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# A Note on Controlling Chaotic Dynamics in the Presence of Climate Change Externalities in the New Keynesian Monetary Economics Model

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**Abstract:** The role of central banks to mitigate the negative impact of climate change risks on economic development has become crucial in the monetary policy agenda. As recently experienced after the COVID-19 pandemic, unexpected shocks in the energy supply sector can destabilize the productive sector, and thus the solvency of the entire financial system. This paper shows the possibility of the emergence of a shock on the equilibrium pattern due to the emergence of chaotic dynamics and suggests the tools devoted to correcting such behavior. The occurrence of chaos in economics is particularly relevant when the policymaker has to choose the appropriate decisions to achieve the desired growth rate of the economy, without suffering the possible unwanted oscillations on national income. The idea is that if the economy is stabilized at its highest growth rate, then any possible low-growth poverty trap is avoided. Our idea is to consider a monetary economics model with an externality factor due to climate impacts on labor force and show the parametric condition for the emergence of chaotic dynamics. We then apply a correction algorithm that allows us to determine the restrictions on the parameter set necessary to eliminate or control the chaotic dynamics and restore the stability of equilibrium. Our findings are finally sustained by real data that show a possible link between adverse climatic scenarios and the intervention of the monetary authority, but also confirm that if the interest rates are raised above a sustainable target, then the economy will start to oscillate around the desired long-run equilibrium, thus experiencing periods of unwanted fall in real output, that are in fact a poverty trap.

**Keywords:** chaotic dynamics, climate externality, monetary model

## 1. Introduction

Nowadays, the role of central banks in facing the risks coming from climate change and energy shortage, with the related negative impacts on job creation and economic development, has become a crucial issue to deal with and is expected to become even more prominent in the future monetary policy agenda, within the mandate of preserving price stability, to promote a sustainable transition of the economic systems [1–4]. The reason for the need of central banks' intervention to support governmental funds to finance new green policies is based on the consideration that the climate-related events and the related shocks in the sector of energy supply might be able to destabilize the financial stability of the current fossil fuel-based productive firms and increase therefore the likelihood of possible defaults in the banking system [5–7]. This may be due to the eventuality of unexpected losses in assets portfolios of non-performing loans in those firms that have not yet completed the process to a green transition in the production processes, and lead to a domino effect that can undermine the solvency of the entire financial system [8–11].

More crucially, the task of price stability in the long run can be compromised when the expected productivity losses that may result if the producing sector is damaged by negative climate events or by a resource scarcity associated to long periods of dry seasons may disrupt the supply-chain and determine an increase in production costs, which finally contribute to exacerbate inflationary pressures and hamper economic growth. In this view, moving towards a low-carbon economy through climate-damaging mitigation efforts may require the promotion of significant adjustments and structural change in various economic sectors [12, 13]. This transition can impact the labor-force employment and productivity, with industries facing unwanted job losses, and can also affect inflation dynamics as prices of goods and services based on carbon-intensive technologies may rise [14, 15]. As a consequence, the pattern towards an intended sustainable equilibrium may be undermined and results instead in a low-growth trapping region eventually associated to an indeterminate long run solution. In the worst scenario, chaotic dynamics can emerge, thus puzzling the correct policy action (both fiscal and monetary) to be adopted to achieve the high-growth steady state [16–18].

In this light, a relevant number of articles in the literature is moving towards a focus on the policy implications due to the emergence of chaotic dynamics in economic systems. A recent paper by Barnett et al. [19] has concentrated its attention on the Shilnikov bifurcation theorem [20] and the possibility of chaos-driven mechanics, that can appear in the standard monetary framework of the New Keynesian economy under an active monetary policy. The generated strange attractor shows a spiraling structure around a homoclinic orbit that results when the three-dimensional dynamic system undergoes a saddle focus equilibrium [21]. These irregular dynamics thus have the possibility to undermine the convergence towards a stable equilibrium [22, 23]. Instead, depending on the initial conditions of the economic variables, it can produce a set of two different equilibria, one of which becomes a low-growth trapping equilibrium [24–26].

Our research question is grounded on this debate and is developed as follows. First, we decide to take advantage of the New Keynesian monetary model framework and introduce a variant in terms of a negative climate externality on the productivity of labor force to prove that a chaotic scenario can eventually emerge. Additionally, we want to show how the chaotic dynamics generated can be controlled to restore local stability of the transitional dynamics, when the Ott et al. algorithm

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[27], henceforth OGY, is applied. In summary, we briefly present in Section 2 the variant of the Barnett et al. model [19], with climate externality on labor force, and derive the three-dimensional system of first-order differential equations. We also characterize the restrictions to the original parametric set at which the model satisfies the requirements of the Shilnikov theorem [20] that guarantee the emergence of chaotic dynamics. In Section 3, we apply the OGY algorithm for controlling the chaotic dynamics and provide a conclusive example. A final concluding section reassesses the main findings of the paper and provides a hint of the policy implications of the study, outlines the possible limitations and suggests the lines for future research.

## 2. The Model

Consider the baseline New Keynesian monetary model which represents a seminal framework in describing the rule followed by a Monetary Authority in conducting its policy to avoid the possible emergence of liquidity traps in connection with standard fiscal policies and the interest rate rule suggested by the Taylor principle. The basic conclusion of this literature suggests that, as far as the economy is approaching a liquidity trap, a large fiscal stimulus will be needed to drive the economy off the low-inflation trapping equilibrium, thus making the trap unsustainable for a representative agent guided by rational expectation [28, 29]. More in detail, the model characterizes the dynamic evolution of the key macroeconomic variables and permits to explain the observed fluctuations in output growth, inflation, and the nominal interest rate produced by any monetary shock that happens in the economic systems. However, it might be interesting to introduce the effect of central bank's wrong or misleading perceptions regarding unobservables, such as the effects of negative externalities on the job market, that can lead to persistent monetary policy errors in sustaining the growth pattern of economic aggregates. This is what we are aimed to exploit in the following discussion.

We consider here the model version described in Barnett et al. [19] and restrict the analysis to a so-called cashless economy (i.e., an economy with no money either in the utility or in the production function). Using the standard Pontryagin maximization principle, the objective of the optimizing strategy is then to maximize the level of utility, U:[1]

$$\underset{c_i,l_i,\pi_i}{Max} \int_0^\infty U(c_i,l_i,\pi_i)e^{-\rho t}dt$$

subject to the constraints on the variation of assets, $a_i$, and the variation of prices in the market, $p_i$.

$$\begin{aligned} \dot{a}_i &= (R-\pi_i)a_i + \tfrac{p_i}{p}y(l_i) - c_i - \tau \\ \dot{p}_i &= \pi_i p_i \\ a_i(0) &= a_{i0} \\ p_i(0) &= p_{i0} \end{aligned}$$

In particular, the aggregate utility is the sum of the stream derived from consumption, $c_i$, net of the impact of disutility of labor, and the inflation gap:

$$U(c_i,l_i,\pi_i) = u(c_i) - f(l_i) - \tfrac{\eta}{2}\left(\pi_i - \pi^*\right)^2 \tag{1}$$

In more detail, $l$ represents the labor force units, $\pi_i - \pi^*$ is the current inflationary gap from its target, $\pi^*$, being $\eta$ the degree to which agents dislike to deviate in the current price-setting from the intended rate of inflation. Moreover, as standard in the literature, the disutility of labor is described by the following form $f(l) = \frac{l^{1+\psi}}{1+\psi}$, where $\psi>0$ is a measure of the preference of leisure in utility. Without any loss of generality, we can assume a standard CES (i.e., with constant elasticity of substitution) utility function of the form $u(c) = \frac{c^{1+\phi}-1}{1+\phi}$, where $\Phi>0$ is the inverse of the intertemporal elasticity of substitution.

Additionally, in defining the budget constraint, variation of real financial wealth, $a_i$, equals real interest earnings on wealth, $(R-\pi_i)a_i$, plus disposable income, $\frac{p_i}{p}y(l_i)$, net of the consumption expenditure, $c_i$, and tax earnings, $\tau$. Here, $R$ is the nominal interest rate, $y(l_i)$ and the amount of goods produced according to a production function using, $\frac{p_i}{p}$ is the share of the price faced by agent $i$ with respect to the average level of price, $P$, and $\tau$ the amount of real lump-sum taxes.

As a novelty of the standard analysis, that considers the production function linear in labor as the only input, we assume instead $y(l_i) = l_i \cdot l_a^\theta$, to characterize the presence of a negative externality factor in the production function, $l_a^\theta$, being $\theta<0$ the externality rate. In detail, we are assuming that there exists a negative factor, that which we call climate change externality, that may negatively affect labor force productivity (in terms of health or work effects) and thus reduce production of goods and therefore expected national income.

Finally, recall that interest rate, $R$, is set according to a standard Taylor principle as in Benhabib et al. [30, 31], to ensure that:

$$R(\pi) = \overline{R}e^{\left(C/\overline{R}\right)(\pi-\pi^*)} \tag{2}$$

being $C = R'\left(\pi^*\right)$ and $\overline{R} = R\left(\pi^*\right)$ two positive constants.

A Ricardian monetary-fiscal regime is also assumed, implying that tax revenues correspond to a fraction of total assets:

$$\tau(a) = \alpha a \tag{3}$$

where the marginal tax rate $\alpha \equiv \tau'(a) \in (0,1)$.[2]

Recalling that in a symmetric equilibrium consumption equals the level of income, $c = y(l)$, it is easy to derive the following three-dimensional system ($S$) of differential equations (see reference by Barnett et al. [19] for the full details of the derivation):

$$\begin{aligned} \dot{\mu}_1 &= (\rho - R + \pi)\mu_1 \\ \eta\dot{\pi} &= \rho(\pi - \pi^*)\eta - c(\mu_1)\left[(1-\phi)\mu_1 + \tfrac{\phi}{1+\vartheta}c(\mu_1)^{\psi - \frac{\theta}{1+\theta}}\right] \\ \dot{a} &= (R-\pi)a - \tau, \end{aligned} \tag{S}$$

It is interesting to observe that when inflation is at the targeted rate, $\pi^*$, then $\mu_1^*$ exists and is unique, it follows that at the steady state $P^* \equiv \left(\mu_1^*, \pi^*, a^*\right)$ the triplet $\left(\mu_1, \pi, a\right)$ is such that $\dot{\mu}_1 = \dot{\pi} = \dot{a} = 0$.

Therefore, we can compute the steady-state level of consumption, that is:

$$c^* = \left[\tfrac{\phi-1}{\phi}(1+\theta)\right]^{\frac{1}{\Psi+\Phi-\frac{\theta}{1+\theta}}} \tag{4}$$

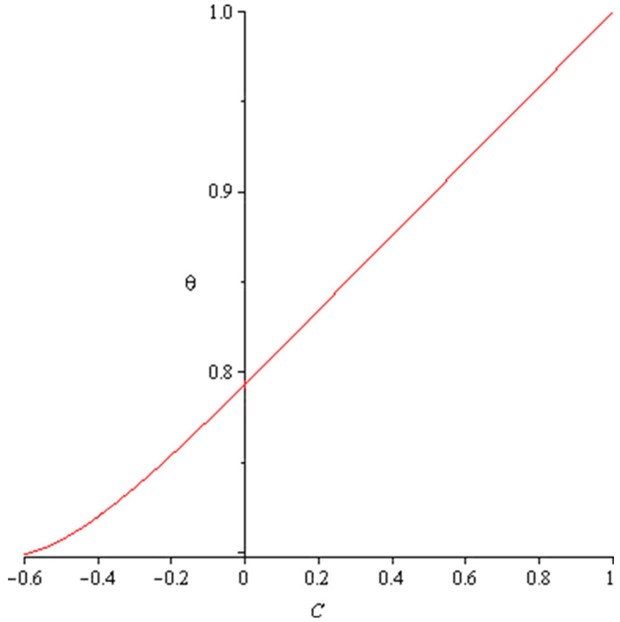
which is clearly an increasing function of the externality, θ, as shown in Figure 1.

But since $\theta$ is constrained to be negative, this confirms that the higher the negative impact (moving on the left of the $x$-axis in Figure 1) on labor productivity due to climate change external effects, the lower the consumption rate, and therefore the lower the growth rate of the

[1] The reference model contains all mathematical derivation.

[2] The fiscal regime is assumed to be Ricardian when it ensures that total government liabilities converge to zero in the present discounted value of the required variables.

**Figure 1**
**The profile of consumption under externality variations**




economy, which captures our starting idea of determining a negative link between an external factor negatively affecting labor productivity and therefore consumption opportunities.

The dynamics around the equilibrium can therefore be studied. Let $\mathcal{J}$ denote the (3×3) Jacobian matrix of system (*S*), evaluated at the steady state, $P^*$. Hence,

$$\mathcal{J}=\begin{bmatrix} 0 & \left(1-R'\left(\pi^*\right)\right)\mu_1^* & 0 \\ j_{21}^* & \rho & 0 \\ 0 & 0 & \overline{R}-\pi^*-\tau'\left(a^*\right) \end{bmatrix} \tag{5}$$

where $j_{21}^* = (1-\phi)\left(c_{\mu_1}^*\mu_1^* - c^*\right) - \frac{\phi[1+\psi(1+\theta)]}{(1+\theta)^2}c^{*\psi\frac{\theta}{1+\theta}}$.

The eigenvalues of $\mathcal{J}$ are the solutions of the characteristic equation:

$$det\,(\lambda\mathbf{I}-\mathcal{J}) = \lambda^3 - Tr(\mathcal{J})\lambda^2 + B(\mathcal{J})\lambda - Det(\mathcal{J})$$

where **I** is the identity matrix and:

$$Tr(\mathcal{J}) = \rho + \overline{R} - \pi^* - \tau'(a^*) \tag{6}$$

$$Det(\mathcal{J}) = [\overline{R} - \pi^* - \tau'(a^*)]\,[R'(\pi^*) - 1]\mu_1^* j_{21}^* \tag{7}$$

$$B(\mathcal{J}) = \left[R'\left(\pi^*\right)-1\right]\mu_1^* j_{21}^* + \left[\overline{R}-\pi^*-\tau'\left(a^*\right)\right]\rho \tag{8}$$

are Trace, Determinant, and Sum of principal minors of $\mathcal{J}$, respectively.

We proceed now to highlight the conditions to obtain a Shilnikov attractor around the steady state, $P^*$.

**Definition 1.** *Consider a generic dynamic system:*

$$\frac{dy}{dt} = g(y,\mu), \quad y \in \mathbb{R}^3, \quad \mu \in \mathbb{R}^1$$

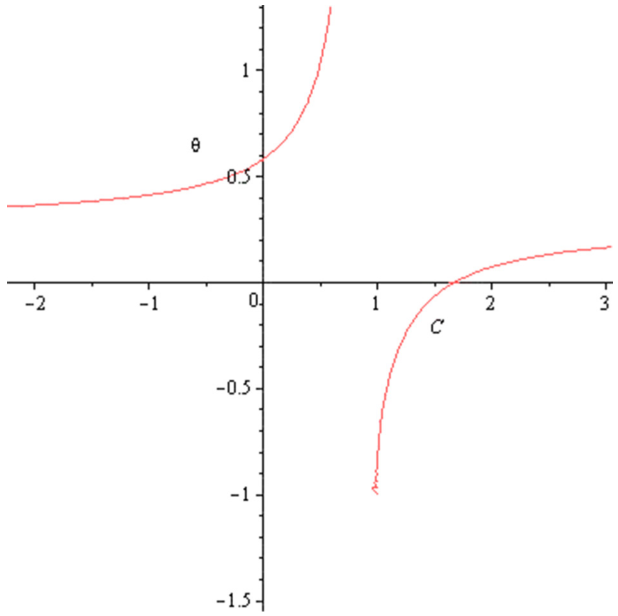
*where g is sufficiently smooth. Assume g has a hyperbolic saddle-focus equilibrium,* $\hat{y}=0$, *at the chosen bifurcation parameter* $\widehat{\mu}$, *such that the eigenvalues of the associated Jacobian matrix,* $\mathcal{J}=Dg$, *are in the form* $\beta$ *and* $\gamma \pm \omega i$, *with* $\beta\gamma<0$, *and the associated saddle quantity is* $\sigma = |\beta| - |\gamma| \neq 0$. *Let:*

$$\Theta \equiv B(\mathcal{J}) + Tr(\mathcal{J})^2 = 0 \tag{9}$$

*be the standard bifurcating condition. If we can determine that* $\frac{\partial\Theta}{\partial\widehat{\mu}} \neq 0$ *then a sequence of chaotic cycles (called Smale horseshoes) emerges around the homoclinic orbit that connects the saddle-focus equilibrium.*[3]

When we apply Equation (9) to system (*S*), we set all parameters as in the standard New Keynesian models, whereas we keep free the pair ($C$, $\theta$), so that Equation (9) reduces to the form $\Theta \equiv \Theta(C, \theta) = 0$, which is represented in Figure 2.

**Figure 2**
**The Shilnikov bifurcation area**




The graph shows that a very narrow combination of the pair of parameters ($C$, $\theta$) exists that allows to be simultaneously in presence of an active monetary policy ($C > 1$) and with a negative climate externality ($\theta < 0$), for the onset of the saddle-focus equilibrium implied by the Shilnikov chaos. Therefore, the region at which a damping oscillation of the equilibrium trajectories occurs when $C \in (1,1.65)$ and $\theta \in (-1,0)$.[4]

Additionally, since Equation (9) monotonically increases in $\theta$ (with $\frac{\partial\Theta}{\partial\theta} > 0$ and $\frac{\partial\sigma}{\partial\theta} > 0$), the requirements of Definition 1 for the equilibrium $P^*$ to exhibit a saddle-focus dynamics along the Shilnikov chaotic attractor are guaranteed in the expected parametric area where $C \in (1,1.65)$. This result suggests some interesting economic insights regarding the restrictions obtained on the chosen parameters. Given the presence of negative climate externality on the labor force input, if the central bank pushes forward its monetary stance with an active

[3] The full derivation of the mathematical conditions for the presence of stable or saddle-focus eigenvalues is outlined in Appendix 1.

[4] An active monetary policy is, by definition, an action that >1% increase in the nominal interest rate, $R$, to a 1% increase in inflation, $\pi$, that can be mathematically represented as a derivative greater than unity, $\frac{\partial R}{\partial \pi} = R'(\pi) = C > 1$. Additionally, assuming a negative climate externality requires a parameter that describes a negative ratio to represent the fractional impact, between 0 and 1 ($-1 < \theta < 0$), of the damage produced to the variable being considered, that is in our case the productivity of labor forces.

monetary policy to lower the expected inflation, thereby increasing the interest rates, the economy will react by reducing production activities, leaving the market with less created jobs. Workers may therefore face a trade-off between achieving a higher wealth today, if a mild intervention on interest rates is conducted, or leaving space to more productions, which also means more severe climate impact on workers' health. A chaotic dynamic wave then appears and might become an issue for policymakers'' agenda devoted to avoiding irregular cycles and implement actions aimed to guarantee economic stability.

The numerical computations of the chaotic attractor via a standard machine software can be lengthy and time consuming. We need then to find a mathematical tool that simplifies the equations to work with. In detail, following Freire et al. [32] and Wiggins [33], we can transform S in the following hypernormal (truncated) form detailed in system *N*:

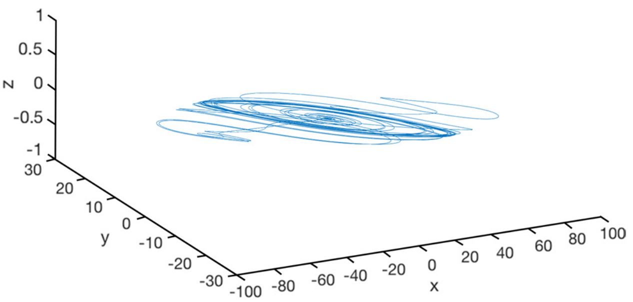
$$\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{pmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ \varepsilon_1 & \varepsilon_2 & \varepsilon_3 \end{bmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ dx^2 + kx^3 \end{pmatrix} \qquad (N)$$

being $(x, y, z)$ the new set of coordinates, whereas $d$ and $k$ are coefficients of the original non-linear terms, and $\varepsilon_1 = \mathrm{Det}(\mathcal{J})$, $\varepsilon_2 = -\mathrm{B}(\mathcal{J})$, $\varepsilon_3 = \mathrm{Tr}(\mathcal{J})$ are the standard so-called unfolding parameters.

We present now an example to corroborate our results.

**Example 1.** *Concentrate on the case where monetary policy is active (i.e., $C > 1$). Consider the set of the parameters $M \equiv \{(\beta, \eta, \kappa, \phi, \psi, \rho, \Phi)\} \equiv (1.975,350,0.90899,21,1,0.018,2)$, $(\overline{R}, \pi^*) = (0.06,0.042)$ and $(\tau', C)$=(0.15,1.5) as in Barnett et al. [19], which are all standard in the related monetary policy literature, and assume $\theta = -0.5$, joint with the initial conditions $(x, y, z) = (0.01,0.01,0.01)$. The structure of eigenvalues is $\lambda_1 = -0.4819999999$ and $\lambda_{2,3} = 0.008999999926 \pm 1.773157988i$, which satisfies the requirements of an equilibrium characterized by a saddle-focus dynamics with a positive saddle quantity described in Definition 1. The attractor generated by this example is represented in the following Figure 3.*

**Figure 3**
**The chaotic attractor**

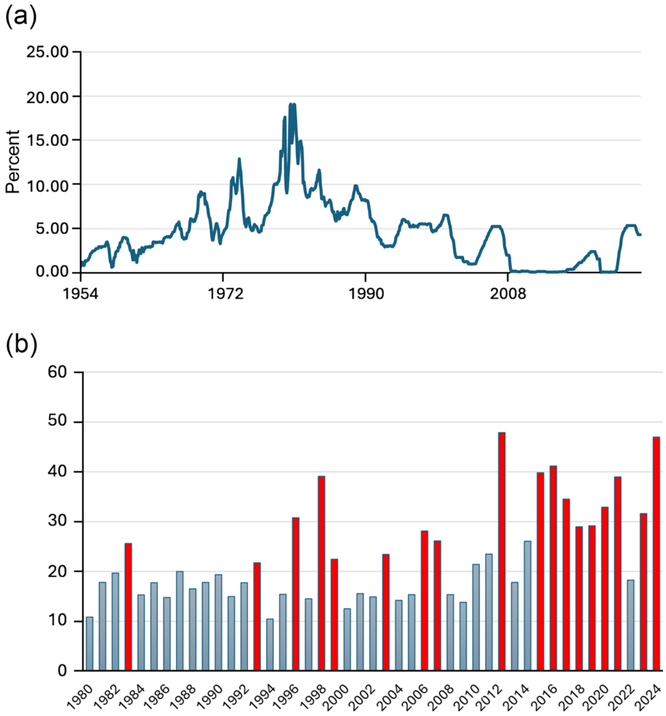



It is interesting to observe the impact of the externality $\theta$ on parameter $C$, which is the measure of the degree of monetary policy. By substituting Equations (6) and (8) into Equation (9), we can easily derive from the Shilnikov condition that:

$$C = 1 + \frac{\left[\rho + \overline{R} - \pi^*(a^*)\right]^2 - \left[\overline{R} - \pi^* - \tau'(a^*)\right]\rho}{c^{*1-\Phi}\left\{\left(\frac{\phi-1}{\Phi}\right) - \frac{\phi[1+\psi(1+\theta)]}{(1+\theta)^2} c^{*\psi-1-\frac{\theta}{1+\theta}}\right\}}$$

where clearly $\frac{\partial c}{\partial \theta} < 0$, meaning that the more negative the externality becomes, the externality the more active monetary policy has to be. Hence, central banks need to respond to unwanted negative effects on labor productivity due to climate change by continuously rising the interest rate, though this might lead to a chaotic scenario. It becomes therefore crucial to determine the condition for avoiding the indeterminacy problem driven by the chaotic outcome. This is done in the following section. To give a hint to a rationale for the model assumption, the diagrams reported in Figure 4 clearly depicts the direct connection that our research question theoretically wants to show throughout the paper. Indeed, all the moments where the monetary policy has been active, with a raising setting of nominal interest rates, can be associated with periods in which the climate indexes for the U.S. economy show an increase in the damaging conditions, as described by the U.S. Climate Extreme index, which describes (in red) the events of climate anomalies, such as rainfalls or drought seasons, that where expected in normal weather forecasts.

**Figure 4**
**(a) The evolution of federal fund rate in U.S. and (b) the climate extreme index**




This basically confirms that if productions are expanded at the expenses of the climate, this may produce a negative impact both on the health conditions of workers but and also on inflation in the market for goods, which consequently implies a move of the central banks to raise the interest rates to cool down the price increase.

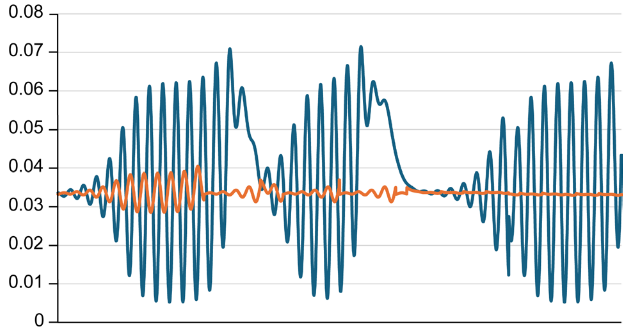
## 3. The Chaos Control Algorithm

As shown by Bella and Mattana [34] for chaos in presence of financial bubbles, one powerful tool for controlling the chaotic dynamics and pushing the economy to the unique (stable) equilibrium is the method proposed by Ott et al. [27], commonly named OGY. This algorithm needs the modification of a chosen control parameter, in our case the externality ($\theta$), to force all eigenvalues of system (*S*) to present a negative real part (see reference by Bella and Mattana [34] for the full details of the mathematical procedure).[5]

At this scope, the two different scenarios related to the profile of inflation are shown in Figure 5, where the time path of $\pi_t$, is reported

[5] A sketch of the mathematical derivation of the OGY algorithm in presented in Appendix 2.

**Figure 5**
**The pattern of uncontrolled and controlled inflation**




both when chaos emerges (the blue curve), when, given all other parameters as in Example 1, $\theta = -0.5$, and when the stabilization is achieved through the OGY algorithm (red curve), when $\theta = -0.0005$ which permits to derive the following desired sequence of real (and negative) eigenvalues: $\lambda_1 = -7.096442388$, $\lambda_2 = -0.4820000001$, $\lambda_3$ $-7.078442388$.

It is noteworthy that once the stabilization of the economy is achieved, the intertemporal profile of inflation exhibits no bursts, and therefore inflation will converge steadily to the long run steady-state solution. In terms of monetary policy action, this might suggest that once control over the negative climate effects is done, the necessity for active monetary policy is mitigated. Thus, interest rates can start to lower because inflation returns under control without negative impacting on the level of investments and production, which also allows to expand the aggregate demand and consumption and restores a balanced growth rate of the economy. The interpretation of this result is straightforward, suggesting that a policy intervention to lower the negative impact of climate change on labor productivity might restore stability by pushing the economic activity even though the monetary policy is aggressive in fighting inflation. This implies that a positive government intervention to subsidize the green sectors to preserve labor force's health may crowd-out the negative impact on the productive sector due to an increase in interest rates to turn down inflation.

## 4. Conclusions

We have shown that the Shilnikov theorem in a New Keynesian economy with negative externality on labor force due to adverse climate change can produce a chaotic attractor in a possible region of the model parameters. This has different policy implications in terms of policy actions to be chosen to restore stability, because they might be misleading and produce instead some non-controllable long-run fluctuations in the transitional dynamics of the model. This sounds as a possible confirmation of the possibility for different countries, similar in their economic fundamentals, to evolve at different growth rates in the long run. In this regard, Figure 2 has identified a critical surface, showing that if parameters are chosen in the narrow region of active monetary policy, $C \in (1,1.65)$, and negative externality, $\theta \in (-1,0)$, then the transitional dynamics towards the equilibrium resembles a saddle-focus with an oscillating pattern that produces a structure of cycling waves around the equilibrium. The global analysis of this phenomenon implies that, for any given set of initial conditions, the economy moves off the targeted equilibrium along the unstable manifold of the saddle-focus and will begin to oscillate around the equilibrium. This possible outcome is grounded on the real-life data showing a possible link of an active intervention by central banks in raising the nominal interest rates and the possibility for adverse climate conditions, as shown in Figure 4.

As it is a common result in the related literature, chaotic dynamics may eventually appear in highly nonlinear models. In this situation, the sensitivity of transitional dynamics to the initial economic fundamentals makes it impossible to the representative agent that forms its decisions under rational expectations to forecast the future outcomes of economic variables. The simulation presented in Figure 3 has shown that equilibrium trajectories may follow different oscillating patterns forming an attracting region around the intended equilibrium, as implied by the critical surface associated to the Shilnikov theorem. This also means also that any governmental intervention that is planned to restore the economic stability may produce completely different scenarios, even if applied to economies with similar initial endowments, and result instead in an undesired trapping region. As suggested in the final section, the achievement of a (stable) unique steady state can be obtained if the economy is be driven back along the stable path of the equilibrium trajectories. To do so, the application of the OGY algorithm has permitted to characterize the level of the chosen policy parameter (in our case, the externality parameter) as the appropriate policy instrument to be implemented for controlling the emergence of aperiodic dynamics, restore saddle-path stability, and avoid the cycles implied by Shilnikov chaos.

Suggestions coming from this study to policymakers facing the event of an economic crisis coming from climate change and energy shortage, with the associated negative impacts on job creation and economic development, joint with the preservation of price stability, if decide to increase the nominal interest rate to favor the national bonds to sustain the economy, which may produce unwanted and unstable fluctuations, would instead first intervene to create the monetary stance to favor the reduction of the negative environmental externality, at the scope of reducing the possibility of a chaotic dynamics around the intended long run equilibrium, which is no more secured by the only active monetary intervention.

A possible evolution of the present version of the model should be aimed to reformulate the analysis by studying the possibility of heterogeneity between the economic agents, and thus avoid the limitations present in the representative agent framework. This may also serve to endogenize the stochastic process of randomly distributed productivities of workers and so investigate the conditions that guarantee the economy to avoid possible unwanted fluctuations. We leave these possible considerations to future research.

## Conflicts of Interest

The author declares that he has no conflicts of interest to this work.

## Data Availability Statement

The Federal Reserve Bank of St. Louis data that support the findings of this study are openly available at https://fred.stlouisfed.org/series/FEDFUNDS. The National Centers for Environmental Information of USA data that support the findings of this study are openly available at https://www.ncei.noaa.gov/access/monitoring/cei/graph.

## Author Contribution Statement

**Giovanni Bella:** Conceptualization, Methodology, Software, Validation, Formal analysis, Investigation, Resources, Data curation, Writing – original draft, Writing - review & editing, Visualization, Supervision, Project administration.

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# APPENDIXES

## Appendix 1: saddle focus eigenvalues

Let $\mathcal{J}$ be the Jacobian matrix of system (*S*), evaluated at the equilibrium. The eigenvalues of $\mathcal{J}$ are the solutions of the following characteristic equation:

$$det(\lambda \mathbf{I} - \mathcal{J}) = \lambda^3 - Tr(\mathcal{J})\lambda^2 + B(\mathcal{J})\lambda - Det(\mathcal{J}) \tag{A.1}$$

which can be solved with the application of Cardano's formula for a cubic equation, producing the following roots:

$$\lambda_1 = -\frac{a}{3} + u + v$$

$$\lambda_{2,3} = -\frac{a}{3} - \frac{u+v}{3} \pm \sqrt{3}\frac{u-v}{2}i$$

where $i = \sqrt{-1}$ is the imaginary unit, while $u = \sqrt[3]{-\frac{q}{2} + \sqrt{\Delta}}$ and $v = \sqrt[3]{-\frac{q}{2} + \sqrt{\Delta}}$, given the discriminant $\Delta = \left(\frac{p}{3}\right)^3 + \left(\frac{q}{2}\right)^2$, with $p = \frac{3b-a^2}{3}$ and $q = c + \frac{2}{27}a^3 - \frac{ab}{3}$, assuming that $a = -Tr(\mathcal{J})$, $b = B(\mathcal{J})$, and $c = -Det(\mathcal{J})$.

The equation in Equation (A.1) exhibits: (*i*) one real root ($\lambda_1$) and a pair of complex conjugate eigenvalues when $\Delta > 0$, and (*ii*) three real distinct solutions if $\Delta < 0$. Case (*i*) is necessary in our paper for the emergence of the saddle-focus equilibrium implied by the Shilnikov theorem. Case (*ii*) can be used for the correction mechanism implied by the OGY algorithm to end the chaotic motion and stabilize to regular frequency the equilibrium dynamics.

## Appendix 2: the OGY algorithm

The algorithm for proving the controllability of a given system requires that the nonlinear system be written in state-space notation. We first put the linear part of system (*S*) in the following form:

$$\dot{\mathbf{w}} = \mathcal{J}\mathbf{w} + \mathbf{MKw} \tag{A.2}$$

where $\mathbf{w} = (w_1, w_2, w_3)^T$, while $\mathcal{J}$ is as in Equation (5). Moreover, $\mathbf{M} = \left(\frac{\partial \dot{w}_1}{\partial \theta}, \frac{\partial \dot{w}_2}{\partial \theta}, \frac{\partial \dot{w}_3}{\partial \theta}\right)^T$, while $\mathbf{K} = (k_1, k_2, k_3)$ is a $(1 \times 3)$ vector. System (A.2) is then put into the following first-companion form:

$$\dot{\boldsymbol{\omega}} = (\mathbf{A} - \mathbf{BK})\boldsymbol{\omega} \tag{A.3}$$

where the vector $\boldsymbol{\omega} = (\omega_1, \omega_2, \omega_3)^T$ results from the transformation $\mathbf{w} = \mathbf{T}\boldsymbol{\omega}$, and $A = \mathbf{T}^{-1}\mathcal{J}\,\mathbf{T}$ is given by:

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ \varepsilon_1 & \varepsilon_2 & \varepsilon_3 \end{bmatrix} \tag{A.4}$$

as in system (*N*), and where $\mathbf{B} = \mathbf{T}^{-1}\mathbf{M}$. In detail, the transformation matrix $\mathbf{T}$ must be chosen to satisfy the product $\mathbf{T} = \mathbf{NW}$, with:

$$\mathbf{N} = [\mathbf{B}, \mathcal{J}\mathbf{B}, \mathcal{J}^2\mathbf{B}] \tag{A.5}$$

and

$$\mathbf{W} = \begin{bmatrix} \varepsilon_2 & \varepsilon_3 & 1 \\ \varepsilon_3 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \tag{A.6}$$

Controllability requires matrix **N** to have full rank. Since, in our case, matrix **A** is non-degenerate, the controllability of system (*N*) by means of changes in $\theta$ is feasible and produces three eigenvalues of the characteristic equation, one of which is negative and two have positive real parts.