplify our notations using the following notation:

$\sigma_i := \sigma(\{l_i\})$, $\sigma_{ij} := \sigma(\{l_i, l_j\})$, $\sigma_{iK} := \sigma(\{l_i\} \cup K)$, $\Psi_i = \Psi(\{l_i\})$, $\Psi_{ij} := \Psi(\{l_i, l_j\})$, $\Psi_{iK} := \Psi(\{l_i\} \cup K)$ *for all* $l_i, l_j \in X$, $i \neq j$ *and for all* $K \subset X$.

The notion CI is a generalization of the WAM in which each subset of the universal set is given a weight using a FM (see, Choquet, 1953; Grabisch, 1996). If the FM is 2 -additive, then the classical CI definition transforms into form which is given in Calvo et al. (2002) and Mayag et al. (2011).

From Grabisch (1996), Calvo et al. (2002) and Mayag et al. (2011), we can see that computing the CI with regard to a FM that is 2-additive just requires interaction indices. As a result, we use interaction indices to tackle pattern recognition issues in Section 4. Now, we introduce new trigonometric SMs.

Definition 1. *Let $X = \{\iota_1, \ldots, \iota_n\}$ be a finite set, let A and B be two SFSs in X, and let σ be a FM on X. A cosine SM based on CI is given with*

$$W_{SFC^1}^{(C,\sigma)}(A,B) := (C)\int_X f_{A,B}^{(1)}(\iota)\, d\sigma, \tag{12}$$

where

$$f_{A,B}^{(1)}(\iota_i) := \frac{\mu_A^2(\iota_i)\mu_B^2(\iota_i) + \eta_A^2(\iota_i)\eta_B^2(\iota_i) + \nu_A^2(\iota_i)\nu_B^2(\iota_i)}{\sqrt{\mu_A^4(\iota_i) + \eta_A^4(\iota_i) + \nu_A^4(\iota_i)}\sqrt{\mu_B^4(\iota_i) + \eta_B^4(\iota_i) + \nu_B^4(\iota_i)}} \tag{13}$$

for $i = 1, \ldots, n$. If σ is 2-additive, then $W_{SFC^1}^{(C_{2-add.},\sigma)}$ is given with

$$W_{SFC^1}^{(C_{2-add.},\sigma)}(A,B) := (C_{2-add.})\int_X f_{A,B}^{(1)}(\iota)\, d\sigma. \tag{14}$$

Proposition 1. *Let $X = \{\iota_1, \ldots, \iota_n\}$ be a finite set, let A and B be two SFSs in X. $W_{SFC^1}^{(C,\sigma)}$ and $W_{SFC^1}^{(C_{2-add.},\sigma)}$ satisfy the following properties:*

(P_1) $0 \leq W_{SFC^1}^{(C,\sigma)}(A,B), W_{SFC^1}^{(C_{2-add.},\sigma)}(A,B) \leq 1$,

(P_2) $W_{SFC^1}^{(C,\sigma)}(A,B) = W_{SFC^1}^{(C,\sigma)}(B,A)$ and $W_{SFC^1}^{(C_{2-add.},\sigma)}(A,B) = W_{SFC^1}^{(C_{2-add.},\sigma)}(B,A)$,

(P_3) *If $A = B$ then $W_{SFC^1}^{(C,\sigma)}(A,B) = 1$ and $W_{SFC^1}^{(C_{2-add.},\sigma)}(A,B) = 1$.*

Proof. (P_1) Since $f_{A,B}^{(1)}(\iota_i) \in [0,1]$ and the CI is monotone, we have $0 \leq W_{SFC^1}^{(C,\sigma)}(A,B) \leq 1$ and $0 \leq W_{SFC^1}^{(C_{2-add.},\sigma)}(A,B) \leq 1$.

(P_2) The proof is trivial since $f_{A,B}^{(1)}(\iota_i) = f_{B,A}^{(1)}(\iota_i)$.

(P_3) If $A = B$, then we have $\mu_A(\iota_i) = \mu_B(\iota_i)$, $\eta_A(\iota_i) = \eta_B(\iota_i)$ and $\nu_A(\iota_i) = \nu_B(\iota_i)$, for $i = 1, \ldots, n$, which yields that $f_{A,B}^{(1)}(\iota_i) = 1$. Hence, we obtain $W_{SFC^1}^{(C,\sigma)}(A,B) = 1$. Moreover, we have $W_{SFC^1}^{(C_{2-add.},\sigma)}(A,B) = \sum_{B \subset X} \Psi(B) = 1$ from item (ii) of Proposition 2 in Chateauneuf and Jaffray (1989). Thus, the proof is completed.

Now, we suggest the following cosine SM based on the CI.

Definition 2. *Let $X = \{\iota_1, \ldots, \iota_n\}$ be a finite set, let A and B be two SFSs in X, and let σ be a FM on X. A cosine SM based on CI is given with*

$$W_{SFC^2}^{(C,\sigma)}(A,B) := (C)\int_X f_{A,B}^{(2)}(\iota)\, d\sigma, \tag{15}$$

where

$$f_{A,B}^{(2)}(\iota_i) : = \frac{\mu_A^2(\iota_i)\mu_B^2(\iota_i) + \eta_A^2(\iota_i)\eta_B^2(\iota_i) + \nu_A^2(\iota_i)\nu_B^2(\iota_i) + \pi_A^2(\iota_i)\pi_B^2(\iota_i)}{\sqrt{\mu_A^4(\iota_i) + \eta_A^4(\iota_i) + \nu_A^4(\iota_i) + \pi_A^4(\iota_i)}\sqrt{\mu_B^4(\iota_i) + \eta_B^4(\iota_i) + \nu_B^4(\iota_i) + \pi_B^4(\iota_i)}},$$

for $i = 1, \ldots, n$. If σ is 2-additive, then this Choquet cosine SM $W_{SFC^2}^{(C_{2-add.},\sigma)}$ is given with

$$W_{SFC^2}^{(C_{2-add.},\sigma)}(A,B) := (C_{2-add.})\int_X f_{A,B}^{(2)}(\iota)\, d\sigma(\iota). \tag{16}$$

Proposition 2. *$W_{SFC^2}^{(C,\sigma)}$ and $W_{SFC^2}^{(C_{2-add.},\sigma)}$ satisfy the conditions $P_1 - P_3$ from* Proposition 1.

Now, we propose more Choquet cosine SMs.

Definition 3. *Let $X = \{\iota_1, \ldots, \iota_n\}$ be a finite set, let A and B be two SFSs in X, and let σ be a FM on X. Two SMs are given with*

$$W_{SFCS^1}^{(C,\sigma)}(A,B) := (C)\int_X g_{A,B}^{(1)}(\iota)\, d\sigma \tag{17}$$

and

$$W_{SFCS^2}^{(C,\sigma)}(A,B) := (C)\int_X g_{A,B}^{(2)}(\iota)\, d\sigma, \tag{18}$$

where

$$g_{A,B}^{(1)}(\iota_i) : = \cos\left[\frac{\pi}{2}\left(|\mu_A^2(\iota_i) - \mu_B^2(\iota_i)| \vee |\eta_A^2(\iota_i) - \eta_B^2(\iota_i)| \vee |\nu_A^2(\iota_i) - \nu_B^2(\iota_i)|\right)\right]$$

and

$$g_{A,B}^{(2)}(\iota_i) : = \cos\left[\frac{\pi}{4}\left(|\mu_A^2(\iota_i) - \mu_B^2(\iota_i)| + |\eta_A^2(\iota_i) - \eta_B^2(\iota_i)| + |\nu_A^2(\iota_i) - \nu_B^2(\iota_i)|\right)\right]$$

for $i = 1, \ldots, n$. If σ is 2-additive, then these Choquet cosine SMs $W_{SFCS^1}^{(C_{2-add.},\sigma)}$ and $W_{SFCS^2}^{(C_{2-add.},\sigma)}$ are given with

$$W_{SFCS^1}^{(C_{2-add.},\sigma)}(A,B) := (C_{2-add.})\int_X g_{A,B}^{(1)}(\iota)\,d\sigma. \tag{19}$$

and

$$W_{SFCS^2}^{(C_{2-add.},\sigma)}(A,B) := (C_{2-add.})\int_X g_{A,B}^{(2)}(\iota)\,d\sigma. \tag{20}$$

Proposition 3. *The Choquet cosine SMs $W_{SFCS^k}^{(C,\sigma)}$ and $W_{SFCS^k}^{(C_{2-add.},\sigma)}$, for $k=1,2$ satisfy P_1, P_2, P_3 and the following properties:*

(P_3') If $W_{SFCS^k}^{(C,\sigma)}(A,B)=1$ and $W_{SFCS^k}^{(C_{2-add.},\sigma)}(A,B)=1$, then $A=B$.

(P_4) If C is an SFS in X and $A\subset B\subset C$, then $W_{SFCS^k}^{(C,\sigma)}(A,C)\le W_{SFCS^k}^{(C,\sigma)}(A,B)$ and $W_{SFCS^k}^{(C,\sigma)}(A,C)\le W_{SFCS^k}^{(C,\sigma)}(B,C)$. Moreover, $W_{SFCS^k}^{(C_{2-add.},\sigma)}(A,C)\le W_{SFCS^k}^{(C_{2-add.},\sigma)}(A,B)$ and $W_{SFCS^k}^{(C_{2-add.},\sigma)}(A,C)\le W_{SFCS^k}^{(C_{2-add.},\sigma)}(B,C)$.

Proof. (P_1), (P_2), and (P_3) can be proved similar to Proposition 1.

(P_3') Let $W_{SFCS^k}^{(C,\sigma)}=1$ and $W_{SFCS^k}^{(C_{2-add.},\sigma)}=1$ for $k=1,2$. Then, since $\cos 0=1$ we have $g_{A,B}^{(k)}(\iota_i)=1$ which yields that $|\mu_A^2(\iota_i)-\mu_B^2(\iota_i)|=0$, $|\eta_A^2(\iota_i)-\eta_B^2(\iota_i)|=0$ and $|\nu_A^2(\iota_i)-\nu_B^2(\iota_i)|=0$, $i=1,2,\ldots,n$. Therefore, we obtain $\mu_A^2(\iota_i)=\mu_B^2(\iota_i)$, $\eta_A^2(\iota_i)=\eta_B^2(\iota_i)$ and $\nu_A^2(\iota_i)=\nu_B^2(\iota_i)$, for $i=1,2,\ldots,n$. Hence, $A=B$.

(P_4) If $A\subset B\subset C$, then $\mu_A(\iota_i)\le\mu_B(\iota_i)\le\mu_C(\iota_i)$, $\eta_A(\iota_i)\ge\eta_B(\iota_i)\ge\eta_C(\iota_i)$ and $\nu_A(\iota_i)\ge\nu_B(\iota_i)\ge\nu_C(\iota_i)$, for $i=1,2,\ldots,n$. Then, $\mu_A^2(\iota_i)\le\mu_B^2(\iota_i)\le\mu_C^2(\iota_i)$, $\eta_A^2(\iota_i)\ge\eta_B^2(\iota_i)\ge\eta_C^2(\iota_i)$ and $\nu_A^2(\iota_i)\ge\nu_B^2(\iota_i)\ge\nu_C^2(\iota_i)$, for $i=1,2,\ldots,n$. Thus, we have

$$\begin{aligned}
|\mu_A^2(\iota_i)-\mu_B^2(\iota_i)| &\le |\mu_A^2(\iota_i)-\mu_C^2(\iota_i)|\\
|\mu_B^2(\iota_i)-\mu_C^2(\iota_i)| &\le |\mu_A^2(\iota_i)-\mu_C^2(\iota_i)|\\
|\eta_A^2(\iota_i)-\eta_B^2(\iota_i)| &\le |\eta_A^2(\iota_i)-\eta_C^2(\iota_i)|\\
|\eta_B^2(\iota_i)-\eta_C^2(\iota_i)| &\le |\eta_A^2(\iota_i)-\eta_C^2(\iota_i)|\\
|\nu_A^2(\iota_i)-\nu_B^2(\iota_i)| &\le |\nu_A^2(\iota_i)-\nu_C^2(\iota_i)|\\
|\nu_B^2(\iota_i)-\nu_C^2(\iota_i)| &\le |\nu_A^2(\iota_i)-\nu_C^2(\iota_i)|.
\end{aligned}$$

So, we obtain $g_{A,C}^{(k)}(\iota_i)\le g_{A,B}^{(k)}(\iota_i)$ and $g_{A,C}^{(k)}(\iota_i)\le g_{B,C}^{(k)}(\iota_i)$ since cosine function is decreasing on $[0,\pi/2]$. Therefore, $W_{SFCS^k}^{(C,\sigma)}(A,C)\le W_{SFCS^k}^{(C,\sigma)}(A,B)$ and $W_{SFCS^k}^{(C,\sigma)}(A,C)\le W_{SFCS^k}^{(C,\sigma)}(B,C)$ for $k=1,2$. If σ is 2-additive, then the proof is similar to proof of $W_{SFCS^k}^{(C,\sigma)}$.

We now propose more Choquet cosine SMs.

Definition 4. *Let $X=\{\iota_1,\ldots,\iota_n\}$ be a finite set, let A and B be two SFSs in X, and let σ be a FM on X. Two SMs based on the cosine function are given with*

$$W_{SFCS^3}^{(C,\sigma)}(A,B) := (C)\int_X g_{A,B}^{(3)}(\iota)\,d\sigma, \tag{21}$$

$$W_{SFCS^4}^{(C,\sigma)}(A,B) := (C)\int_X g_{A,B}^{(4)}(\iota)\,d\sigma, \tag{22}$$

where

$$g_{A,B}^{(3)}(\iota_i): = \cos\left[\frac{\pi}{2}\begin{pmatrix} |\mu_A^2(\iota_i)-\mu_B^2(\iota_i)|\vee|\eta_A^2(\iota_i)-\eta_B^2(\iota_i)|\vee\\ |\nu_A^2(\iota_i)-\nu_B^2(\iota_i)|\vee|\pi_A^2(\iota_i)-\pi_B^2(\iota_i)| \end{pmatrix}\right]$$

and

$$g_{A,B}^{(4)}(\iota_i): = \cos\left[\frac{\pi}{4}\begin{pmatrix} |\mu_A^2(\iota_i)-\mu_B^2(\iota_i)|+|\eta_A^2(\iota_i)-\eta_B^2(\iota_i)|\\ +|\nu_A^2(\iota_i)-\nu_B^2(\iota_i)||\pi_A^2(\iota_i)-\pi_B^2(\iota_i)| \end{pmatrix}\right]$$

for $i=1,\ldots,n$. If σ is 2-additive, then these SMs $W_{SFCS^3}^{(C_{2-add.},\sigma)}$ and $W_{SFCS^4}^{(C_{2-add.},\sigma)}$ are given with

$$W_{SFCS^3}^{(C_{2-add.},\sigma)}(A,B) := (C_{2-add.})\int_X g_{A,B}^{(3)}(\iota)\,d\sigma. \tag{23}$$

and

$$W_{SFCS^4}^{(C_{2-add.},\sigma)}(A,B) := (C_{2-add.})\int_X g_{A,B}^{(4)}(\iota)\,d\sigma. \tag{24}$$

Proposition 4. *$W_{SFCS^3}^{(C,\sigma)}$, $W_{SFCS^4}^{(C,\sigma)}$, $W_{SFCS^3}^{(C_{2-add.},\sigma)}$, and $W_{SFCS^4}^{(C_{2-add.},\sigma)}$ satisfy the conditions P_1,P_2,P_3,P_3', and P_4 in the same way as Proposition 3 does.*

Moreover, we propose more Choquet SMs as follows.

Definition 5. *Let $X=\{\iota_1,\ldots,\iota_n\}$ be a finite set, let A and B be two SFSs in X, and let σ be a FM on X. Two SMs based on the cotangent function are given with*

$$W_{SFCT^1}^{(C,\sigma)}(A,B) := (C)\int_X h_{A,B}^{(1)}(\iota)\,d\sigma \tag{25}$$

and

$$W_{SFCT^2}^{(C,\sigma)}(A,B) := (C)\int_X h_{A,B}^{(2)}(\iota)\,d\sigma, \tag{26}$$

where

$$h_{A,B}^{(1)}(\iota_i): = \cot\left[\frac{\pi}{4}+\frac{\pi}{4}\left(|\mu_A^2(\iota_i)-\mu_B^2(\iota_i)|\vee|\eta_A^2(\iota_i)-\eta_B^2(\iota_i)|\vee|\nu_A^2(\iota_i)-\nu_B^2(\iota_i)|\right)\right]$$

and

$$h_{A,B}^{(2)}(\iota_i): = \cot\left[\frac{\pi}{4}+\frac{\pi}{8}\left(|\mu_A^2(\iota_i)-\mu_B^2(\iota_i)|+|\eta_A^2(\iota_i)-\eta_B^2(\iota_i)|+|\nu_A^2(\iota_i)-\nu_B^2(\iota_i)|\right)\right]$$

for $i=1,\ldots,n$. If σ is 2-additive, then these SMs $W_{SFCT^1}^{(C_{2-add.},\sigma)}$ and $W_{SFCT^2}^{(C_{2-add.},\sigma)}$ are given with

$$W_{SFCT^1}^{(C_{2-add.},\sigma)}(A,B) := (C_{2-add.})\int_X h_{A,B}^{(1)}(\iota)\,d\sigma \tag{27}$$

and

$$W_{SFCT^2}^{(C_{2-add.},\sigma)}(A,B) := (C_{2-add.})\int_X h_{A,B}^{(2)}(\iota)\,d\sigma. \quad (28)$$

Finally, we propose more Choquet SMs as follows:

Definition 6. *Let $X = \{\iota_1, \ldots, \iota_n\}$ be a finite set, let A and B be two SFSs in X, and let σ be a FM on X. Two SMs based on the cotangent function are given with*

$$W_{SFCT^3}^{(C,\sigma)}(A,B) := (C)\int_X h_{A,B}^{(3)}(\iota)\,d\sigma \quad (29)$$

and

$$W_{SFCT^4}^{(C,\sigma)}(A,B) := (C)\int_X h_{A,B}^{(4)}(\iota)\,d\sigma, \quad (30)$$

where

$$h_{A,B}^{(3)}(\iota_i): = \cot\left[\frac{\pi}{4}+\frac{\pi}{4}\left(\begin{array}{c}|\mu_A^2(\iota_i)-\mu_B^2(\iota_i)|\vee|\eta_A^2(\iota_i)-\eta_B^2(\iota_i)|\\ \vee|\nu_A^2(\iota_i)-\nu_B^2(\iota_i)|\vee|\pi_A^2(\iota_i)-\pi_B^2(\iota_i)|\end{array}\right)\right]$$

and

$$h_{A,B}^{(4)}(\iota_i): = \cot\left[\frac{\pi}{4}+\frac{\pi}{8}\left(\begin{array}{c}|\mu_A^2(\iota_i)-\mu_B^2(\iota_i)|+|\eta_A^2(\iota_i)-\eta_B^2(\iota_i)|\\ +|\nu_A^2(\iota_i)-\nu_B^2(\iota_i)|+|\pi_A^2(\iota_i)-\pi_B^2(\iota_i)|\end{array}\right)\right]$$

for $i = 1, \ldots, n$. If σ is 2-additive, then these SMs $W_{SFCT^3}^{(C_{2-add.},\sigma)}$ and $W_{SFCT^4}^{(C_{2-add.},\sigma)}$ are given with

$$W_{SFCT^3}^{(C_{2-add.},\sigma)}(A,B) := (C_{2-add.})\int_X h_{A,B}^{(3)}(\iota)\,d\sigma \quad (31)$$

and

$$W_{SFCT^4}^{(C_{2-add.},\sigma)}(A,B) := (C_{2-add.})\int_X h_{A,B}^{(4)}(\iota)\,d\sigma. \quad (32)$$

Proposition 5. *The cotangent SMs $W_{SFCT^k}^{(C,\sigma)}$, $(k = 1,2,3,4)$ satisfy P_1, P_2, P_3, P_3', and P_4.*

Proof. It can be proved in the same way as Proposition 3.

It is worth noting that if we use a measure that is additive rather than a FM, the SMs proposed in Definitions 1 – 6 become (1) – (10), respectively.

4. Pattern Recognition Applications

The suggested SMs for SFSs are implemented to the evaluation of certain pattern recognition problems in this part to demonstrate their out-performance and appropriateness.

4.1. A cleaner production problem

The following cleaner production problem is adapted from Rafiq et al. (2019). The introduction of cleaner production is effective in resolving the contradiction between economic growth and the environmental crisis. Cleaner production has been developed in many gold mines to achieve sustainable development goals and to avoid the destruction of the ecological environment when mining natural resources. In this problem, Rafiq et al. (2019) obtained that pattern $\curlyvee$ belongs to class $\curlyvee_4$, using SMs recalled in (2) – (10) and pattern $\curlyvee$ belongs to class $\curlyvee_1$ using SMs recalled in (1), [see, Table 3 of Rafiq et al., 2019].

Example 1. In this example, a company wants to invest money in a gold furnace. We analyze the problem of cleaner production pattern recognition. A well-functioning furnace with a cleaner production unit offers more benefits compared to other furnaces. Consider the set of the criteria:

$$X = \left\{\begin{array}{l}\wp_1(Management), \wp_2(Production), \wp_3(Resource),\\ \wp_4(Waste\ Utilization), \wp_5(Environment)\end{array}\right\}.$$

Moreover, each furnace $\curlyvee_i$ for $i = 1,2,3,4,5$ and an unknown furnace $\curlyvee$ are given as an SFS with respect to all of the criteria (see, Table 1 and Table 2 of Rafiq et al., 2019), respectively.

The goal is to classify $\curlyvee$ into one of the furnaces $\curlyvee_i$ for $i = 1,2,3,4$ with respect to the criteria. For this purpose, we use the proposed Choquet SMs and so we need a FM. We consider the weights $l = (0.124, 0.216, 0.274, 0.154, 0.232)^T$ of the criteria given in Rafiq et al. (2019) to obtain a λ-FM. We construct this promised λ-FM σ by taking $\lambda = 0.75$ (see; Takahagi, 2000) in Table 2.

Now, we can calculate the similarities using Definition 1–6. For example, we obtain the cosine values as $f_{\curlyvee_1,\curlyvee}^{(1)}(\wp_1) = 0.9624$, $f_{\curlyvee_1,\curlyvee}^{(1)}(\wp_2) = 0.9350$, $f_{\curlyvee_1,\curlyvee}^{(1)}(\wp_3) = 0.9089$, $f_{\curlyvee_1,\curlyvee}^{(1)}(\wp_4) = 0.9721$, $f_{\curlyvee_1,\curlyvee}^{(1)}(\wp_5) = 0.9535$, from (13), for $\curlyvee_1$ pattern. Therefore, we get

Table 2
λ-FM

$\sigma(\emptyset) = 0$	$\sigma(\{\wp_1\}) = 0.095809$	$\sigma(\{\wp_2\}) = 0.171315$
$\sigma(\{\wp_3\}) = 0.220954$	$\sigma(\{\wp_4\}) = 0.120005$	$\sigma(\{\wp_5\}) = 0.184848$
$\sigma(\{\wp_1,\wp_2\}) = 0.279434$	$\sigma(\{\wp_1,\wp_3\}) = 0.332639$	$\sigma(\{\wp_1,\wp_4\}) = 0.224437$
$\sigma(\{\wp_1,\wp_5\}) = 0.293939$	$\sigma(\{\wp_2,\wp_3\}) = 0.420658$	$\sigma(\{\wp_2,\wp_4\}) = 0.306738$
$\sigma(\{\wp_2,\wp_5\}) = 0.379913$	$\sigma(\{\wp_3,\wp_4\}) = 0.360845$	$\sigma(\{\wp_3,\wp_5\}) = 0.436433$
$\sigma(\{\wp_4,\wp_5\}) = 0.321489$	$\sigma(\{\wp_1,\wp_2,\wp_3\}) = 0.546694$	$\sigma(\{\wp_1,\wp_2,\wp_4\}) = 0.424589$
$\sigma(\{\wp_1,\wp_2,\wp_5\}) = 0.503021$	$\sigma(\{\wp_1,\wp_3,\wp_4\}) = 0.482583$	$\sigma(\{\wp_1,\wp_3,\wp_5\}) = 0.563603$
$\sigma(\{\wp_1,\wp_4,\wp_5\}) = 0.440399$	$\sigma(\{\wp_2,\wp_3,\wp_4\}) = 0.578523$	$\sigma(\{\wp_2,\wp_3,\wp_5\}) = 0.663824$
$\sigma(\{\wp_2,\wp_4,\wp_5\}) = 0.534111$	$\sigma(\{\wp_3,\wp_4,\wp_5\}) = 0.595718$	$\sigma(\{\wp_1,\wp_2,\wp_3,\wp_4\}) = 0.715903$
$\sigma(\{\wp_1,\wp_2,\wp_3,\wp_5\}) = 0.807333$	$\sigma(\{\wp_1,\wp_2,\wp_4,\wp_5\}) = 0.668299$	$\sigma(\{\wp_1,\wp_3,\wp_4,\wp_5\}) = 0.734334$
$\sigma(\{\wp_2,\wp_3,\wp_4,\wp_5\}) = 0.843575$	$\sigma(\{\wp_1,\wp_2,\wp_3,\wp_4,\wp_5\}) = 1$	

Table 3
Comparison of the results of Example 1

	Similarity Measures	Similarity scores				Best Selections
		(Υ_1,Υ)	(Υ_2,Υ)	(Υ_3,Υ)	(Υ_4,Υ)	
The results of Rafiq et al. (2019)	$WSFC^1$	**0.9413**	0.9331	0.8984	0.9162	Υ_1
	$WSFC^2$	0.8294	0.8566	0.8432	**0.9293**	Υ_4
	$WSFCS^1$	0.9418	0.9077	0.9222	**0.9682**	Υ_4
	$WSFCS^2$	0.9603	0.9528	0.9641	**0.9758**	Υ_4
	$WSFCS^3$	0.8950	0.9007	0.9121	**0.9559**	Υ_4
	$WSFCS^4$	0.8861	0.8912	0.8975	**0.9495**	Υ_4
	$WSFCT^1$	0.7158	0.6722	0.6920	**0.7760**	Υ_4
	$WSFCT^2$	0.7595	0.7530	0.7816	**0.8060**	Υ_4
	$WSFCT^3$	0.6555	0.6513	0.6725	**0.7451**	Υ_4
	$WSFCT^4$	0.6366	0.6294	0.6533	**0.7337**	Υ_4
The results of proposed Choquet trigonometric similarity measures	$W^{(C,\sigma)}_{SFC^1}$	**0.9376**	0.9265	0.8818	0.9082	Υ_1
	$W^{(C,\sigma)}_{SFC^2}$	0.8121	0.8488	0.8248	**0.9244**	Υ_4
	$W^{(C,\sigma)}_{SFCS^1}$	0.9376	0.8961	0.9141	**0.9672**	Υ_4
	$W^{(C,\sigma)}_{SFCS^2}$	0.9574	0.9471	0.9609	**0.9742**	Υ_4
	$W^{(C,\sigma)}_{SFCS^3}$	0.8849	0.8923	0.9002	**0.9515**	Υ_4
	$W^{(C,\sigma)}_{SFCS^4}$	0.8750	0.8828	0.8868	**0.9439**	Υ_4
	$W^{(C,\sigma)}_{SFCT^1}$	0.7056	0.6530	0.6745	**0.7732**	Υ_4
	$W^{(C,\sigma)}_{SFCT^2}$	0.7510	0.7398	0.7694	**0.7999**	Υ_4
	$W^{(C,\sigma)}_{SFCT^3}$	0.6380	0.6412	0.6456	**0.7343**	Υ_4
	$W^{(C,\sigma)}_{SFCT^4}$	0.6181	0.6191	0.6271	**0.7211**	Υ_4

*Bold values indicate the maximum of the corresponding line.

$f^{(1)}_{\Upsilon_1,\Upsilon}(\wp_3) \le f^{(1)}_{\Upsilon_1,\Upsilon}(\wp_2) \le f^{(1)}_{\Upsilon_1,\Upsilon}(\wp_5) \le f^{(1)}_{\Upsilon_1,\Upsilon}(\wp_1) \le f_{\Upsilon_1,\Upsilon}(\wp_4)$ and so we have

$$W^{(C,\sigma)}_{SFC^1}(\Upsilon_1,\Upsilon) = (C)\int_X f^{(1)}_{\Upsilon_1,\Upsilon}(\wp)\,d\sigma(\wp) = \sum_{k=1}^{5}\left(f^{(1)}_{\Upsilon_1,\Upsilon}(\wp_{(k)}) - f^{(1)}_{\Upsilon_1,\Upsilon}(\wp_{(k-1)})\right)\sigma(E_{(k)})$$
$$= \left(f^{(1)}_{\Upsilon_1,\Upsilon}(\wp_3) - f^{(1)}_{\Upsilon_1,\Upsilon}(\wp_0)\right)\sigma(E_{(1)}) + \left(f^{(1)}_{\Upsilon_1,\Upsilon}(\wp_2) - f^{(1)}_{\Upsilon_1,\Upsilon}(\wp_3)\right)\sigma(E_{(2)})$$
$$+ \left(f^{(1)}_{\Upsilon_1,\Upsilon}(\wp_5) - f^{(1)}_{\Upsilon_1,\Upsilon}(\wp_2)\right)\sigma(E_{(3)}) + \left(f^{(1)}_{\Upsilon_1,\Upsilon}(\wp_1) - f^{(1)}_{\Upsilon_1,\Upsilon}(\wp_5)\right)\sigma(E_{(4)})$$
$$+ (f_{\Upsilon_1,\Upsilon}(\wp_4) - f_{\Upsilon_1,\Upsilon}(\wp_1))\sigma(E_{(5)}) = 0.9089 \times 1 + (0.9350 - 0.9089)$$
$$\times 0.668299 + (0.9535 - 0.9350) \times 0.440399 + (0.9624 - 0.9535)$$
$$\times 0.224437 + (0.9721 - 0.9624) \times 0.120005 = 0.9376.$$

The numerical results are given in Table 3. The proposed trigonometric SMs appoint the unknown furnace Υ to the known furnace Υ_4 except for the $W^{(C,\sigma)}_{SFC^1}(\Upsilon_i,\Upsilon)(i = 1,2,3,4)$. This result is in agreement with the one obtained in Rafiq et al. (2019).

4.2. A medical diagnosis problem

A medical diagnosis problem adopted from Rafiq et al. (2019) is used to explain the feasibility of the proposed SM and provide a comparative analysis. The process of determining which ailment is responsible for a patient's symptoms is known as medical diagnosis. In this process, many illnesses are compared to the symptoms of the target patient. Rafiq et al. (2019) obtained that target patient P belongs to class λ_1 by using SMs recalled in (2) – (10) and pattern P belongs to class λ_4 using SMs recalled in (1) [see, Table 5 of Rafiq et al. (2019)].

Example 2. Let us consider the set of diagnosis

$$D = \left\{ \begin{array}{c} \lambda_1(\text{Viral fever}), \lambda_2(\text{Malaria}), \\ \lambda_3(\text{Typhoid}), \lambda_4(\text{Stomach problem}) \end{array} \right\}$$

and symptoms

$$S = \left\{ \begin{array}{c} s_1(\text{Temperature}), s_2(\text{Headache}), s_3(\text{Stomach pain}), \\ s_4(\text{Cough}), s_5(\text{Chest pain}) \end{array} \right\}.$$

Each diagnosis λ_i, $i = 1,2,3,4$ and the patient P that have all the symptoms are given as SFSs in Rafiq et al. (2019) (see, subsection 4.3 in Rafiq et al. (2019)).

The purpose is to classify Υ as one of the diagnostic λ_i for $i = 1,2,3,4$ in terms of symptoms. To do this, we need a FM. We build a 2-additive FM to determine the interaction between symptoms. We know that the Möbius inverse of subsets of two elements equals the interaction index whenever measure is 2-additive (see (11)). To determine the FMs of the remaining two-element subsets, we use the Möbius form of the FM. We use the FM of singletons as the weight vector of symptoms $(0.124, 0.216, 0.274, 0.154, 0.232)^T$ described in Rafiq et al. (2019) since the Möbius inverse of singletons is identical to the FM. Furthermore, because the sum of Möbius inverses of singletons is one, we can deduce from (ii) of Proposition 2 in Chateauneuf and Jaffray (1989) that the sum of Möbius inverses of subsets containing two elements should be zero. Now, taking into account the interaction of symptoms, we have the Möbius inverse shown in Table 4:

Table 4
Möbius inverse of the FM

$\Psi_1 = 0.124$	$\Psi_2 = 0.216$	$\Psi_3 = 0.274$	$\Psi_4 = 0.154$
$\Psi_5 = 0.232$	$\Psi_{1,2} = 0$	$\Psi_{1,3} = 0$	$\Psi_{1,4} = 0.2$
$\Psi_{1,5} = 0$	$\Psi_{2,3} = -0.1$	$\Psi_{2,4} = 0.1$	$\Psi_{2,5} = 0$
$\Psi_{3,4} = 0$	$\Psi_{3,5} = -0.2$	$\Psi_{4,5} = 0$	

Table 5
Comparison of classification result of Example 2

	Similarity measures	Similarity scores $(⅄_1, \Upsilon)$	$(⅄_2, \Upsilon)$	$(⅄_3, \Upsilon)$	$(⅄_4, \Upsilon)$	Best selections
The results of Rafiq et al. (2019)	$WSFC^1$	0.9137	0.8962	0.9337	**0.9511**	$⅄_4$
	$WSFC^2$	**0.9231**	0.7600	0.8141	0.8180	$⅄_1$
	$WSFCS^1$	**0.9682**	0.8729	0.9544	0.9497	$⅄_1$
	$WSFCS^2$	**0.9727**	0.9344	0.9578	0.9613	$⅄_1$
	$WSFCS^3$	**0.9551**	0.8462	0.8905	0.8817	$⅄_1$
	$WSFCS^4$	**0.9430**	0.8238	0.8690	0.8682	$⅄_1$
	$WSFCT^1$	**0.7760**	0.6198	0.7570	0.7339	$⅄_1$
	$WSFCT^2$	**0.7940**	0.7219	0.7783	0.7811	$⅄_1$
	$WSFCT^3$	**0.7430**	0.5895	0.6748	0.6312	$⅄_1$
	$WSFCT^4$	**0.7220**	0.5675	0.6388	0.6203	$⅄_1$
The results of proposed 2-additive Choquet trigonometric similarity measures	$W_{SFC^1}^{(C_{2-add.},\xi)}$	0.9276	0.9350	0.9319	**0.9525**	$⅄_4$
	$W_{SFC^2}^{(C_{2-add.},\xi)}$	0.9055	0.8504	0.8124	**0.9087**	$⅄_4$
	$W_{SFCS^1}^{(C_{2-add.},\xi)}$	0.9621	0.8987	0.9469	**0.9635**	$⅄_4$
	$W_{SFCS^2}^{(C_{2-add.},\xi)}$	0.9689	0.9505	0.9446	**0.9857**	$⅄_4$
	$W_{SFCS^3}^{(C_{2-add.},\xi)}$	0.9402	0.8897	0.8986	**0.9441**	$⅄_4$
	$W_{SFC^4}^{(C_{2-add.},\xi)}$	0.9181	0.8789	0.8498	**0.9456**	$⅄_4$
	$W_{SFCT^1}^{(C_{2-add.},\xi)}$	0.7587	0.6732	0.7434	**0.7614**	$⅄_4$
	$W_{SFCT^2}^{(C_{2-add.},\xi)}$	0.7827	0.7736	0.7461	**0.8412**	$⅄_4$
	$W_{SFCT^3}^{(C_{2-add.},\xi)}$	**0.7109**	0.6624	0.6799	0.7105	$⅄_1$
	$W_{SFCT^4}^{(C_{2-add.},\xi)}$	0.6728	0.6500	0.6127	**0.7117**	$⅄_4$

*Best selections in bold indicate disagreement results between proposed SMs and Rafiq et al. (2019).

For example, since $\Psi_1 < \Psi_4 < \Psi_2 < \Psi_5 < \Psi_3$, we consider that the severe stomach pain may cause chest pain and so interaction between these two symptoms should be negative. That is, there is a redundancy between the symptoms s_3, s_5 and we assign a negative value for $I_{3,5} = \Psi_{3,5}$. Moreover, we know that temperature may cause headache. However, we consider that such a small weight assigned temperature could not cause headache and so there is no relationship between these two symptoms. That is, s_1 and s_2 are independent from each other and so we assign zero for $I_{1,2} = \Psi_{1,2}$. Similarly, we know that when temperature and cough come together, it may be a sign of many viral diseases. Therefore, we assign a positive value for $I_{1,4} = \Psi_{1,4}$. Now, we calculate the proposed SMs between patients and diseases with respect to given symptoms.

For example, we obtain the cosine values as $f^{(1)}_{⅄_1,\Upsilon}(s_1) = 0.9330$, $f^{(1)}_{⅄_1,\Upsilon}(s_2) = 0.8911$, $f^{(1)}_{⅄_1,\Upsilon}(s_3) = 0.8562$, $f^{(1)}_{⅄_1,\Upsilon}(s_4) = 0.9086$, $f^{(1)}_{⅄_1,\Upsilon}(s_5) = 0.9958$ for $⅄_1$ disease. Therefore, we have

$$W_{SFC^1}^{(C_{2-add.},\xi)}(⅄_1,\Upsilon) = (C_{2-add.})\int_S f^{(1)}_{⅄_1,\Upsilon}(s)\,d\xi(s) = \sum_{s_i\in S}\Psi(\{s_i\})f^{(1)}_{⅄_1,\Upsilon}(s_i)$$
$$+\sum_{\{s_i,s_j\}\subseteq S}\Psi(\{s_i,s_j\})\,min(f^{(1)}_{⅄_1,\Upsilon}(s_i),f^{(1)}_{⅄_1,\Upsilon}(s_j)) = \Psi(\{s_1\})\times f^{(1)}_{⅄_1,\Upsilon}(s_1)$$
$$+\Psi(\{s_2\})\times f^{(1)}_{⅄_1,\Upsilon}(s_2)+\Psi(\{s_3\})\times f^{(1)}_{⅄_1,\Upsilon}(s_3)+\Psi(\{s_4\})\times f^{(1)}_{⅄_1,\Upsilon}(s_4)$$
$$+\Psi(\{s_5\})\times f^{(1)}_{⅄_1,\Upsilon}(s_5)+\Psi(\{s_1,s_4\})\times \min(f^{(1)}_{⅄_1,\Upsilon}(s_1),f^{(1)}_{⅄_1,\Upsilon}(s_4))$$
$$+\Psi(\{s_2,s_3\})\times \min(f^{(1)}_{⅄_1,\Upsilon}(s_2),f^{(1)}_{⅄_1,\Upsilon}(s_3))+\Psi(\{s_2,s_4\})$$
$$\times \min(f^{(1)}_{⅄_1,\Upsilon}(s_2),f^{(1)}_{⅄_1,\Upsilon}(s_4))+\Psi(\{s_3,s_5\})\times \min(f^{(1)}_{⅄_1,\Upsilon}(s_3),f^{(1)}_{⅄_1,\Upsilon}(s_5))$$
$$= 0.124\times 0.9330+0.216\times 0.8911+0.274\times 0.8562+0.154$$
$$\times 0.9086+0.232\times 0.9958+0.2\times \min(0.9330,0.9086)-0.1$$
$$\times \min(0.8911,0.8562)+0.1\times \min(0.8911,0.9086)-0.2$$
$$\times \min(0.8562,0.9958) = 0.9276.$$

It is shown that nine SMs allocate the unknown patient P to the known class $⅄_4$ in Table 5.

4.3. A pattern recognition problem

In this subsection, we examine a pattern recognition which is adopted from Wei et al. (2019).

Example 3. Let $X = \{\iota_1, \iota_2, \iota_3\}$ be a finite set and let $\ltimes_1, \ltimes_2$ and $\ltimes_3$ be three patterns which are given in Wei et al. (2019). Moreover, $\ltimes$ be a pattern that should be classified into one of the $\ltimes_1, \ltimes_2,$ and $\ltimes_3$ classes which is given in Wei et al. (2019). We use a hypothetical FM. We use the virtual weights of Wei et al. (2019) as a singleton FM and use the monotonous characteristics of the FM to generate the remaining measures (see Table 6).

Table 6 reflects the positive interaction between ι_1 and ι_2, negative interaction between ι_2 and ι_3 criteria, and no interaction between ι_1 and ι_3 which are used to construct the FM (Grabisch, 1997). Now, we calculate the SMs using Definition (1–6).

Note that incompatible results with the past results may cause because of the sensitivity of the FM theory (see Table 7).

4.4. A medical diagnosis problem

In this subsection, we examine a medical diagnosis problem which is adopted from Wei et al. (2019).

Example 4. Let us consider the set D of the diagnoses used in Example 2 with one additional diagnosis chest problem $⅄_5$ and let the set S of symptoms remains same as Example 2. Moreover, each diagnosis $⅄_i$, $i = 1, 2, 3, 4, 5$ and the patient Υ that have all the symptoms are given as SFSs in Wei et al. (2019) (see, Example 2 in Wei et al. (2019)).

We again use the concept of Möbius inverse to determine the FM. Since the Möbius inverse of singletons is equal to the FM,

Table 6
FM

$\delta(\varnothing) = 0$	$\delta(\{\iota_1\}) = 0.2$	$\delta(\{\iota_2\}) = 0.3$	$\delta(\{\iota_3\}) = 0.5$
$\delta(\{\iota_1, \iota_2\}) = 0.9$	$\delta(\{\iota_1, \iota_3\}) = 0.7$	$\delta(\{\iota_2, \iota_3\}) = 0.5$	$\delta(\{\iota_1, \iota_2, \iota_3\}) = 1$

Table 7
Comparison of classification result of Example 3

	Similarity measures	Similarity scores $(\ltimes_1, \ltimes)$	$(\ltimes_2, \ltimes)$	$(\ltimes_3, \ltimes)$	Best selection
The results of Wei et al. (2019)	$WSFC^1$	0.9367	0.9406	**0.9434**	$\ltimes_3$
	$WSFC^2$	0.8504	0.8144	**0.9195**	$\ltimes_3$
	$WSFCS^1$	0.8656	0.8919	**0.9135**	$\ltimes_3$
	$WSFCS^2$	0.9257	**0.9440**	0.9419	$\ltimes_2$
	$WSFCS^3$	0.8620	0.8919	**0.9135**	$\ltimes_3$
	$WSFCS^4$	0.8357	0.8631	**0.8953**	$\ltimes_3$
	$WSFCT^1$	0.6221	0.6152	**0.6606**	$\ltimes_3$
	$WSFCT^2$	0.6767	**0.7202**	0.7141	$\ltimes_2$
	$WSFCT^3$	0.6109	0.6152	**0.6606**	$\ltimes_3$
	$WSFCT^4$	0.5549	0.5763	**0.6337**	$\ltimes_3$
The results of proposed Choquet trigonometric similarity measures	$W^{(C,\delta)}_{SFC^1}$	**0.9423**	0.8700	0.9311	$\ltimes_1$
	$W^{(C,\delta)}_{SFC^2}$	0.8566	0.8014	**0.9207**	$\ltimes_3$
	$W^{(C,\delta)}_{SFCS^1}$	**0.9488**	0.9032	0.9348	$\ltimes_1$
	$W^{(C,\delta)}_{SFCS^2}$	0.9434	0.9274	**0.9498**	$\ltimes_3$
	$W^{(C,\delta)}_{SFCS^3}$	**0.9418**	0.9032	0.9348	$\ltimes_1$
	$W^{(C,\delta)}_{SFCS^4}$	0.8879	0.8761	**0.9134**	$\ltimes_3$
	$W^{(C,\delta)}_{SFCT^1}$	**0.7479**	0.6324	0.6957	$\ltimes_1$
	$W^{(C,\delta)}_{SFCT^2}$	0.7108	0.6896	**0.7302**	$\ltimes_3$
	$W^{(C,\delta)}_{SFCT^3}$	**0.7267**	0.6324	0.6957	$\ltimes_1$
	$W^{(C,\delta)}_{SFCT^4}$	0.6156	0.5928	**0.6590**	$\ltimes_3$

*Best selections in bold indicate disagreement results between proposed SMs and Wei et al. (2019).

Table 8
Möbius inverse of the FM

$\Psi_1 = 0.25$	$\Psi_2 = 0.15$	$\Psi_3 = 0.15$	$\Psi_4 = 0.25$	$\Psi_5 = 0.20$
$\Psi_{1,2} = -0.01$	$\Psi_{1,3} = 0$	$\Psi_{1,4} = 0.02$	$\Psi_{1,5} = 0$	$\Psi_{2,3} = 0$
$\Psi_{2,4} = 0$	$\Psi_{2,5} = 0$	$\Psi_{3,4} = 0$	$\Psi_{3,5} = 0$	$\Psi_{4,5} = -0.01$

we take the FM of singletons as the weight vector of symptoms given in Wei et al. (2019): $(0.25, 0.15, 0.15, 0.25, 0.20)^T$.

Now, considering interaction of the symptoms similar to Example 2 we have the Möbius inverse given in Table 8.

For example, since $\Psi_2 \leq \Psi_3 < \Psi_5 < \Psi_1 \leq \Psi_4$, we consider that the severe temperature may cause headache and so interaction between these two symptoms should be negative. That is, there is a redundancy between the symptoms s_1, s_2 and we assign a negative value for $I_{1,2} = \Psi_{1,2}$. Moreover, we know that when temperature and cough come together, it may be a sign of many viral diseases. Therefore, similar to Example 2 we assign a positive value for $I_{1,4} = \Psi_{1,4}$. In addition, we consider that if the symptoms are independent from each other, then we assign zero interaction.

Now, we compute the proposed SMs between patients and diseases in terms of the given symptoms (see, Table 9).

It is shown that the proposed 2-additive Choquet SMs appoint the target patient $\curlyvee$ to the disease $\curlywedge_3$ in Table 9. This result is in agreement with the one obtained in Wei et al. (2019).

5. Comparison Analysis

In this part, we examine how the paper's findings compare to those found in the literature.

5.1. Consistency analysis with Spearman's Rank Correlation Coefficient (SRCC)

Using SRCC (Spearman, 1987), we assess the ranking consistency of Examples 1 and 4. The SRCC ρ is shown below, and the test results are shown in Tables 3 and 9:

$$\rho =: 1 - \frac{6}{n(n^2 - 1)} \sum_{i=1}^{n} d_i^2, \qquad (33)$$

where n is the number of results examined, and d_i denotes the difference in the results' ranks. Figure 1 demonstrates the SRCC which shows the consistency between result of Rafiq et al. (2019) and us for Example 1.

Table 9
Comparison of classification result of Example 4

	Similarity measures	Similarity scores					Best selection
		$(⅄_1, Y)$	$(⅄_2, Y)$	$(⅄_3, Y)$	$(⅄_4, Y)$	$(⅄_5, Y)$	
The results of Wei et al. (2019)	$WSFC^1$	0.8220	0.7872	**0.9575**	0.6836	0.7537	$⅄_3$
	$WSFC^2$	0.8506	0.8432	**0.9310**	0.7312	0.8215	$⅄_3$
	$WSFCS^1$	0.9378	0.9453	**0.9833**	0.8824	0.9168	$⅄_3$
	$WSFCS^2$	0.9509	0.9474	**0.9813**	0.9171	0.9448	$⅄_3$
	$WSFCS^3$	0.9105	0.9062	**0.9619**	0.8329	0.8887	$⅄_3$
	$WSFCS^4$	0.8968	0.8806	**0.9491**	0.7971	0.8741	$⅄_3$
	$WSFCT^1$	0.7239	0.7240	**0.8358**	0.6070	0.6595	$⅄_3$
	$WSFCT^2$	0.7441	0.7279	**0.8257**	0.6608	0.7271	$⅄_3$
	$WSFCT^3$	0.6712	0.6411	**0.7784**	0.5567	0.6132	$⅄_3$
	$WSFCT^4$	0.6381	0.6007	**0.7337**	0.5174	0.5952	$⅄_3$
The results of proposed 2-additive Choquet trigonometric similarity measures	$W^{(C_{2-add.},\zeta)}_{SFC^1}$	0.8192	0.7825	**0.9578**	0.6840	0.7485	$⅄_3$
	$W^{(C_{2-add.},\zeta)}_{SFC^2}$	0.8504	0.8430	**0.9395**	0.7326	0.8215	$⅄_3$
	$W^{(C_{2-add.},\zeta)}_{SFCS^1}$	0.9366	0.9450	**0.9830**	0.8826	0.9303	$⅄_3$
	$W^{(C_{2-add.},\zeta)}_{SFCS^2}$	0.9502	0.9469	**0.9812**	0.9169	0.9438	$⅄_3$
	$W^{(C_{2-add.},\zeta)}_{SFCS^3}$	0.9106	0.9062	**0.9622**	0.8534	0.9024	$⅄_3$
	$W^{(C_{2-add.},\zeta)}_{SFCS^4}$	0.8969	0.8802	**0.9491**	0.7986	0.8737	$⅄_3$
	$W^{(C_{2-add.},\zeta)}_{SFCT^1}$	0.7217	0.7235	**0.8351**	0.6073	0.6818	$⅄_3$
	$W^{(C_{2-add.},\zeta)}_{SFCT^2}$	0.7427	0.7269	**0.8257**	0.6606	0.7254	$⅄_3$
	$W^{(C_{2-add.},\zeta)}_{SFCT^3}$	0.6713	0.6411	**0.7790**	0.5776	0.6356	$⅄_3$
	$W^{(C_{2-add.},\zeta)}_{SFCT^4}$	0.6382	0.6002	**0.7339**	0.5188	0.5947	$⅄_3$

*Bold values indicate the maximum of the corresponding line.

Figure 1
The SRCC for Example 1

	$WSFC^1$	$WSFC^2$	$WSFCS^1$	$WSFCS^2$	$WSFCS^3$	$WSFCS^4$	$WSFCT^1$	$WSFCT^2$	$WSFCT^3$	$WSFCT^4$
$W^{(C,\sigma)}_{SFC^1}$	1	-0.4	0	-0.6	-0.8	-0.8	0	-0.6	-0.6	-0.6
$W^{(C,\sigma)}_{SFC^2}$	-0.4	1	0.2	0.4	0.8	0.8	0.2	0.4	0.4	0.4
$W^{(C,\sigma)}_{SFCS^1}$	0	0.2	1	0.8	0.4	0.4	1	0.8	0.8	0.8
$W^{(C,\sigma)}_{SFCS^2}$	-0.6	0.4	0.8	1	0.8	0.8	0.8	1	1	1
$W^{(C,\sigma)}_{SFCS^3}$	-0.8	0.8	0.4	0.8	1	1	0.4	0.8	0.8	0.8
$W^{(C,\sigma)}_{SFCS^4}$	-0.8	0.8	0.4	0.8	1	1	0.4	0.8	0.8	0.8
$W^{(C,\sigma)}_{SFCT^1}$	0	0.2	1	0.8	0.4	0.4	1	0.8	0.8	0.8
$W^{(C,\sigma)}_{SFCT^2}$	-0.6	0.4	0.8	1	0.8	0.8	0.8	1	1	1
$W^{(C,\sigma)}_{SFCT^3}$	-0.8	0.8	0.4	0.8	1	1	0.4	0.8	0.8	0.8
$W^{(C,\sigma)}_{SFCT^4}$	-0.8	0.8	0.4	0.8	1	1	0.4	0.8	0.8	0.8

Similarly, we obtain consistent results between (Wei, 2017) and Example 4. Figure 2 demonstrates the SRCC which shows the consistency between result of Wei et al. (2019) and our results in Example 4.

5.2. Contributions of the proposed CI model

The important contributions of the proposed CI model are summarized below:

Since the proposed SMs for SFSs are based on the CI, they are generalizations of the SMs that were given with the arithmetic and weighted average in the literature.

Unlike the WAM, the CI models the interaction between criteria. Therefore, proposed SMs are more sensitive.

2-additive FMs model the interaction between the criteria with the help of the Möbius inverse. Therefore, the direction and the magnitude of the interaction can be determined. In other words, the Möbius inverse of a 2-additive FM in the fuzzy environment corresponds to the correlation coefficient in the real environment.

While calculating the proposed 2-additive Choquet SMs, the decision-making process becomes shorter and easier, since 2-additive FMs facilitate the process of determining the measure.

Figure 2
The SRCC for Example 4

	$WSFC^1$	$WSFC^2$	$WSFCS^1$	$WSFCS^2$	$WSFCS^3$	$WSFCS^4$	$WSFCT^1$	$WSFCT^2$	$WSFCT^3$	$WSFCT^4$
$W_{SFC^1}^{(C_2-add,\varsigma)}$	1	1	0.9	1	1	1	0.9	1	1	1
$W_{SFC^1}^{(C_2-add,\varsigma)}$	1	1	0.9	1	1	1	0.9	1	1	1
$W_{SFCS^1}^{(C_2-add,\varsigma)}$	0.9	0.9	1	0.9	0.9	0.9	1	0.9	0.9	0.9
$W_{SFCS^1}^{(C_2-add,\varsigma)}$	1	1	0.9	1	1	1	0.9	1	1	1
$W_{SFCS^3}^{(C_2-add,\varsigma)}$	1	1	0.9	1	1	1	0.9	1	1	1
$W_{SFCS^4}^{(C_2-add,\varsigma)}$	1	1	0.9	1	1	1	0.9	1	1	1
$W_{SFCT^1}^{(C_2-add,\varsigma)}$	0.9	0.9	1	0.9	0.9	0.9	1	0.9	0.9	0.9
$W_{SFCT^1}^{(C_2-add,\varsigma)}$	1	1	0.9	1	1	1	0.9	1	1	1
$W_{SFCT^3}^{(C_2-add,\varsigma)}$	1	1	0.9	1	1	1	0.9	1	1	1
$W_{SFCT^4}^{(C_2-add,\varsigma)}$	1	1	0.9	1	1	1	0.9	1	1	1

6. Conclusion

This paper proposes new SMs based on the CI. Here, we use FMs to characterize the interactions between the elements of a particular universe of SFS and use the CI model instead of the weighted average model to calculate the dimensions. To demonstrate the efficiency of the proposed Choquet SMs, we also apply them to pattern recognition problems from the literature and then compare the result with the existing result. Most results are consistent with past results. Consistency between these results is supported by Spearman's correlation coefficient. In the future, similar SMs can be considered for other types of FSs and more SMs such as Dice Similarity can be given in the same environment.

Acknowledgement

The research of Mehmet Ünver and Murat Olgun has been supported by The Scientific and Technological Research Council of Turkey (TÜBİTAK) Grant 121F007.

The research of Ezgi Türkarslan has been supported by Turkish Scientific and Technological Research Council (TÜBİTAK) Programme 2221 and Grant 121F007.

The authors are grateful to the reviewers for carefully reading the manuscript and for offering many suggestions which resulted in an improved presentation.

Conflicts of Interest

The authors declare that they have no conflicts of interest to this work.

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How to Cite: Ünver, M., Olgun, M., & Türkarslan, E. (2022). Cosine and Cotangent Similarity Measures Based on Choquet Integral for Spherical Fuzzy Sets and Applications to Pattern Recognition. *Journal of Computational and Cognitive Engineering* 1(1), 21–31, https://doi.org/10.47852/bonviewJCCE2022010105