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RESEARCH ARTICLE

Prioritized Muirhead Mean Aggregation Operators under the Complex Single-Valued Neutrosophic Settings and Their Application in Multi-Attribute Decision-Making

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Abstract: Two critical tasks in multi-attribute decision-making (MADM) are to describe criterion values and to aggregate the described information to generate a ranking of alternatives. A flexible and superior tool for the first task is complex single-valued neutrosophic (CSVN) setting, and a powerful device for the subsequent assignment is aggregation operator. Up until this point, almost 30 diverse aggregation operators of CSVN have been introduced. Every operator has its unmistakable qualities and can function admirably for explicit reasons. Notwithstanding, there is not yet an operator that can give helpful consensus and adaptability in conglomerating rule esteems, managing the heterogeneous interrelationships among models, and decreasing the impact of outrageous basis esteems. In genuine decision-making interaction, there are cases that the interrelationships of contentions do not exist in each one of the contentions, however, in piece of the contentions. Subsequently, there is a need to parcel the contentions into various parts. For this, the technique of prioritized Muirhead mean (PMM) aggregation operator is massive, dominant, and more flexible to investigate the interrelationships between any numbers of objects. The goal of this study is to initiate the CSVN setting and to determine their important algebraic laws. Moreover, to provide such an aggregation operator, the principle of CSVN PMM (CSVNPMM) operator and CSVN prioritized dual Muirhead mean (CSVNPDMM) operator is elaborated, and their particular cases are discussed. Further, based on these operators, we presented a new method to deal with the MADM problems under the fuzzy environment. Finally, we used some practical examples to illustrate the validity and superiority of the proposed method by comparing with other existing methods.

Keywords: complex single-valued neutrosophic sets, prioritized Muirhead mean operators, decision-making techniques

1. Introduction

Multi-attribute decision-making (MADM) is the fundamental importance of the decision-making (DM) science whose expectation is to perceive the best option(s) from the pack of likely ones. In genuine DM, the person needs to assess the given choices by various classes such as single, span, and so on, for assessment purposes. Nonetheless, in different erratic conditions, it is normally trying for the pioneer to deliver their decisions as a fresh number. To handle such nature of worries, the phenomena of the fuzzy set (FS) was elaborated by Zadeh (1965). FS is the modified technique of crisp set, which covers the truth grade (TG) belonging to unit interval instead of two opinions "0" or "1." Sometimes, the theory of FS has been neglected, for illustration, if an intellectual gives the data in the shape of "yes" or "no." To handle such sort of data, the theory of FS has not been able to resolve it. For this, Atanassov (1986) initiated the technique of intuitionistic FS (IFS). An IFS covers two sorts of data such as TG and falsity grade (FG) with the condition that the sum of the duplet lies between "0" and "1." Due to its shape, the principle of IFS has gotten massive attraction from the different intellectuals. For illustration, Karaaslan and Karatas (2015) presented the bipolar soft sets. Liu et al. (2021) explored some operators under the interval-valued IFSs. Thao (2021) initiated numerous sorts of measures by using IFSs. Gao et al. (2021) elaborated the MADM technique under the IFSs. Karmakar et al. (2021) presented the type-2 intuitionistic fuzzy matrix game and its applications. Türk et al. (2021) discussed solar site selection problems based on an IFS. Yang and Yao (2021) developed three-way decisions under the IFSs. Jana and Pal (2018) explored the bipolar intuitionistic soft sets and their application in DM troubles.

The theory of an IFS also cannot work in numerous situations. For illustration, if an individual gives information in the shape of "yes," "abstinence," and "no," then the principle of an IFS has been neglected. For this, the principle of the neutrosophic set (NS) was developed by Smarandache (1998). NS covers the TG $\mathcal{M}_{\overline{\overline{\mathcal{T}\mathcal{I}_C}}}(\overline{\overline{\Xi}})$, abstinence grade (AG) $\mathcal{A}_{\overline{\overline{\mathcal{T}\mathcal{I}_C}}}(\overline{\overline{\Xi}})$, and FG $\mathcal{N}_{\overline{\overline{\mathcal{T}\mathcal{I}_C}}}(\overline{\overline{\Xi}})$

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belonging to $]0^-,1^+[$ with a well-known characteristic $0^- \leq \mathcal{M}_{\overline{\overline{\mathcal{TI}}}_C}(\Xi) + \mathcal{A}_{\overline{\overline{\mathcal{TI}}}_C}(\Xi) + \mathcal{N}_{\overline{\overline{\mathcal{TI}}}_C}(\Xi) \leq 3^+$. Further, Wang et al. (2010) modified the NS to initiate the single-valued NS (SVNS), which covers the TG $\mathcal{M}_{\overline{\overline{\mathcal{TI}}}_C}(\Xi)$, AG $\mathcal{A}_{\overline{\overline{\mathcal{TI}}}_C}(\Xi)$, and FG $\mathcal{N}_{\overline{\overline{\mathcal{TI}}}_C}(\Xi)$ belonging to [0,1] with a well-known characteristic $0 \leq \mathcal{M}_{\overline{\overline{\mathcal{TI}}}_C}(\Xi) + \mathcal{A}_{\overline{\overline{\mathcal{TI}}}_C}(\Xi) + \mathcal{N}_{\overline{\overline{\mathcal{TI}}}_C}(\Xi) \leq 3$. The principles of SVNS and NS have gotten massive attraction from the different intellectuals. For example, Ye (2014) presented certain measures for interval-valued NSs. Yang et al. (2017) utilized the principle of rough set in the environment of SVNS. Ji et al. (2018) explored the frank prioritized Bonferroni mean operators for SVNS. Sahin and Kucuk (2015) initiated the subsethood measures for SVNS. Ye (2014) elaborated the correlation measures under the SVNS. Peng et al. (2014) proposed decision-making method for SVNS. Saqlain et al. (2020) developed the tangent measures for SVNS. Kandasamy (2018) developed the double-valued NS and their applications. Chai et al. (2021) initiated the measures for neutrosophic soft sets. Qin and Wang (2020) explored the entropy measures for SVNS. Chatterjee et al. (2016) presented the similarity measures for SVNSs.

To handle awkward and ambiguous data in genuine life dilemmas, the principle of FS has been neglected in some cases, for illustration, if an intellectual provides two-dimensional data in the shape of single sets. For this, Ramot et al. (2002) elaborated the principle of complex FS (CFS), which covers the TG $\mathcal{M}_{\overline{\overline{\mathcal{TI}}}_C}(\Xi) = \mathcal{M}_{\overline{\overline{\mathcal{TI}}}_R}(\Xi)e^{\text{ff}2\pi\left(\mathcal{M}_{\overline{\overline{\mathcal{TI}}}_I}(\Xi)\right)}$ with the rule $0 \leq \mathcal{M}_{\overline{\overline{\mathcal{TI}}}_R}(\Xi), \mathcal{M}_{\overline{\overline{\mathcal{TI}}}_I}(\Xi) \leq 1$. Moreover, Liu et al. (2020) explored certain sorts of measures for CFSs. Further, Alkouri and Salleh (2012) developed the complex IFS (CIFS), by including the FG $\mathcal{N}_{\overline{\overline{\mathcal{TI}}}_C}(\Xi) = \mathcal{N}_{\overline{\overline{\mathcal{TI}}}_R}(\Xi)e^{\text{ff}2\pi\left(\mathcal{N}_{\overline{\overline{\mathcal{TI}}}_I}(\Xi)\right)}$ in the environment of CFS with the condition $0 \leq \mathcal{M}_{\overline{\overline{\mathcal{TI}}}_R} + \mathcal{N}_{\overline{\overline{\mathcal{TI}}}_R} \leq 1$ and $0 \leq \mathcal{M}_{\overline{\overline{\mathcal{TI}}}_I} + \mathcal{N}_{\overline{\overline{\mathcal{TI}}}_I} \leq 1$. Due to its structure, certain people exploited it in the natural environment of separated regions. For instance, Gulzar et al. (2020) initiated the CIFSs. Yaqoob et al. (2019) proposed CIF graphs. Garg and Rani (2019) introduced the complex interval-valued IFSs. Kumar and Bajaj (2014) developed CIF soft sets. Ngan et al. (2020) explored group based on CIFS. Yaqoob et al. (2019) initiated the CIF graphs.

Certain intellectuals have utilized the theories of IFSs, NSs, SVNSs, and CIFSs in the environment of distinct regions. But in some cases, these existing theories are not able to handle awkward and complicated data in genuine life troubles. For illustration, if an individual gives two-dimensional information in the shape of TG, AG, and FG, then the principle of CIFS has been neglected. For this, the principle of complex NS (CNS) was developed by Ali and Smarandache (2017). CNS covers the TG $\mathcal{M}_{\overline{\overline{\mathcal{TI}}}}(\Xi) = \mathcal{M}_{\overline{\overline{\mathcal{TI}}}_R}(\Xi)e^{\text{ff}2\pi\left(\mathcal{M}_{\overline{\overline{\mathcal{TI}}}_I}(\Xi)\right)}$, abstinence grade (AG) $\mathcal{A}_{\overline{\overline{\mathcal{TI}}}_C}(\Xi) = \mathcal{A}_{\overline{\overline{\mathcal{TI}}}_R}(\Xi)e^{\text{ff}2\pi\left(\mathcal{A}_{\overline{\overline{\mathcal{TI}}}_I}(\Xi)\right)}$, and FG $\mathcal{N}_{\overline{\overline{\mathcal{TI}}}_C}(\Xi) = \mathcal{N}_{\overline{\overline{\mathcal{TI}}}_R}(\Xi)e^{\text{ff}2\pi\left(\mathcal{N}_{\overline{\overline{\mathcal{TI}}}_I}(\Xi)\right)}$, whose real and unreal parts belonging to $]0^-,1^+[$ with a well-known characteristic $0^- \leq \mathcal{M}_{\overline{\overline{\mathcal{TI}}}_R} + \mathcal{A}_{\overline{\overline{\mathcal{TI}}}_R} + \mathcal{N}_{\overline{\overline{\mathcal{TI}}}_R} \leq 3^+$ and $0^- \leq \mathcal{M}_{\overline{\overline{\mathcal{TI}}}_I} + \mathcal{A}_{\overline{\overline{\mathcal{TI}}}_I} + \mathcal{N}_{\overline{\overline{\mathcal{TI}}}_I} \leq 3^+$. Moreover, Singh (2018) initiated the complex neutrosophic lattice. Al-Quran and Alkhazaleh (2018) developed the relationship among CNSs and their applications. But, if an intellectual gives data in the shape of $\mathcal{M}_{\overline{\overline{\mathcal{TI}}}_C}(\Xi) = \mathcal{M}_{\overline{\overline{\mathcal{TI}}}_R}(\Xi)e^{\text{ff}2\pi\left(\mathcal{M}_{\overline{\overline{\mathcal{TI}}}_I}(\Xi)\right)}$, $\mathcal{A}_{\overline{\overline{\mathcal{TI}}}_C}(\Xi) = \mathcal{A}_{\overline{\overline{\mathcal{TI}}}_R}(\Xi)e^{\text{ff}2\pi\left(\mathcal{A}_{\overline{\overline{\mathcal{TI}}}_I}(\Xi)\right)}$, and $\mathcal{N}_{\overline{\overline{\mathcal{TI}}}_C}(\Xi) = \mathcal{N}_{\overline{\overline{\mathcal{TI}}}_R}(\Xi)e^{\text{ff}2\pi\left(\mathcal{N}_{\overline{\overline{\mathcal{TI}}}_I}(\Xi)\right)}$, where the values of real and unreal parts belong to standard unit interval, i.e., [0,1] with $0 \leq \mathcal{M}_{\overline{\overline{\mathcal{TI}}}_R} + \mathcal{A}_{\overline{\overline{\mathcal{TI}}}_R} + \mathcal{N}_{\overline{\overline{\mathcal{TI}}}_R} \leq 3$ and $0 \leq \mathcal{M}_{\overline{\overline{\mathcal{TI}}}_I} + \mathcal{A}_{\overline{\overline{\mathcal{TI}}}_I} + \mathcal{N}_{\overline{\overline{\mathcal{TI}}}_I} \leq 3$, then the theory of CNS has been neglected. For this, in this study, we try to present the principle of complex single-valued neutrosophic sets (CSVNS) and to determine their algebraic laws. In the SVNS hypothesis, just the level of the assets is considered during the examination, which might bring about loss of data under some specific cases, while the factor of periodicity is totally overlooked. To keep away from such a deficiency of data, there is a need to add both the variables into the examination. To additionally delineate the idea of stage terms, we give a model. Assume that an organization XYZ needs to buy a vehicle from a carmaker ABC. The carmaker ABC gives the organization XYZ data with respect to models of vehicles and their relating creation dates. The assignment of the organization is to choose the most ideal model of the vehicle with its creation date all the while. Hence, here the issue is two-dimensional, in particular (i) model of vehicle and (ii) creation date of the vehicle. It is clearly seen that such kind of issues cannot be demonstrated precisely by considering both the measurements at the same time utilizing the customary speculations. Consequently, the most ideal way of addressing all the data given by the specialists is by utilizing the CSVNS hypothesis. The plentiful terms in CSVNS might be utilized to give an organization's choice with respect to the model of vehicles, and the stage terms might be utilized to address organization's choice in regard to the creation date of vehicles. Keeping the advantages of the CSVNS, we examined the primary goal of this analysis as illustrated below.

1. To initiate the CSVNS and to determine their important algebraic laws.
2. To present CSVNPMM operator and CSVNPDMM operator are elaborated and their particular cases are discussed.
3. To propose an MADM procedure under the presented operators.
4. To initiate numerous examples to determine the advantages, sensitive analysis, and geometrical expressions of the proposed works to find the supremacy and flexibility of the initiated works.

The remainder of this paper is formed as follows: in Section II, we review the basic principle of SVNSs and their algebraic laws. The principle of SVNPWA operator, SVNPGA operator, Muirhead mean (MM) operator, and their specific cases are reviewed. In Section III, we initiated the CSVNS and determine their important algebraic laws. In Section IV, CSVNPMM operator and CSVNPDMM operator are elaborated and their particular cases are discussed. In Section V, an MADM technique is presented based on investigated operators. In Section VI, we present the conclusion of this study.

2. Preliminaries

In this analysis, we review the basic principle of SVNSs and their algebraic laws. The principles of SVNPWA operator, SVNPGA operator, MM operator, and their specific cases are reviewed. The term $\overline{\overline{\mathfrak{X}}}$ stated the universal sets.

Definition 1: (Wang et al., 2010) A SVNS $\overline{\overline{\mathcal{TI}}}_{CN}$ is stated by

$$\overline{\overline{\mathcal{TI}}}_{CN}=\left\{\left(\mathcal{M}_{\overline{\overline{\mathcal{TI}}}_{C}}\left(\overline{\overline{\Xi}}\right),\mathcal{A}_{\overline{\overline{\mathcal{TI}}}_{C}}\left(\overline{\overline{\Xi}}\right),\mathcal{N}_{\overline{\overline{\mathcal{TI}}}_{C}}\left(\overline{\overline{\Xi}}\right)\right):\overline{\overline{\Xi}}\in\overline{\overline{\mathfrak{X}}}\right\} \tag{1}$$

where $\mathcal{M}_{\overline{\overline{\mathcal{TI}}}_{C}}\left(\overline{\overline{\Xi}}\right)$, $\mathcal{A}_{\overline{\overline{\mathcal{TI}}}_{C}}\left(\overline{\overline{\Xi}}\right)$, and $\mathcal{N}_{\overline{\overline{\mathcal{TI}}}_{C}}\left(\overline{\overline{\Xi}}\right)$ belong to [0,1] with $0\leq\mathcal{M}_{\overline{\overline{\mathcal{TI}}}_{C}}\left(\overline{\overline{\Xi}}\right)+\mathcal{A}_{\overline{\overline{\mathcal{TI}}}_{C}}\left(\overline{\overline{\Xi}}\right)+\mathcal{N}_{\overline{\overline{\mathcal{TI}}}_{C}}\left(\overline{\overline{\Xi}}\right)\leq 3$. The object $\overline{\overline{\mathcal{TI}}}_{CN-\mathfrak{ff}}=\left(\mathcal{M}_{\overline{\overline{\mathcal{TI}}}_{C-\mathfrak{ff}}},\mathcal{A}_{\overline{\overline{\mathcal{TI}}}_{C-\mathfrak{ff}}},\mathcal{N}_{\overline{\overline{\mathcal{TI}}}_{C-\mathfrak{ff}}}\right),\mathfrak{ff}=1,2,\ldots,\mu,$ stated the SVNNs.

Definition 2: (Wang et al., 2010) Suppose $\overline{\overline{\mathcal{TI}}}_{CN-1}=\left(\mathcal{M}_{\overline{\overline{\mathcal{TI}}}_{C-1}},\mathcal{A}_{\overline{\overline{\mathcal{TI}}}_{C-1}},\mathcal{N}_{\overline{\overline{\mathcal{TI}}}_{C-1}}\right)$ be any SVNN. The score value (SV) is stated by:

$$\overline{\overline{\mathcal{S}}}\left(\overline{\overline{\mathcal{TI}}}_{CN-\mathfrak{ff}}\right)=\frac{1+\left(\mathcal{M}_{\overline{\overline{\mathcal{TI}}}_{C-\mathfrak{ff}}}-2\mathcal{A}_{\overline{\overline{\mathcal{TI}}}_{C-\mathfrak{ff}}}-\mathcal{N}_{\overline{\overline{\mathcal{TI}}}_{C-\mathfrak{ff}}}\right)\left(2-\mathcal{M}_{\overline{\overline{\mathcal{TI}}}_{C-\mathfrak{ff}}}-\mathcal{N}_{\overline{\overline{\mathcal{TI}}}_{C-\mathfrak{ff}}}\right)}{2} \tag{2}$$

Definition 3: (Wang et al., 2010) Suppose $\overline{\overline{\mathcal{TI}}}_{CN-1}=\left(\mathcal{M}_{\overline{\overline{\mathcal{TI}}}_{C-1}},\mathcal{A}_{\overline{\overline{\mathcal{TI}}}_{C-1}},\mathcal{N}_{\overline{\overline{\mathcal{TI}}}_{C-1}}\right)$ be any SVNN. The accuracy value (AV) is stated by:

$$\overline{\overline{\mathcal{H}}}\left(\overline{\overline{\mathcal{TI}}}_{CN-\mathfrak{ff}}\right)=\frac{\mathcal{M}_{\overline{\overline{\mathcal{TI}}}_{C-\mathfrak{ff}}}+\mathcal{A}_{\overline{\overline{\mathcal{TI}}}_{C-\mathfrak{ff}}}+\mathcal{N}_{\overline{\overline{\mathcal{TI}}}_{C-\mathfrak{ff}}}}{3} \tag{3}$$

where $\overline{\overline{\mathcal{H}}}\left(\overline{\overline{\mathcal{TI}}}_{CN-\mathfrak{ff}}\right)\in[0,1]$.

For any two SVNNs $\overline{\overline{\mathcal{TI}}}_{CN-1}=\left(\mathcal{M}_{\overline{\overline{\mathcal{TI}}}_{C-1}},\mathcal{A}_{\overline{\overline{\mathcal{TI}}}_{C-1}},\mathcal{N}_{\overline{\overline{\mathcal{TI}}}_{C-1}}\right)$ and $\overline{\overline{\mathcal{TI}}}_{CN-2}=\left(\mathcal{M}_{\overline{\overline{\mathcal{TI}}}_{C-2}},\mathcal{A}_{\overline{\overline{\mathcal{TI}}}_{C-2}},\mathcal{N}_{\overline{\overline{\mathcal{TI}}}_{C-2}}\right)$,

1. $\Longrightarrow\overline{\overline{\mathcal{S}}}\left(\overline{\overline{\mathcal{TI}}}_{CN-1}\right)>\overline{\overline{\mathcal{S}}}\left(\overline{\overline{\mathcal{TI}}}_{CN-2}\right)\Longrightarrow\overline{\overline{\mathcal{TI}}}_{CN-1}>\overline{\overline{\mathcal{TI}}}_{CN-2}$;
2. $\Longrightarrow\overline{\overline{\mathcal{S}}}\left(\overline{\overline{\mathcal{TI}}}_{CN-1}\right)<\overline{\overline{\mathcal{S}}}\left(\overline{\overline{\mathcal{TI}}}_{CN-2}\right)\Longrightarrow\overline{\overline{\mathcal{TI}}}_{CN-1}<\overline{\overline{\mathcal{TI}}}_{CN-2}$;
3. $\Longrightarrow\overline{\overline{\mathcal{S}}}\left(\overline{\overline{\mathcal{TI}}}_{CN-1}\right)=\overline{\overline{\mathcal{S}}}\left(\overline{\overline{\mathcal{TI}}}_{CN-2}\right)\Longrightarrow$.
   (i) $\Longrightarrow\overline{\overline{\mathcal{H}}}\left(\overline{\overline{\mathcal{TI}}}_{CN-1}\right)>\overline{\overline{\mathcal{H}}}\left(\overline{\overline{\mathcal{TI}}}_{CN-2}\right)\Longrightarrow\overline{\overline{\mathcal{TI}}}_{CN-1}>\overline{\overline{\mathcal{TI}}}_{CN-2}$;
   (ii) $\Longrightarrow\overline{\overline{\mathcal{H}}}\left(\overline{\overline{\mathcal{TI}}}_{CN-1}\right)<\overline{\overline{\mathcal{H}}}\left(\overline{\overline{\mathcal{TI}}}_{CN-2}\right)\Longrightarrow\overline{\overline{\mathcal{TI}}}_{CN-1}<\overline{\overline{\mathcal{TI}}}_{CN-2}$;
   (iii) $\Longrightarrow\overline{\overline{\mathcal{H}}}\left(\overline{\overline{\mathcal{TI}}}_{CN-1}\right)=\overline{\overline{\mathcal{H}}}\left(\overline{\overline{\mathcal{TI}}}_{CN-2}\right)\Longrightarrow\overline{\overline{\mathcal{TI}}}_{CN-1}=\overline{\overline{\mathcal{TI}}}_{CN-2}$.

Definition 4: (Wang et al., 2010) Suppose $\overline{\overline{\mathcal{TI}}}_{CN-1}=\left(\mathcal{M}_{\overline{\overline{\mathcal{TI}}}_{C-1}},\mathcal{A}_{\overline{\overline{\mathcal{TI}}}_{C-1}},\mathcal{N}_{\overline{\overline{\mathcal{TI}}}_{C-1}}\right)$ and $\overline{\overline{\mathcal{TI}}}_{CN-2}=\left(\mathcal{M}_{\overline{\overline{\mathcal{TI}}}_{C-2}},\mathcal{A}_{\overline{\overline{\mathcal{TI}}}_{C-2}},\mathcal{N}_{\overline{\overline{\mathcal{TI}}}_{C-2}}\right)$ be any two SVNNs. Then

$$\overline{\overline{\mathcal{TI}}}_{CN-1}\oplus\overline{\overline{\mathcal{TI}}}_{CN-2}=\left(\mathcal{M}_{\overline{\overline{\mathcal{TI}}}_{C-1}}+\mathcal{M}_{\overline{\overline{\mathcal{TI}}}_{C-2}}-\mathcal{M}_{\overline{\overline{\mathcal{TI}}}_{C-1}}\mathcal{M}_{\overline{\overline{\mathcal{TI}}}_{C-2}},\mathcal{A}_{\overline{\overline{\mathcal{TI}}}_{C-1}}\mathcal{A}_{\overline{\overline{\mathcal{TI}}}_{C-2}},\mathcal{N}_{\overline{\overline{\mathcal{TI}}}_{C-1}}\mathcal{N}_{\overline{\overline{\mathcal{TI}}}_{C-2}}\right) \tag{4}$$

$$\overline{\overline{\mathcal{TI}}}_{CN-1}\otimes\overline{\overline{\mathcal{TI}}}_{CN-2}=\begin{pmatrix}\mathcal{M}_{\overline{\overline{\mathcal{TI}}}_{C-1}}\mathcal{M}_{\overline{\overline{\mathcal{TI}}}_{C-2}},\mathcal{A}_{\overline{\overline{\mathcal{TI}}}_{C-1}}+\mathcal{A}_{\overline{\overline{\mathcal{TI}}}_{C-2}}-\mathcal{A}_{\overline{\overline{\mathcal{TI}}}_{C-1}}\mathcal{A}_{\overline{\overline{\mathcal{TI}}}_{C-2}},\\ \mathcal{N}_{\overline{\overline{\mathcal{TI}}}_{C-1}}+\mathcal{N}_{\overline{\overline{\mathcal{TI}}}_{C-2}}-\mathcal{N}_{\overline{\overline{\mathcal{TI}}}_{C-1}}\mathcal{N}_{\overline{\overline{\mathcal{TI}}}_{C-2}}\end{pmatrix} \tag{5}$$

$$\delta_S\overline{\overline{\mathcal{TI}}}_{CN-1}=\left(1-\left(1-\mathcal{M}_{\overline{\overline{\mathcal{TI}}}_{C-1}}\right)^{\overline{\overline{\delta}}_S},\mathcal{A}^{\overline{\overline{\delta}}_S}_{\overline{\overline{\mathcal{TI}}}_{C-1}},\mathcal{N}^{\overline{\overline{\delta}}_S}_{\overline{\overline{\mathcal{TI}}}_{C-1}}\right) \tag{6}$$

$$\overline{\overline{\mathcal{TI}}}_{CN-1}^{\overline{\overline{\delta}}_S}=\left(\mathcal{M}^{\overline{\overline{\delta}}_S}_{\overline{\overline{\mathcal{TI}}}_{C-1}},1-\left(1-\mathcal{A}_{\overline{\overline{\mathcal{TI}}}_{C-\mathfrak{ff}}}\right)^{\overline{\overline{\delta}}_S},1-\left(1-\mathcal{N}_{\overline{\overline{\mathcal{TI}}}_{C-\mathfrak{ff}}}\right)^{\overline{\overline{\delta}}_S}\right) \tag{7}$$

Definition 5: (Garg, 2018) Suppose $\overline{\overline{\mathcal{I}}}_{CN-\mathfrak{ff}}=\left(\mathcal{M}_{\overline{\overline{\mathcal{TI}}}_{C-\mathfrak{ff}}},\mathcal{A}_{\overline{\overline{\mathcal{TI}}}_{C-\mathfrak{ff}}},\mathcal{N}_{\overline{\overline{\mathcal{TI}}}_{C-\mathfrak{ff}}}\right),\mathfrak{ff}=1,2,\ldots,\mu,$ be any group of SVNNs. The SVNPWA operator is stated by

$$SVNPWA\left(\overline{\overline{\mathcal{TI}}}_{CN-1},\overline{\overline{\mathcal{TI}}}_{CN-2},\ldots,\overline{\overline{\mathcal{TI}}}_{CN-\mu}\right)=\left(1-\prod_{\mathfrak{ff}=1}^{\mu}\left(1-\mathcal{M}_{\overline{\overline{\mathcal{TI}}}_{C-\mathfrak{ff}}}\right)^{\frac{\mathbb{H}_{\mathfrak{ff}}}{\sum_{\mathfrak{ff}=1}^{\mu}\mathbb{H}_{\mathfrak{ff}}}},\prod_{\mathfrak{ff}=1}^{\mu}\mathcal{A}_{\overline{\overline{\mathcal{TI}}}_{C-\mathfrak{ff}}}^{\frac{\mathbb{H}_{\mathfrak{ff}}}{\sum_{\mathfrak{ff}=1}^{\mu}\mathbb{H}_{\mathfrak{ff}}}},\prod_{\mathfrak{ff}=1}^{\mu}\mathcal{N}_{\overline{\overline{\mathcal{TI}}}_{C-\mathfrak{ff}}}^{\frac{\mathbb{H}_{\mathfrak{ff}}}{\sum_{\mathfrak{ff}=1}^{\mu}\mathbb{H}_{\mathfrak{ff}}}}\right) \tag{8}$$

where $\mathbb{H}_1=1$ and $\mathbb{H}_{\mathfrak{ff}}=\prod_{k=1}^{\mathfrak{ff}-1}\overline{\overline{\mathcal{S}}}\left(\overline{\overline{\mathcal{TI}}}_{CN-k}\right)$.

Definition 6: (Garg, 2018) Suppose $\overline{\overline{\mathcal{TI}}}_{CN-\mathfrak{ff}}=\left(\mathcal{M}_{\overline{\overline{\mathcal{TI}}}_{C-\mathfrak{ff}}},\mathcal{A}_{\overline{\overline{\mathcal{TI}}}_{C-\mathfrak{ff}}},\mathcal{N}_{\overline{\overline{\mathcal{TI}}}_{C-\mathfrak{ff}}}\right),\mathfrak{ff}=1,2,\ldots,\mu,$ be any group of SVNNs. The SVNPGA operator is stated by

$$SVNPGA\left(\overline{\overline{\mathcal{TI}}}_{CN-1},\overline{\overline{\mathcal{TI}}}_{CN-2},\ldots,\overline{\overline{\mathcal{TI}}}_{CN-\mu}\right)=\left(\prod_{\mathfrak{ff}=1}^{\mu}\mathcal{M}_{\overline{\overline{\mathcal{TI}}}_{C-\mathfrak{ff}}}^{\frac{\mathbb{H}_{\mathfrak{ff}}}{\sum_{\mathfrak{ff}=1}^{\mu}\mathbb{H}_{\mathfrak{ff}}}},1-\prod_{\mathfrak{ff}=1}^{\mu}\left(1-\mathcal{A}_{\overline{\overline{\mathcal{TI}}}_{C-\mathfrak{ff}}}\right)^{\frac{\mathbb{H}_{\mathfrak{ff}}}{\sum_{\mathfrak{ff}=1}^{\mu}\mathbb{H}_{\mathfrak{ff}}}},1-\prod_{\mathfrak{ff}=1}^{\mu}\left(1-\mathcal{N}_{\overline{\overline{\mathcal{TI}}}_{C-\mathfrak{ff}}}\right)^{\frac{\mathbb{H}_{\mathfrak{ff}}}{\sum_{\mathfrak{ff}=1}^{\mu}\mathbb{H}_{\mathfrak{ff}}}}\right) \tag{9}$$

where $\mathbb{H}_1$ and $\mathbb{H}_{\mathfrak{ff}}=\prod_{k=1}^{\mathfrak{ff}-1}\overline{\overline{\mathcal{S}}}\left(\overline{\overline{\mathcal{TI}}}_{CN-k}\right)$.

Definition 7: (Muirhead, 1902) Suppose $\overline{\overline{\mathcal{TI}}}_{CN-\mathfrak{ff}},\mathfrak{ff}=1,2,\ldots,\mu,$ be any group of positive terms with parameters $\overline{\overline{\mathcal{P}}}=\left(\overline{\overline{\mathcal{P}}}_1,\overline{\overline{\mathcal{P}}}_2,\ldots,\overline{\overline{\mathcal{P}}}_\mu\right)\in R^{\mu}$. The MM operator is stated by

$$MM^{\overline{\overline{\mathcal{P}}}}\left(\overline{\overline{\mathcal{TI}}}_{CN-1},\overline{\overline{\mathcal{TI}}}_{CN-2},\ldots,\overline{\overline{\mathcal{TI}}}_{CN-\mu}\right)=\left(\frac{1}{\mu!}\sum_{\sigma\in\mathbb{S}_\mu}\prod_{\mathfrak{ff}=1}^{\mu}\overline{\overline{\mathcal{TI}}}_{CN-\sigma(\mathfrak{ff})}^{\overline{\overline{\mathcal{P}}}_{\mathfrak{ff}}}\right)^{\frac{1}{\sum_{\mathfrak{ff}=1}^{\mu}\overline{\overline{\mathcal{P}}}_{\mathfrak{ff}}}} \tag{10}$$

where σ is the permutation of $(\mathfrak{ff}=1,2,\ldots,\mu)$ and $\mathbb{S}_\mu$ is the group of permutations of $\mathfrak{ff}=1,2,\ldots,\mu$. For different values of $\overline{\overline{\mathcal{P}}}=\left(\overline{\overline{\mathcal{P}}}_1,\overline{\overline{\mathcal{P}}}_2,\ldots,\overline{\overline{\mathcal{P}}}_\mu\right)$, certain specific cases are discussed below.

1. For $\overline{\overline{\mathcal{P}}} = (1, 0, \ldots, 0)$, Equation (10) is changed to

$$MM^{(1,0,\ldots,0)}\left(\overline{\overline{\mathcal{TI}}}_{CN-1}, \overline{\overline{\mathcal{TI}}}_{CN-2}, \ldots, \overline{\overline{\mathcal{TI}}}_{CN-\mu}\right) = \frac{1}{\mu}\prod_{\mathrm{ff}=1}^{\mu}\overline{\overline{\mathcal{TI}}}_{CN-\mathrm{ff}} \tag{11}$$

Which is expressed as the AA operator (AAO).

2. For $\overline{\overline{\mathcal{P}}} = \left(\frac{1}{\mu}, \frac{1}{\mu}, \ldots, \frac{1}{\mu}\right)$, Equation (10) is changed to

$$MM^{\left(\frac{1}{\mu}, \frac{1}{\mu}, \ldots, \frac{1}{\mu}\right)}\left(\overline{\overline{\mathcal{TI}}}_{CN-1}, \overline{\overline{\mathcal{TI}}}_{CN-2}, \ldots, \overline{\overline{\mathcal{TI}}}_{CN-\mu}\right) = \prod_{\mathrm{ff}=1}^{\mu}\overline{\overline{\mathcal{TI}}}_{CN-\mathrm{ff}}^{\frac{1}{\mu}} \tag{12}$$

Is expressed as the GA operator (GAO).

3. For $\overline{\overline{\mathcal{P}}} = (1, 1, 0, 0, \ldots, 0)$, Equation (10) is changed to

$$MM^{(1,1,0,0,\ldots,0)}\left(\overline{\overline{\mathcal{TI}}}_{CN-1}, \overline{\overline{\mathcal{TI}}}_{CN-2}, \ldots, \overline{\overline{\mathcal{TI}}}_{CN-\mu}\right) = \left(\frac{1}{\mu(\mu+1)}\sum_{\substack{\mathrm{ff}, j = 1, \\ \mathrm{ff} \neq j}}^{\mu}\overline{\overline{\mathcal{TI}}}_{CN-\mathrm{ff}} * \overline{\overline{\mathcal{TI}}}_{CN-j}\right)^{\frac{1}{2}} \tag{13}$$

Is expressed as the BM operator (BMO).

4. For $\overline{\overline{\mathcal{P}}} = \left(\overbrace{1, 1, \ldots, 1}^{k}, \overbrace{0, 0, \ldots, 0}^{\mu-k}\right)$, Equation (10) is changed to

$$MM^{\left(\overbrace{1, 1, \ldots, 1}^{k}, \overbrace{0, 0, \ldots, 0}^{\mu-k}\right)}\left(\overline{\overline{\mathcal{TI}}}_{CN-1}, \overline{\overline{\mathcal{TI}}}_{CN-2}, \ldots, \overline{\overline{\mathcal{TI}}}_{CN-\mu}\right) = \left(\frac{1}{C_k^{\mu}}\sum_{1 \leq j_1 <, \ldots, < j_k \leq \mu}\prod_{\mathrm{ff}=1}^{k}\overline{\overline{\mathcal{TI}}}_{CN-j_{\mathrm{ff}}}\right)^{\frac{1}{k}} \tag{14}$$

Is expressed as the MSM operator (MSMO).

3. Complex Single-Valued Neutrosophic Sets

In this study, we combine two distinct principles such as SVNS and CFS to initiate the novel principle of CSVNSs and to develop their algebraic laws.

Definition 8: A CSVNS $\overline{\overline{\mathcal{TI}}}_{CN}$ is stated by

$$\overline{\overline{\mathcal{TI}}}_{CN} = \left\{\left(\mathcal{M}_{\overline{\overline{\mathcal{TI}}}_C}(\Xi), \mathcal{A}_{\overline{\overline{\mathcal{TI}}}_C}(\Xi), \mathcal{N}_{\overline{\overline{\mathcal{TI}}}_C}(\Xi)\right) : \Xi \in \overline{\overline{\mathfrak{X}}}\right\} \tag{15}$$

where $\mathcal{M}_{\overline{\overline{\mathcal{TI}}}_C}(\Xi) = \mathcal{M}_{\overline{\overline{\mathcal{TI}}}_R}(\Xi)e^{\mathrm{ff}2\pi\left(\mathcal{M}_{\overline{\overline{\mathcal{TI}}}_I}(\Xi)\right)}$, $\mathcal{A}_{\overline{\overline{\mathcal{TI}}}_C}(\Xi) = \mathcal{A}_{\overline{\overline{\mathcal{TI}}}_R}(\Xi)e^{\mathrm{ff}2\pi\left(\mathcal{A}_{\overline{\overline{\mathcal{TI}}}_I}(\Xi)\right)}$, and $\mathcal{N}_{\overline{\overline{\mathcal{TI}}}_C}(\Xi) = \mathcal{N}_{\overline{\overline{\mathcal{TI}}}_R}(\Xi)e^{\mathrm{ff}2\pi\left(\mathcal{N}_{\overline{\overline{\mathcal{TI}}}_I}(\Xi)\right)}$ with $0 \leq \mathcal{M}_{\overline{\overline{\mathcal{TI}}}_R} + \mathcal{A}_{\overline{\overline{\mathcal{TI}}}_R} + \mathcal{N}_{\overline{\overline{\mathcal{TI}}}_R} \leq 3$ and $0 \leq \mathcal{M}_{\overline{\overline{\mathcal{TI}}}_I} + \mathcal{A}_{\overline{\overline{\mathcal{TI}}}_I} + \mathcal{N}_{\overline{\overline{\mathcal{TI}}}_I} \leq 3$. The object

$$\overline{\overline{\mathcal{TI}}}_{CN-\mathrm{ff}} = \left(\mathcal{M}_{\overline{\overline{\mathcal{TI}}}_{R-\mathrm{ff}}}e^{\mathrm{ff}2\pi\left(\mathcal{M}_{\overline{\overline{\mathcal{TI}}}_{I-\mathrm{ff}}}\right)}, \mathcal{A}_{\overline{\overline{\mathcal{TI}}}_{R-\mathrm{ff}}}e^{\mathrm{ff}2\pi\left(\mathcal{A}_{\overline{\overline{\mathcal{TI}}}_{I-\mathrm{ff}}}\right)}, \mathcal{N}_{\overline{\overline{\mathcal{TI}}}_{R-\mathrm{ff}}}e^{\mathrm{ff}2\pi\left(\mathcal{N}_{\overline{\overline{\mathcal{TI}}}_{I-\mathrm{ff}}}\right)}\right),$$

$\mathrm{ff} = 1, 2, \ldots, \mu$, stated the CSVNNs.

Definition 9: Suppose $\overline{\overline{\mathcal{TI}}}_{CN-1} = \left(\mathcal{M}_{\overline{\overline{\mathcal{TI}}}_{R-1}}e^{\mathrm{ff}2\pi\left(\mathcal{M}_{\overline{\overline{\mathcal{TI}}}_{I-1}}\right)}, \mathcal{A}_{\overline{\overline{\mathcal{TI}}}_{R-1}}e^{\mathrm{ff}2\pi\left(\mathcal{A}_{\overline{\overline{\mathcal{TI}}}_{I-1}}\right)}, \mathcal{N}_{\overline{\overline{\mathcal{TI}}}_{R-1}}e^{\mathrm{ff}2\pi\left(\mathcal{N}_{\overline{\overline{\mathcal{TI}}}_{I-1}}\right)}\right)$ be any CSVNN. The SV is stated by

$$\overline{\overline{\mathcal{S}}}\left(\overline{\overline{\mathcal{TI}}}_{CN-\mathrm{ff}}\right) = \frac{\left|\mathcal{M}_{\overline{\overline{\mathcal{TI}}}_R} - \mathcal{A}_{\overline{\overline{\mathcal{TI}}}_R} - \mathcal{N}_{\overline{\overline{\mathcal{TI}}}_R} + \mathcal{M}_{\overline{\overline{\mathcal{TI}}}_I} - \mathcal{A}_{\overline{\overline{\mathcal{TI}}}_I} - \mathcal{N}_{\overline{\overline{\mathcal{TI}}}_I}\right|}{3} \tag{16}$$

Definition 10: Suppose $\overline{\overline{\mathcal{TI}}}_{N-1} = \left(\mathcal{M}_{\overline{\overline{\mathcal{TI}}}_{R-1}}e^{\mathrm{ff}2\pi\left(\mathcal{M}_{\overline{\overline{\mathcal{TI}}}_{I-1}}\right)}, \mathcal{A}_{\overline{\overline{\mathcal{TI}}}_{R-1}}e^{\mathrm{ff}2\pi\left(\mathcal{A}_{\overline{\overline{\mathcal{TI}}}_{I-1}}\right)}, \mathcal{N}_{\overline{\overline{\mathcal{TI}}}_{R-1}}e^{\mathrm{ff}2\pi\left(\mathcal{N}_{\overline{\overline{\mathcal{TI}}}_{I-1}}\right)}\right)$ be any CSVNN. The AV is stated by

$$\overline{\overline{\mathcal{H}}}\left(\overline{\overline{\mathcal{TI}}}_{CN-\mathrm{ff}}\right) = \frac{\mathcal{M}_{\overline{\overline{\mathcal{TI}}}_R} + \mathcal{A}_{\overline{\overline{\mathcal{TI}}}_R} + \mathcal{N}_{\overline{\overline{\mathcal{TI}}}_R} + \mathcal{M}_{\overline{\overline{\mathcal{TI}}}_I} + \mathcal{A}_{\overline{\overline{\mathcal{TI}}}_I} + \mathcal{N}_{\overline{\overline{\mathcal{TI}}}_I}}{3} \tag{17}$$

where $\overline{\overline{\mathcal{H}}}\left(\overline{\overline{\mathcal{TI}}}_{CN-\mathrm{ff}}\right) \in [0, 1]$.

For any two CSVNNs $\overline{\overline{\mathcal{TI}}}_{CN-\mathrm{ff}} = \left(\mathcal{M}_{\overline{\overline{\mathcal{TI}}}_{R-\mathrm{ff}}}e^{\mathrm{ff}2\pi\left(\mathcal{M}_{\overline{\overline{\mathcal{TI}}}_{I-\mathrm{ff}}}\right)}, \mathcal{A}_{\overline{\overline{\mathcal{TI}}}_{R-\mathrm{ff}}}e^{\mathrm{ff}2\pi\left(\mathcal{A}_{\overline{\overline{\mathcal{TI}}}_{I-\mathrm{ff}}}\right)}, \mathcal{N}_{\overline{\overline{\mathcal{TI}}}_{R-\mathrm{ff}}}e^{\mathrm{ff}2\pi\left(\mathcal{N}_{\overline{\overline{\mathcal{TI}}}_{I-\mathrm{ff}}}\right)}\right), \mathrm{ff} = 1, 2,$

1. $\Longrightarrow \overline{\overline{\mathcal{S}}}\left(\overline{\overline{\mathcal{TI}}}_{CN-1}\right) > \overline{\overline{\mathcal{S}}}\left(\overline{\overline{\mathcal{TI}}}_{CN-2}\right) \Longrightarrow \overline{\overline{\mathcal{TI}}}_{CN-1} > \overline{\overline{\mathcal{TI}}}_{CN-2}$;
2. $\Longrightarrow \overline{\overline{\mathcal{S}}}\left(\overline{\overline{\mathcal{TI}}}_{CN-1}\right) < \overline{\overline{\mathcal{S}}}\left(\overline{\overline{\mathcal{TI}}}_{CN-2}\right) \Longrightarrow \overline{\overline{\mathcal{TI}}}_{CN-1} < \overline{\overline{\mathcal{TI}}}_{CN-2}$;
3. $\Longrightarrow \overline{\overline{\mathcal{S}}}\left(\overline{\overline{\mathcal{TI}}}_{CN-1}\right) = \overline{\overline{\mathcal{S}}}\left(\overline{\overline{\mathcal{TI}}}_{CN-2}\right) \Longrightarrow$.
   i) $\Longrightarrow \overline{\overline{\mathcal{H}}}\left(\overline{\overline{\mathcal{TI}}}_{CN-1}\right) > \overline{\overline{\mathcal{H}}}\left(\overline{\overline{\mathcal{TI}}}_{CN-2}\right) \Longrightarrow \overline{\overline{\mathcal{TI}}}_{CN-1} > \overline{\overline{\mathcal{TI}}}_{CN-2}$;
   ii) $\Longrightarrow \overline{\overline{\mathcal{H}}}\left(\overline{\overline{\mathcal{TI}}}_{CN-1}\right) < \overline{\overline{\mathcal{H}}}\left(\overline{\overline{\mathcal{TI}}}_{CN-2}\right) \Longrightarrow \overline{\overline{\mathcal{TI}}}_{CN-1} < \overline{\overline{\mathcal{TI}}}_{CN-2}$;
   iii) $\Longrightarrow \overline{\overline{\mathcal{H}}}\left(\overline{\overline{\mathcal{TI}}}_{CN-1}\right) = \overline{\overline{\mathcal{H}}}\left(\overline{\overline{\mathcal{TI}}}_{CN-2}\right) \Longrightarrow \overline{\overline{\mathcal{TI}}}_{CN-1} = \overline{\overline{\mathcal{TI}}}_{CN-2}$.

Definition 11: Suppose $\overline{\overline{\mathcal{TI}}}_{CN-1} = \left(\mathcal{M}_{\overline{\overline{\mathcal{TI}}}_{R-1}}e^{\mathrm{ff}2\pi\left(\mathcal{M}_{\overline{\overline{\mathcal{TI}}}_{I-1}}\right)}, \mathcal{A}_{\overline{\overline{\mathcal{TI}}}_{R-1}}e^{\mathrm{ff}2\pi\left(\mathcal{A}_{\overline{\overline{\mathcal{TI}}}_{I-1}}\right)}, \mathcal{N}_{\overline{\overline{\mathcal{TI}}}_{R-1}}e^{\mathrm{ff}2\pi\left(\mathcal{N}_{\overline{\overline{\mathcal{TI}}}_{I-1}}\right)}\right)$ and $\overline{\overline{\mathcal{TI}}}_{CN-2} = \left(\mathcal{M}_{\overline{\overline{\mathcal{TI}}}_{R-2}}e^{\mathrm{ff}2\pi\left(\mathcal{M}_{\overline{\overline{\mathcal{TI}}}_{I-2}}\right)}, \mathcal{A}_{\overline{\overline{\mathcal{TI}}}_{R-2}}e^{\mathrm{ff}2\pi\left(\mathcal{A}_{\overline{\overline{\mathcal{TI}}}_{I-2}}\right)}, \mathcal{N}_{\overline{\overline{\mathcal{TI}}}_{R-2}}e^{\mathrm{ff}2\pi\left(\mathcal{N}_{\overline{\overline{\mathcal{TI}}}_{I-2}}\right)}\right)$ be any two CSVNNs. Then

$$\overline{\overline{\mathcal{TI}}}_{CN-1} \oplus \overline{\overline{\mathcal{TI}}}_{CN-2} = \begin{pmatrix} \left(\mathcal{M}_{\overline{\overline{\mathcal{TI}}}_{R-1}} + \mathcal{M}_{\overline{\overline{\mathcal{TI}}}_{R-2}} - \mathcal{M}_{\overline{\overline{\mathcal{TI}}}_{R-1}}\mathcal{M}_{\overline{\overline{\mathcal{TI}}}_{R-2}}\right)e^{\mathrm{ff}2\pi\left(\mathcal{M}_{\overline{\overline{\mathcal{TI}}}_{I-1}} + \mathcal{M}_{\overline{\overline{\mathcal{TI}}}_{I-2}} - \mathcal{M}_{\overline{\overline{\mathcal{TI}}}_{I-1}}\mathcal{M}_{\overline{\overline{\mathcal{TI}}}_{I-2}}\right)}, \\ \left(\mathcal{A}_{\overline{\overline{\mathcal{TI}}}_{R-1}}\mathcal{A}_{\overline{\overline{\mathcal{TI}}}_{R-2}}\right)e^{\mathrm{ff}2\pi\left(\mathcal{A}_{\overline{\overline{\mathcal{TI}}}_{I-1}}\mathcal{A}_{\overline{\overline{\mathcal{TI}}}_{I-2}}\right)}, \mathcal{N}_{\overline{\overline{\mathcal{TI}}}_{R-1}}\mathcal{N}_{\overline{\overline{\mathcal{TI}}}_{R-2}}e^{\mathrm{ff}2\pi\left(\mathcal{N}_{\overline{\overline{\mathcal{TI}}}_{I-1}}\mathcal{N}_{\overline{\overline{\mathcal{TI}}}_{I-2}}\right)} \end{pmatrix} \tag{18}$$

$$\overline{\overline{\mathcal{TI}_{CN-1}}} \otimes \overline{\overline{\mathcal{TI}_{CN-2}}} = \begin{pmatrix} \left(\mathcal{M}_{\overline{\overline{\mathcal{TI}_{R-1}}}}\mathcal{M}_{\overline{\overline{\mathcal{TI}_{R-2}}}}\right)e^{\text{ff}2\pi\left(\mathcal{M}_{\overline{\overline{\mathcal{TI}_{I-1}}}}\mathcal{M}_{\overline{\overline{\mathcal{TI}_{I-2}}}}\right)}, \\ \left(\mathcal{A}_{\overline{\overline{\mathcal{TI}_{R-1}}}}+\mathcal{A}_{\overline{\overline{\mathcal{TI}_{R-2}}}}-\mathcal{A}_{\overline{\overline{\mathcal{TI}_{R-1}}}}\mathcal{A}_{\overline{\overline{\mathcal{TI}_{R-2}}}}\right)e^{\text{ff}2\pi\left(\mathcal{A}_{\overline{\overline{\mathcal{TI}_{I-1}}}}+\mathcal{A}_{\overline{\overline{\mathcal{TI}_{I-2}}}}-\mathcal{A}_{\overline{\overline{\mathcal{TI}_{I-1}}}}\mathcal{A}_{\overline{\overline{\mathcal{TI}_{I-2}}}}\right)}, \\ \left(\mathcal{N}_{\overline{\overline{\mathcal{TI}_{R-1}}}}+\mathcal{N}_{\overline{\overline{\mathcal{TI}_{R-2}}}}-\mathcal{N}_{\overline{\overline{\mathcal{TI}_{R-1}}}}\mathcal{N}_{\overline{\overline{\mathcal{TI}_{R-2}}}}\right)e^{\text{ff}2\pi\left(\mathcal{N}_{\overline{\overline{\mathcal{TI}_{I-1}}}}+\mathcal{N}_{\overline{\overline{\mathcal{TI}_{I-2}}}}-\mathcal{N}_{\overline{\overline{\mathcal{TI}_{I-1}}}}\mathcal{N}_{\overline{\overline{\mathcal{TI}_{I-2}}}}\right)} \end{pmatrix} \tag{19}$$

$$\widetilde{\delta_S}\overline{\overline{\mathcal{TI}_{CN-1}}} = \begin{pmatrix} 1-\left(1-\mathcal{M}_{\overline{\overline{\mathcal{TI}_{R-1}}}}\right)^{\widetilde{\delta_S}}e^{\text{ff}2\pi\left(1-\left(1-\mathcal{M}_{\overline{\overline{\mathcal{TI}_{I-1}}}}\right)^{\widetilde{\delta_S}}\right)}, \\ \mathcal{A}^{\widetilde{\delta_S}}_{\overline{\overline{\mathcal{TI}_{R-1}}}}e^{\text{ff}2\pi\left(\mathcal{A}^{\widetilde{\delta_S}}_{\overline{\overline{\mathcal{TI}_{I-1}}}}\right)}, \mathcal{N}^{\widetilde{\delta_S}}_{\overline{\overline{\mathcal{TI}_{R-1}}}}e^{\text{ff}2\pi\left(\mathcal{N}^{\widetilde{\delta_S}}_{\overline{\overline{\mathcal{TI}_{I-1}}}}\right)} \end{pmatrix} \tag{20}$$

$$\overline{\overline{\mathcal{TI}_{CN-1}}}^{\widetilde{\delta_S}} = \begin{pmatrix} \mathcal{M}^{\widetilde{\delta_S}}_{\overline{\overline{\mathcal{TI}_{R-1}}}}e^{\text{ff}2\pi\left(\mathcal{M}^{\widetilde{\delta_S}}_{\overline{\overline{\mathcal{TI}_{I-1}}}}\right)}, 1-\left(1-\mathcal{A}_{\overline{\overline{\mathcal{TI}_{R-1}}}}\right)^{\widetilde{\delta_S}}e^{\text{ff}2\pi\left(1-\left(1-\mathcal{A}_{\overline{\overline{\mathcal{TI}_{I-1}}}}\right)^{\widetilde{\delta_S}}\right)}, \\ 1-\left(1-\mathcal{N}_{\overline{\overline{\mathcal{TI}_{R-1}}}}\right)^{\widetilde{\delta_S}}e^{\text{ff}2\pi\left(1-\left(1-\mathcal{N}_{\overline{\overline{\mathcal{TI}_{I-1}}}}\right)^{\widetilde{\delta_S}}\right)} \end{pmatrix} \tag{21}$$

Definition 12: Suppose $\overline{\overline{\mathcal{TI}_{CN-\text{ff}}}} = \left(\mathcal{M}_{\overline{\overline{\mathcal{TI}_{R-\text{ff}}}}}e^{\text{ff}2\pi\left(\mathcal{M}_{\overline{\overline{\mathcal{TI}_{I-\text{ff}}}}}\right)}, \mathcal{A}_{\overline{\overline{\mathcal{TI}_{R-\text{ff}}}}}e^{\text{ff}2\pi\left(\mathcal{A}_{\overline{\overline{\mathcal{TI}_{I-\text{ff}}}}}\right)}, \mathcal{N}_{\overline{\overline{\mathcal{TI}_{R-\text{ff}}}}}e^{\text{ff}2\pi\left(\mathcal{N}_{\overline{\overline{\mathcal{TI}_{I-\text{ff}}}}}\right)}\right)$, $\text{ff} = 1, 2, \ldots, \mu$, be any group of CSVNNs. The CSVNPWA operator is stated by

$$CSVNPWA\left(\overline{\overline{\mathcal{TI}_{CN-1}}}, \overline{\overline{\mathcal{TI}_{CN-2}}}, \ldots, \overline{\overline{\mathcal{TI}_{CN-\mu}}}\right) = \begin{pmatrix} 1-\prod_{\text{ff}=1}^{\mu}\left(1-\mathcal{M}_{\overline{\overline{\mathcal{TI}_{R-\text{ff}}}}}\right)^{\frac{\mathbb{H}_{\text{ff}}}{\sum_{\text{ff}=1}^{\mu}\mathbb{H}_{\text{ff}}}}e^{\text{ff}2\pi\left(1-\prod_{\text{ff}=1}^{\mu}\left(1-\mathcal{M}_{\overline{\overline{\mathcal{TI}_{I-\text{ff}}}}}\right)^{\frac{\mathbb{H}_{\text{ff}}}{\sum_{\text{ff}=1}^{\mu}\mathbb{H}_{\text{ff}}}}\right)}, \\ \prod_{\text{ff}=1}^{\mu}\mathcal{A}^{\frac{\mathbb{H}_{\text{ff}}}{\sum_{\text{ff}=1}^{\mu}\mathbb{H}_{\text{ff}}}}_{\overline{\overline{\mathcal{TI}_{R-\text{ff}}}}}e^{\text{ff}2\pi\left(\prod_{\text{ff}=1}^{\mu}\mathcal{A}^{\frac{\mathbb{H}_{\text{ff}}}{\sum_{\text{ff}=1}^{\mu}\mathbb{H}_{\text{ff}}}}_{\overline{\overline{\mathcal{TI}_{I-\text{ff}}}}}\right)}, \prod_{\text{ff}=1}^{\mu}\mathcal{N}^{\frac{\mathbb{H}_{\text{ff}}}{\sum_{\text{ff}=1}^{\mu}\mathbb{H}_{\text{ff}}}}_{\overline{\overline{\mathcal{TI}_{R-\text{ff}}}}}e^{\text{ff}2\pi\left(\prod_{\text{ff}=1}^{\mu}\mathcal{N}^{\frac{\mathbb{H}_{\text{ff}}}{\sum_{\text{ff}=1}^{\mu}\mathbb{H}_{\text{ff}}}}_{\overline{\overline{\mathcal{TI}_{I-\text{ff}}}}}\right)} \end{pmatrix} \tag{22}$$

where $\mathbb{H}_1$ and $\mathbb{H}_{\text{ff}} = \prod_{k=1}^{\text{ff}-1}\overline{\overline{\mathcal{S}}}\left(\overline{\overline{\mathcal{TI}_{CN-k}}}\right)$.

Definition 13: Suppose $\overline{\overline{\mathcal{TI}_{CN-\text{ff}}}} = \left(\mathcal{M}_{\overline{\overline{\mathcal{TI}_{R-\text{ff}}}}}e^{\text{ff}2\pi\left(\mathcal{M}_{\overline{\overline{\mathcal{TI}_{I-\text{ff}}}}}\right)}, \mathcal{A}_{\overline{\overline{\mathcal{TI}_{R-\text{ff}}}}}e^{\text{ff}2\pi\left(\mathcal{A}_{\overline{\overline{\mathcal{TI}_{I-\text{ff}}}}}\right)}, \mathcal{N}_{\overline{\overline{\mathcal{TI}_{R-\text{ff}}}}}e^{\text{ff}2\pi\left(\mathcal{N}_{\overline{\overline{\mathcal{TI}_{I-\text{ff}}}}}\right)}\right)$, $\text{ff} = 1, 2, \ldots, \mu$, be any group of CSVNNs. The CSVNPGA operator is stated by

$$CSVNPGA\left(\overline{\overline{\mathcal{TI}_{CN-1}}}, \overline{\overline{\mathcal{TI}_{CN-2}}}, \ldots, \overline{\overline{\mathcal{TI}_{CN-\mu}}}\right) = \begin{pmatrix} \prod_{\text{ff}=1}^{\mu}\mathcal{M}^{\frac{\mathbb{H}_{\text{ff}}}{\sum_{\text{ff}=1}^{\mu}\mathbb{H}_{\text{ff}}}}_{\overline{\overline{\mathcal{TI}_{R-\text{ff}}}}}e^{\text{ff}2\pi\left(\prod_{\text{ff}=1}^{\mu}\mathcal{M}^{\frac{\mathbb{H}_{\text{ff}}}{\sum_{\text{ff}=1}^{\mu}\mathbb{H}_{\text{ff}}}}_{\overline{\overline{\mathcal{TI}_{I-\text{ff}}}}}\right)}, \\ 1-\prod_{\text{ff}=1}^{\mu}\left(1-\mathcal{A}_{\overline{\overline{\mathcal{TI}_{R-\text{ff}}}}}\right)^{\frac{\mathbb{H}_{\text{ff}}}{\sum_{\text{ff}=1}^{\mu}\mathbb{H}_{\text{ff}}}}e^{\text{ff}2\pi\left(1-\prod_{\text{ff}=1}^{\mu}\left(1-\mathcal{A}_{\overline{\overline{\mathcal{TI}_{I-\text{ff}}}}}\right)^{\frac{\mathbb{H}_{\text{ff}}}{\sum_{\text{ff}=1}^{\mu}\mathbb{H}_{\text{ff}}}}\right)}, \\ 1-\prod_{\text{ff}=1}^{\mu}\left(1-\mathcal{N}_{\overline{\overline{\mathcal{TI}_{R-\text{ff}}}}}\right)^{\frac{\mathbb{H}_{\text{ff}}}{\sum_{\text{ff}=1}^{\mu}\mathbb{H}_{\text{ff}}}}e^{\text{ff}2\pi\left(1-\prod_{\text{ff}=1}^{\mu}\left(1-\mathcal{N}_{\overline{\overline{\mathcal{TI}_{I-\text{ff}}}}}\right)^{\frac{\mathbb{H}_{\text{ff}}}{\sum_{\text{ff}=1}^{\mu}\mathbb{H}_{\text{ff}}}}\right)} \end{pmatrix} \tag{23}$$

where $\mathbb{H}_1$ and $\mathbb{H}_{\text{ff}} = \prod_{k=1}^{\text{ff}-1}\overline{\overline{\mathcal{S}}}\left(\overline{\overline{\mathcal{TI}_{CN-k}}}\right)$.

4. Prioritized Muirhead Mean Operators Based on CSVNSs

The goal of this study is to initiate the CSVNS and to determine their important algebraic laws. Moreover, the principle of CSVNPMM operator and CSVNPDMM operator is elaborated and their particular cases are discussed. The technique of PMM aggregation operator is massive, dominant, and more flexible to investigate the interrelationships between any number of objects.

Definition 14: Suppose $\overline{\overline{\mathcal{TI}_{CN-\text{ff}}}} = \left(\mathcal{M}_{\overline{\overline{\mathcal{TI}_{R-\text{ff}}}}}e^{\text{ff}2\pi\left(\mathcal{M}_{\overline{\overline{\mathcal{TI}_{I-\text{ff}}}}}\right)}, \mathcal{A}_{\overline{\overline{\mathcal{TI}_{R-\text{ff}}}}}e^{\text{ff}2\left(\mathcal{A}_{\overline{\overline{\mathcal{TI}_{I-\text{ff}}}}}\right)}, \mathcal{N}_{\overline{\overline{\mathcal{TI}_{R-\text{ff}}}}}e^{\text{ff}2\pi\left(\mathcal{N}_{\overline{\overline{\mathcal{TI}_{I-\text{ff}}}}}\right)}\right)$, $\text{ff} = 1, 2, \ldots, \mu$, be any group of CSVNNs. The CSVNPMM operator is stated by

$$CSVNPMM\left(\overline{\overline{\mathcal{TI}_{CN-1}}}, \overline{\overline{\mathcal{TI}_{CN-2}}}, \ldots, \overline{\overline{\mathcal{TI}_{CN-\mu}}}\right) = \left(\frac{1}{\mu!}\oplus_{\sigma\in\ddot{\mathbb{S}}_{\mu}}\prod_{\text{ff}=1}^{\mu}\left(\mu\frac{\mathbb{H}_{\sigma(\text{ff})}}{\sum_{\text{ff}=1}^{\mu}\mathbb{H}_{\text{ff}}}\overline{\overline{\mathcal{TI}_{CN-\sigma(\text{ff})}}}\right)^{\overline{\overline{\mathcal{P}_{\text{ff}}}}}\right)^{\frac{1}{\sum_{\text{ff}=1}^{\mu}\overline{\overline{\mathcal{P}_{\text{ff}}}}}} \tag{24}$$

where $\mathbb{H}_1$ and $\mathbb{H}_{\text{ff}} = \prod_{k=1}^{\text{ff}-1}\overline{\overline{\mathcal{S}}}\left(\overline{\overline{\mathcal{TI}_{CN-k}}}\right)$, and σ is the permutation (PM) of $(\text{ff} = 1, 2, \ldots, \mu)$ and $\ddot{\mathbb{S}}_{\mu}$ is the group of PMs of $\text{ff} = 1, 2, \ldots, \mu$. For different values of $\overline{\overline{\mathcal{P}}} = \left(\overline{\overline{\mathcal{P}_1}}, \overline{\overline{\mathcal{P}_2}}, \ldots, \overline{\overline{\mathcal{P}_{\mu}}}\right) \in R^{\mu}$, certain specific cases are discussed below.

Theorem 1: Suppose $\overline{\overline{\mathcal{TI}_{CN-\text{ff}}}} = \left(\mathcal{M}_{\overline{\overline{\mathcal{TI}_{R-\text{ff}}}}}e^{\text{ff}2\pi\left(\mathcal{M}_{\overline{\overline{\mathcal{TI}_{I-\text{ff}}}}}\right)}, \mathcal{A}_{\overline{\overline{\mathcal{TI}_{R-\text{ff}}}}}e^{\text{ff}2\pi\left(\mathcal{A}_{\overline{\overline{\mathcal{TI}_{I-\text{ff}}}}}\right)}, \mathcal{N}_{\overline{\overline{\mathcal{TI}_{R-\text{ff}}}}}e^{\text{ff}2\pi\left(\mathcal{N}_{\overline{\overline{\mathcal{TI}_{I-\text{ff}}}}}\right)}\right)$, $\text{ff} = 1, 2, \ldots, \mu$, be any group of CSVNNs. Then by using Equation (24), we determine

$$
\begin{aligned}
&CSVNPMM\left(\overline{\overline{\mathcal{TI}_{CN-1}}},\overline{\overline{\mathcal{TI}_{CN-2}}},\ldots,\overline{\overline{\mathcal{TI}_{CN-\mu}}}\right)\\
&=\left(\begin{array}{c}
\left(1-\left(\prod_{\sigma\in\breve{\mathbb{S}}}\left(1-\prod_{\mathfrak{ff}=1}^{\mu}\left(1-\left(1-\mathcal{M}_{\overline{\overline{\mathcal{TI}_{R-\sigma(\mathfrak{ff})}}}}\right)^{\mu\frac{\mathbb{H}_{\sigma(\mathfrak{ff})}}{\sum_{\mathfrak{ff}=1}^{\mu}\mathbb{H}_{\mathfrak{ff}}}}\right)^{\overline{\overline{\mathcal{P}_{\mathfrak{ff}}}}}\right)\right)^{\frac{1}{\mu!}}\right)^{\frac{1}{\sum_{\mathfrak{ff}=1}^{\mu}\overline{\overline{\mathcal{P}_{\mathfrak{ff}}}}}}\\
e^{\mathfrak{ff}2\pi\left(1-\left(\prod_{\sigma\in\breve{\mathbb{S}}_{\mu}}\left(1-\prod_{\mathfrak{ff}=1}^{\mu}\left(1-\left(1-\mathcal{M}_{\overline{\overline{\mathcal{TI}_{I-\sigma(\mathfrak{ff})}}}}\right)^{\mu\frac{\mathbb{H}_{\sigma(\mathfrak{ff})}}{\sum_{\mathfrak{ff}=1}^{\mu}\mathbb{H}_{\mathfrak{ff}}}}\right)^{\overline{\overline{\mathcal{P}_{\mathfrak{ff}}}}}\right)\right)^{\frac{1}{\mu!}}\right)^{\frac{1}{\sum_{\mathfrak{ff}=1}^{\mu}\overline{\overline{\mathcal{P}_{\mathfrak{ff}}}}}}},\\
1-\left(1-\left(\prod_{\sigma\in\breve{\mathbb{S}}_{\mu}}\left(1-\prod_{\mathfrak{ff}=1}^{\mu}\left(1-\left(1-\mathcal{A}_{\overline{\overline{\mathcal{TI}_{R-\sigma(\mathfrak{ff})}}}}^{\mu\frac{\mathbb{H}_{\sigma(\mathfrak{ff})}}{\sum_{\mathfrak{ff}=1}^{\mu}\mathbb{H}_{\mathfrak{ff}}}}\right)\right)^{\overline{\overline{\mathcal{P}_{\mathfrak{ff}}}}}\right)\right)^{\frac{1}{\mu!}}\right)^{\frac{1}{\sum_{\mathfrak{ff}=1}^{\mu}\overline{\overline{\mathcal{P}_{\mathfrak{ff}}}}}}\\
e^{\mathfrak{ff}2\pi\left(1-\left(1-\left(\prod_{\sigma\in\breve{\mathbb{S}}_{\mu}}\left(1-\prod_{\mathfrak{ff}=1}^{\mu}\left(1-\left(1-\mathcal{A}_{\overline{\overline{\mathcal{TI}_{I-\sigma(\mathfrak{ff})}}}}^{\mu\frac{\mathbb{H}_{\sigma(\mathfrak{ff})}}{\sum_{\mathfrak{ff}=1}^{\mu}\mathbb{H}_{\mathfrak{ff}}}}\right)\right)^{\overline{\overline{\mathcal{P}_{\mathfrak{ff}}}}}\right)\right)^{\frac{1}{\mu!}}\right)^{\frac{1}{\sum_{\mathfrak{ff}=1}^{\mu}\overline{\overline{\mathcal{P}_{\mathfrak{ff}}}}}}\right)},\\
1-\left(1-\left(\prod_{\sigma\in\breve{\mathbb{S}}_{\mu}}\left(1-\prod_{\mathfrak{ff}=1}^{\mu}\left(1-\left(1-\mathcal{N}_{\overline{\overline{\mathcal{TI}_{R-\sigma(\mathfrak{ff})}}}}^{\mu\frac{\mathbb{H}_{\sigma(\mathfrak{ff})}}{\sum_{\mathfrak{ff}=1}^{\mu}\mathbb{H}_{\mathfrak{ff}}}}\right)\right)^{\overline{\overline{\mathcal{P}_{\mathfrak{ff}}}}}\right)\right)^{\frac{1}{\mu!}}\right)^{\frac{1}{\sum_{\mathfrak{ff}=1}^{\mu}\overline{\overline{\mathcal{P}_{\mathfrak{ff}}}}}}\\
e^{\mathfrak{ff}2\pi\left(1-\left(1-\left(\prod_{\sigma\in\breve{\mathbb{S}}_{\mu}}\left(1-\prod_{\mathfrak{ff}=1}^{\mu}\left(1-\left(1-\mathcal{N}_{\overline{\overline{\mathcal{TI}_{I-\sigma(\mathfrak{ff})}}}}^{\mu\frac{\mathbb{H}_{\sigma(\mathfrak{ff})}}{\sum_{\mathfrak{ff}=1}^{\mu}\mathbb{H}_{\mathfrak{ff}}}}\right)\right)^{\overline{\overline{\mathcal{P}_{\mathfrak{ff}}}}}\right)\right)^{\frac{1}{\mu!}}\right)^{\frac{1}{\sum_{\mathfrak{ff}=1}^{\mu}\overline{\overline{\mathcal{P}_{\mathfrak{ff}}}}}}\right)}
\end{array}\right)
\end{aligned}
\tag{25}
$$

Proof: Suppose $\overline{\overline{\mathcal{TI}_{CN-\mathfrak{ff}}}}=\left(\mathcal{M}_{\overline{\overline{\mathcal{TI}_{R-\mathfrak{ff}}}}}e^{\mathfrak{ff}2\pi\left(\mathcal{M}_{\overline{\overline{\mathcal{TI}_{I-\mathfrak{ff}}}}}\right)},\mathcal{A}_{\overline{\overline{\mathcal{TI}_{R-\mathfrak{ff}}}}}e^{\mathfrak{ff}2\pi\left(\mathcal{A}_{\overline{\overline{\mathcal{TI}_{I-\mathfrak{ff}}}}}\right)},\mathcal{N}_{\overline{\overline{\mathcal{TI}_{R-\mathfrak{ff}}}}}e^{\mathfrak{ff}2\pi\left(\mathcal{N}_{\overline{\overline{\mathcal{TI}_{I-\mathfrak{ff}}}}}\right)}\right),\mathfrak{ff}=1,2,\ldots,\mu,$ be any group of CSVNNs. Then by using Definition (11), we have

$$
\mu\frac{\mathbb{H}_{\sigma(\mathfrak{ff})}}{\sum_{\mathfrak{ff}=1}^{\mu}\mathbb{H}_{\mathfrak{ff}}}\overline{\overline{\mathcal{TI}_{CN-\sigma(\mathfrak{ff})}}}=\left(\begin{array}{c}
1-\left(1-\mathcal{M}_{\overline{\overline{\mathcal{TI}_{R-\sigma(\mathfrak{ff})}}}}\right)^{\frac{\mathbb{H}_{\sigma(\mathfrak{ff})}}{\sum_{\mathfrak{ff}=1}^{\mu}\mathbb{H}_{\mathfrak{ff}}}}e^{\mathfrak{ff}2\pi\left(1-\left(1-\mathcal{M}_{\overline{\overline{\mathcal{TI}_{I-\sigma(\mathfrak{ff})}}}}\right)^{\mu\frac{\mathbb{H}_{\sigma(\mathfrak{ff})}}{\sum_{\mathfrak{ff}=1}^{\mu}\mathbb{H}_{\mathfrak{ff}}}}\right)},\\
\mathcal{A}_{\overline{\overline{\mathcal{TI}_{R-\sigma(\mathfrak{ff})}}}}^{\mu\frac{\mathbb{H}_{\sigma(\mathfrak{ff})}}{\sum_{\mathfrak{ff}=1}^{\mu}\mathbb{H}_{\mathfrak{ff}}}}e^{\mathfrak{ff}2\pi\left(\mathcal{A}_{\overline{\overline{\mathcal{TI}_{I-\sigma(\mathfrak{ff})}}}}^{\mu\frac{\mathbb{H}_{\sigma(\mathfrak{ff})}}{\sum_{\mathfrak{ff}=1}^{\mu}\mathbb{H}_{\mathfrak{ff}}}}\right)},\mathcal{N}_{\overline{\overline{\mathcal{TI}_{R-\sigma(\mathfrak{ff})}}}}^{\mu\frac{\mathbb{H}_{\sigma(\mathfrak{ff})}}{\sum_{\mathfrak{ff}=1}^{\mu}\mathbb{H}_{\mathfrak{ff}}}}e^{\mathfrak{ff}2\pi\left(\mathcal{N}_{\overline{\overline{\mathcal{TI}_{I-\sigma(\mathfrak{ff})}}}}^{\mu\frac{\mathbb{H}_{\sigma(\mathfrak{ff})}}{\sum_{\mathfrak{ff}=1}^{\mu}\mathbb{H}_{\mathfrak{ff}}}}\right)}
\end{array}\right)
$$

Then,

$$\left(\mu\frac{\mathbb{H}_{\sigma(ff)}}{\sum_{ff=1}^{\mu}\mathbb{H}_{ff}}\overline{\overline{\mathcal{TI}_{CN-\sigma(ff)}}}\right)^{\overline{\overline{\mathcal{P}_{ff}}}}$$

$$=\begin{pmatrix}\left(1-\left(1-\mathcal{M}_{\overline{\overline{\mathcal{TI}_{R-\sigma(ff)}}}}\right)^{\mu\frac{\mathbb{H}_{\sigma(ff)}}{\sum_{ff=1}^{\mu}\mathbb{H}_{ff}}}\right)^{\overline{\overline{\mathcal{P}_{ff}}}}e^{ff2\pi\left(1-\left(1-\mathcal{M}_{\overline{\overline{\mathcal{TI}_{I-\sigma(ff)}}}}\right)^{\mu\frac{\mathbb{H}_{\sigma(ff)}}{\sum_{ff=1}^{\mu}\mathbb{H}_{ff}}}\right)^{\overline{\overline{\mathcal{P}_{ff}}}}},\\ \left(1-\left(1-\mathcal{A}^{\mu\frac{\mathbb{H}_{\sigma(ff)}}{\sum_{ff=1}^{\mu}\mathbb{H}_{ff}}}_{\overline{\overline{\mathcal{TI}_{R-\sigma(ff)}}}}\right)\right)^{\overline{\overline{\mathcal{P}_{ff}}}}e^{ff2\pi\left(1-\left(1-\mathcal{A}^{\mu\frac{\mathbb{H}_{\sigma(ff)}}{\sum_{ff=1}^{\mu}\mathbb{H}_{ff}}}_{\overline{\overline{\mathcal{TI}_{I-\sigma(ff)}}}}\right)\right)^{\overline{\overline{\mathcal{P}_{ff}}}}},\\ \left(1-\left(1-\mathcal{N}^{\mu\frac{\mathbb{H}_{\sigma(ff)}}{\sum_{ff=1}^{\mu}\mathbb{H}_{ff}}}_{\overline{\overline{\mathcal{TI}_{R-\sigma(ff)}}}}\right)\right)^{\overline{\overline{\mathcal{P}_{ff}}}}e^{ff2\pi\left(1-\left(1-\mathcal{N}^{\mu\frac{\mathbb{H}_{\sigma(ff)}}{\sum_{ff=1}^{\mu}\mathbb{H}_{ff}}}_{\overline{\overline{\mathcal{TI}_{I-\sigma(ff)}}}}\right)\right)^{\overline{\overline{\mathcal{P}_{ff}}}}}\end{pmatrix}$$

Thus,

$$\oplus_{\sigma\in\mathbb{S}_{\mu}}\prod_{ff=1}^{\mu}\left(\frac{\mathbb{H}_{\sigma(ff)}}{\sum_{ff=1}^{\mu}\mathbb{H}_{ff}}\overline{\overline{\mathcal{TI}_{CN-\sigma(ff)}}}\right)^{\overline{\overline{\mathcal{P}_{ff}}}}$$

$$=\begin{pmatrix}\left(1-\left(\prod_{\sigma\in\mathbb{S}_{\mu}}\left(1-\prod_{ff=1}^{\mu}\left(1-\left(1-\mathcal{M}_{\overline{\overline{\mathcal{TI}_{R-\sigma(ff)}}}}\right)^{\mu\frac{\mathbb{H}_{\sigma(ff)}}{\sum_{ff=1}^{\mu}\mathbb{H}_{ff}}}\right)^{\overline{\overline{\mathcal{P}_{ff}}}}\right)\right)\right)\\ e^{ff2\pi\left(1-\left(\prod_{\sigma\in\mathbb{S}_{\mu}}\left(1-\prod_{ff=1}^{\mu}\left(1-\left(1-\mathcal{M}_{\overline{\overline{\mathcal{TI}_{I-\sigma(ff)}}}}\right)^{\mu\frac{\mathbb{H}_{\sigma(ff)}}{\sum_{ff=1}^{\mu}\mathbb{H}_{ff}}}\right)^{\overline{\overline{\mathcal{P}_{ff}}}}\right)\right)\right)},\\ \prod_{\sigma\in\mathbb{S}_{\mu}}\left(1-\prod_{ff=1}^{\mu}\left(\left(1-\mathcal{A}^{\mu\frac{\mathbb{H}_{\sigma(ff)}}{\sum_{ff=1}^{\mu}\mathbb{H}_{ff}}}_{\overline{\overline{\mathcal{TI}_{R-\sigma(ff)}}}}\right)\right)^{\overline{\overline{\mathcal{P}_{ff}}}}\right)e^{ff2\pi\left(\prod_{\sigma\in\mathbb{S}_{\mu}}\left(1-\prod_{ff=1}^{\mu}\left(\left(1-\mathcal{A}^{\mu\frac{\mathbb{H}_{\sigma(ff)}}{\sum_{ff=1}^{\mu}\mathbb{H}_{ff}}}_{\overline{\overline{\mathcal{TI}_{I-\sigma(ff)}}}}\right)\right)^{\overline{\overline{\mathcal{P}_{ff}}}}\right)\right)},\\ \prod_{\sigma\in\mathbb{S}_{\mu}}\left(1-\prod_{ff=1}^{\mu}\left(\left(1-\mathcal{N}^{\mu\frac{\mathbb{H}_{\sigma(ff)}}{\sum_{ff=1}^{\mu}\mathbb{H}_{ff}}}_{\overline{\overline{\mathcal{TI}_{R-\sigma(ff)}}}}\right)\right)^{\overline{\overline{\mathcal{P}_{ff}}}}\right)e^{ff2\pi\left(\prod_{\sigma\in\mathbb{S}_{\mu}}\left(1-\prod_{ff=1}^{\mu}\left(\left(1-\mathcal{N}^{\mu\frac{\mathbb{H}_{\sigma(ff)}}{\sum_{ff=1}^{\mu}\mathbb{H}_{ff}}}_{\overline{\overline{\mathcal{TI}_{I-\sigma(ff)}}}}\right)\right)^{\overline{\overline{\mathcal{P}_{ff}}}}\right)\right)}\end{pmatrix}$$

Then,

$$\left(\frac{1}{\mu!}\oplus_{\sigma\in\mathbb{S}_\mu}\prod_{ff=1}^{\mu}\left(\mu\frac{\mathbb{H}_{\sigma(ff)}}{\sum_{ff=1}^{\mu}\mathbb{H}_{ff}}\overline{\overline{\mathcal{TI}_{CN-\sigma(ff)}}}\right)^{\overline{\overline{\mathcal{P}_{ff}}}}\right)^{\frac{1}{\sum_{ff=1}^{\mu}\overline{\overline{\mathcal{P}_{ff}}}}}$$

$$=\begin{pmatrix}\left(1-\left(\prod_{\sigma\in\mathbb{S}_\mu}\left(1-\prod_{ff=1}^{\mu}\left(1-\left(1-\mathcal{M}_{\overline{\overline{\mathcal{TI}_{R-\sigma(ff)}}}}\right)^{\mu\frac{\mathbb{H}_{\sigma(ff)}}{\sum_{ff=1}^{\mu}\mathbb{H}_{ff}}}\right)^{\overline{\overline{\mathcal{P}_{ff}}}}\right)\right)^{\frac{1}{\mu!}}\right)^{\frac{1}{\sum_{ff=1}^{\mu}\overline{\overline{\mathcal{P}_{ff}}}}} \\ e^{ff2\pi\left(1-\left(\prod_{\sigma\in\mathbb{S}_\mu}\left(1-\prod_{ff=1}^{\mu}\left(1-\left(1-\mathcal{M}_{\overline{\overline{\mathcal{TI}_{I-\sigma(ff)}}}}\right)^{\mu\frac{\mathbb{H}_{\sigma(ff)}}{\sum_{ff=1}^{\mu}\mathbb{H}_{ff}}}\right)^{\overline{\overline{\mathcal{P}_{ff}}}}\right)\right)^{\frac{1}{\mu!}}\right)^{\frac{1}{\sum_{ff=1}^{\mu}\overline{\overline{\mathcal{P}_{ff}}}}}}, \\ 1-\left(1-\left(\prod_{\sigma\in\mathbb{S}_\mu}\left(1-\prod_{ff=1}^{\mu}\left(1-\left(1-\mathcal{A}_{\overline{\overline{\mathcal{TI}_{R-\sigma(ff)}}}}^{\mu\frac{\mathbb{H}_{\sigma(ff)}}{\sum_{ff=1}^{\mu}\mathbb{H}_{ff}}}\right)\right)^{\overline{\overline{\mathcal{P}_{ff}}}}\right)\right)^{\frac{1}{\mu!}}\right)^{\frac{1}{\sum_{ff=1}^{\mu}\overline{\overline{\mathcal{P}_{ff}}}}} \\ e^{ff2\pi\left(1-\left(1-\left(\prod_{\sigma\in\mathbb{S}_\mu}\left(1-\prod_{ff=1}^{\mu}\left(1-\left(1-\mathcal{A}_{\overline{\overline{\mathcal{TI}_{I-\sigma(ff)}}}}^{\mu\frac{\mathbb{H}_{\sigma(ff)}}{\sum_{ff=1}^{\mu}\mathbb{H}_{ff}}}\right)\right)^{\overline{\overline{\mathcal{P}_{ff}}}}\right)\right)^{\frac{1}{\mu!}}\right)^{\frac{1}{\sum_{ff=1}^{\mu}\overline{\overline{\mathcal{P}_{ff}}}}}\right)}, \\ 1-\left(1-\left(\prod_{\sigma\in\mathbb{S}_\mu}\left(1-\prod_{ff=1}^{\mu}\left(1-\left(1-\mathcal{N}_{\overline{\overline{\mathcal{TI}_{R-\sigma(ff)}}}}^{\mu\frac{\mathbb{H}_{\sigma(ff)}}{\sum_{ff=1}^{\mu}\mathbb{H}_{ff}}}\right)\right)^{\overline{\overline{\mathcal{P}_{ff}}}}\right)\right)^{\frac{1}{\mu!}}\right)^{\frac{1}{\sum_{ff=1}^{\mu}\overline{\overline{\mathcal{P}_{ff}}}}} \\ e^{ff2\pi\left(1-\left(1-\left(\prod_{\sigma\in\mathbb{S}_\mu}\left(1-\prod_{ff=1}^{\mu}\left(1-\left(1-\mathcal{N}_{\overline{\overline{\mathcal{TI}_{I-\sigma(ff)}}}}^{\mu\frac{\mathbb{H}_{\sigma(ff)}}{\sum_{ff=1}^{\mu}\mathbb{H}_{ff}}}\right)\right)^{\overline{\overline{\mathcal{P}_{ff}}}}\right)\right)^{\frac{1}{\mu!}}\right)^{\frac{1}{\sum_{ff=1}^{\mu}\overline{\overline{\mathcal{P}_{ff}}}}}\right)}\end{pmatrix}$$

Hence proof.

Moreover, by using the presented operators, we elaborate on the principle of monotonicity, boundedness, and idempotency.

Theorem 2: Let $\overline{\overline{\mathcal{TI}_{CN-ff}}}=\left(\mathcal{M}_{\overline{\overline{\mathcal{TI}_{R-ff}}}}e^{ff2\pi\left(\mathcal{M}_{\overline{\overline{\mathcal{TI}_{I-ff}}}}\right)},\mathcal{A}_{\overline{\overline{\mathcal{TI}_{R-ff}}}}e^{ff2\pi\left(\mathcal{A}_{\overline{\overline{\mathcal{TI}_{I-ff}}}}\right)},\mathcal{N}_{\overline{\overline{\mathcal{TI}_{R-ff}}}}e^{ff2\pi\left(\mathcal{N}_{\overline{\overline{\mathcal{TI}_{I-ff}}}}\right)}\right), ff=1,2,\ldots,\mu,$ be any group of CSVNNs. If $\overline{\overline{\mathcal{TI}_{CN-ff}}}\leq\overline{\overline{\mathcal{TI}_{CN-ff}}}^{*}$, i.e., $\mathcal{M}_{\overline{\overline{\mathcal{TI}_{R-ff}}}}\leq\mathcal{M}^{*}_{\overline{\overline{\mathcal{TI}_{R-ff}}}},\mathcal{A}_{\overline{\overline{\mathcal{TI}_{R-ff}}}}\geq\mathcal{A}^{*}_{\overline{\overline{\mathcal{TI}_{R-ff}}}},\mathcal{N}_{\overline{\overline{\mathcal{TI}_{R-ff}}}}\geq\mathcal{N}^{*}_{\overline{\overline{\mathcal{TI}_{R-ff}}}}$ and $\mathcal{M}_{\overline{\overline{\mathcal{TI}_{I-ff}}}}\leq\mathcal{M}^{*}_{\overline{\overline{\mathcal{TI}_{I-ff}}}},\mathcal{A}_{\overline{\overline{\mathcal{TI}_{I-ff}}}}\geq\mathcal{A}^{*}_{\overline{\overline{\mathcal{TI}_{I-ff}}}},\mathcal{N}_{\overline{\overline{\mathcal{TI}_{I-ff}}}}\geq\mathcal{N}^{*}_{\overline{\overline{\mathcal{TI}_{I-ff}}}}$, then

$$\begin{aligned}&CSVNPMM\left(\overline{\overline{\mathcal{TI}_{CN-1}}},\overline{\overline{\mathcal{TI}_{CN-2}}},\ldots,\overline{\overline{\mathcal{TI}_{CN-\mu}}}\right)\\&\leq CSVNPMM\left(\overline{\overline{\mathcal{TI}_{CN-1}}}^{*},\overline{\overline{\mathcal{TI}_{CN-2}}}^{*},\ldots,\overline{\overline{\mathcal{TI}_{CN-\mu}}}^{*}\right)\end{aligned}\tag{26}$$

Proof: Suppose $\overline{\overline{\mathcal{TI}_{CN-ff}}}\leq\overline{\overline{\mathcal{TI}_{CN-ff}}}^{*}$, i.e., $\mathcal{M}_{\overline{\overline{\mathcal{TI}_{R-ff}}}}\leq\mathcal{M}^{*}_{\overline{\overline{\mathcal{TI}_{R-ff}}}},\mathcal{A}_{\overline{\overline{\mathcal{TI}_{R-ff}}}}\geq\mathcal{A}^{*}_{\overline{\overline{\mathcal{TI}_{R-ff}}}},\mathcal{N}_{\overline{\overline{\mathcal{TI}_{R-ff}}}}\geq\mathcal{N}^{*}_{\overline{\overline{\mathcal{TI}_{R-ff}}}}$ and $\mathcal{M}_{\overline{\overline{\mathcal{TI}_{I-ff}}}}\leq\mathcal{M}^{*}_{\overline{\overline{\mathcal{TI}_{I-ff}}}},\mathcal{A}_{\overline{\overline{\mathcal{TI}_{I-ff}}}}\geq\mathcal{A}^{*}_{\overline{\overline{\mathcal{TI}_{I-ff}}}},\mathcal{N}_{\overline{\overline{\mathcal{TI}_{I-ff}}}}\geq\mathcal{N}^{*}_{\overline{\overline{\mathcal{TI}_{I-ff}}}}$, then we prove that Equation (26) is true. First, if $\mathcal{M}_{\overline{\overline{\mathcal{TI}_{R-ff}}}}\leq\mathcal{M}^{*}_{\overline{\overline{\mathcal{TI}_{R-ff}}}}$, then

$$\left(1-\mathcal{M}_{\overline{\overline{\mathcal{TI}_{R-\sigma(ff)}}}}\right)^{\mu\frac{\mathbb{H}_{\sigma(ff)}}{\sum_{ff=1}^{\mu}\mathbb{H}_{ff}}}\geq\left(1-\mathcal{M}^{*}_{\overline{\overline{\mathcal{TI}_{R-\sigma(ff)}}}}\right)^{\mu\frac{\mathbb{H}_{\sigma(ff)}}{\sum_{ff=1}^{\mu}\mathbb{H}_{ff}}},\quad\text{where}$$

$\mathbb{H}_1=1$ and $\mathbb{H}_{ff}=\prod_{k=1}^{ff-1}\overline{\overline{\mathcal{S}}}\left(\overline{\overline{\mathcal{TI}_{CN-k}}}\right)$, thus,

$$1-\left(1-\mathcal{M}_{\overline{\overline{\mathcal{TI}_{R-\sigma(ff)}}}}\right)^{\mu\frac{\mathbb{H}_{\sigma(ff)}}{\sum_{ff=1}^{\mu}\mathbb{H}_{ff}}}\leq1-\left(1-\mathcal{M}^{*}_{\overline{\overline{\mathcal{TI}_{R-\sigma(ff)}}}}\right)^{\mu\frac{\mathbb{H}_{\sigma(ff)}}{\sum_{ff=1}^{\mu}\mathbb{H}_{ff}}},$$

then,

$$1-\prod_{\text{ff}=1}^{\mu}\left(1-\left(1-\mathcal{M}_{\overline{\overline{\mathcal{TI}_{R-\sigma(\text{f})}}}}\right)^{\mu\frac{\mathbb{H}_{\sigma(\text{ff})}}{\sum_{\text{ff}=1}^{\mu}\mathbb{H}_{\text{ff}}}}\right)^{\overline{\overline{\mathcal{P}_{\text{ff}}}}}$$
$$\geq 1-\prod_{\text{ff}=1}^{\mu}\left(1-\left(1-\mathcal{M}^{*}_{\overline{\overline{\mathcal{TI}_{R-\sigma(\text{ff})}}}}\right)^{\mu\frac{\mathbb{H}_{\sigma(\text{ff})}}{\sum_{\text{ff}=1}^{\mu}\mathbb{H}_{\text{ff}}}}\right)^{\overline{\overline{\mathcal{P}_{\text{ff}}}}},$$

where σ is the permutation of $(\text{ff}=1,2,\ldots,\mu)$. Therefore

$$\left(1-\left(\prod_{\sigma\in\breve{\mathbb{S}}_{\mu}}\left(1-\prod_{\text{ff}=1}^{\mu}\left(1-\left(1-\mathcal{M}_{\overline{\overline{\mathcal{TI}_{R-\sigma(\text{ff})}}}}\right)^{\mu\frac{\mathbb{H}_{\sigma(\text{ff})}}{\sum_{\text{ff}=1}^{\mu}\mathbb{H}_{\text{ff}}}}\right)^{\overline{\overline{\mathcal{P}_{\text{ff}}}}}\right)\right)^{\frac{1}{\mu!}}\right)^{\frac{1}{\sum_{\text{ff}=1}^{\mu}\overline{\overline{\mathcal{P}_{\text{ff}}}}}}$$
$$\leq\left(1-\left(\prod_{\sigma\in\breve{\mathbb{S}}_{\mu}}\left(1-\prod_{\text{ff}=1}^{\mu}\left(1-\left(1-\mathcal{M}_{\overline{\overline{\mathcal{TI}_{R-\sigma(\text{ff})}}}}\right)^{\mu\frac{\mathbb{H}_{\sigma(\text{ff})}}{\sum_{\text{ff}=1}^{\mu}\mathbb{H}_{\text{ff}}}}\right)^{\overline{\overline{\mathcal{P}_{\text{ff}}}}}\right)\right)^{\frac{1}{\mu!}}\right)^{\frac{1}{\sum_{\text{ff}=1}^{\mu}\overline{\overline{\mathcal{P}_{\text{ff}}}}}},$$

where $\breve{\mathbb{S}}_{\mu}$ is the group of permutations of $\text{ff}=1;2;\ldots,\mu$, for different values of $\overline{\overline{\mathcal{P}}}=\left(\overline{\overline{\mathcal{P}_1}},\overline{\overline{\mathcal{P}_2}},\ldots,\overline{\overline{\mathcal{P}_{\mu}}}\right)\in R^{\mu}$. Similarly, we determine for the unreal part of TG, such that

$$\left(1-\left(\prod_{\sigma\in\breve{\mathbb{S}}_{\mu}}\left(1-\prod_{\text{ff}=1}^{\mu}\left(1-\left(1-\mathcal{M}_{\overline{\overline{\mathcal{TI}_{I-\sigma(\text{ff})}}}}\right)^{\mu\frac{\mathbb{H}_{\sigma(\text{ff})}}{\sum_{\text{ff}=1}^{\mu}\mathbb{H}_{\text{ff}}}}\right)^{\overline{\overline{\mathcal{P}_{\text{ff}}}}}\right)\right)^{\frac{1}{\mu!}}\right)^{\frac{1}{\sum_{\text{ff}=1}^{\mu}\overline{\overline{\mathcal{P}_{\text{ff}}}}}}$$
$$\leq\left(1-\left(\prod_{\sigma\in\breve{\mathbb{S}}_{\mu}}\left(1-\prod_{\text{ff}=1}^{\mu}\left(1-\left(1-\mathcal{M}^{*}_{\overline{\overline{\mathcal{TI}_{I-\sigma(\text{ff})}}}}\right)^{\mu\frac{\mathbb{H}_{\sigma(\text{ff})}}{\sum_{\text{ff}=1}^{\mu}\mathbb{H}_{\text{ff}}}}\right)^{\overline{\overline{\mathcal{P}_{\text{ff}}}}}\right)\right)^{\frac{1}{\mu!}}\right)^{\frac{1}{\sum_{\text{ff}=1}^{\mu}\overline{\overline{\mathcal{P}_{\text{ff}}}}}}.$$

In another case, suppose $\mathcal{A}_{\overline{\overline{\mathcal{TI}_{R-\text{ff}}}}}\geq\mathcal{A}^{*}_{\overline{\overline{\mathcal{TI}_{R-\text{ff}}}}}$, then obviously,

$$1-\left(1-\left(\prod_{\sigma\in\breve{\mathbb{S}}_{\mu}}\left(1-\prod_{\text{f}=1}^{\mu}\left(1-\left(1-\mathcal{A}_{\overline{\overline{\mathcal{TI}_{R-\sigma(\text{ff})}}}}^{\mu\frac{\mathbb{H}_{\sigma(\text{ff})}}{\sum_{\text{ff}=1}^{\mu}\mathbb{H}_{\text{ff}}}}\right)\right)^{\overline{\overline{\mathcal{P}_{\text{ff}}}}}\right)\right)^{\frac{1}{\mu!}}\right)^{\frac{1}{\sum_{\text{ff}=1}^{\mu}\overline{\overline{\mathcal{P}_{\text{ff}}}}}}$$
$$\geq 1-\left(1-\left(\prod_{\sigma\in\breve{\mathbb{S}}_{\mu}}\left(1-\prod_{\text{f}=1}^{\mu}\left(1-\left(1-\mathcal{A}^{*\,\mu\frac{\mathbb{H}_{\sigma(\text{ff})}}{\sum_{\text{ff}=1}^{\mu}\mathbb{H}_{\text{ff}}}}_{\overline{\overline{\mathcal{TI}_{R-\sigma(\text{ff})}}}}\right)\right)^{\overline{\overline{\mathcal{P}_{\text{ff}}}}}\right)\right)^{\frac{1}{\mu!}}\right)^{\frac{1}{\sum_{\text{ff}=1}^{\mu}\overline{\overline{\mathcal{P}_{\text{ff}}}}}},$$

and,

$$1-\left(1-\left(\prod_{\sigma\in\breve{\mathbb{S}}_{\mu}}\left(1-\prod_{\text{ff}=1}^{\mu}\left(1-\left(1-\mathcal{A}^{*\,\mu\frac{\mathbb{H}_{\sigma(\text{ff})}}{\sum_{\text{ff}=1}^{\mu}\mathbb{H}_{\text{ff}}}}_{\overline{\overline{\mathcal{TI}_{I-\sigma(\text{ff})}}}}\right)\right)^{\overline{\overline{\mathcal{P}_{\text{ff}}}}}\right)\right)^{\frac{1}{\mu!}}\right)^{\frac{1}{\sum_{\text{ff}=1}^{\mu}\overline{\overline{\mathcal{P}_{\text{ff}}}}}}$$
$$\geq 1-\left(1-\left(\prod_{\sigma\in\breve{\mathbb{S}}_{\mu}}\left(1-\prod_{\text{ff}=1}^{\mu}\left(1-\left(1-\mathcal{A}^{*\,\mu\frac{\mathbb{H}_{\sigma(\text{ff})}}{\sum_{\text{ff}=1}^{\mu}\mathbb{H}_{\text{ff}}}}_{\overline{\overline{\mathcal{TI}_{I-\sigma(\text{ff})}}}}\right)\right)^{\overline{\overline{\mathcal{P}_{\text{ff}}}}}\right)\right)^{\frac{1}{\mu!}}\right)^{\frac{1}{\sum_{\text{ff}=1}^{\mu}\overline{\overline{\mathcal{P}_{\text{ff}}}}}}$$

For FG, we have

$$1-\left(1-\left(\prod_{\sigma\in\breve{\mathbb{S}}_{\mu}}\left(1-\prod_{\text{f}=1}^{\mu}\left(1-\left(1-\mathcal{N}_{\overline{\overline{\mathcal{TI}_{R-\sigma(\text{ff})}}}}^{\mu\frac{\mathbb{H}_{\sigma(\text{ff})}}{\sum_{\text{ff}=1}^{\mu}\mathbb{H}_{\text{ff}}}}\right)\right)^{\overline{\overline{\mathcal{P}_{\text{ff}}}}}\right)\right)^{\frac{1}{\mu!}}\right)^{\frac{1}{\sum_{\text{ff}=1}^{\mu}\overline{\overline{\mathcal{P}_{\text{ff}}}}}}$$
$$\geq 1-\left(1-\left(\prod_{\sigma\in\breve{\mathbb{S}}_{\mu}}\left(1-\prod_{\text{ff}=1}^{\mu}\left(1-\left(1-\mathcal{N}^{*\,\mu\frac{\mathbb{H}_{\sigma(\text{ff})}}{\sum_{\text{ff}=1}^{\mu}\mathbb{H}_{\text{ff}}}}_{\overline{\overline{\mathcal{TI}_{R-\sigma(\text{ff})}}}}\right)\right)^{\overline{\overline{\mathcal{P}_{\text{ff}}}}}\right)\right)^{\frac{1}{\mu!}}\right)^{\frac{1}{\sum_{\text{ff}=1}^{\mu}\overline{\overline{\mathcal{P}_{\text{ff}}}}}},$$

and,

$$1-\left(1-\left(\prod_{\sigma\in\breve{\mathbb{S}}_{\mu}}\left(1-\prod_{\text{ff}=1}^{\mu}\left(1-\left(1-\mathcal{N}_{\overline{\overline{\mathcal{TI}_{I-\sigma(\text{ff})}}}}^{\mu\frac{\mathbb{H}_{\sigma(\text{ff})}}{\sum_{\text{ff}=1}^{\mu}\mathbb{H}_{\text{ff}}}}\right)\right)^{\overline{\overline{\mathcal{P}_{\text{ff}}}}}\right)\right)^{\frac{1}{\mu!}}\right)^{\frac{1}{\sum_{\text{ff}=1}^{\mu}\overline{\overline{\mathcal{P}_{\text{ff}}}}}}$$
$$\geq 1-\left(1-\left(\prod_{\sigma\in\breve{\mathbb{S}}_{\mu}}\left(1-\prod_{\text{ff}=1}^{\mu}\left(1-\left(1-\mathcal{N}^{*\,\mu\frac{\mathbb{H}_{\sigma(\text{ff})}}{\sum_{\text{ff}=1}^{\mu}\mathbb{H}_{\text{ff}}}}_{\overline{\overline{\mathcal{TI}_{I-\sigma(\text{ff})}}}}\right)\right)^{\overline{\overline{\mathcal{P}_{\text{ff}}}}}\right)\right)^{\frac{1}{\mu!}}\right)^{\frac{1}{\sum_{\text{ff}=1}^{\mu}\overline{\overline{\mathcal{P}_{\text{ff}}}}}}.$$

Then by using Definition 4, we have

$$CSVNPMM\left(\overline{\overline{\mathcal{TI}_{CN-1}}},\overline{\overline{\mathcal{TI}_{CN-2}}},\ldots,\overline{\overline{\mathcal{TI}_{CN-\mu}}}\right)$$
$$\leq CSVNPMM\left(\overline{\overline{\mathcal{TI}_{CN-1}}}^{*},\overline{\overline{\mathcal{TI}_{CN-2}}}^{*},\ldots,\overline{\overline{\mathcal{TI}_{CN-\mu}}}^{*}\right).$$

Theorem 3: Let $\overline{\overline{\mathcal{TI}_{CN-\text{ff}}}}=\left(\mathcal{M}_{\overline{\overline{\mathcal{TI}_{R-\text{ff}}}}}e^{\text{ff}2\pi\left(\mathcal{M}_{\overline{\overline{\mathcal{TI}_{I-\text{ff}}}}}\right)},\mathcal{A}_{\overline{\overline{\mathcal{TI}_{R-\text{ff}}}}}e^{\text{ff}2\pi\left(\mathcal{A}_{\overline{\overline{\mathcal{TI}_{I-\text{ff}}}}}\right)},\mathcal{N}_{\overline{\overline{\mathcal{TI}_{R-\text{ff}}}}}e^{\text{ff}2\pi\left(\mathcal{N}_{\overline{\overline{\mathcal{TI}_{I-\text{ff}}}}}\right)}\right),\text{ff}=1,2,\ldots,\mu,$ be any group of CSVNNs. If $\overline{\overline{\mathcal{I}_{CN-\text{ff}}}}^{-}=\left(\min_{\text{ff}}\mathcal{M}_{\overline{\overline{\mathcal{TI}_{R-\text{ff}}}}}e^{\text{ff}2\pi\left(\min_{\text{ff}}\mathcal{M}_{\overline{\overline{\mathcal{TI}_{I-\text{ff}}}}}\right)},\max_{\text{ff}}\mathcal{A}_{\overline{\overline{\mathcal{TI}_{R-\text{ff}}}}}e^{\text{ff}2\pi\left(\max_{\text{ff}}\mathcal{A}_{\overline{\overline{\mathcal{TI}_{I-\text{ff}}}}}\right)},\right.$ $\left.\max_{\text{ff}}\mathcal{N}_{\overline{\overline{\mathcal{TI}_{R-\text{ff}}}}}e^{\text{ff}2\pi\left(\max_{\text{ff}}\mathcal{N}_{\overline{\overline{\mathcal{TI}_{I-\text{ff}}}}}\right)}\right)=\left(\mathcal{M}^{-}_{\overline{\overline{\mathcal{TI}_{R}}}}e^{\text{ff}2\pi\left(\mathcal{M}^{-}_{\overline{\overline{\mathcal{TI}_{I}}}}\right)},\mathcal{A}^{-}_{\overline{\overline{\mathcal{TI}_{R}}}}e^{\text{ff}2\pi\left(\mathcal{A}^{-}_{\overline{\overline{\mathcal{TI}_{I}}}}\right)},\mathcal{N}^{-}_{\overline{\overline{\mathcal{TI}_{R}}}}e^{\text{ff}2\pi\left(\mathcal{N}^{-}_{\overline{\overline{\mathcal{TI}_{I}}}}\right)}\right),$
and

$\min_{\text{ff}}\mathcal{A}_{\overline{\overline{\mathcal{TI}_{R-\text{ff}}}}}e^{\text{ff}2\pi\left(\min_{\text{ff}}\mathcal{A}_{\overline{\overline{\mathcal{TI}_{I-\text{ff}}}}}\right)},\min_{\text{ff}}\mathcal{N}_{\overline{\overline{\mathcal{TI}_{R-\text{ff}}}}}e^{\text{ff}2\pi\left(\min_{\text{ff}}\mathcal{N}_{\overline{\overline{\mathcal{TI}_{I-\text{ff}}}}}\right)}$), then

$$\overline{\overline{\mathcal{TI}_{CN-\text{ff}}}}^{-}\leq CSVNPMM\left(\overline{\overline{\mathcal{TI}_{CN-1}}},\overline{\overline{\mathcal{TI}_{CN-2}}},\ldots,\overline{\overline{\mathcal{TI}_{CN-\mu}}}\right)\leq\overline{\overline{\mathcal{TI}_{CN-\text{ff}}}}^{+} \tag{27}$$

Proof: Suppose, $\min_{\text{ff}}\mathcal{M}_{\overline{\overline{\mathcal{TI}_{R-\text{ff}}}}}\leq\mathcal{M}_{\overline{\overline{\mathcal{TI}_{R-\text{ff}}}}}$, thus, $\min_{\text{ff}}\mathcal{M}_{\overline{\overline{\mathcal{TI}_{R-\text{ff}}}}}\leq\mathcal{M}_{\overline{\overline{\mathcal{TI}_{R-\sigma(\text{ff})}}}}$, then

$$\left(1-\min_{\text{ff}}\mathcal{M}_{\overline{\overline{\mathcal{TI}_{R-\text{ff}}}}}\right)^{\mu\frac{\mathbb{H}_{\sigma(\text{ff})}}{\sum_{\text{ff}=1}^{\mu}\mathbb{H}_{\text{ff}}}}\geq\left(1-\mathcal{M}_{\overline{\overline{\mathcal{TI}_{R-\sigma(\text{ff})}}}}\right)^{\mu\frac{\mathbb{H}_{\sigma(\text{ff})}}{\sum_{\text{ff}=1}^{\mu}\mathbb{H}_{\text{ff}}}},$$

then,

$$1-\left(1-\min_{\text{ff}}\mathcal{M}_{\overline{\overline{\mathcal{TI}_{R-\text{ff}}}}}\right)^{\mu\frac{\mathbb{H}_{\sigma(\text{ff})}}{\sum_{\text{ff}=1}^{\mu}\mathbb{H}_{\text{ff}}}}\leq 1-\left(1-\mathcal{M}_{\overline{\overline{\mathcal{TI}_{R-\sigma(\text{ff})}}}}\right)^{\mu\frac{\mathbb{H}_{\sigma(\text{ff})}}{\sum_{\text{ff}=1}^{\mu}\mathbb{H}_{\text{ff}}}},$$

thus,

$$\left(\prod_{\mathfrak{f}\mathfrak{f}=1}^{\mu}\left(1-\left(1-\min_{\mathfrak{f}\mathfrak{f}}\mathcal{M}_{\overline{\overline{\mathcal{T}\mathcal{I}_{R-\mathfrak{f}\mathfrak{f}}}}}\right)^{\mu\frac{\mathbb{H}_{\sigma(\mathfrak{f}\mathfrak{f})}}{\sum_{\mathfrak{f}\mathfrak{f}=1}^{\mu}\mathbb{H}_{\mathfrak{f}\mathfrak{f}}}}\right)^{\overline{\overline{\mathcal{P}_{\mathfrak{f}\mathfrak{f}}}}}\right)$$

$$\leq\left(\prod_{\mathfrak{f}\mathfrak{f}=1}^{\mu}\left(1-\left(1-\mathcal{M}_{\overline{\overline{\mathcal{T}\mathcal{I}_{R-\sigma(\mathfrak{f}\mathfrak{f})}}}}\right)^{\mu\frac{\mathbb{H}_{\sigma(\mathfrak{f}\mathfrak{f})}}{\sum_{\mathfrak{f}\mathfrak{f}=1}^{\mu}\mathbb{H}_{\mathfrak{f}\mathfrak{f}}}}\right)^{\overline{\overline{\mathcal{P}_{\mathfrak{f}\mathfrak{f}}}}}\right),$$

then,

$$\prod_{\sigma\in\breve{\mathbb{S}}_{\mu}}\left(1-\prod_{\mathfrak{f}=1}^{\mu}\left(1-\left(1-\min_{\mathfrak{f}\mathfrak{f}}\mathcal{M}_{\overline{\overline{\mathcal{T}\mathcal{I}_{R-\mathfrak{f}\mathfrak{f}}}}}\right)^{\mu\frac{\mathbb{H}_{\sigma(\mathfrak{f}\mathfrak{f})}}{\sum_{\mathfrak{f}\mathfrak{f}=1}^{\mu}\mathbb{H}_{\mathfrak{f}\mathfrak{f}}}}\right)^{\overline{\overline{\mathcal{P}_{\mathfrak{f}\mathfrak{f}}}}}\right)$$

$$\geq\prod_{\sigma\in\breve{\mathbb{S}}_{\mu}}\left(1-\prod_{\mathfrak{f}\mathfrak{f}=1}^{\mu}\left(1-\left(1-\mathcal{M}_{\overline{\overline{\mathcal{T}\mathcal{I}_{R-\sigma(\mathfrak{f}\mathfrak{f})}}}}\right)^{\mu\frac{\mathbb{H}_{\sigma(\mathfrak{f}\mathfrak{f})}}{\sum_{\mathfrak{f}\mathfrak{f}=1}^{\mu}\mathbb{H}_{\mathfrak{f}\mathfrak{f}}}}\right)^{\overline{\overline{\mathcal{P}_{\mathfrak{f}\mathfrak{f}}}}}\right)$$

Therefore,

$$\left(1-\left(\prod_{\sigma\in\breve{\mathbb{S}}_{\mu}}\left(1-\prod_{\mathfrak{f}\mathfrak{f}=1}^{\mu}\left(1-\left(1-\min_{\mathfrak{f}\mathfrak{f}}\mathcal{M}_{\overline{\overline{\mathcal{T}\mathcal{I}_{R-\mathfrak{f}\mathfrak{f}}}}}\right)^{\mu\frac{\mathbb{H}_{\sigma(\mathfrak{f}\mathfrak{f})}}{\sum_{\mathfrak{f}\mathfrak{f}=1}^{\mu}\mathbb{H}_{\mathfrak{f}\mathfrak{f}}}}\right)^{\overline{\overline{\mathcal{P}_{\mathfrak{f}\mathfrak{f}}}}}\right)\right)^{\frac{1}{\mu!}}\right)^{\frac{1}{\sum_{\mathfrak{f}\mathfrak{f}=1}^{\mu}\overline{\overline{\mathcal{P}_{\mathfrak{f}\mathfrak{f}}}}}}$$

$$\leq\left(1-\left(\prod_{\sigma\in\breve{\mathbb{S}}_{\mu}}\left(1-\prod_{\mathfrak{f}\mathfrak{f}=1}^{\mu}\left(1-\left(1-\mathcal{M}_{\overline{\overline{\mathcal{T}\mathcal{I}_{R-\sigma(\mathfrak{f}\mathfrak{f})}}}}\right)^{\mu\frac{\mathbb{H}_{\sigma(\mathfrak{f}\mathfrak{f})}}{\sum_{\mathfrak{f}\mathfrak{f}=1}^{\mu}\mathbb{H}_{\mathfrak{f}\mathfrak{f}}}}\right)^{\overline{\overline{\mathcal{P}_{\mathfrak{f}\mathfrak{f}}}}}\right)\right)^{\frac{1}{\mu!}}\right)^{\frac{1}{\sum_{\mathfrak{f}\mathfrak{f}=1}^{\mu}\overline{\overline{\mathcal{P}_{\mathfrak{f}\mathfrak{f}}}}}}$$

implies that

$$\mathcal{M}^{-}_{\overline{\overline{\mathcal{T}\mathcal{I}_{R-\mathfrak{f}\mathfrak{f}}}}}\leq\left(1-\left(\prod_{\sigma\in\breve{\mathbb{S}}_{\mu}}\left(1-\prod_{\mathfrak{f}\mathfrak{f}=1}^{\mu}\left(1-\left(1-\mathcal{M}_{\overline{\overline{\mathcal{T}\mathcal{I}_{R-\sigma(\mathfrak{f}\mathfrak{f})}}}}\right)^{\mu\frac{\mathbb{H}_{\sigma(\mathfrak{f}\mathfrak{f})}}{\sum_{\mathfrak{f}\mathfrak{f}=1}^{\mu}\mathbb{H}_{\mathfrak{f}\mathfrak{f}}}}\right)^{\overline{\overline{\mathcal{P}_{\mathfrak{f}\mathfrak{f}}}}}\right)\right)^{\frac{1}{\mu!}}\right)^{\frac{1}{\sum_{\mathfrak{f}\mathfrak{f}=1}^{\mu}\overline{\overline{\mathcal{P}_{\mathfrak{f}\mathfrak{f}}}}}}$$

Similarly,

$$\mathcal{M}^{-}_{\overline{\overline{\mathcal{T}\mathcal{I}_{I}}}}\leq\left(1-\left(\prod_{\sigma\in\breve{\mathbb{S}}_{\mu}}\left(1-\prod_{\mathfrak{f}\mathfrak{f}=1}^{\mu}\left(1-\left(1-\mathcal{M}_{\overline{\overline{\mathcal{T}\mathcal{I}_{I-\sigma(\mathfrak{f}\mathfrak{f})}}}}\right)^{\mu\frac{\mathbb{H}_{\sigma(\mathfrak{f}\mathfrak{f})}}{\sum_{\mathfrak{f}\mathfrak{f}=1}^{\mu}\mathbb{H}_{\mathfrak{f}\mathfrak{f}}}}\right)^{\overline{\overline{\mathcal{P}_{\mathfrak{f}\mathfrak{f}}}}}\right)\right)^{\frac{1}{\mu!}}\right)^{\frac{1}{\sum_{\mathfrak{f}\mathfrak{f}=1}^{\mu}\overline{\overline{\mathcal{P}_{\mathfrak{f}\mathfrak{f}}}}}}$$

In the same way, we get

$$\mathcal{A}^{-}_{\overline{\overline{\mathcal{T}\mathcal{I}_{R}}}}\geq1-\left(1-\left(\prod_{\sigma\in\breve{\mathbb{S}}_{\mu}}\left(1-\prod_{\mathfrak{f}\mathfrak{f}=1}^{\mu}\left(1-\left(1-\mathcal{A}_{\overline{\overline{\mathcal{T}\mathcal{I}_{R-\sigma(\mathfrak{f}\mathfrak{f})}}}}^{\mu\frac{\mathbb{H}_{\sigma(\mathfrak{f}\mathfrak{f})}}{\sum_{\mathfrak{f}\mathfrak{f}=1}^{\mu}\mathbb{H}_{\mathfrak{f}\mathfrak{f}}}}\right)\right)^{\overline{\overline{\mathcal{P}_{\mathfrak{f}\mathfrak{f}}}}}\right)\right)^{\frac{1}{\mu!}}\right)^{\frac{1}{\sum_{\mathfrak{f}\mathfrak{f}=1}^{\mu}\overline{\overline{\mathcal{P}_{\mathfrak{f}\mathfrak{f}}}}}}$$

$$\mathcal{A}^{-}_{\overline{\overline{\mathcal{T}\mathcal{I}_{I}}}}\geq1-\left(1-\left(\prod_{\sigma\in\breve{\mathbb{S}}_{\mu}}\left(1-\prod_{\mathfrak{f}\mathfrak{f}=1}^{\mu}\left(1-\left(1-\mathcal{A}_{\overline{\overline{\mathcal{T}\mathcal{I}_{I-\sigma(\mathfrak{f}\mathfrak{f})}}}}^{\mu\frac{\mathbb{H}_{\sigma(\mathfrak{f}\mathfrak{f})}}{\sum_{\mathfrak{f}\mathfrak{f}=1}^{\mu}\mathbb{H}_{\mathfrak{f}\mathfrak{f}}}}\right)\right)^{\overline{\overline{\mathcal{P}_{\mathfrak{f}\mathfrak{f}}}}}\right)\right)^{\frac{1}{\mu!}}\right)^{\frac{1}{\sum_{\mathfrak{f}\mathfrak{f}=1}^{\mu}\overline{\overline{\mathcal{P}_{\mathfrak{f}\mathfrak{f}}}}}},$$

and

$$\mathcal{N}^{-}_{\overline{\overline{\mathcal{T}\mathcal{I}_{R}}}}\geq1-\left(1-\left(\prod_{\sigma\in\breve{\mathbb{S}}_{\mu}}\left(1-\prod_{\mathfrak{f}\mathfrak{f}=1}^{\mu}\left(1-\left(1-\mathcal{N}_{\overline{\overline{\mathcal{T}\mathcal{I}_{R-\sigma(\mathfrak{f}\mathfrak{f})}}}}^{\mu\frac{\mathbb{H}_{\sigma(\mathfrak{f}\mathfrak{f})}}{\sum_{\mathfrak{f}\mathfrak{f}=1}^{\mu}\mathbb{H}_{\mathfrak{f}\mathfrak{f}}}}\right)\right)^{\overline{\overline{\mathcal{P}_{\mathfrak{f}\mathfrak{f}}}}}\right)\right)^{\frac{1}{\mu!}}\right)^{\frac{1}{\sum_{\mathfrak{f}\mathfrak{f}=1}^{\mu}\overline{\overline{\mathcal{P}_{\mathfrak{f}\mathfrak{f}}}}}}$$

$$\mathcal{N}^{-}_{\overline{\overline{\mathcal{T}\mathcal{I}_{I}}}}\geq1-\left(1-\left(\prod_{\sigma\in\breve{\mathbb{S}}_{\mu}}\left(1-\prod_{\mathfrak{f}\mathfrak{f}=1}^{\mu}\left(1-\left(1-\mathcal{N}_{\overline{\overline{\mathcal{T}\mathcal{I}_{I-\sigma(\mathfrak{f}\mathfrak{f})}}}}^{\mu\frac{\mathbb{H}_{\sigma(\mathfrak{f}\mathfrak{f})}}{\sum_{\mathfrak{f}\mathfrak{f}=1}^{\mu}\mathbb{H}_{\mathfrak{f}\mathfrak{f}}}}\right)\right)^{\overline{\overline{\mathcal{P}_{\mathfrak{f}\mathfrak{f}}}}}\right)\right)^{\frac{1}{\mu!}}\right)^{\frac{1}{\sum_{\mathfrak{f}\mathfrak{f}=1}^{\mu}\overline{\overline{\mathcal{P}_{\mathfrak{f}\mathfrak{f}}}}}}.$$

Then,

$$\left(\mathcal{M}^{-}_{\overline{\overline{\mathcal{T}\mathcal{I}_{R}}}}e^{\mathfrak{f}2\pi\left(\mathcal{M}^{-}_{\overline{\overline{\mathcal{T}\mathcal{I}_{I}}}}\right)},\mathcal{A}^{-}_{\overline{\overline{\mathcal{T}\mathcal{I}_{R}}}}e^{\mathfrak{f}\mathfrak{f}2\pi\left(\mathcal{A}^{-}_{\overline{\overline{\mathcal{T}\mathcal{I}_{I}}}}\right)},\mathcal{N}^{-}_{\overline{\overline{\mathcal{T}\mathcal{I}_{R}}}}e^{\mathfrak{f}\mathfrak{f}2\pi\left(\mathcal{N}^{-}_{\overline{\overline{\mathcal{T}\mathcal{I}_{I}}}}\right)}\right)$$

$$\leq CSVNPMM\left(\overline{\overline{\mathcal{T}\mathcal{I}_{CN-1}}},\overline{\overline{\mathcal{T}\mathcal{I}_{CN-2}}},\ldots,\overline{\overline{\mathcal{T}\mathcal{I}_{CN-\mu}}}\right)$$

Similarly, we determine

$$CSVNPMM\left(\overline{\overline{\mathcal{T}\mathcal{I}_{CN-1}}},\overline{\overline{\mathcal{T}\mathcal{I}_{CN-2}}},\ldots,\overline{\overline{\mathcal{T}\mathcal{I}_{CN-\mu}}}\right)$$

$$\leq\left(\mathcal{M}^{+}_{\overline{\overline{\mathcal{T}\mathcal{I}_{R}}}}e^{\mathfrak{f}\mathfrak{f}2\pi\left(\mathcal{M}^{+}_{\overline{\overline{\mathcal{T}\mathcal{I}_{I}}}}\right)},\mathcal{A}^{+}_{\overline{\overline{\mathcal{T}\mathcal{I}_{R}}}}e^{\mathfrak{f}\mathfrak{f}2\pi\left(\mathcal{A}^{+}_{\overline{\overline{\mathcal{T}\mathcal{I}_{I}}}}\right)},\mathcal{N}^{+}_{\overline{\overline{\mathcal{T}\mathcal{I}_{R}}}}e^{\mathfrak{f}\mathfrak{f}2\pi\left(\mathcal{N}^{+}_{\overline{\overline{\mathcal{T}\mathcal{I}_{I}}}}\right)}\right)$$

From the above information, we determine

$$\overline{\overline{\mathcal{T}\mathcal{I}_{CN-\mathfrak{f}\mathfrak{f}}}}^{-}\leq CSVNPMM\left(\overline{\overline{\mathcal{T}\mathcal{I}_{CN-1}}},\overline{\overline{\mathcal{T}\mathcal{I}_{CN-2}}},\ldots,\overline{\overline{\mathcal{T}\mathcal{I}_{CN-\mu}}}\right)\leq\overline{\overline{\mathcal{T}\mathcal{I}_{CN-\mathfrak{f}\mathfrak{f}}}}^{+}$$

Theorem 4: Let $\overline{\overline{\mathcal{T}\mathcal{I}_{CN-\mathfrak{f}\mathfrak{f}}}}=\left(\mathcal{M}_{\overline{\overline{\mathcal{T}\mathcal{I}_{R-\mathfrak{f}\mathfrak{f}}}}}e^{\mathfrak{f}\mathfrak{f}2\pi\left(\mathcal{M}_{\overline{\overline{\mathcal{T}\mathcal{I}_{I-\mathfrak{f}\mathfrak{f}}}}}\right)},\mathcal{A}_{\overline{\overline{\mathcal{T}\mathcal{I}_{R-\mathfrak{f}\mathfrak{f}}}}}e^{\mathfrak{f}\mathfrak{f}2\pi\left(\mathcal{A}_{\overline{\overline{\mathcal{T}\mathcal{I}_{I-\mathfrak{f}\mathfrak{f}}}}}\right)},\mathcal{N}_{\overline{\overline{\mathcal{T}\mathcal{I}_{R-\mathfrak{f}\mathfrak{f}}}}}e^{\mathfrak{f}\mathfrak{f}2\pi\left(\mathcal{N}_{\overline{\overline{\mathcal{T}\mathcal{I}_{I-\mathfrak{f}\mathfrak{f}}}}}\right)}\right),\mathfrak{f}\mathfrak{f}=1,2,\ldots,\mu$, be any group of CSVNNs. If $\overline{\overline{\mathcal{T}\mathcal{I}_{CN-\mathfrak{f}\mathfrak{f}}}}=\overline{\overline{\mathcal{T}\mathcal{I}_{CN}}}$, then

$$CSVNPMM\left(\overline{\overline{\mathcal{T}\mathcal{I}_{CN-1}}},\overline{\overline{\mathcal{T}\mathcal{I}_{CN-2}}},\ldots,\overline{\overline{\mathcal{T}\mathcal{I}_{CN-\mu}}}\right)=\overline{\overline{\mathcal{T}\mathcal{I}_{CN}}}\quad(28)$$

Proof: Suppose $\overline{\overline{\mathcal{I}_{CN-\text{ff}}}} = \overline{\overline{\mathcal{TI}_{CN}}} = \left(\mathcal{M}_{\overline{\overline{\mathcal{TI}_R}}} e^{\text{ff}2\pi\left(\mathcal{M}_{\overline{\overline{\mathcal{TI}_I}}}\right)}, \mathcal{A}_{\overline{\overline{\mathcal{TI}_R}}} e^{\text{ff}2\pi\left(\mathcal{A}_{\overline{\overline{\mathcal{TI}_I}}}\right)}, \mathcal{N}_{\overline{\overline{\mathcal{TI}_R}}} e^{\text{ff}2\pi\left(\mathcal{N}_{\overline{\overline{\mathcal{TI}_I}}}\right)}\right)$,

then by using Equation (24), such that

$$CSVNPMM\left(\overline{\overline{\mathcal{TI}_{CN-1}}}, \overline{\overline{\mathcal{TI}_{CN-2}}}, \ldots, \overline{\overline{\mathcal{TI}_{CN-\mu}}}\right)$$

$$= \left(\frac{1}{\mu!} \oplus_{\sigma\in\mathbb{S}_\mu} \prod_{\text{ff}=1}^{\mu} \left(\mu \frac{\mathbb{H}_{\sigma(\text{ff})}}{\sum_{\text{ff}=1}^{\mu} \mathbb{H}_{\text{ff}}} \overline{\overline{\mathcal{TI}_{CN-\sigma(\text{ff})}}}\right)^{\overline{\overline{\mathcal{P}_{\text{ff}}}}}\right)^{\frac{1}{\sum_{\text{ff}=1}^{\mu} \overline{\overline{\mathcal{P}_{\text{ff}}}}}}$$

$$= \left(\frac{1}{\mu!} \oplus_{\sigma\in\mathbb{S}_\mu} \prod_{\text{ff}=1}^{\mu} \left(\mu \frac{\mathbb{H}}{\mathbb{H}} \overline{\overline{\mathcal{TI}_{CN}}}\right)^{\overline{\overline{\mathcal{P}_{\text{ff}}}}}\right)^{\frac{1}{\sum_{\text{ff}=1}^{\mu} \overline{\overline{\mathcal{P}_{\text{ff}}}}}} = \left(\frac{1}{\mu}\left(\mu \overline{\overline{\mathcal{TI}_{CN}}}\right)^{\overline{\overline{\mathcal{P}}}}\right)^{\frac{1}{\mathcal{P}}} = \overline{\overline{\mathcal{TI}_{CN}}}$$

Moreover, based on Equation (24), we elaborate different specific cases of the initiated works by using the value of parameters $\overline{\overline{\mathcal{P}}} = \left(\overline{\overline{\mathcal{P}_1}}, \overline{\overline{\mathcal{P}_2}}, \ldots, \overline{\overline{\mathcal{P}_\mu}}\right) \in R^\mu$.

1. For $\overline{\overline{\mathcal{P}}} = (\gamma, 0, \ldots, 0)$, Eq. (24) is

$$CSVNPMM\left(\overline{\overline{\mathcal{TI}_{CN-1}}}, \overline{\overline{\mathcal{TI}_{CN-2}}}, \ldots, \overline{\overline{\mathcal{TI}_{CN-\mu}}}\right)$$

$$= \left(\frac{1}{\mu!} \oplus_{\sigma\in\mathbb{S}_\mu} \left(\mu \frac{\mathbb{H}_{\sigma(1)}}{\sum_{\text{ff}=1}^{\mu} \mathbb{H}_{\text{ff}}} \overline{\overline{\mathcal{TI}_{CN-\sigma(1)}}}\right)\right)^{\frac{1}{\sum_{\text{ff}=1}^{\mu} \overline{\overline{\mathcal{P}_{\text{ff}}}}}}$$

$$= \oplus_{\text{ff}=1}^{\mu} \left(\frac{\mathbb{H}_{\text{ff}}}{\sum_{\text{ff}=1}^{\mu} \mathbb{H}_{\text{ff}}} \overline{\overline{\mathcal{TI}_{CN-\text{ff}}}}\right) \tag{29}$$

which is called CSVN prioritized weighted averaging (CSVNPWA) operator.

2. For $\overline{\overline{\mathcal{P}}} = (1, 0, \ldots, 0)$, Equation (24) is

$$CSVNPMM\left(\overline{\overline{\mathcal{TI}_{CN-1}}}, \overline{\overline{\mathcal{TI}_{CN-2}}}, \ldots, \overline{\overline{\mathcal{TI}_{CN-\mu}}}\right)$$

$$= \left(\frac{1}{\mu!} \oplus_{\sigma\in\mathbb{S}_\mu} \left(\mu \frac{\mathbb{H}_{\sigma(1)}}{\sum_{\text{ff}=1}^{\mu} \mathbb{H}_{\text{ff}}} \overline{\overline{\mathcal{TI}_{CN-\sigma(1)}}}\right)^{\gamma}\right)^{\frac{1}{\gamma}}$$

$$= \left(\frac{1}{\mu} \oplus_{\text{ff}=1}^{\mu} \left(\mu \frac{\mathbb{H}_{\text{ff}}}{\sum_{\text{ff}=1}^{\mu} \mathbb{H}_{\text{ff}}} \overline{\overline{\mathcal{TI}_{CN-\text{ff}}}}\right)^{\gamma}\right)^{\frac{1}{\gamma}} \tag{30}$$

which is called CSVN generalized hybrid prioritized weighted averaging (CSVNGHPWA) operator.

3. For $\overline{\overline{\mathcal{P}}} = (1, 1, 0, \ldots, 0)$, Equation (24) is

$$CSVNPMM\left(\overline{\overline{\mathcal{TI}_{CN-1}}}, \overline{\overline{\mathcal{TI}_{CN-2}}}, \ldots, \overline{\overline{\mathcal{TI}_{CN-\mu}}}\right)$$

$$= \left(\frac{1}{\mu!} \oplus_{\sigma\in\mathbb{S}_\mu} \left(\mu \frac{\mathbb{H}_{\sigma(1)}}{\sum_{\text{ff}=1}^{\mu} \mathbb{H}_{\text{ff}}} \overline{\overline{\mathcal{TI}_{CN-\sigma(1)}}}\right)\left(\mu \frac{\mathbb{H}_{\sigma(2)}}{\sum_{\text{ff}=1}^{\mu} \mathbb{H}_{\text{ff}}} \overline{\overline{\mathcal{TI}_{CN-\sigma(2)}}}\right)\right)^{\frac{1}{2}}$$

$$= \left(\frac{\mu^2}{\mu!} \oplus_{\substack{\text{ff}, j=1, \\ \text{ff}\neq j}}^{\mu} \left(\mu \frac{\mathbb{H}_{\text{ff}}}{\sum_{\text{ff}=1}^{\mu} \mathbb{H}_{\text{ff}}} \overline{\overline{\mathcal{TI}_{CN-\text{ff}}}}\right)\left(\mu \frac{\mathbb{H}_j}{\sum_{j=1}^{\mu} \mathbb{H}_j} \overline{\overline{\mathcal{TI}_{CN-j}}}\right)\right)^{\frac{1}{2}} \tag{31}$$

which is called CSVN prioritized BM (CSVNPBM) operator.

4. For $\overline{\overline{\mathcal{P}}} = \left(\overbrace{1, 1, \ldots, 1}^{t\ terms}, \overbrace{0, 0, \ldots, 0}^{\mu - tterms}\right)$, Equation (24) is

$$CSVNPMM\left(\overline{\overline{\mathcal{TI}_{CN-1}}}, \overline{\overline{\mathcal{TI}_{CN-2}}}, \ldots, \overline{\overline{\mathcal{TI}_{CN-\mu}}}\right)$$

$$= \left(\frac{2\mu^t t}{\mu!} \oplus_{1<\le\text{ff}_1\le\text{ff}_2\le\ldots\le\text{ff}_t<\mu} \otimes_{j=1}^{t} \left(\frac{\mathbb{H}_{\text{ff}_j}}{\sum_{s=1}^{\mu} \mathbb{H}_s} \overline{\overline{\mathcal{TI}_{CN-\text{ff}_j}}}\right)\right)^{\frac{1}{t}} \tag{32}$$

which is called CSVN prioritized MSM (CSVNPMSM) operator.

Definition 15: Let $\overline{\overline{\mathcal{TI}_{CN-\text{ff}}}} = \left(\mathcal{M}_{\overline{\overline{\mathcal{TI}_{R-\text{ff}}}}} e^{\text{ff}2\pi\left(\mathcal{M}_{\overline{\overline{\mathcal{TI}_{I-\text{ff}}}}}\right)}, \mathcal{A}_{\overline{\overline{\mathcal{TI}_{R-\text{ff}}}}} e^{\text{ff}2\pi\left(\mathcal{A}_{\overline{\overline{\mathcal{TI}_{I-\text{ff}}}}}\right)}, \mathcal{N}_{\overline{\overline{\mathcal{TI}_{R-\text{ff}}}}} e^{\text{ff}2\pi\left(\mathcal{N}_{\overline{\overline{\mathcal{TI}_{I-\text{ff}}}}}\right)}\right), \text{ff} = 1, 2, \ldots, \mu,$ be any group of CSVNNs. The CSVNPDMM operator is stated by

$$CSVNPDMM\left(\overline{\overline{\mathcal{TI}_{CN-1}}}, \overline{\overline{\mathcal{TI}_{CN-2}}}, \ldots, \overline{\overline{\mathcal{TI}_{CN-\mu}}}\right)$$

$$= \frac{1}{\sum_{\text{ff}=1}^{\mu} \overline{\overline{\mathcal{P}_{\text{ff}}}}} \left(\prod_{\sigma\in\mathbb{S}_\mu} \oplus_{\text{ff}=1}^{\mu} \left(\overline{\overline{\mathcal{P}_{\text{ff}}}}\, \overline{\overline{\mathcal{TI}_{CN-\sigma(\text{ff})}}}\right)^{\frac{\mathbb{H}_{\sigma(\text{ff})}}{\sum_{\text{ff}=1}^{\mu} \mathbb{H}_{\text{ff}}}}\right)^{\frac{1}{\mu!}} \tag{33}$$

Theorem 5: Let $\overline{\overline{\mathcal{TI}_{CN-\text{ff}}}} = \left(\mathcal{M}_{\overline{\overline{\mathcal{TI}_{R-\text{ff}}}}} e^{\text{ff}2\pi\left(\mathcal{M}_{\overline{\overline{\mathcal{TI}_{I-\text{ff}}}}}\right)}, \mathcal{A}_{\overline{\overline{\mathcal{TI}_{R-\text{ff}}}}} e^{\text{ff}2\pi\left(\mathcal{A}_{\overline{\overline{\mathcal{TI}_{I-\text{ff}}}}}\right)}, \mathcal{N}_{\overline{\overline{\mathcal{TI}_{R-\text{ff}}}}} e^{\text{ff}2\pi\left(\mathcal{N}_{\overline{\overline{\mathcal{TI}_{I-\text{ff}}}}}\right)}\right), \text{ff} = 1, 2, \ldots, \mu,$ be any group of CSVNNs. Then by using Equation (33), we determine

$$
\begin{aligned}
&CSVNPMM\left(\overline{\overline{\mathcal{TI}_{CN-1}}}, \overline{\overline{\mathcal{TI}_{CN-2}}}, \ldots, \overline{\overline{\mathcal{TI}_{CN-\mu}}}\right)\\
&= \begin{pmatrix}
1-\left(1-\left(\prod_{\sigma\in\ddot{\mathbb{S}}_\mu}\left(1-\prod_{\mathfrak{ff}=1}^{\mu}\left(1-\left(1-\mathcal{M}_{\overline{\overline{\mathcal{TI}_{R-\sigma(\mathfrak{ff})}}}}\right)^{\mu\frac{\mathbb{H}_{\sigma(\mathfrak{ff})}}{\sum_{\mathfrak{ff}=1}^{\mu}\mathbb{H}_{\mathfrak{ff}}}}\right)\right)^{\overline{\overline{\mathcal{P}_{\mathfrak{ff}}}}}\right)^{\frac{1}{\mu!}}\right)^{\frac{1}{\sum_{\mathfrak{ff}=1}^{\mu}\frac{1}{\overline{\overline{\mathcal{P}_{\mathfrak{ff}}}}}}}\\
e^{\mathfrak{ff}2\pi\left(1-\left(1-\left(\prod_{\sigma\in\ddot{\mathbb{S}}_\mu}\left(1-\prod_{\mathfrak{ff}=1}^{\mu}\left(1-\left(1-\mathcal{M}_{\overline{\overline{\mathcal{TI}_{I-\sigma(\mathfrak{ff})}}}}\right)^{\mu\frac{\mathbb{H}_{\sigma(\mathfrak{ff})}}{\sum_{\mathfrak{ff}=1}^{\mu}\mathbb{H}_{\mathfrak{ff}}}}\right)\right)^{\overline{\overline{\mathcal{P}_{\mathfrak{ff}}}}}\right)^{\frac{1}{\mu!}}\right)^{\frac{1}{\sum_{\mathfrak{ff}=1}^{\mu}\frac{1}{\overline{\overline{\mathcal{P}_{\mathfrak{ff}}}}}}}\right)},\\
\left(1-\left(\prod_{\sigma\in\ddot{\mathbb{S}}_\mu}\left(1-\prod_{\mathfrak{ff}=1}^{\mu}\left(1-\left(1-\mathcal{A}_{\overline{\overline{\mathcal{TI}_{R-\sigma(\mathfrak{ff})}}}}\right)^{\mu\frac{\mathbb{H}_{\sigma(\mathfrak{ff})}}{\sum_{\mathfrak{ff}=1}^{\mu}\mathbb{H}_{\mathfrak{ff}}}}\right)^{\overline{\overline{\mathcal{P}_{\mathfrak{ff}}}}}\right)\right)^{\frac{1}{\mu!}}\right)^{\frac{1}{\sum_{\mathfrak{ff}=1}^{\mu}\frac{1}{\overline{\overline{\mathcal{P}_{\mathfrak{ff}}}}}}}\\
e^{\mathfrak{ff}2\pi\left(1-\left(\prod_{\sigma\in\ddot{\mathbb{S}}_\mu}\left(1-\prod_{\mathfrak{ff}=1}^{\mu}\left(1-\left(1-\mathcal{A}_{\overline{\overline{\mathcal{TI}_{I-\sigma(\mathfrak{ff})}}}}\right)^{\mu\frac{\mathbb{H}_{\sigma(\mathfrak{ff})}}{\sum_{\mathfrak{ff}=1}^{\mu}\mathbb{H}_{\mathfrak{ff}}}}\right)^{\overline{\overline{\mathcal{P}_{\mathfrak{ff}}}}}\right)\right)^{\frac{1}{\mu!}}\right)^{\frac{1}{\sum_{\mathfrak{ff}=1}^{\mu}\frac{1}{\overline{\overline{\mathcal{P}_{\mathfrak{ff}}}}}}}},\\
\left(1-\left(\prod_{\sigma\in\ddot{\mathbb{S}}_\mu}\left(1-\prod_{\mathfrak{ff}=1}^{\mu}\left(1-\left(1-\mathcal{N}_{\overline{\overline{\mathcal{TI}_{R-\sigma(\mathfrak{ff})}}}}\right)^{\mu\frac{\mathbb{H}_{\sigma(\mathfrak{ff})}}{\sum_{\mathfrak{ff}=1}^{\mu}\mathbb{H}_{\mathfrak{ff}}}}\right)^{\overline{\overline{\mathcal{P}_{\mathfrak{ff}}}}}\right)\right)^{\frac{1}{\mu!}}\right)^{\frac{1}{\sum_{\mathfrak{ff}=1}^{\mu}\frac{1}{\overline{\overline{\mathcal{P}_{\mathfrak{ff}}}}}}}\\
e^{\mathfrak{ff}2\pi\left(1-\left(\prod_{\sigma\in\ddot{\mathbb{S}}_\mu}\left(1-\prod_{\mathfrak{ff}=1}^{\mu}\left(1-\left(1-\mathcal{N}_{\overline{\overline{\mathcal{TI}_{I-\sigma(\mathfrak{ff})}}}}\right)^{\mu\frac{\mathbb{H}_{\sigma(\mathfrak{ff})}}{\sum_{\mathfrak{ff}=1}^{\mu}\mathbb{H}_{\mathfrak{ff}}}}\right)^{\overline{\overline{\mathcal{P}_{\mathfrak{ff}}}}}\right)\right)^{\frac{1}{\mu!}}\right)^{\frac{1}{\sum_{\mathfrak{ff}=1}^{\mu}\frac{1}{\overline{\overline{\mathcal{P}_{\mathfrak{ff}}}}}}}}
\end{pmatrix}
\end{aligned}
\tag{34}
$$

Proof: Omitted.

Moreover, by using the presented operators, we elaborate on the principle of monotonicity, boundedness, and idempotency.

Theorem 6: Let $\overline{\overline{\mathcal{TI}_{CN-\mathfrak{ff}}}} = \left(\mathcal{M}_{\overline{\overline{\mathcal{TI}_{R-\mathfrak{ff}}}}} e^{\mathfrak{ff}2\pi\left(\mathcal{M}_{\overline{\overline{\mathcal{TI}_{I-\mathfrak{ff}}}}}\right)}, \mathcal{A}_{\overline{\overline{\mathcal{TI}_{R-\mathfrak{ff}}}}} e^{\mathfrak{ff}2\pi\left(\mathcal{A}_{\overline{\overline{\mathcal{TI}_{I-\mathfrak{ff}}}}}\right)}, \mathcal{N}_{\overline{\overline{\mathcal{TI}_{R-\mathfrak{ff}}}}} e^{\mathfrak{ff}2\pi\left(\mathcal{N}_{\overline{\overline{\mathcal{TI}_{I-\mathfrak{ff}}}}}\right)}\right), \mathfrak{ff} = 1, 2, \ldots,$ be any group of CSVNNs. If $\overline{\overline{\mathcal{TI}_{CN-\mathfrak{ff}}}} \le \overline{\overline{\mathcal{TI}_{CN-\mathfrak{ff}}}}^*$, i.e., $\mathcal{M}_{\overline{\overline{\mathcal{TI}_{R-\mathfrak{ff}}}}} \le \mathcal{M}^*_{\overline{\overline{\mathcal{TI}_{R-\mathfrak{ff}}}}}, \mathcal{A}_{\overline{\overline{\mathcal{TI}_{R-\mathfrak{ff}}}}} \ge \mathcal{A}^*_{\overline{\overline{\mathcal{TI}_{R-\mathfrak{ff}}}}}, \mathcal{N}_{\overline{\overline{\mathcal{TI}_{R-\mathfrak{ff}}}}} \ge \mathcal{N}^*_{\overline{\overline{\mathcal{TI}_{R-\mathfrak{ff}}}}}$ and $\mathcal{M}_{\overline{\overline{\mathcal{TI}_{I-\mathfrak{ff}}}}} \le \mathcal{M}^*_{\overline{\overline{\mathcal{TI}_{I-\mathfrak{ff}}}}}, \mathcal{A}_{\overline{\overline{\mathcal{TI}_{I-\mathfrak{ff}}}}} \ge \mathcal{A}^*_{\overline{\overline{\mathcal{TI}_{I-\mathfrak{ff}}}}}, \mathcal{N}_{\overline{\overline{\mathcal{TI}_{I-\mathfrak{ff}}}}} \ge \mathcal{N}^*_{\overline{\overline{\mathcal{TI}_{I-\mathfrak{ff}}}}}$, then

$$
\begin{aligned}
&CSVNPDMM\left(\overline{\overline{\mathcal{TI}_{CN-1}}}, \overline{\overline{\mathcal{TI}_{CN-2}}}, \ldots, \overline{\overline{\mathcal{TI}_{CN-\mu}}}\right)\\
&\quad\le CSVNPDMM\left(\overline{\overline{\mathcal{TI}_{CN-1}}}^*, \overline{\overline{\mathcal{TI}_{CN-2}}}^*, \ldots, \overline{\overline{\mathcal{TI}_{CN-\mu}}}^*\right)
\end{aligned}
\tag{35}
$$

Proof: Omitted.

Theorem 7: Let $\overline{\overline{\mathcal{TI}_{CN-\mathfrak{ff}}}} = \left(\mathcal{M}_{\overline{\overline{\mathcal{TI}_{R-\mathfrak{ff}}}}} e^{\mathfrak{ff}2\pi\left(\mathcal{M}_{\overline{\overline{\mathcal{TI}_{I-\mathfrak{ff}}}}}\right)}, \mathcal{A}_{\overline{\overline{\mathcal{TI}_{R-\mathfrak{ff}}}}} e^{\mathfrak{ff}2\pi\left(\mathcal{A}_{\overline{\overline{\mathcal{TI}_{I-\mathfrak{ff}}}}}\right)}, \mathcal{N}_{\overline{\overline{\mathcal{TI}_{R-\mathfrak{ff}}}}} e^{\mathfrak{ff}2\pi\left(\mathcal{N}_{\overline{\overline{\mathcal{TI}_{I-\mathfrak{ff}}}}}\right)}\right), \mathfrak{ff} = 1, 2, \ldots, \mu,$ be any group of CSVNNs. If $\overline{\overline{\mathcal{TI}_{CN-\mathfrak{ff}}}}^- = \Big(\min_{\mathfrak{ff}} \mathcal{M}_{\overline{\overline{\mathcal{TI}_{R-\mathfrak{ff}}}}}\, e^{\mathfrak{ff}2\pi\left(\min_{\mathfrak{ff}} \mathcal{M}_{\overline{\overline{\mathcal{TI}_{I-\mathfrak{ff}}}}}\right)}, \max_{\mathfrak{ff}} \mathcal{A}_{\overline{\overline{\mathcal{TI}_{R-\mathfrak{ff}}}}}\, e^{\mathfrak{ff}2\pi\left(\max_{\mathfrak{ff}} \mathcal{A}_{\overline{\overline{\mathcal{TI}_{I-\mathfrak{ff}}}}}\right)}, \max_{\mathfrak{ff}} \mathcal{N}_{\overline{\overline{\mathcal{TI}_{R-\mathfrak{ff}}}}}\, e^{\mathfrak{ff}2\pi\left(\max_{\mathfrak{ff}} \mathcal{N}_{\overline{\overline{\mathcal{TI}_{I-\mathfrak{ff}}}}}\right)}\Big) = \left(\mathcal{M}^-_{\overline{\overline{\mathcal{TI}_R}}}\, e^{\mathfrak{ff}2\pi\left(\mathcal{M}^-_{\overline{\overline{\mathcal{TI}_I}}}\right)}, \mathcal{A}^-_{\overline{\overline{\mathcal{TI}_R}}}\, e^{\mathfrak{ff}2\pi\left(\mathcal{A}^-_{\overline{\overline{\mathcal{TI}_I}}}\right)}, \mathcal{N}^-_{\overline{\overline{\mathcal{TI}_R}}}\, e^{\mathfrak{ff}2\pi\left(\mathcal{N}^-_{\overline{\overline{\mathcal{TI}_I}}}\right)}\right)$, and $\overline{\overline{\mathcal{TI}_{CN-\mathfrak{ff}}}}^+ = \left(\mathcal{M}^+_{\overline{\overline{\mathcal{TI}_R}}} e^{\mathfrak{ff}2\pi\left(\mathcal{M}^+_{\overline{\overline{\mathcal{TI}_I}}}\right)}, \mathcal{A}^+_{\overline{\overline{\mathcal{TI}_R}}} e^{\mathfrak{ff}2\pi\left(\mathcal{A}^+_{\overline{\overline{\mathcal{TI}_I}}}\right)}, \mathcal{N}^+_{\overline{\overline{\mathcal{TI}_R}}} e^{\mathfrak{ff}2\pi\left(\mathcal{N}^+_{\overline{\overline{\mathcal{TI}_I}}}\right)}\right) = \left(\max_{\mathfrak{ff}} \mathcal{M}_{\overline{\overline{\mathcal{TI}_{R-\mathfrak{ff}}}}} e^{\mathfrak{ff}2\pi\left(\max_{\mathfrak{ff}} \mathcal{M}_{\overline{\overline{\mathcal{TI}_{I-\mathfrak{ff}}}}}\right)}, \min_{\mathfrak{ff}} \mathcal{A}_{\overline{\overline{\mathcal{TI}_{R-\mathfrak{ff}}}}} e^{\mathfrak{ff}2\pi\left(\min_{\mathfrak{ff}} \mathcal{A}_{\overline{\overline{\mathcal{TI}_{I-\mathfrak{ff}}}}}\right)}, \min_{\mathfrak{ff}} \mathcal{N}_{\overline{\overline{\mathcal{TI}_{R-\mathfrak{ff}}}}} e^{\mathfrak{ff}2\pi\left(\min_{\mathfrak{ff}} \mathcal{N}_{\overline{\overline{\mathcal{TI}_{I-\mathfrak{ff}}}}}\right)}\right)$, then

$$\overline{\overline{\mathcal{TI}_{CN-\mathfrak{ff}}}}^{-} \leq CSVNPDMM\left(\overline{\overline{\mathcal{TI}_{CN-1}}}, \overline{\overline{\mathcal{TI}_{CN-2}}}, \ldots, \overline{\overline{\mathcal{TI}_{CN-\mu}}}\right) \leq \overline{\overline{\mathcal{TI}_{CN-\mathfrak{ff}}}}^{+} \quad (36)$$

Proof: Omitted.

Theorem 8: Let $\overline{\overline{\mathcal{TI}_{C-\mathfrak{ff}}}} = \left(\mathcal{M}_{\overline{\overline{\mathcal{TI}_{R-\mathfrak{ff}}}}} e^{\mathfrak{ff}2\pi\left(\mathcal{M}_{\overline{\overline{\mathcal{TI}_{I-\mathfrak{ff}}}}}\right)}, \mathcal{A}_{\overline{\overline{\mathcal{TI}_{R-\mathfrak{ff}}}}} e^{\mathfrak{ff}2\pi\left(\mathcal{A}_{\overline{\overline{\mathcal{TI}_{I-\mathfrak{ff}}}}}\right)}, \mathcal{N}_{\overline{\overline{\mathcal{TI}_{R-\mathfrak{ff}}}}} e^{\mathfrak{ff}2\pi\left(\mathcal{N}_{\overline{\overline{\mathcal{TI}_{I-\mathfrak{ff}}}}}\right)}\right), \mathfrak{ff} = 1, 2, \ldots, \mu,$ be any group of CSVNNs. If $\overline{\overline{\mathcal{TI}_{CN-\mathfrak{ff}}}} = \overline{\overline{\mathcal{TI}_{CN}}}$, then

$$CSVNPDMM\left(\overline{\overline{\mathcal{TI}_{CN-1}}}, \overline{\overline{\mathcal{TI}_{CN-2}}}, \ldots, \overline{\overline{\mathcal{TI}_{CN-\mu}}}\right) = \overline{\overline{\mathcal{TI}_{CN}}} \quad (37)$$

Proof: Omitted.

5. MADM Method Based on CSVNSs

In this analysis, we elaborate an MADM technique by using the investigated works under the CSVNSs to resolve a realistic DM dilemma. The genuine life example is illustrated below based on the initiated operators under the CSVNSs.

5.1. Decision-making techniques

To handle inconsistent and ambiguous data in genuine life dilemmas, we take a group of alternatives and their attributes in the shape of $\overline{\overline{\mathcal{TI}_{AT}}} = \left\{\overline{\overline{\mathcal{TI}_{AT-1}}}, \overline{\overline{\mathcal{TI}_{AT-2}}}, \ldots, \overline{\overline{\mathcal{TI}_{AT-m}}}\right\}$ and $\overline{\overline{\mathcal{TI}_{AL}}} = \left\{\overline{\overline{\mathcal{TI}_{AL-1}}}, \overline{\overline{\mathcal{TI}_{AL-2}}}, \ldots, \overline{\overline{\mathcal{TI}_{AL-\mu}}}\right\}$. The experts provide data in the shape of $\overline{\overline{\mathcal{TI}_{CN-\mathfrak{ff}}}} = \left(\mathcal{M}_{\overline{\overline{\mathcal{TI}_{R-\mathfrak{ff}}}}} e^{\mathfrak{ff}2\pi\left(\mathcal{M}_{\overline{\overline{\mathcal{TI}_{I-\mathfrak{ff}}}}}\right)}, \mathcal{A}_{\overline{\overline{\mathcal{TI}_{R-\mathfrak{ff}}}}} e^{\mathfrak{ff}2\pi\left(\mathcal{A}_{\overline{\overline{\mathcal{TI}_{I-\mathfrak{ff}}}}}\right)}, \mathcal{N}_{\overline{\overline{\mathcal{TI}_{R-\mathfrak{ff}}}}} e^{\mathfrak{ff}2\pi\left(\mathcal{N}_{\overline{\overline{\mathcal{TI}_{I-\mathfrak{ff}}}}}\right)}\right), \mathfrak{ff} = 1, 2, \ldots, \mu,$ that stated the CSVNSs, where $\mathcal{M}_{\overline{\overline{\mathcal{TI}_C}}}\left(\overline{\overline{\Xi}}\right) = \mathcal{M}_{\overline{\overline{\mathcal{TI}_R}}}\left(\overline{\overline{\Xi}}\right) e^{\mathfrak{ff}2\pi\left(\mathcal{M}_{\overline{\overline{\mathcal{TI}_I}}}\left(\overline{\overline{\Xi}}\right)\right)}, \mathcal{A}_{\overline{\overline{\mathcal{TI}_C}}}\left(\overline{\overline{\Xi}}\right) = \mathcal{A}_{\overline{\overline{\mathcal{TI}_R}}}\left(\overline{\overline{\Xi}}\right) e^{\mathfrak{ff}2\pi\left(\mathcal{A}_{\overline{\overline{\mathcal{TI}_I}}}\left(\overline{\overline{\Xi}}\right)\right)},$ and $\mathcal{N}_{\overline{\overline{\mathcal{TI}_C}}}\left(\overline{\overline{\Xi}}\right) = \mathcal{N}_{\overline{\overline{\mathcal{TI}_R}}}\left(\overline{\overline{\Xi}}\right) e^{\mathfrak{ff}2\pi\left(\mathcal{N}_{\overline{\overline{\mathcal{TI}_I}}}\left(\overline{\overline{\Xi}}\right)\right)}$ with $0 \leq \mathcal{M}_{\overline{\overline{\mathcal{TI}_R}}} + \mathcal{A}_{\overline{\overline{\mathcal{TI}_R}}} + \mathcal{N}_{\overline{\overline{\mathcal{TI}_R}}} \leq 3$ and $0 \leq \mathcal{M}_{\overline{\overline{\mathcal{TI}_I}}} + \mathcal{A}_{\overline{\overline{\mathcal{TI}_I}}} + \mathcal{N}_{\overline{\overline{\mathcal{TI}_I}}} \leq 3$. Based on the study, we elaborate a DM procedure, whose stages are illustrated below.

Stage 1: Initiated the matrix in the shape of CSVNSs. If the data are in the shape of benefits, then it is ok, but if the data are in the shape of cost types, then the matrix is normalized by using Equation (38), we have

$$\overline{\overline{\mathcal{TI}_{CN}}} = \begin{cases} \left(\mathcal{M}_{\overline{\overline{\mathcal{TI}_{R-\mathfrak{ff}}}}} e^{\mathfrak{ff}2\pi\left(\mathcal{M}_{\overline{\overline{\mathcal{TI}_{I-\mathfrak{ff}}}}}\right)}, \mathcal{A}_{\overline{\overline{\mathcal{TI}_{R-\mathfrak{ff}}}}} e^{\mathfrak{ff}2\pi\left(\mathcal{A}_{\overline{\overline{\mathcal{TI}_{I-\mathfrak{ff}}}}}\right)}, \mathcal{N}_{\overline{\overline{\mathcal{TI}_{R-\mathfrak{ff}}}}} e^{\mathfrak{ff}2\pi\left(\mathcal{N}_{\overline{\overline{\mathcal{TI}_{I-\mathfrak{ff}}}}}\right)}\right) & \textit{for benefit types} \\ \left(\mathcal{N}_{\overline{\overline{\mathcal{TI}_{R-\mathfrak{ff}}}}} e^{\mathfrak{ff}2\pi\left(\mathcal{N}_{\overline{\overline{\mathcal{TI}_{I-\mathfrak{ff}}}}}\right)}, \mathcal{A}_{\overline{\overline{\mathcal{TI}_{R-\mathfrak{ff}}}}} e^{\mathfrak{ff}2\pi\left(\mathcal{A}_{\overline{\overline{\mathcal{TI}_{I-\mathfrak{ff}}}}}\right)}, \mathcal{M}_{\overline{\overline{\mathcal{TI}_{R-\mathfrak{ff}}}}} e^{\mathfrak{ff}2\pi\left(\mathcal{M}_{\overline{\overline{\mathcal{TI}_{I-\mathfrak{ff}}}}}\right)}\right) & \textit{for cost types} \end{cases} \quad (38)$$

Stage 2: Find the $\mathbb{H}_{\mathfrak{ff}j}, \mathfrak{ff} = 1, 2, \ldots, m,$, by using Equation (39), such that

$$\mathbb{H}_{\mathfrak{ff}j} = \begin{cases} 1 & j = 1 \\ \prod_{k=1}^{j-1} \overline{\overline{\mathcal{S}}}\left(\overline{\overline{\mathcal{TI}_{CN-k}}}\right) & j = 1, 2, \ldots, \mu \end{cases} \quad (39)$$

Stage 3: Under the principle of CSVNPMM operator and CSVNPDMM operator, we determine the aggregated values of the original matrix.

Stage 4: Investigated the SV of the accumulated values.

Stage 5: Determining the ranking values of the SV is to examine the best optimal.

5.2. Illustrated example

The information of this numerical is taken from Garg and Rani (2019). Let we choose the five alternatives such that Zensar Tech ($\overline{\overline{\mathcal{TI}_{AT-1}}}$), NIIT Tech ($\overline{\overline{\mathcal{TI}_{AT-2}}}$), HCL Tech ($\overline{\overline{\mathcal{TI}_{AT-3}}}$), Hexaware Tech ($\overline{\overline{\mathcal{TI}_{AT-4}}}$), and Tech Mahindra ($\overline{\overline{\mathcal{TI}_{AT-5}}}$), and the determination is held based on the various models, in particular, innovation skills ($\overline{\overline{\mathcal{TI}_{AL-1}}}$), administration quality ($\overline{\overline{\mathcal{TI}_{AL-2}}}$) project executives ($\overline{\overline{\mathcal{TI}_{AL-3}}}$) and industry experience ($\overline{\overline{\mathcal{TI}_{AL-4}}}$). Under the above consideration, we elaborated a DM procedure, whose stages are illustrated below.

Stage 1: Initiated the matrix in the shape of CSVNSs as in Table 1. We know that Table 1 covers all the benefit types of data, so no need to be normalized.

Table 1
Original decision matrix

	$\overline{\overline{\mathfrak{C}_{AL-1}}}$	$\overline{\overline{\mathfrak{C}_{AL-2}}}$
$\overline{\overline{\mathfrak{C}_{AT-1}}}$	$\left(0.9e^{\mathfrak{ff}2\pi(0.8)}, 0.8e^{\mathfrak{ff}2\pi(0.7)}, 0.7e^{\mathfrak{ff}2\pi(0.6)}\right)$	$\left(0.91e^{\mathfrak{ff}2\pi(0.81)}, 0.81e^{\mathfrak{ff}2\pi(0.71)}, 0.71e^{\mathfrak{ff}2\pi(0.61)}\right)$
$\overline{\overline{\mathfrak{C}_{AT-2}}}$	$\left(0.8e^{\mathfrak{ff}2\pi(0.6)}, 0.5e^{\mathfrak{ff}2\pi(0.2)}, 0.7e^{\mathfrak{ff}2\pi(0.4)}\right)$	$\left(0.81e^{\mathfrak{ff}2\pi(0.61)}, 0.51e^{\mathfrak{ff}2\pi(0.21)}, 0.71e^{\mathfrak{ff}2\pi(0.41)}\right)$
$\overline{\overline{\mathfrak{C}_{AT-3}}}$	$\left(0.9e^{\mathfrak{ff}2\pi(0.8)}, 0.1e^{\mathfrak{ff}2\pi(0.2)}, 0.4e^{\mathfrak{ff}2\pi(0.3)}\right)$	$\left(0.91e^{\mathfrak{ff}2\pi(0.81)}, 0.11e^{\mathfrak{ff}2\pi(0.21)}, 0.41e^{\mathfrak{ff}2\pi(0.31)}\right)$
$\overline{\overline{\mathfrak{C}_{AT-4}}}$	$\left(0.7e^{\mathfrak{ff}2\pi(0.6)}, 0.5e^{\mathfrak{ff}2\pi(0.3)}, 0.4e^{\mathfrak{ff}2\pi(0.4)}\right)$	$\left(0.71e^{\mathfrak{ff}2\pi(0.61)}, 0.51e^{\mathfrak{ff}2\pi(0.31)}, 0.41e^{\mathfrak{ff}2\pi(0.41)}\right)$
$\overline{\overline{\mathfrak{C}_{AT-5}}}$	$\left(0.7e^{\mathfrak{ff}2\pi(0.5)}, 0.5e^{\mathfrak{ff}2\pi(0.4)}, 0.6e^{\mathfrak{ff}2\pi(0.4)}\right)$	$\left(0.71e^{\mathfrak{ff}2\pi(0.51)}, 0.51e^{\mathfrak{ff}2\pi(0.41)}, 0.61e^{\mathfrak{ff}2\pi(0.41)}\right)$
	$\overline{\overline{\mathcal{TI}_{AL-3}}}$	$\overline{\overline{\mathcal{TI}_{AL-4}}}$
$\overline{\overline{\mathfrak{C}_{AT-1}}}$	$\left(0.92e^{\mathfrak{ff}2\pi(0.82)}, 0.82e^{\mathfrak{ff}2\pi(0.72)}, 0.72e^{\mathfrak{ff}2\pi(0.62)}\right)$	$\left(0.93e^{\mathfrak{ff}2\pi(0.83)}, 0.83e^{\mathfrak{ff}2\pi(0.73)}, 0.73e^{\mathfrak{ff}2\pi(0.63)}\right)$
$\overline{\overline{\mathfrak{C}_{AT-2}}}$	$\left(0.82e^{\mathfrak{ff}2\pi(0.62)}, 0.52e^{\mathfrak{ff}2\pi(0.22)}, 0.72e^{\mathfrak{ff}2\pi(0.42)}\right)$	$\left(0.83e^{\mathfrak{ff}2\pi(0.63)}, 0.53e^{\mathfrak{ff}2\pi(0.23)}, 0.73e^{\mathfrak{ff}2\pi(0.43)}\right)$
$\overline{\overline{\mathfrak{C}_{AT-3}}}$	$\left(0.92e^{\mathfrak{ff}2\pi(0.82)}, 0.12e^{\mathfrak{ff}2\pi(0.22)}, 0.42e^{\mathfrak{ff}2\pi(0.32)}\right)$	$\left(0.93e^{\mathfrak{ff}2\pi(0.83)}, 0.13e^{\mathfrak{ff}2\pi(0.23)}, 0.43e^{\mathfrak{ff}2\pi(0.33)}\right)$
$\overline{\overline{\mathfrak{C}_{AT-4}}}$	$\left(0.72e^{\mathfrak{ff}2\pi(0.62)}, 0.52e^{\mathfrak{ff}2\pi(0.32)}, 0.42e^{\mathfrak{ff}2\pi(0.42)}\right)$	$\left(0.73e^{\mathfrak{ff}2\pi(0.63)}, 0.53e^{\mathfrak{ff}2\pi(0.33)}, 0.43e^{\mathfrak{ff}2\pi(0.43)}\right)$
$\overline{\overline{\mathfrak{C}_{AT-5}}}$	$\left(0.72e^{\mathfrak{ff}2\pi(0.52)}, 0.52e^{\mathfrak{ff}2\pi(0.42)}, 0.62e^{\mathfrak{ff}2\pi(0.42)}\right)$	$\left(0.73e^{\mathfrak{ff}2\pi(0.53)}, 0.53e^{\mathfrak{ff}2\pi(0.43)}, 0.63^{\mathfrak{ff}2\pi(0.43)}\right)$

Stage 2: Find the $\mathbb{H}_{\mathfrak{ff}j}, \mathfrak{ff} = 1, 2, \ldots, m,$, by using Equation (39), such that

$$\mathbb{H}_{\mathfrak{ff}j} = \begin{bmatrix} 1 & 0.3666678 & 0.136889 & 0.052018 \\ 1 & 0.133333 & 0.018667 & 0.002738 \\ 1 & 0.233333 & 0.052889 & 0.011636 \\ 1 & 0.1 & 0.010667 & 0.001209 \\ 1 & 0.233333 & 0.056 & 0.013813 \end{bmatrix}$$

Stage 3: Under the principle of CSVNPMM operator and CSVNPDMM operator, we determine the aggregated values of the original matrix that are discussed in Table 2

Table 2
Expression of the aggregated values

	CSVNPMM operator	CSVNPDMM operator
$\overline{\overline{\mathfrak{C}}}_{AT-1}$	$\left(0.3480e^{\mathfrak{ff}2\pi(0.3236)}, 0.6043e^{\mathfrak{ff}2\pi(0.6334)}, 0.6334e^{\mathfrak{ff}2\pi(0.6577)}\right)$	$\left(0.5628e^{\mathfrak{ff}2\pi(0.6043)}, 0.3236e^{\mathfrak{ff}2\pi(0.3061)}, 0.3061e^{\mathfrak{ff}2\pi(0.2906)}\right)$
$\overline{\overline{\mathfrak{C}}}_{AT-2}$	$\left(0.2402e^{\mathfrak{ff}2\pi(0.2203)}, 0.6808e^{\mathfrak{ff}2\pi(0.7517)}, 0.6343e^{\mathfrak{ff}2\pi(0.7027)}\right)$	$\left(0.6054e^{\mathfrak{ff}2\pi(0.6585)}, 0.2111e^{\mathfrak{ff}2\pi(0.1766)}, 0.2297e^{\mathfrak{ff}2\pi(0.2013)}\right)$
$\overline{\overline{\mathfrak{C}}}_{AT-3}$	$\left(0.2965e^{\mathfrak{ff}2\pi(0.2785)}, 0.7858e^{\mathfrak{ff}2\pi(0.7513)}, 0.7024e^{\mathfrak{ff}2\pi(0.7253)}\right)$	$\left(0.5640e^{\mathfrak{ff}2\pi(0.6050)}, 0.1770e^{\mathfrak{ff}2\pi(0.1999)}, 0.2299e^{\mathfrak{ff}2\pi(0.2163)}\right)$
$\overline{\overline{\mathfrak{C}}}_{AT-4}$	$\left(0.2125e^{\mathfrak{ff}2\pi(0.2041)}, 0.6809e^{\mathfrak{ff}2\pi(0.7257)}, 0.7028e^{\mathfrak{ff}2\pi(0.7028)}\right)$	$\left(0.6344e^{\mathfrak{ff}2\pi(0.6586)}, 0.1958e^{\mathfrak{ff}2\pi(0.1771)}, 0.1870e^{\mathfrak{ff}2\pi(0.1870)}\right)$
$\overline{\overline{\mathfrak{C}}}_{AT-5}$	$\left(0.2689e^{\mathfrak{ff}2\pi(0.2450)}, 0.6805e^{\mathfrak{ff}2\pi(0.7024)}, 0.6582e^{\mathfrak{ff}2\pi(0.7024)}\right)$	$\left(0.6339e^{\mathfrak{ff}2\pi(0.6805)}, 0.2450e^{\mathfrak{ff}2\pi(0.2326)}, 0.2568e^{\mathfrak{ff}2\pi(0.2326)}\right)$

for $\overline{\overline{\mathcal{P}}} = (0.1, 0.1, 0.1, 0.1)$.

Table 3
Expression of the SV of the accumulated values

	CSVNPMM operator	CSVNPDMM operator
$\overline{\overline{\mathfrak{C}}}_{AT-1}$	0.6190	0.0197
$\overline{\overline{\mathfrak{C}}}_{AT-2}$	0.7697	0.1483
$\overline{\overline{\mathfrak{C}}}_{AT-3}$	0.7966	0.1152
$\overline{\overline{\mathfrak{C}}}_{AT-4}$	0.7985	0.1819
$\overline{\overline{\mathfrak{C}}}_{AT-5}$	0.7432	0.1158

Stage 4: Investigated the SV of the accumulated values that are illustrated in Table 3.

Stage 5: Determining the ranking values of the score values is to examine the best optimal, which are discussed below.

For CSVNPMM operator:

$$\overline{\overline{\mathcal{TI}}}_{AT-4} \geq \overline{\overline{\mathcal{TI}}}_{AT-3} \geq \overline{\overline{\mathcal{TI}}}_{AT-2} \geq \overline{\overline{\mathcal{TI}}}_{AT-5} \geq \overline{\overline{\mathcal{TI}}}_{AT-1}$$

For CSVNPMM operator:

$$\overline{\overline{\mathcal{TI}}}_{AT-4} \geq \overline{\overline{\mathcal{TI}}}_{AT-2} \geq \overline{\overline{\mathcal{TI}}}_{AT-5} \geq \overline{\overline{\mathcal{TI}}}_{A-3} \geq \overline{\overline{\mathcal{TI}}}_{AT-1}$$

The best optimal is $\overline{\overline{\mathcal{TI}}}_{AT-4}$. Under the presented works, both operators are given the same results. Moreover, to determine the consistency and flexibility of the initiated works based on CSVNSs, we compare the presented operators with certain existing operators in the next section.

5.3. Sensitive analysis

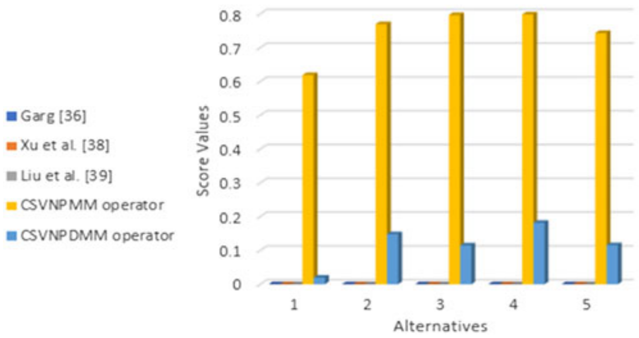
To handle ambiguity and inconsistent data, the elaborated operators are massive, dominant, and more flexible compared with other theories. To prove that the initiated operators are massively superior to the existing operators, we choose some prevailing ideas based on IFSs, CIFSs, SVNSs, and CNSs to prove that the initiated principles are massive and dominant. For this, we choose the following prevailing operators: Garg (2018) initiated the PMM operators for NSs, Xu et al. (2019) developed the power MM operators for interval-valued IFSs, and Liu et al. (2019) elaborated the power MM operators for SVNSs. The accumulated result is discussed in Table 4.

The prevailing operators under the IFSs, SVNSs, and interval-valued IFSs are not able to determine the information in Table 1. The geometrical form of the data in Table 4 is provided in Figure 1.

Table 4
Expression of the sensitive analysis for the data in Table 1

Methods	Score values	Ranking values
Garg (2018)	Cannot be Calculated	Cannot be Calculated
Xu et al. (2019)	Cannot be Calculated	Cannot be Calculated
Liu et al. (2019)	Cannot be Calculated	Cannot be Calculated
CSVNPMM operator	0.6190, 0.7697, 0.7966, 0.7985, 0.7432	$\overline{\overline{\mathcal{TI}}}_{AT-4} \geq \overline{\overline{\mathcal{TI}}}_{AT-3} \geq \overline{\overline{\mathcal{TI}}}_{AT-2} \geq \overline{\overline{\mathcal{TI}}}_{AT-5} \geq \overline{\overline{\mathcal{TI}}}_{AT-1}$
CSVNPDMM operator	0.0197, 0.1483, 0.1152, 0.1819, 0.1158	$\overline{\overline{\mathcal{TI}}}_{AT-4} \geq \overline{\overline{\mathcal{TI}}}_{AT-2} \geq \overline{\overline{\mathcal{TI}}}_{AT-5} \geq \overline{\overline{\mathcal{TI}}}_{AT-3} \geq \overline{\overline{\mathcal{TI}}}_{AT-1}$

Figure 1
Graphical shown for the data given in Table 4



If we consider the value of unreal parts is equal to zero in Table 1, then the accumulated result is discussed in Table 5.

5.4. Advantages

The principle of CSVNS is massively extended than the prevailing theories. In this analysis, we illustrated the specific cases of the initiated CSVNSs, which are discussed below.

1. For
$$\mathcal{A}_{\overline{\overline{\mathcal{TI}}}_C}\left(\overline{\overline{\Xi}}\right) = \mathcal{A}_{\overline{\overline{\mathcal{TI}}}_R}\left(\overline{\overline{\Xi}}\right) e^{\mathfrak{i}2\pi\left(\mathcal{A}_{\overline{\overline{\mathcal{TI}}}_I}\left(\overline{\overline{\Xi}}\right)\right)} = 0.e^{\mathfrak{i}2\pi(0)} = 0.1 = 0,$$
the CSVNS is changed to CIFSs.

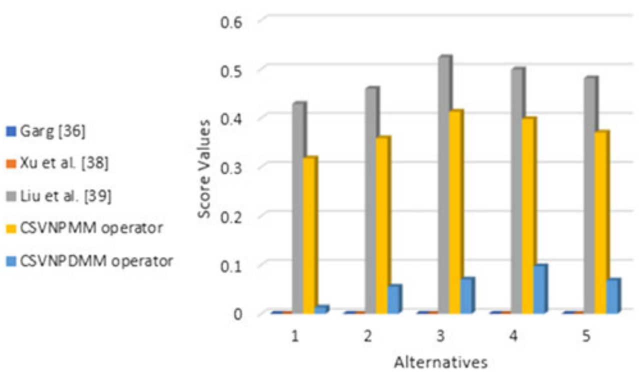
2. For
$$\mathcal{A}_{\overline{\overline{\mathcal{TI}}}_C}\left(\overline{\overline{\Xi}}\right) = \mathcal{A}_{\overline{\overline{\mathcal{TI}}}_R}\left(\overline{\overline{\Xi}}\right) e^{\mathfrak{i}2\pi\left(\mathcal{A}_{\overline{\overline{\mathcal{TI}}}_I}\left(\overline{\overline{\Xi}}\right)\right)} = 0.e^{\mathfrak{i}2\pi(0)} = 0.1 = 0,$$
and
$$\mathcal{N}_{\overline{\overline{\mathcal{TI}}}_C}\left(\overline{\overline{\Xi}}\right) = \mathcal{N}_{\overline{\overline{\mathcal{TI}}}_R}\left(\overline{\overline{\Xi}}\right) e^{\mathfrak{i}2\pi\left(\mathcal{N}_{\overline{\overline{\mathcal{TI}}}_I}\left(\overline{\overline{\Xi}}\right)\right)} = 0.e^{\mathfrak{i}2\pi(0)} = 0.1 = 0,$$
the CSVNS is changed to CFSs.

Table 5
Expression of the comparative analysis for the information in Table 1 (without imaginary parts)

Methods	Score values	Ranking values
Garg (2018)	Cannot be calculated	Cannot be calculated
Xu et al. (2019)	Cannot be calculated	Cannot be calculated
Liu et al. (2019)	0.4288, 0.4598, 0.5239, 0.4992, 0.4813	$\overline{\overline{\mathcal{TI}}}_{AT-3} \geq \overline{\overline{\mathcal{TI}}}_{AT-4} \geq \overline{\overline{\mathcal{TI}}}_{AT-5} \geq \overline{\overline{\mathcal{TI}}}_{AT-2} \geq \overline{\overline{\mathcal{TI}}}_{AT-1}$
CSVNPMM operator	0.3177, 0.3587, 0.4129, 0.3981, 0.3702	$\overline{\overline{\mathcal{TI}}}_{AT-3} \geq \overline{\overline{\mathcal{TI}}}_{AT-4} \geq \overline{\overline{\mathcal{TI}}}_{AT-5} \geq \overline{\overline{\mathcal{TI}}}_{AT-2} \geq \overline{\overline{\mathcal{TI}}}_{AT-1}$
CSVNPDMM operator	0.0130, 0.0555, 0.0703, 0.0971, 0.0683	$\overline{\overline{\mathcal{TI}}}_{AT-4} \geq \overline{\overline{\mathcal{TI}}}_{AT-3} \geq \overline{\overline{\mathcal{TI}}}_{AT-5} \geq \overline{\overline{\mathcal{TI}}}_{AT-2} \geq \overline{\overline{\mathcal{TI}}}_{AT-1}$

The best optimal is $\overline{\overline{\mathcal{TI}}}_{AT-3}$ by using Liu et al. (2019) and CSVNPMM operator. The CSVNPDMM operator gives a different result, which is $\overline{\overline{\mathcal{TI}}}_{AT-4}$. Moreover, we explained the advantage of the developed operators with the help of their structures. The geometrical form of the data in Table 5 is provided in Figure 2.

Figure 2
Graphical shown for the data given in Table 5



3. If we choose $]0^-, 1^+[$, $[0^-, 3^+]$ instead of $[0,1]$, $[0,1]$, then the CSVNS is changed to CNSs.
4. For $\mathcal{M}_{\overline{\overline{\mathcal{TI}}}_I} = \mathcal{A}_{\overline{\overline{\mathcal{TI}}}_I} = \mathcal{N}_{\overline{\overline{\mathcal{TI}}}_I} = 0$, the CSVNS is changed to SVNSs.
5. For $\mathcal{M}_{\overline{\overline{\mathcal{TI}}}_I} = \mathcal{A}_{\overline{\overline{\mathcal{TI}}}_I} = \mathcal{N}_{\overline{\overline{\mathcal{TI}}}_I} = 0$, and $\mathcal{A}_{\overline{\overline{\mathcal{TI}}}_R}\left(\overline{\overline{\Xi}}\right) = 0$, the CSVNS is changed to IFSs.
6. For $\mathcal{M}_{\overline{\overline{\mathcal{TI}}}_I} = \mathcal{A}_{\overline{\overline{\mathcal{TI}}}_I} = \mathcal{N}_{\overline{\overline{\mathcal{TI}}}_I} = 0$, and $\mathcal{A}_{\overline{\overline{\mathcal{TI}}}_R}\left(\overline{\overline{\Xi}}\right) = \mathcal{N}_{\overline{\overline{\mathcal{TI}}}_R}\left(\overline{\overline{\Xi}}\right) = 0$, the CSVNS is changed to FSs.

Under the above points, the principles of IFSs, NSs, SVNSs, CIFSs, and CNSs are the specific cases of the initiated CSVNSs. Therefore, the elaborated works based on CSVNS are massive, attractive, and more dominant to determine the supremacy of the initiated works.

6. Conclusion

The certain individual has employed the principle of PMM aggregation operator in the environment of distinct regions. The main goal of this study is discussed below.

1. We initiated the CSVNS and determined their important algebraic laws.

2. The principle of CSVNPMM operator and CSVNPDMM operator is elaborated and their particular cases are discussed.
3. A MADM procedure is explored under the presented operators by using the CSVNSs.
4. Numerous examples are illustrated to determine the advantages, sensitive analysis, and geometrical expressions of the proposed works to find the supremacy and flexibility of the initiated works.

In the future, we will adjust the hypothesis of complex q-rung orthopair fuzzy sets (Ali et al., 2020), complex spherical fuzzy sets (Ali et al., 2020), complex T-spherical fuzzy sets (Ali et al., 2020), linear Diophantine fuzzy sets (Riaz & Hashmi, 2019), Pythagorean m-polar fuzzy sets (Riaz & Hashmi, 2020), and T-spherical fuzzy sets (Balin, 2020; Guleria & Bajaj, 2020; Liu et al., 2019; Mahmood et al., 2019; Riaz et al., 2021; Wu et al., 2020) to advance the excellence of the created works.

Data Availability

The data used to support the findings of this study can be used by anyone without prior permission of the authors by just citing this article.

Conflicts of Interest

The authors declare that they have no conflicts of interest to this work.

Ethics Declaration Statement

The authors declare that this is their original work, and it is neither submitted nor under consideration in any other journal simultaneously.

Informed Consent

All the authors agreed and informed to submit this paper in the journal "Soft Computing" for possible publication.

Authors' Contributions

All authors contributed equally and significantly in writing this article. All authors read and approved the final manuscript.

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