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Intelligent System of Estimation of Total Factor Productivity (TFP) and Investment Efficiency in the Economy with External Technology Gaps

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Abstract: This paper is the first to propose an aggregate S-trend factor production function to estimate total factor productivity (TFP) and investment efficiency in an economy. This function implements Charles R. Hulten's organizing principle: to what extent the growth of the economy is due to an increase in "productivity" (progress in technology and organization of production) and to what extent to "capital formation" (increased investment in human capital, knowledge, and fixed capital). Estimation of future members of the series is usually done by a forecast model. It is a model that approximates a trend. The Verhulst's S-curve $y'(\mathrm{t}) = u + A(1 + B * \exp(-a(\mathrm{t} - m)))^{-1}$ is used as the approximation function. Here, A, B, and a are the parameters that change the shape of the S-curve, and u and m are the parameters that change the position of the S-curve in the first quarter, $t \geq 0$. By aggregate S-trend production function, we mean a two-factor production function of the form $Z(t) = P(t)y'(t) = P(t) * S(x(t), A, B, a, m, u)$. Here, the function $S(x(t), A, B, a, m, u)$ is a $S-$ curve trend with the factor $x(t) \equiv t$. It represents the GDP growth rate over a time interval equal to the product of the S-curve elasticity over the growth rate of n(t) over that interval and takes into account all factors affecting *S*-curve elasticity, including, for example, labor and capital. The value of the elasticity affects the value of TFP ($P(t)$), but not vice versa. In this sense, the trend forecasting model $S(t)$ is certainly broader than the concept of "capital formation". The error of approximation is quantitatively measured by the MAPE criterion.

Keywords: deterministic trends, one factor S-trend PF, aggregate S-trend PF, TFP, Charles R. Hulten principle, investment efficiency, external technology gaps

1. Introduction

In modern economics, growth of capital, labor, and technical progress are the three main sources of economic growth of a country and a region. The rate of labor growth is constrained by the rate of population growth, especially in industrialized countries, where the population rarely grows by more than 2% per year, even taking into account international migration [1]. Consequently, the growth rate of capital (physical and human) and technological progress are the main sources of much of economic growth. This fact emphasizes the relevance of finding the level of total factor productivity (hereinafter – TFP), which is one of the key indicators of production efficiency both at the level of individual firms and at the level of industries, regions, and countries. There is a wide range of methods that allow calculating TFP.

Historically, the calculation of TFP has been based on the notion of a production function. In economics, a production function gives the technological relation between quantities of physical inputs and quantities of output of goods.

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2. Literature Review

2.1. Cobb–Douglas era

The concept of production function was formulated in the 30s of the 19th century by Cobb and Douglas. In its most standard form for production of a single good with two factors, it is given by the formula (without the statistical components):

$$Y(t) = AK^{a}L^{1-b} \tag{1}$$

$Y(t)$ – total output (the real value of all goods produced in a year); $L(t)$ – labor input (person-hours worked in a year); $K(t)$ – capital input (a measure of all machinery, equipment, and buildings; the value of capital input divided by the price of capital); A – constant – efficiency coefficient; $0 < a < 1$ and $0 < 1 - a < 1$ are the output elasticities of capital and labor, respectively. These values are constants determined by available technology.

The production function Equation (1) can be written in the intensive form

$$y(t) = Ak(t)^{a} \tag{2}$$

here

$y(t) = \frac{Y(t)}{L(t)}$ – output per man;

$k(t) = \frac{K(t)}{L(t)}$ – capital per man.

Advantages and disadvantages of production function Equation (1) may be found in Bhanu Murthy [2].

2.2. Solow era

A number of works by prominent economists, such as Abramovitz [3], Jorgenson and Griliches [4], Jorgenson et al. [5], and Kuznets [6], are devoted to the study of the sources of economic growth.

The Solow model (Solow–Swan model) – a model of exogenous economic growth is based on the works of Solow [7] and Swan [8]. The concept of TFP, called the Solow residual, was introduced by Solow [9] and proposed on the basis of production function Equation (1) an aggregated production function of the form

$$y(t) = P(t)k(t)^a,\ t = 1, \ldots, T \tag{3}$$

Here, the TFP coefficient $P(t)$ measures the cumulated effect of shifts over time.

The variables $\frac{\Delta y}{y}$, $\frac{\Delta P}{P}$, $a\frac{d\Delta k}{dk}$ are the growth or decay rates of the variables $y(t),\ p(t),\ k(t)$ over the interval $[t_i,\ t_{i+1}]$ in fractions or percentages. We will denote them $G(y(t)),\ G(p(t)),\ G(k(t))$, respectively. Then, Equation (3) will be rewritten in the form

$$G(y(t)) = G(p(t)) + G(k(t))$$

Since the series $G(y(t))$, $G(k(t))$ can be determined from the original data series, it is easy to determine the components

$$G(p(t)) = G(y(t)) - G(k(t)$$

By coefficients $G(p)$, the TFP coefficients $P(t)$ are easily determined. To some extent, the disadvantages of production function Equations (1) and (2) are inherent in the aggregate production function Equation (3). Nevertheless, the model Equation (3) is considered the starting point of all modern models of economic growth. Solow's residual is still, after many decades, the workhorse of empirical growth analysis. For an introduction to the problem, we refer you to Hulten et al. [10] and Hulten [11], which provides an extensive bibliography between 1956 and 2001.

2.3. Era of aggregate production functions

At the current stage of economic development, it is necessary to find new approaches to modeling economic growth. More complex models of economic growth are required, taking into account a large number of factors and based on the newest achievements in the field of econometrics and forecasting.

The concept of production function (*PF*) is basic in economic theory. Formally, the production function looks like this:

$$y(t) = f(x_1(t), \ldots, x_n(t), a_1, \ldots, a_m)$$

$y(t)$ – volume (quantity) of output; $x_1(t), \ldots, x_n(t)$ – quantities of inputs (used); vector $(x_1(t), \ldots, x_n(t))$) is called the resource configuration, $x_1(t) > 0, \ldots, x_{n1}(t) > 0$; $a_1, \ldots, a_m$ – parameters; the symbol f, called the *PF* characteristic, shows how the quantity of a resource is formally transformed into the volume of output.

Some scientists [12] define the production function as an economic and mathematical expression of the dependence of the result of production activity on the factors conditioning it. Formally, the aggregate production function (based on *PF*) looks as follows:

$$y(t) = P(t)f(x_1(t), \ldots, x_n(t), a_1, \ldots, a_m)$$

$P(t)$ – total factor productivity coefficient.

2.4. A selection of the most relevant to building new aggregate production functions over the past 5 years

Generalization and further development of methods measuring TFP:

Tsionas and Polemis [13], Tsounis and Steedman [14], Francis et al. [15], Whelan [16], Li and Li [17], Harb and Bassil [18].

A criticism

Felipe and McCombi [19].

Last but not least

The US Bureau of Labor Statistics (BLS) [20]. This article defines key terms and concepts that are central to understanding how the BLS produces measures of productivity for different levels of the US economy. Conceptually, our approach is close to the BLS concept. This topic is discussed in detail in the section Discussion of results.

3. Problem Statement

3.1. S-trend production function

Let the time series under study be

$$y(t) = y(t_1), \ldots, y(t_T) \tag{4}$$

For example, it can be the GDP per capita of a country. For time series, it is customary to consider its levels as a mixture of four components – trend, cyclical, seasonal, and random components that cannot be measured [21].

$$y(t_i) = T(t_i) + C(t_i) + S(t_i) + \varepsilon(t_i)$$

$T(t_i)$ is a trend, the main tendency in the development of the process under study over time. This trend is a deterministic component, independent of cyclical, seasonal, and random components. It can be represented as a more or less smooth curve.

The components of the time series $T(t_i)$ are not observable. They are theoretical values. The estimation of future time series components is usually done using a predictive model. A predictive model is a model that approximates a trend. We choose the S-shaped Verhulst curve as a trend forecasting model (TFM)

$$y'(t) = u + \frac{A}{1 + B * \exp(-a(t - m))} = S(t, A, B, a, m, u) \tag{5}$$

Corollary 1. The type of forecasting model can be determined by the graph ($y(t)$, t) of the original series. Thus, the original data Equation (4) should be approximated by the TFM Equation (5).

Corollary 2. The accuracy of the approximation of the series Equation (4) by the TFM Equations (2) and (9) is estimated by the MAPE criterion.

$$\text{MAPE}(y(t), y'(t)) = \frac{100\%}{\text{N}} \sum_{t=1}^{N} \left| \frac{y(t) - y'(t)}{y(t)} \right| \tag{6}$$

$y(t)$ are the coordinates of the point plot of the original series Equation (4), and $y'(t)$ are the coordinates of the TFM Equation (5) being constructed. These coordinates are determined by the choice of the vector of parameters A, B, a,m,u in Equation (5).

A series of similar approximations is performed until the smallest MAPE value is obtained. The MAPE criterion is easy to interpret. For example, MAPE = 14% means that the average difference between the predicted value and the actual value is 14%. MAPE < 10% is considered an excellent result, and 10% < MAPE < 20% is considered a good result in Equation (6).

Remark 2. The development of the Solow model based on S-curves is given in our papers [22, 23].

3.2. Aggregate S-trend production function

The purpose of this paper is to develop a new two-factor aggregate production function of the form

$$z(t) = P(t)\, S(x(t), A, B, a, m, u),\ t = 1, \ldots, T \tag{7}$$

where $P(t)$ – TFP coefficient, and $S(t, A, B, a, m,u)$ is the TFM of the series under study: factor $x(t) = t$. In this case, the main factor that characterizes the economic output is time with a step of 1 year (the usual step of statistical tables).

The production function Equation (7) makes no assumptions about the factors affecting economic output. It allows to realize on its basis [11] organizing principle: to what extent the growth of the economy is due to an increase in "productivity" (progress in technology and organization of production) and to what extent to "capital formation" (investment in human capital, knowledge, and fixed capital).

4. Methodology

4.1. Diagram of technological mode of GDP per capita in Germany 1972–2018 data source

"World Development Indicators." World Bank [24].

The initial data (see Table 1) are represented by Figure 1. The diagram is broken down into rising and falling sections, which we call cycles, (see Figure 3) by means of increasing and decreasing Verhulst. trends (see Figure 2). There are a total of five cycles available.

Figure 1
Dot plot of raw data GDP per capita in Germany (1972–2018)

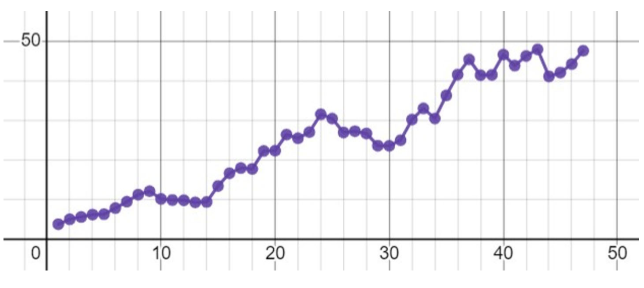


Table 1
Here $y(t)$ – GDP per capita in thousands of dollars (in current US$)

No.	Year	y	No.	Year	y	No.	Year	y
1	1972	3.81	17	1988	17.99	33	2004	33.04
2	1973	5.05	18	1989	17.70	34	2005	30.51
3	1974	5.63	19	1990	22.30	35	2006	36.32
4	1975	6.24	20	1991	22.38	36	2007	41.59
5	1976	6.33	21	1992	26.44	37	2008	45.43
6	1977	7.88	22	1993	25.52	38	2009	41.44
7	1978	9.48	23	1994	27.08	39	2010	41.53
8	1979	11.28	24	1995	31.57	40	2011	46.64
9	1980	12.14	25	1996	30.49	41	2012	43.86
10	1981	10.20	26	1997	26.98	42	2013	46.28
11	1982	9.91	27	1998	27.29	43	2014	47.96
12	1983	9.86	28	1999	26.75	44	2015	41.14
13	1984	9.31	29	2000	23.64	45	2016	42.10
14	1985	9.43	30	2001	23.61	46	2017	44.24
15	1986	13.46	31	2002	25.03	47	2018	47.60
16	1987	16.67	32	2003	30.24			

Figure 2
If $a > 0$, we have an increasing trend; if $a < 0$, we have a decreasing trend

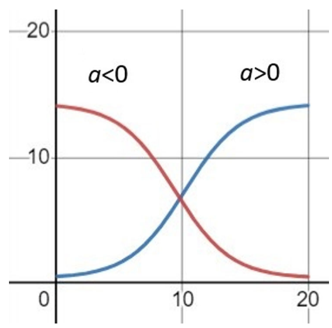
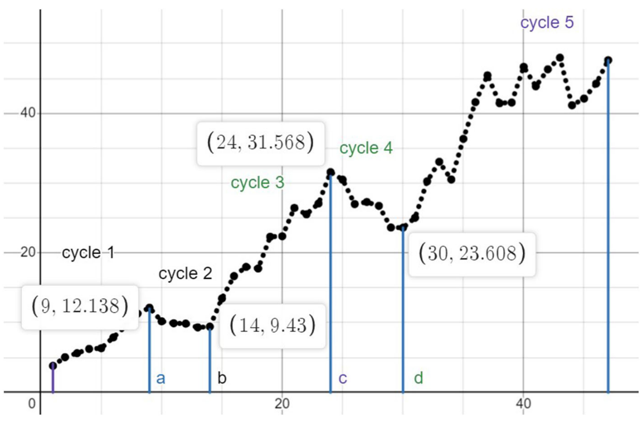


Figure 3
There are $S_i(t)$ – trend, $i = 1, \ldots 5$ that are defined by their lower and upper asymptotes



Description of S-trends of cycles
Approximation diagram of the technological pattern of GDP per capita in Germany 1972–2018 by S-trends of cycles

There are three increasing trends:

$$Theta\ 1 \le S_1(t) \le Theta\ 3$$

$$Theta\ 2 \le S_3(t) \le Theta\ 5$$

$$Theta\ 4 \le S_5(t) \le Theta\ 6$$

and two decreasing trends:

$$Theta\ 2 \le S_2(t) \le Theta\ 3$$

$$Theta\ 4 \le S_4(t) \le Theta\ 5$$

Decreasing trends are exogenous, as they are caused by changes in the world economy (wars, economic crises such as financial crises, and the like). Predicting such trends is a challenging task that lies outside the scope of this paper. Table 3 illustrates the large losses caused by decreasing trends.

Table 2
Cycles are defined by their lower and upper asymptotes

Theta	1	2	3	4	5	6
Value	3.81	12.1	9.4	31.6	23.6	45.4

Table 3
Decreasing trends are characterized by two parameters (see Table 2)

Technological gap in time	Technological results gap
$b - a = 5\ years$	Theta 2 –Theta 3 $= -3{,}000$
$c - d = 5\ years$	Theta 4 –Theta 5 $= -8{,}000$

Mathematics. Transition from the aggregate S-trend production function Equation (5) **to the production function in rates of growth of variables (continuous version)**

Let us compute the differential of the function $y(t)$ in relation Equation (7)

$$dy(t) = \frac{\partial y(t)}{\partial P(t)} dP(t) * S(x(t)) + P(t) * \frac{\partial y(t)}{\partial S(t)} \frac{\partial S(t)}{\partial x(t)} dx(t) \quad (8)$$

Let us divide both parts of Equation (8) by $y(t)$

$$\frac{dy(t)}{y(t)} = S(x(t))dP(t) * \frac{1}{P(t)*S(t)} + \frac{1}{P(t)*S(t)} * P(t) \frac{\partial S(t)}{\partial (x(t))} dx(t)$$

After reducing the multipliers in the numerator and denominator, we obtain the production function Equation (7) in the growth rates of the variables

$$\frac{dy(t)}{y\ (t)} = \frac{dP(t)}{P(t)} + \frac{\partial S(t)}{\partial x(t)} \frac{x(t)}{S(t)} \frac{dx(t)}{x(t)}$$

or

$$\frac{dy(t)}{y\ (t)} = \frac{dP(t)}{P(t)} + E_s(x(t)) \frac{dx(t)}{x(t)} \quad (9)$$

Here $E_s(x(t)) = \frac{\partial S(x(t))}{\partial (x(t))} \frac{x(t)}{S(t)}$ – elasticity of $S(x(t))$ with respect to factor $x(t)$.

Aggregate S-trend production function in growth rates of variables (discrete variant). Economic analysis

Let us convert to finite differences in relation Equation (9)

$$\frac{\Delta y(t)}{y\ (t)} = \frac{\Delta P(t)}{P(t)} + E_s(x(t)) \frac{\Delta x(t)}{x(t)} \quad (10)$$

We keep the following notations. The rate of increase or decrease of the variable $y\ (t)$ on the interval $[t,\ t+1]$.

$G_y(t) = \frac{y(t+1)}{y(t)} - 1$ in fractions or percent $G_y(t)\%$.

On the other hand, the new growth theory and another strand of neoclassical economics – the theory of capital and investment – prioritize increased investment in human capital, knowledge, and fixed capital. This component is called "capital formation." The component $G_s(t)$ characterizes the increase in $G_y(t)$ caused by "capital formation," and $G_p(t)$ characterizes the increase in $G_y(t)$ caused by "productivity." Let us rewrite the relation Equation (10) in the form:

$$G_y(t) = G_p(t) + G_s(t).$$

The components $G_y(t),\ G_s(t)$ are observable as they can be found from the original statistical data. The component $G_p(t)$ is unobservable and is found from the equation:

$$G_p(t) = G_y(t) - G_s(t).$$

5. Component Assessment of the Aggregate S-trend Production Function in the Growth Rates of Variables from the Initial Data

5.1. Component assessment by way of example Cycle_1

The Table 4 is a summary table for Cycle 1. It contains the trend vector $y'(t)$. The accuracy of approximation of the original data $y(t)$ by this vector is $MAPE\ (y, y')\% = 7.73\%$. The aggregate production function (2) has the form $Z\ (t) = P(t)y'(t)$. Here, P(t) is a productivity coefficient (TFP coefficient), determined by the recurrent formula $P\,(t+1) = (1 + G_p\,(t))P\,(t)$ [9].

The component $G_p(t)$ characterizes the increase in $P(t)$ (see Table 5). It is assumed that $P\,(1) = 1$, By varying the coefficient $P(1) = 1$ in the neighborhood of 1 we will achieve the minimum value criterion $MAPE(y(t), z(t)y')\% = 3.38\%$ at $P(1) = 1.10$.

Conclusion

For aggregate S-trend production functions Equation (8), the expected value reliability is (100–3.38) = 92.62%,

For S-trend production functions, the expected reliability is (100–7.73) = 92.27%.

$$G_y\ (t) = \frac{y(t+1) - y(t)}{y(t)} \text{ may be both positive or negative}$$

$$G_n\ (t) = \frac{n(t+1) - n(t)}{n(t)} = \frac{1}{t} \text{ always positive}$$

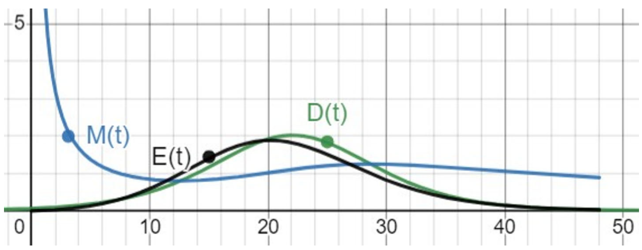
Table 4
(Cycle_1) Accuracy of cycle approximation using S-trend and aggregate S-trend production functions

A1	B1	a1	m1	u1			
10	984	0.57	−6.7	3			
A	B	C	D	E	F	G	H
N	Year	*y(t)*	*y'(t)*	Er	*P(t)*	*Z(t)*	Er
1	1972	3.81	3.76	0.01	1.10	4.13	0.08
2	1973	5.05	4.26	0.15	1.22	5.19	0.03
3	1974	5.63	5.04	0.11	1.10	5.54	0.02
4	1975	6.24	6.12	0.02	0.98	5.99	0.04
5	1976	6.33	7.45	0.18	0.79	5.92	0.07
6	1977	7.88	8.86	0.12	0.85	7.56	0.04
7	1978	9.48	10.15	0.07	0.92	9.30	0.02
8	1979	11.28	11.16	0.01	1.01	11.28	0.00
9	1980	12.14	11.87	0.02	1.03	12.25	0.01
			sum	0.70		sum	0.30
			MAPE	0.08		MAPE	0.03
			MAPE%	7.73		MAPE%	3.38

Table 5
(Cycle_1) Calculation of the components of the velocities $Gy(t)$, $Gn(t)$, $Es(t)$, $Gs(t)$, $Gp(t)$

A	B	I	J	K	L	M
ND	Year	*Gn(t)*	*Es(t)*	*Gy(t)*	*Gs(t)*	*Gp(t)*
1	1972					
2	1973	1.00	0.22	0.32	0.22	0.11
3	1974	0.50	0.43	0.12	0.21	−0.10
4	1975	0.33	0.65	0.11	0.22	−0.11
5	1976	0.25	0.82	0.02	0.20	−0.19
6	1977	0.20	0.85	0.24	0.17	0.07
7	1978	0.17	0.77	0.20	0.13	0.08
8	1979	0.14	0.61	0.19	0.09	0.10
9	1980	0.13	0.44	0.08	0.06	0.02

Figure 4
(Cycle_1) Calculation of elasticity



$E_s(t)$ is an elasticity of the S-curve with respect to t may be both positive or negative.

For the calculation, we will use the DESMOS graphing calculator. We plot the derivative $D(t) = \frac{dS(t)}{dt}$ of the function $S(t$, the mean of the function $M(t) = \frac{S(t)}{t}$, then $E_s(t) = \frac{D(t)}{M(t)}$

1) $G_s(t) = G_n(t)E_s(t)$ may be both positive or negative
2) $G_y(t) = G_p(t) + G_s(t)$
3) $G_p(t) = G_y(t) - G_s(t)$may be both positive or negative

Figure 5
(Cycle_1) The proportions of the contribution *"productivity"* and contribution *"capital formation"* in $G_y(t)$%

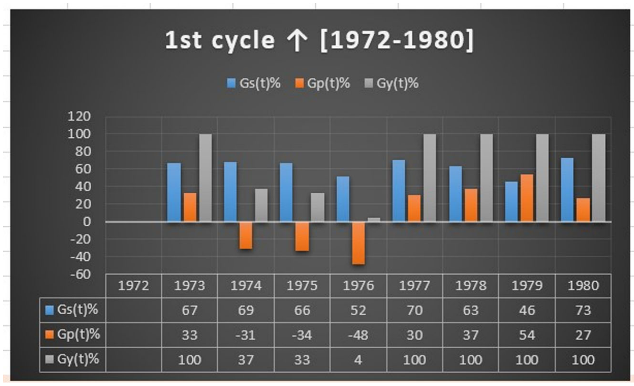


Table 6
(Cycle_1) Growth rate contribution diagram $G_y(t)$%. $G_s(t)$%. $G_p(t)$

A	B	N	O	P	Q	R	S
N	Year	*Gs(t)* %	*Gp(t)* %	*Gy(t)* %	ABS *Gs(t)*	ABS *Gp(t)*	Factor
1	1972						
2	1973	67	33	100	0.22	0.11	0.32
3	1974	69	−31	37	0.21	0.10	0.31
4	1975	66	−34	33	0.22	0.11	0.33
5	1976	52	−48	4	0.20	0.19	0.39
6	1977	70	30	100	0.17	0.07	0.24
7	1978	63	37	100	0.13	0.08	0.20
8	1979	46	54	100	0.09	0.10	0.19
9	1980	73	27	100	0.06	0.02	0.08

The formulas of Table 6 are also valid for Tables 9, 12, 15, 18, 19.

4) $\text{factor} = \text{ABS}(G_s(t)) + \text{ABS}(G_p(t))$
5) $G_s(t)\% = 100 * G_s(t)/\text{factor}$
6) $G_p(t)\% = 100 * G_p(t)/\text{factor}$
7) $G_y(t)\% = 100 * G_y(t)/\text{factor}$

Remark 3. For all cycles 1–5 in the diagram $G_s(t)$% is labeled in blue, $G_p(t)$% is labeled in orange, and $G_y(t)$% is labeled in gray. By definition: $G_y(t) = G_p(t) + G_s(t)$.

6. Results

The Table 7 is a summary table for Cycle 2. It contains the trend vector $y'(t)$. The accuracy of approximation of the original data y(t) by this vector is $MAPE(y, y')\% = 4.05\%$. The aggregate production function (2) has the form $Z(t) = P(t)y'(t)$. Here, P(t) is a productivity coefficient (TFP coefficient), determined by the recurrent formula $P(t+1) = (1 + G_p(t))P(t)$ [9].

The component $G_p(t)$ characterizes the increase in *P*(*t*) (see Table 8). It is assumed that *P*(1)=1, By varying the coefficient *P*(1) = 1 in the neighborhood of 1 we will achieve the minimum value criterion $MAPE(y(t), z(t)y')\% = 5.38\%$ at $P(1) = 0.89$.

Conclusion

For aggregate S-trend production functions, the expected value reliability is (100–4.05) = 95.95%.

Table 7
(Cycle_2) Accuracy of cycle approximation using S-trend and aggregate S-trend production functions

A2	B2	a2	m2	u2			
2.80	70	−2.1	12	9.30			
A	B	C	D	E	F	G	H
N	Year	*y*(*t*)	*y*'(*t*)	Er	*P*(*t*)	*Z*(*t*)	Er
9	1980	12.14	11.78	0.03	0.89	10.49	0.14
10	1981	10.20	10.67	0.05	0.96	10.24	0.00
11	1982	9.91	9.59	0.03	1.06	10.13	0.02
12	1983	9.86	9.34	0.05	1.02	9.53	0.03
13	1984	9.32	9.30	0.00	1.02	9.50	0.02
			sum	0.16		sum	0.22
			MAPE	0.04		MAPE	0.05
			MAPE%	4.05		MAPE%	5.38

Table 8
(Cycle_2) Calculation of the components of the velocities
$Gy(t)$, $Gn(t)$, $Es(t)$, $Gs(t)$, $Gp(t)$

A	B	I	J	K	L	M
N	Year	*Gn*(*t*)	*Es*(*t*)	*Gy*(*t*)	*Gs*(*t*)	*Gp*(*t*)
9	1980					
10	1981	0.11	0.20	−0.16	−0.02	−0.14
11	1982	0.10	1.07	−0.03	−0.11	0.08
12	1983	0.09	1.16	0.00	−0.11	0.10
13	1984	0.08	0.26	−0.06	−0.02	0.03

Table 9
(Cycle_2) Growth rate contribution diagram
$G_y(t)\%$, $G_s(t)\%$, $G_p(t)\%$

A	B	N	O	P	Q	R	S
N	Year	*Gs*(*t*) %	*Gp*(*t*) %	*Gy*(*t*) %	ABS *Gs*(*t*)	ABS *Gp*(*t*)	Factor
9	1980						
10	1981	−14	−86	−100	0.02	0.14	0.16
11	1982	−58	42	−15	0.11	0.08	0.19
12	1983	−51	49	−2	0.11	0.10	0.21
13	1984	−39	−61	−100	0.02	0.03	0.06

For S-trend production functions, the expected reliability is (100–5.38) = 94.62%.

Cycle 3

The Table 10 is a summary table for Cyle 3. It contains the trend vector $y'(t)$. The accuracy of approximation of the original data $y(t)$ by this vector is $MAPE\,(y, y')\% = 13\%$. The aggregate production function (2) has the form $z(t) = P(t)y'(t)$. Here, *P*(*t*) is a productivity coefficient (TFP coefficient), determined by the recurrent formula $P(t+1) = (1 + G_p(t))P(t)$ [9].

The component $G_p(t)$ characterizes the increase in *P*(*t*) (see Table 12). It is assumed that *P*(1) = 1, By varying the coefficient *P*(1) = 1 in the neighborhood of 1 we will achieve the minimum value criterion $MAPE\,(y(t), z(t)y')\% = 2.87\%$ at *P*(1) = 1.

For aggregate S-trend production functions, the expected value reliability is (100–2.9) = 97.1%.

For S-trend production functions, the expected reliability is (100–13) = 98.6%.

Conclusion

Table 10
(Cycle_3) Accuracy of cycle approximation using S-trend and aggregate S-trend production functions

A3	B3	a3	m3	u3			
22.50	1600.00	0.70	8.20	9.10			
A	B	C	D	E	F	G	H
N	Year	*y*(*t*)	*y*′(*t*)	Er	*P*(*t*)	*Z*(*t*)	Er
14	1985	9.43	9.89	0.05	1.00	9.89	0.05
15	1986	13.46	10.63	0.21	1.33	14.10	0.05
16	1987	16.67	11.98	0.28	1.43	17.19	0.03
17	1988	17.99	14.24	0.21	1.25	17.80	0.01
18	1989	17.76	17.50	0.01	0.95	16.71	0.06
19	1990	22.03	21.37	0.03	1.00	21.37	0.03
20	1991	22.38	25.01	0.12	0.89	22.37	0.00
21	1992	26.44	27.76	0.05	0.94	26.09	0.01
22	1993	25.52	29.52	0.16	0.86	25.48	0.00
23	1994	27.08	30.51	0.13	0.89	27.29	0.01
24	1995	31.57	31.05	0.02	1.05	32.72	0.04
			sum	1.26		sum	0.29
			MAPE	0.13		MAPE	0.03
			% MAPE	13%		% MAPE	2.87

Table 11
(Cycle_3) Calculation of the components of the velocities
$Gy(t)$, $Gn(t)$, $Es(t)$, $Gs(t)$, $Gp(t)$

A	B	J	J	K	L	M
N	Year	*Gn*(*t*)	*Es*(*t*)	*Gy*(*t*)	*Gs*(*t*)	*Gp*(*t*)
14	1985					
15	1986	0.071	1.41	0.43	0.10	0.33
16	1987	0.067	2.36	0.24	0.16	0.08
17	1988	0.063	3.32	0.08	0.21	−0.13
18	1989	0.059	3.80	−0.01	0.22	−0.24
19	1990	0.056	3.48	0.24	0.19	0.05
20	1991	0.053	2.30	0.02	0.12	−0.11
21	1992	0.050	2.61	0.18	0.13	0.05
22	1993	0.048	0.99	−0.03	0.05	−0.08
23	1994	0.045	0.55	0.06	0.02	0.04
24	1995	0.043	0.29	0.17	−0.01	0.18

Table 12
(Cycle_3) Growth rate contribution diagram
$G_y(t)\%$, $G_s(t)\%$, $G_p(t\%)$

A	B	N	O	P	Q	R	S
N	Year	*Gs*(*t*) %	*Gp*(*t*) %	*Gy*(*t*) %	ABS *Gs*(*t*)	ABS *Gp*(*t*)	Factor
14	1985						
15	1986	24	76	100	0,10	0.33	0.43
16	1987	66	34	100	0.16	0.08	0.24
17	1988	62	−38	24	0.21	0.13	0.34
18	1989	49	−51	−3	0.22	0.24	0.46
19	1990	80	0	100	0.19	0.05	0.24
20	1991	53	−47	7	0.12	0.11	0.23
21	1992	72	28	100	0.13	0.05	0.18
22	1993	37	−63	−27	0.05	0.08	0.13
23	1994	41	59	100	0.02	0.04	0.06
24	1995	−7	93	87	0.01	0.18	0.19

Cycle 4

The Table 13 is a summary table for Cycle 4. It contains the trend vector $y'(t)$. The accuracy of approximation of the original data $y(t)$ by this vector is $MAPE\,(y,\ y'(t))\% = 5.12\%$. The aggregate production function (2) has the form $z(t) = P(t)y'(t)$. Here, $P(t)$ is a productivity coefficient (TFP coefficient), determined by the recurrent formula $P(t+1) = (1 + G_p(t))P(t)$ [9].

Table 13
(Cycle_4) Accuracy of cycle approximation using S-trend and aggregate S-trend production functions

A4	B4	a1	m1	u1			
8	570	−2.1	30	23.40			
A	B	C	D	E	F	G	H
N	Year	$y(t)$	$y'(t)$	Er	$P(t)$	$Z(t)$	Er
24	1995	31.57	31.38	0.01	1.00	31.38	0.01
25	1996	30.49	31.28	0.03	0.97	30.47	0.00
26	1997	26.98	30.49	0.13	0.92	28.01	0.04
27	1998	27.29	27.31	0.00	1.08	29.37	0.08
28	1999	26.75	24.24	0.09	1.13	27.31	0.02
29	2000	23.64	23.51	0.01	1.01	23.68	0.00
			sum	0.26		sum	0.14
			MAPE	0.05		MAPE	0.03
			MAPE%	5.12		MAPE%	2.75

The component $G_p(t)$ characterizes the increase in $P(t)$ (see Table 14). It is assumed that $P(1)$=1, By varying the coefficient $P(1) = 1$ in the neighborhood of 1 we will achieve the minimum value criterion $MAPE\,(y(t),\ z(t)y')\% = 2.75\%$ at $P(1) = 1$.

Conclusion

Table 14
(Cycle_4) Calculation of the components of the velocities $Gy(t),\ Gn(t),\ Es(t),\ Gs(t),\ Gp(t)$

	B	I	J	K	L	M
N	Year	$Gn(t)$	$Es(t)$	$Gy(t)$	$Gs(t)$	$Gp(t)$
24	1995			A		
25	1996	0.0417	0.2043	−0.03	−0.01	−0.03
26	1997	0.0400	1.4426	−0.11	−0.06	−0.06
27	1998	0.0385	4.1504	0.01	−0.16	0.17
28	1999	0.0370	1.8198	−0.02	−0.07	0.05
29	2000	0.0357	0.2885	−0.12	−0.01	−0.11

Table 15
(Cycle_4) Growth rate contribution diagram $G_y(t)\%,\ G_s(t)\%,\ G_p(t)\%$

A	B	O	P	Q	R	S	
N	Year	$Gs(t)$ %	$Gp(t)$ %	$Gy(t)$ %	ABS $Gs(t)$	ABS $Gp(t)$	Factor
24	1995						
25	1996	−25	−3	−27	0.01	0.03	0.03
26	1997	−50	−50	−100	0.06	0.06	0.11
27	1998	−48	52	3	0.16	0.17	0.33
28	1999	−59	41	−17	0.07	0.05	0.12
29	2000	−9	−91	−100	0.01	0.11	0.12

For aggregate S-trend production functions, the expected value reliability is (100–2.75) = 97.25%.

For S-trend production functions, the expected reliability is (100–5.12) = 94.88%.

Cycle 5

The Table 16 is a summary table for Cycle 5. It contains the trend vector $y'(t)$. The accuracy of approximation of the original data $y(t)$ by this vector is $MAPE\,(y, y')\% = 6.47\%$. The aggregate production function (2) has the form $z(t) = P(t)y'(t)$. Here, $P(t)$ is a productivity coefficient (TFP coefficient), determined by the recurrent formula $P(t+1) = (1 + G_p(t))P(t)$ [9].

Table 16
(Cycle_5) Accuracy of cycle approximation using S-trend and aggregate S-trend production functions

A5	B5	a5	m5	u5			
22	84	1	29.7	23.60			
A	B	C	D	E	F	G	H
N	Year	$y(t)$	$y'(t)$	Er	$P(t)$	$Z(t)$	Er
30	2001	23.70	23.95	0.01	0.97	23.23	0.02
31	2002	25.03	24.52	0.02	0.99	24.23	0.03
32	2003	30.24	25.94	0.14	1.11	28.82	0.05
33	2004	33.04	28.97	0.12	1.09	31.52	0.05
34	2005	30.51	33.88	0.11	0.82	27.86	0.09
35	2006	36.32	39.10	0.08	0.88	34.38	0.05
36	2007	41.86	42.66	0.02	0.96	40.91	0.02
37	2008	45.43	44.42	0.02	1.02	45.13	0.01
38	2009	41.44	45.15	0.09	0.92	41.38	0.00
39	2010	41.53	45.43	0.09	0.92	41.58	0.00
40	2011	46.64	45.54	0.02	1.03	46.74	0.00
41	2012	43.86	45.58	0.04	0.96	43.97	0.00
42	2013	46.28	45.59	0.01	1.02	46.40	0.00
43	2014	47.96	45.60	0.05	1.05	48.07	0.00
44	2015	41.14	45.60	0.11	0.90	41.23	0.00
45	2016	42.10	45.60	0.08	0.93	42.19	0.00
46	2017	44.24	45.60	0.03	0.97	44.34	0.00
47	2018	47.60	45.60	0.04	1.05	47.71	0.00
			Sum	1.10		Sum	0.34
			MAPE	0.06		MAPE	0.02
			MAPE%	6.47		MAPE%	1.97

The component $G_p(t)$ characterizes the increase in $P(t)$ (see Table 17). It is assumed that $P(1)$=1, By varying the coefficient $P(1) = 1$ in the neighborhood of 1 we will achieve the minimum value criterion $MAPE\,(y(t), z(t)y')\% = 2\%$ at $P(1) = 0.97$.

Conclusion

For aggregate S-trend production functions, the expected value reliability is (100–) = 98%.

For S-trend production functions, the expected reliability is (100–6.47) = 95.53%.

7. Discussion of Results

1) We are not aware of any work on constructing a production function similar to aggregate S-trend production function Equation (6).
2) Conceptually, our approach is close to the US BLS [20] concept.

Table 17
(Cycle_5) Calculation of the components of the velocities $Gy(t)$, $Gn(t)$, $Es(t)$, $Gs(t)$, $Gp(t)$

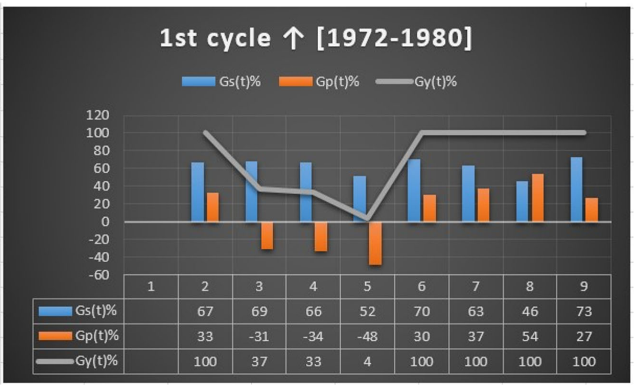
A	B	I	J	K	L	M
N	Year	*Gn*(*t*)	*Es*(*t*)	*Gy*(*t*)	*Gs*(*t*)	*Gp*(*t*)
30	2001					
31	2002	0.0333	1.1245	0.04	0.04	0.01
32	2003	0.0323	2.5953	0.17	0.08	0.08
33	2004	0.0313	3.6324	0.09	0.11	−0.03
34	2005	0.0303	5.5283	−0.08	0.17	−0.24
35	2006	0.0294	4.1201	0.17	0.12	0.05
36	2007	0.0286	2.1596	0.16	0.06	0.10
37	2008	0.0278	0.9354	0.10	0.03	0.07
38	2009	0.0270	0.3725	−0.11	0.01	−0.12
39	2010	0.0263	0.1435	0.00	0.00	0.00
40	2011	0.0256	0.0545	0.13	0.00	0.13
41	2012	0.0250	0.0206	−0.07	0.00	−0.07
42	2013	0.0244	0.0078	0.06	0.00	0.06
43	2014	0.0238	0.0193	0.04	0.00	0.04
44	2015	0.0233	0.0011	−0.16	0.00	−0.16
45	2016	0.0227	0.0004	0.02	0.00	0.02
46	2017	0.0222	0.0002	0.05	0.00	0.05
47	2018	0.0217	0.0001	0.07	0.00	0.07

A characteristic feature of increasing cycles is that the rate of change of the variable *s*(*t*) is always greater than zero Gs% >= 0 (see Figures 5, 6, 9, 10, and 15). A characteristic feature of decreasing cycles is that the rate of change of the variable *s*(*t*) is always less than zero Gs%, <= 0 (see Figures 7, 8, 11, and 12). If the growth rate Gp% > 0, it is postponed above the *t* axis. If the growth rate Gp%< 0, it is plotted below the *t*-axis. The result Gy% = Gs% + Gp% is plotted above or below the *t*-axis depending on the sign of the indicated sum.

The change in the variable $y(t+1)$ is defined by the formula $y(t+1) = (1+Gy)\,y(t) = (1+Gs+Gp)\,y(t)$. When Gy>0 the variable $y(t+1)$ increases, and when Gy<0 it decreases. See Tables 5, 8, 11, 14, and 17 for the values of *Gy, Gs, Gp*.

3) A wider range of application (e.g., GDP problems for different countries); it is not necessary to introduce a priori factors that affect economic output and which, as a rule, are not known; time elasticity is not constant; aggregate S-trend production does not have a set of disadvantages inherent in the Solow model.

Figure 6
(Cycle_1) Output $G_y(t)\%$, as sum of *"productivity"* and *"capital formation"*



4) This is the first time exogenous trends have been studied.

They are the cause of large economic gaps causing great damage to the economy of the country. They are caused by changes in the world economy (wars, economic crises such as financial crises, and

Figure 7
(Cycle_2) The proportions of the contribution *"productivity"* and contribution *"capital formation" in $G_y(t)\%$*

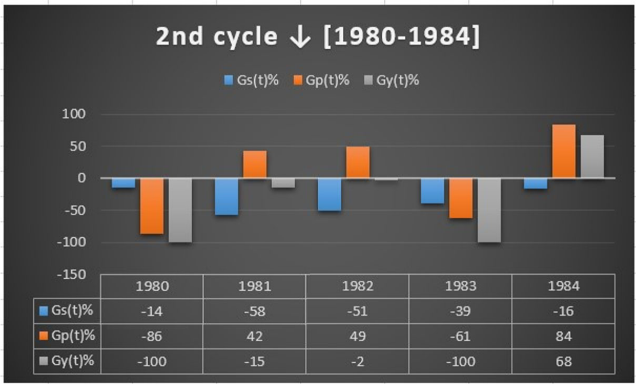


Figure 8
(Cycle_2) Output $G_y(t)\%$, as sum of *"productivity"* and *"capital formation"*

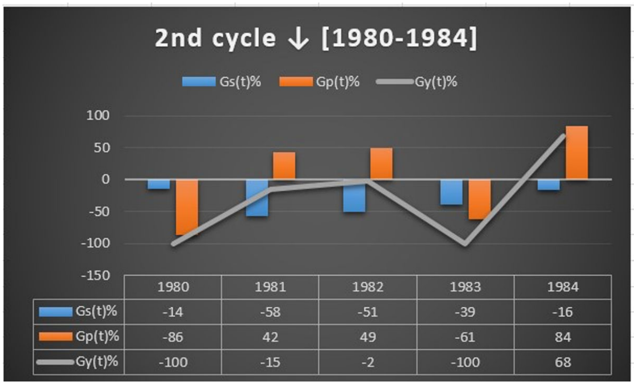
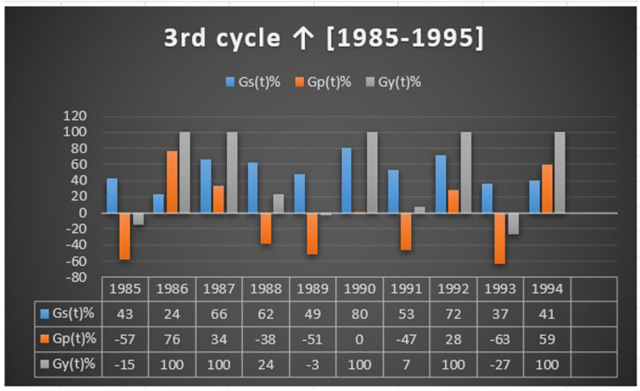


Figure 9
(Cycle_3) The proportions of the contribution *"productivity"* and contribution *"capital formation" in $G_y(t)\%$*



the like). Predicting such trends is a challenging task that lies outside the scope of this paper.

5) Prerequisites for the use of empirical data: the original time series $y(t)$ is objective, i.e., it does not contain errors.

Figure 10
(Cycle_3) Output $G_y(t)$%, as sum of *"productivity" and "capital formation"*

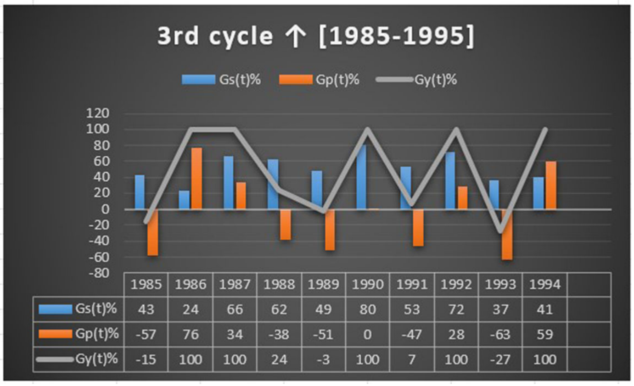


Figure 11
(Cycle_4) The proportions of the contribution of *"productivity"* and *"capital formation" in $G_y(t)$%*

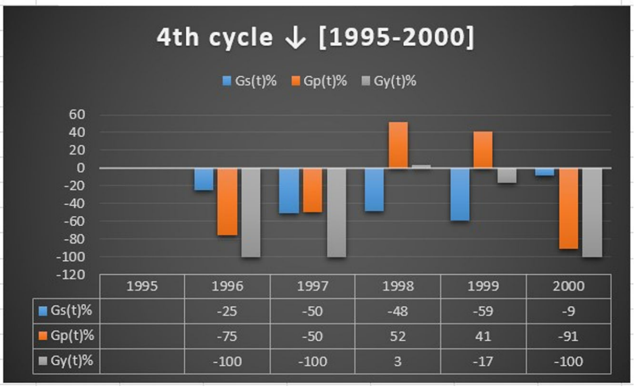


Figure 12
(Cycle_4) Output $G_y(t)$%, as sum of *"productivity" and "capital formation"*

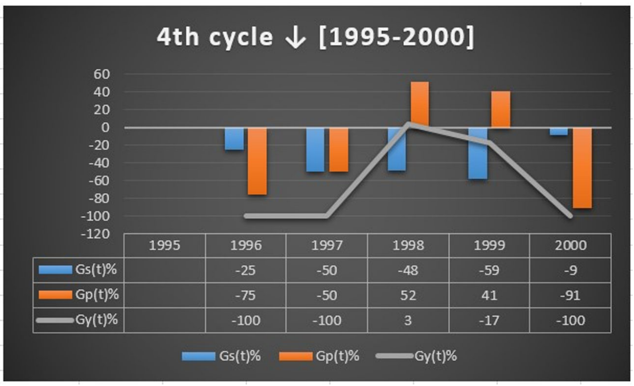


Figure 13
(Cycle_5) The proportions of the contribution of *productivity"* and *"capital formation" in $G_y'; (t)$%*

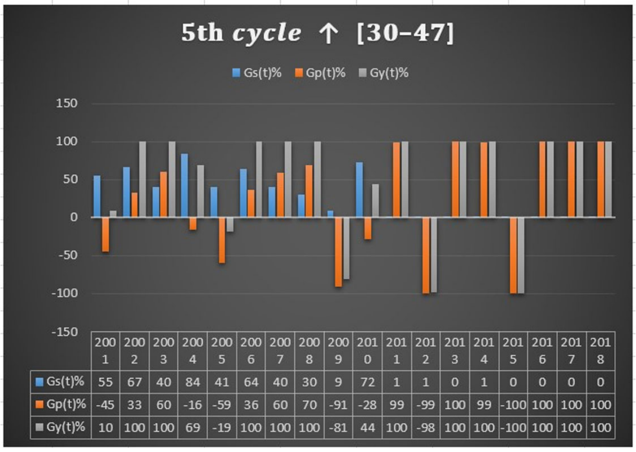


Figure 14
(Cycle_5) Output $G_y(t)$%, as sum of *"productivity" and "capital formation"*

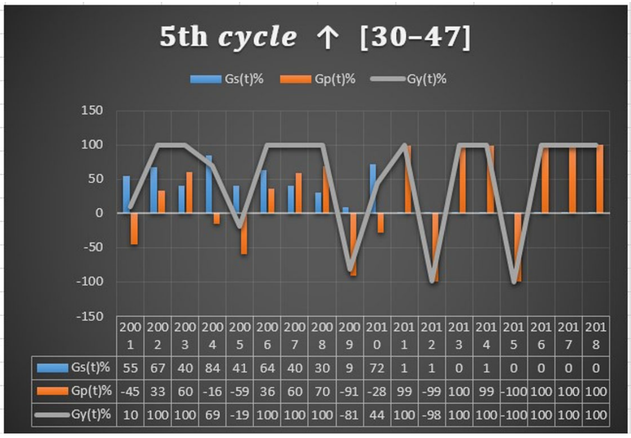


Figure 15
Structure of the German economy (1972–2018)

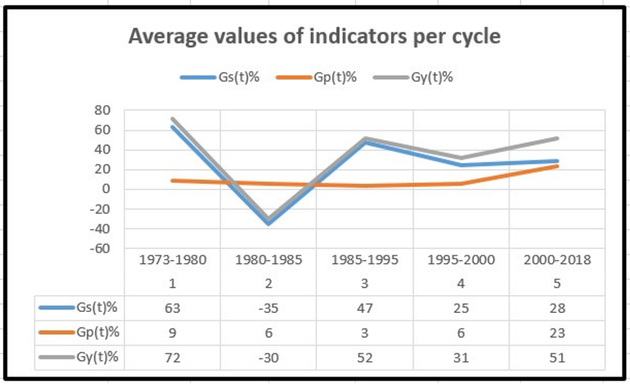


Table 18
(Cycle_5) Growth rate contribution diagram $G_y(t)\%$, $G_s(t)\%$, $G_p(t)\%$

A Num	B Year	N *Gs*(*t*)%	O *Gp*(*t*)%	P *Gy*(*t*)%	Q *ABS*(*Gs*(*t*))	R *ABS*(*Gp*(*t*))	S Factor
30	2001						
31	2002	67	12	100	0.04	0.02	0.06
32	2003	40	41	100	0.08	0.12	0.21
33	2004	84	−19	69	0.11	0.02	0.13
34	2005	41	−59	−19	0.17	0.24	0.41
35	2006	64	26	100	0.12	0.07	0.19
36	2007	40	63	100	0.06	0.09	0.15
37	2008	30	86	100	0.03	0.06	0.09
38	2009	9	−109	−81	0.01	0.10	0.11
39	2010	72	−24	44	0.00	0.00	0.01
40	2011	1	105	100	0.00	0.12	0.12
41	2012	1	−115	−98	0.00	0.06	0.06
42	2013	0	107	100	0.00	0.06	0.06
43	2014	1	109	100	0.00	0.04	0.04
44	2015	0	−112	−100	0.00	0.14	0.14
45	2016	0	93	100	0.00	0.02	0.02
46	2017	0	94	100	0.00	0.05	0.05
47	2018	0	96	100	0.00	0.08	0.08

Table 19
Structure of the German economy (1972–2018) through average values of indicators per cycle

ND	Year	*Gs*(*t*)%	*Gp*(*t*)%	*Gy*(*t*)%
1	1973–1980	63	9	72
2	1980–1985	−35	6	−30
3	1985–1995	47	3	52
4	1995–2000	25	6	31
5	2000–2018	28	23	51

The best results in the development of the German economy come from the cycles: Cycle 1 1973-1980; Cycle 5 1973-1980; Cycle 3 1985-1993; Cycle 4 1995-2000 Worst results: Cycle 2 1980-1985.

6) The posed economic task is solved in this paper by constructing a proper intelligent system. Intelligent system is a technical or software system capable of solving tasks traditionally considered creative, belonging to a specific subject area, the knowledge of which is stored in the memory of such a systemmaker (DM) as opposed to an intelligentised system in which an operator is present.

Ethical Statement

This study does not contain any studies with human or animal subjects performed by the author.

Conflicts of Interest

The author declares that he has no conflicts of interest to this work.

Data Availability Statement

Data sharing is not applicable to this article as no new data were created or analyzed in this study.

Author Contribution Statement

Alexey Lopatin: Conceptualization, Methodology, Software, Validation, Formal analysis, Investigation, Resources, Data curation, Writing – original draft, Writing – review & editing, Visualization, Supervision, Project administration.

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