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About One Representation-Interpreter of a Monotone Measure

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Abstract: We consider the problem of presenting a nonclassical measure (nonadditive, but monotonous), so-called fuzzy (monotone) measure, by a classical measure, particularly by the Murofushi–Sugeno-type new representation-interpreter. The theorems on the universal interpreter of a monotone (fuzzy) measure in the Choquet integral environment and second-order dual capacities are considered.

Keywords: fuzzy measure, classical measure, fuzzy measure representation–interpreter, Choquet integral, second-order capacity

1. Introduction

Our main focus is on multicriteria decision making and the aggregation of inputs, two areas where a finite set of inputs need to be combined into the overall representative value. In contrast to many alternative ways of aggregating inputs, such as weighted means, aggregations based on fuzzy measures allow to incorporate mutual dependency of the inputs, their redundancy, and complementarity. This makes fuzzy measures a valuable tool for modeling systems where the inputs such as decision criteria are correlated.

Beliakov and others in Grabisch (1997) say: "What makes fuzzy measures so valuable is their ability to model the various ways inputs can interact, by assigning importance weights not just to individual inputs, but to all coalitions *C*. Thus, an input may be unimportant individually but gain importance in the presence of other inputs, and vice versa. The central notion of monotonicity has important semantics: increasing the value of any criterion (e.g., utility, preference) cannot decrease the total aggregate value. The flexibility of fuzzy measures when modeling interaction comes at a significant cost: the exponential number of coalitions whose contributions need to be quantified. This gives rise to two problems: their interpretation and elicitation. If a fuzzy measure-based model is to be understood by domain experts, the large number of capacity values need to be combined into some sort of characteristic indices, such as the overall importance of an input in all coalitions, or the overall interaction of a pair of inputs. On the other hand, if a fuzzy measure is to be specified, either by the experts or by machine learning techniques, it has to be done through a few desirability criteria and in a computationally efficient way".

Various simplifications exist that reduce the large number of parameters that characterize fuzzy measures. There are: 1. Symmetric fuzzy measures. In this case, the Choquet integral becomes the popular Ordered Weighted Averaging function. The Sugeno integral with respect to symmetric fuzzy measures also coincides with a special class of functions called the ordered weighted maximum and minimum. 2. Authors sometimes present a range of other simplification strategies called, collectively, k-order fuzzy measures. Here, the interaction among the inputs (in one sense or another) is limited to coalitions of smaller cardinalities (up to k elements). This technique reduces the number of parameters to be specified or learned, and sometimes reduces the number of monotonicity constraints. The latter is crucial for the development of efficient computational algorithms. The problem of learning fuzzy measures from observed or desired data is discussed and translated into optimization problems. In particular, due to very large numbers of monotonicity and other constraints, we prefer the formulation of the learning problem as a linear programming problem. In this setting, we make use of efficient numerical methods, which handle large and sparse matrices of constraints. Still, larger numbers of decision criteria require simplification strategies, and we present learning methods based on k-order simplifications.

It is clear that often more convenient is to use nonadditive but monotonous measures for presenting subjective expert assessments (Denneberg, 1994; Dubois & Prade, 1988; Shafer, 1976; Sirbiladze & Sikharulidze, 2003; Sirbiladze, 2013; Sugeno, 1974). The nonadditivity of a fuzzy measure distinguishes it from the classical measure, namely the probabilistic measure, by lacking many important properties. The authors are researching the "probabilistic properties" of a fuzzy measure (Ban & Gal, 2002; Sirbiladze, 2013, 2020; Sirbiladze & Gachechiladze, 2005; Sirbiladze & Zaporozhets, 2002), and probabilistic representations (Campos Ibañez & Carmona, 1989; Murofushi & Sugeno, 1989; Sirbiladze, 2013, 2020; Sirbiladze & Gachechiladze, 2005; Sirbiladze & Zaporozhets, 2002), which give rise to new perspectives on the use of a monotone (fuzzy)

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measure. With this in mind, we consider a new measure, the Murofushi–Sugeno probabilistic type representation–interpreter (Murofushi & Sugeno, 1989).

Briefly, we consider a review of papers on fuzzy measure non-additive characteristics: representations, identifications, non-additivity, and deffectness indexes.

However, the aggregation process in MCDM is based on the assumption that the criteria (attributes) or preferences of DMPs are independent, and the aggregation operators are linear operators based on additive measures, which are characterized by an independence axiom (Dubois & Prade, 1988; Wakker, 1999). For real decision-making problems, there is a phenomenon ensuring that there exists some degree of interdependent or interactive characteristics between criteria (Grabisch, 1995; Grabisch et al., 2000; Wang & Klir, 1992). For a decision problem, DMPs invited usually come from the same or similar fields. They have almost similar knowledge, social status, and preferences. DMPs' subjective preferences always show non-linearity. Independence phenomena among these criteria and mutual preferential independence of DMPs are violated. In 1974, Sugeno (1974) introduced the concept of non-additive measure (monotone or fuzzy measure), which only requires monotonicity instead of additivity property. It is the most effective tool to model interaction phenomena (Grabisch, 1995, 1996; Ishii & Sugeno, 1985; Kojadinovic, 2002) and deal with decision problems (Grabisch, 1995, 1997; Grabisch et al., 2000) where a fuzzy measure is used instead of a probability one. A review on analyzing decision makers' behavior using fuzzy measure theory can be seen in (Liginlal & Ow, 2006).

There are several methods for the determination or identification of a fuzzy measure. For instance, linear methods (Marichal & Roubens, 1998), quadratic methods (Grabisch, 1996, 1996), methods based on Sugeno's λ-additive measures (Larbani et al., 2011; Wang et al., 1998), heuristic-based methods (Grabisch, 1995), genetic algorithms (Wang et al., 1998) and so on. In Grabisch et al. (2008) the discussion is focused on the usage of Choquet integral in some investigation of fuzzy measure identification method. The robust optimization problems for the fuzzy measure identification are presented in Timonin (2013). The latest papers on some of the proposed methods to reduce the complexity of identifying some of the fuzzy measure values are reviewed in Krishnan et al. (2015).

In Campos Ibañez & Carmona (1989) and Campos Ibañez et al. (1990) a method to study a fuzzy measure utilizing certain sets of associated probabilities has been developed. Distances on fuzzy measures are defined through distances between associated probabilities. Some important properties of Choquet finite integral in terms of associated probabilities are proved.

The non-additivity index is a competent indicator of depicting the kind and intensity of interaction among decision criteria. In this paper, we focus on using the non-additivity index to represent the decision maker's preference information as well as the process of transforming them into standard capacity. In Wu and Beliakov (2018) and Wu and Beliakov (2020) authors discuss the comparison and range representation forms of decision preference information in terms of the non-additivity index and update the inconsistency recognition models and adjustment strategies. Then authors establish a non-additivity index oriented multiple goal linear programming algorithm to find out the minimum deviation capacities with relatively fewer concerns and efforts on inconsistency adjustment. The illustrative example demonstrates the feasibility and the flexibility of the proposed scheme and methods. In Huang et al. (2020) authors adopt a kind of explicit interaction index, the non-additivity index, to construct two types of quasi-random generation methods of capacity under a given decision interaction preference. Compared to the existing random generation algorithms, the methods have relatively satisfactory performance on the statistics characteristic of generated capacities but need rather less calculation effort on the generation process. The authors also show the effectiveness of proposed quasi-random generation methods by an illustrative decision example. Paper (Beliakov & Divakov, 2020) examines the marginal contribution representation of fuzzy measures, used to construct fuzzy measures from empirical data through an optimization process. Authors have shown that the number of variables can be drastically reduced, and the constraints are simplified by using an alternative representation. This technique makes optimizing fitting criteria more efficient numerically and allows one to tackle learning problems with a higher number of correlated decision criteria. In Generation of Capacities and its Application in Comprehensive Decision Aiding (2020) authors have proposed the concepts of k-order additive fuzzy measure, including usual additive measures and fuzzy measures. It was proved that every finite fuzzy measure is a k-order additive fuzzy measure for a unique k. A related topic of the fuzzy measure is to introduce an alternative representation of fuzzy measures, called the interaction representation, which extends the Shapley value and interaction index, proposed by Murofushi.

In Section 2, necessary introductory definitions and some fundamental facts are presented. New results by theorems on the universal representation–interpreter of a fuzzy measure in the environment of the Choquet integral (CI) and second-order dual capacities are considered in Section 3. In Section 4, the main results and future directions of studies of the presented problems are discussed.

2. Preliminaries

Definition 1. (Sugeno, 1974). Let (X, F) be any measurable space. A monotone (fuzzy) measure is called a set function with real non-negative values

$$\nu : \mathrm{F} \to R_0^+$$

having the following properties:

(i) $\nu(\emptyset) = 0, \quad \nu(X) < \infty,$

(ii) *If* $H, K \in \mathrm{F}$, $H \subset K$, *then* $\nu(H) \leq \nu(K)$,

(iii) *For any monotone sequence* $\{H_n\}, H_n \in \mathrm{F}, \lim_{n\to\infty} \nu(B_n) = \nu\left(\lim_{n\to\infty} H_n\right)$.

Definition 2. (Sugeno, 1974). A fuzzy measure $\nu^* : \mathrm{F} \to R_0^+$ is called dual to the fuzzy measure ν if $\forall B \in \mathrm{F}$

$$\nu^*(B) = \nu(X) - \nu(B^C).$$

Denote by $FM(\mathrm{F})$ a set of fuzzy measures defined on space (X, F) and denote by $CM(\mathrm{F})$ a set of classical measures defined on the same space, while $M(\mathrm{F})$ denotes F-measurable functions and $M(\mathrm{F})^+$ denotes nonnegative measurable functions.

Definition 3. (Choquet, 1954). Let $g \in M(\mathrm{F})^+$ be a function and $\nu \in FM(\mathrm{F})$ be a fuzzy measure. The CI is defined as

$$(CI) \int g d\nu \overset{def}{=} \int_0^{+\infty} \nu_g(r) dr, \tag{1}$$

where $\nu_g(r) = \nu(\{y \in X/g(y) \geq r\}),\quad r \geq 0,$ *dr is the Lebesgue measure on* $[0; +\infty[$.

Definition 4. (Choquet, 1954). *Let* $g \in M(\mathrm{F})$ *be a function and* $\nu \in FM(\mathrm{F})$ *be a fuzzy measure. The CI is defined as*

$$(CI)\int g d\nu = (CI)\int g^{+} d\nu - (CI)\int g^{-} d\nu^{*}, \tag{2}$$

where $g^{+} = g \vee 0,\ g^{-} = -(g \wedge 0)$.

Remark 1. If the difference in formula (2) represents $\infty - \infty$, then the CI is not defined.

Definition 5. (Denneberg, 1994).

1. *The fuzzy measure* $\nu : \mathrm{F} \to R_0^{+}$ *is called k-monotonous* $(k \geq 2,\quad k \in N)$*, if for any sets* $B_1, \ldots, B_k \in \mathrm{F}$ *the following inequality*

$$\nu\left(\bigcup_{l=1}^{k} B_l\right) + \sum_{J \subseteq \{1,\ldots,k\}} (-1)^{|J|} \nu\left(\bigcap_{l \in I} B_l\right) \geq 0 \tag{3}$$

   is valid.

2. *A measure* ν *is called completely monotonous if* $\forall k \in N,\ k \geq 2$*, it is k-monotonous.*

Definition 6. (Choquet, 1954). The $k = 2$*-monotonous fuzzy measure is called the Choquet second-order lower capacity (SOLC), and its dual fuzzy measure* ν^{*} *is called the upper capacity (SOUP).*

Clearly, the Choquet second-order dual capacities satisfy the following inequalities $\forall H, K \in \mathrm{F}$

$$\begin{aligned} \nu(H \cup K) + \nu(H \cap K) &\geq \nu(H) + \nu(K), \\ \nu^{*}(H \cup K) + \nu^{*}(H \cap K) &\leq \nu^{*}(H) + \nu^{*}(K). \end{aligned} \tag{4}$$

Definition 7. (Murofushi & Sugeno, 1989). Let (X, F_X) and (Y, F_y) be measurable spaces.

(1) A mapping $\Theta : \mathrm{F}_X \to \mathrm{F}_Y$ is called an interpreter from F_X to F_Y, if the following conditions are true
  (i) $\Theta(\oslash) = \oslash$,
  (ii) *If* $H, K \in \mathrm{F}_X,\ H \subset K$*, then* $\Theta(H) \subset \Theta(K)$.

(2) *A triplet* $(Y, \mathrm{F}_Y, \Theta)$ *is called a frame of the space* (X, F_X)*, if* Θ *is an interpreter from* F_X to F_Y.

(3) *Let* (X, F_X, ν) *be any fuzzy measure space. A cortege* $(Y, \mathrm{F}_Y, m, \Theta)$ *is called a representation–interpreter of a fuzzy measure* ν *from* F_X to F_Y *if* $m : \mathrm{F}_Y \to [0; +\infty)$ *is a classical measure and*

$$\nu = m \circ \Theta\ (\forall C \in \mathrm{F}_X,\ \nu(C) = m(\Theta(C))). \tag{5}$$

Definition 8. (Murofushi & Sugeno, 1989). *Let* Ξ *be some nonempty class of sets from* (X, F_X)*.* Ξ *is a semifilter if it satisfies the following conditions:*

(i) $\oslash \notin \Xi$,

(ii) *If* $H \in \Xi$ *and* $(H \subset K \in \mathrm{F}_X)$*, then* $K \in \Xi$.

Let S_X denote a set of all semi-filters in (X, F_X). Let us introduce a mapping $\Theta_X : \mathrm{F}_X \to 2^{S_X}$ as follows: $\forall C \in \mathrm{F}_X$

$$\Theta_X(C) = \{\theta \in S_X / C \in \theta\}. \tag{6}$$

Definition 9. *A cortege* $(S_X, 2^{S_X}, \Theta_X)$ *is called a universal frame of the space* (X, F_X).

Let us consider some theorems that are proved in Murofushi and Sugeno (1989).

Theorem 1. (Murofushi & Sugeno, 1989). Let $\nu \in FM(\mathrm{F}_X)$ be a fuzzy measure. There exists a classical measure $m \in M(2^{S_X})$, for which $(S_X, 2^{S_X}, m, \Theta_X)$ is a representation–interpreter of a fuzzy measure ν.

Definition 10. *Let* $(Y, \mathrm{F}_y, m, \Theta)$ *be a representation–interpreter of a fuzzy measure* $\nu \in FM(\mathrm{F}_X)$*;* $g \in M(\mathrm{F}_X)^{+}$ *be a nonnegative function. An interpreter of a function g induced by mapping* Ψ *is called a measurable function* F_X*:*

$$\begin{aligned} i_g : Y \to R_0^{+}, \forall y \in Y, i_g(y) \\ = \sup\{r \geq 0 / y \in \Theta(\{x \in X / g(x) \geq r\})\}. \end{aligned} \tag{7}$$

Theorem 2. (Murofushi & Sugeno, 1989). Let $(Y, \mathrm{F}_y, m, \Theta)$ be a representation–interpreter of a fuzzy measure $\nu \in FM(\mathrm{F}_X)$; $g \in M(\mathrm{F}_X)^{+}$ be any function and i_g be its interpreter induced by a mapping Θ, then

$$(CI)\int g\, d\nu = \int i_g\, dm. \tag{8}$$

Remark 2. *(8) represents an interpretation of the CI by the Lebesgue integral.*

Corollary 1. *Let* $\nu \in FM(\mathrm{F}_X)$ *be a fuzzy measure. For* ν*, there exists a classical measure* $m \equiv m_X\ m_X : 2^{S_X} \to R_0^{+}$*, such that for any function* $g \in M(\mathrm{F}_X)^{+}$

$$\nu = m_X \circ \Theta_X, \quad (CI)\int g\, d\nu = \int i_g dm_X. \tag{9}$$

3. Universal Representation–Interpreter of a Fuzzy Measure

Definition 11. *For any classical measure* $m_X \in M(\mathrm{F}_X)$*, a representation*

$$(S_X, 2^{S_X}, m_X, \Theta_X) \tag{10}$$

is called universal representation–interpreter of a fuzzy measure ν.

Remark 3. *In the universal representation from the quadruplet, only* m_X *depends on* ν *by the equality:* $\forall B \in \mathrm{F}_X,\ \nu(B) = m_x(\Theta_X(B))$.

In this article, we aim at consideration of some properties of universal representation–interpreter. Let us consider some theorems, particular cases of which are shown in Sirbiladze and Zaporozhets (2002) and Sirbiladze (2013).

Let (X, F, ν) be a space of fuzzy measures. Let $CM(\mathrm{F}, \nu)$ be a set of all classical measures $m_x : 2^{S_X} \to R_0^+$ from the interpreter (10) and $CM(\mathrm{F}, \nu^*)$ be a set of classical measures m_X^* corresponding to a dual measure ν^*.

Theorem 3. *Fuzzy measures $\nu,\nu^* \in FM(\mathrm{F})$ are dual if and only if for any classical measures $\forall m_X \in CM(\mathrm{F}_X, \nu)$* and *$\forall m_X^* \in CM(\mathrm{F}_X, \nu^*)$ and $\forall B \in \mathrm{F}$:*

$$m_X(\Psi_X(B)) = m_X^*(\Theta_X(B^C)). \tag{11}$$

Theorem 4. *A pair of dual fuzzy measures $\nu,\nu^* \in FM(\mathrm{F})$ are accordingly Choquet SOLC and SOUC if and only if $\forall m_X \in CM(\mathrm{F}, \nu)$ and $\forall m_X^* \in CM(\mathrm{F}, \nu^*)$ classical measures and $\forall H, K \in \mathrm{F}$:*

$$\begin{aligned} m_X(\Theta_X(H \cup K)) &\geq m_X(\Theta_X(H) \cup \Theta_X(K)), \\ m_X^*(\Theta_X(H \cup K)) &\leq m_X^*(\Theta_X(H) \cup \Theta_X(K)). \end{aligned} \tag{12}$$

Theorem 5. *Let ν, $\nu^* \in FM(\mathrm{F})$ be a pair of dual fuzzy measures and $g_1, g_2 \in M(\mathrm{F})$ be any functions for which there exists the CI with respect to fuzzy measures ν and ν^*. Then,*

(1) *There exists the CI for function $g_1 + g_2$;*

(2) *The dual fuzzy measures ν and ν^* are the Choquet SOLC and SOUC then, and only then, if:*

$$\begin{aligned} (CI)\int(g_1+g_2)d\nu &\geq (CI)\int g_1 d\nu + (CI)\int g_2 d\nu, \\ (CI)\int(g_1+g_2)d\nu^* &\leq (CI)\int g_1 d\nu^* + (CI)\int g_2 d\nu^*. \end{aligned} \tag{13}$$

Remark 4. *If for a fuzzy measure $\nu \in FM(\mathrm{F}_X)$ the normalization condition is $\nu(X) = 1$, then the classical measure in its representation–interpreter is a probability measure $(m(X) = 1)$.*

4. Conclusions

For any fuzzy measure, its universal interpreter-representations $(S_X, 2^{S_X}, m_X, \Theta_X)$ turned out to be a class that fully describes the important properties of a fuzzy measure as well as the fairly common measures – the Choquet second-order capacities. In order to develop future research, we will discuss a new representation-interpreter entropy and its main features in the study of expert knowledge streams.

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Conflicts of Interest

The authors declare that they have no conflicts of interest to this work.

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