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# A Comparative Study of Penalized Regression Methods in Estimating and Predicting Economic Growth in Nigeria

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**Abstract:** This study investigated macroeconomic indicators-internal debt (INDT), external debt (EXDT), interest rate (RINR), exchange rate (REXR), and economy openness (OPEN) in modeling, estimating, and predicting Nigeria's economic growth (RGDP). To address multicollinearity and outlier problems, penalized least squares regression methods (LASSO, ridge, and elastic net) were employed. Results from the LASSO model indicated that INDT, RINR, REXR, and OPEN positively influenced RGDP, which contributed 4.27%, 0.40%, 0.49%, and 0.52%, respectively, whereas the influence of EXDT was negative (−0.97%). Ridge and elastic net estimations supported these results, with slight variations in coefficient magnitudes. All models emphasized the adverse effect of EXDT on RGDP. The evaluation and predictive power of the model metrics revealed that LASSO outperformed the other methods, revealing minimum root mean square error (0.2895), mean absolute error (0.2174), and mean absolute percentage error (2.13%), while explaining 91.9% variations in RGDP. Consequently, the results affirmed LASSO penalized technique as the most efficient variable selection with stable prediction under correlated and highly extreme dataset. Overall, the findings highlighted the crucial roles of internal borrowing and economy openness as growth-driven predictors, while stressing the adverse implications of reliance on foreign loans. Hence, the government should ensure proper management of the internally borrowed funds and channeled such funds by investing heavily in infrastructure, technology advancement, and innovation systems to maximize the growth benefits of the loans; application of penalized regression methods, particularly LASSO, should be mainstreamed into economic forecasting units within the government and research institutions to improve evidence-based policy formulation.

**Keywords:** economic growth, debt, monetary policy, trade openness, penalized regression methods

## 1. Introduction

The economy of Nigeria has experienced several downturns, and as such, economic predictions by scholars and researchers are made very difficult. This is fueled by the effect of structural characteristics, outcomes, and contradictions inherent in the economy [1]. In 1970s, the neglect of an agricultural-based economy for crude oil as the economic growth drivers and the intension economic diversification for better productivity and sustainability failed to yield the desired result. The status of the country based on developmental indices indicated that the citizens still lack basic necessities for living (NBS, 2023). According to [1], it is opined that the economy is challenged by the unsustainable and inconsistent growth that is subject to the uncertainty and crashes experienced in the prices of crude oil on the world market, which is the mainstay of the Nigerian economy. It is on the basis of this crude oil price that the debt, either external or internal, exchange rate, interest rate, and the openness of the economy measured by the import-export ratio are determined. The inconsistency, fluctuation, and the dynamic nature of the aforementioned macroeconomic variables affect economic growth.

Despite the proliferation of plans, policies, visions, and reforms targeting key macroeconomic variables such as trade openness, interest, and exchange rates, as well as external and domestic debt management introduced at various times with high hopes, their practical outcomes have remained limited or negligible. This persistent gap between policy formulation and implementation highlights the level of economic maladministration and rooted corruption, which collectively contribute to the country's weak economic performance. Poverty, therefore, has become a persistent and largely avoidable condition that continues to affect the majority of Nigerians. The failure of political leaders and policymakers to demonstrate sustained commitment, discipline, and pragmatic approaches has further constrained the prospects for meaningful economic growth and development [2].

[3–5] emphasized that policymakers and regulatory authorities can enhance economic performance by adopting appropriate exchange rate management. It must be noted that interest rate is one of the main drivers of the economy, although indirectly through investment. Thus, it can be opined that both interest rate and exchange rate enhance economic competitiveness, and as

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such, a feasible policy formulation and direction for better performance are required. It can be further emphasized that monetary policy rates are poised to boost the economy, thereby a call to adopt an appropriate monetary policy to drive the economy.

Also, [6–8] posited that external debt accumulation and debt servicing affected the economy unfavorably and the need for the government to address external debt acquisition diverted away from infrastructure development to grow the economy. It is further stressed that trade policy liberalization must be pursued to ensure sustainable growth. [9, 10] opined that trade openness is a good driver of the economy with a positive impact. Also asserted is the need for export-led diversification to enhance growth when importation exceeds exportation. Thus, the dependency of the macroeconomic variables such as debts, exchange rate, interest rate, and openness economy on each other makes prediction of the economic growth unstable and unreliable using the least squares method. In this situation, an alternative estimation technique such as the penalized least squares method is proposed to efficiently predict the economic growth.

## 2. Literature Review

According to [11, 12], penalized regression methods have high prediction accuracy and computational efficiency that make it appropriate for this study. It was also opined that in similarity with the ordinary least squares method, the residual sum of squares of the penalized least squares regression methods is minimized to obtain estimated coefficients of the model by placing a constraint on the size of the regression coefficients. Thus, the constraint or penalty on the size of the regression causes biased estimates that can lead to improvement in the overall predictive error of the fitted model and reduce the variance of the estimates. [12–14] posited that penalized least square deals with the selection of model subsets to improve predictive accuracy by sacrificing a small quantity of the bias for discarding variable(s). Thus, this implies a continuous process of variable or model selection approach known as regularization or shrinkage method that comprises of ridge least squares method, least absolute shrinkage and selection operator (LASSO), and elastic net least squares method.

[15] focused on identifying and retaining factors that contribute immensely to the economic growth in Nigeria based on some variable selection methods such as stepwise regressions, partial least squares regression, and LASSO. In this study, twelve factors were available to predict economic growth. The results revealed that stepwise regressions were efficient in the presence of multicollinearity. Also, it was observed that partial least squares and the LASSO model efficiently and significantly identified the positive impact of oil revenue, non-oil revenue, and capital expenditure on transfers as Nigeria's economic growth predictors and, as such, retained them. [16] assessed the impact of tariff reductions on fluctuations in customs revenues in Vietnam using the data collected from the government's web portal and the World Bank's website between (2002 and 2017). The LASSO regression method was employed to estimate and predict the relationship between tariff reductions and customs revenue. The results revealed that tariff reductions positively contributed to customs revenues. It was also found that a reduction in tariffs led to an increase in import turnover and the level of compliance with tax laws by import–export enterprises and a decrease in smuggling and trade fraud.

[17] examined the factors influencing the profitability indicators of the banking sector in Turkey. The study considered variables such as securities portfolio, equity, NPL ratio, asset share, deposit, inflation, and gross domestic product as factors affecting the return on assets (ROA) and return on equity (ROE) ratios for the top 10 deposit banks in 2020, based on their asset size. Using the LASSO regression method, it was found that all variables considered as factors, except deposit, did not had significant effect on the ROA and ROE ratios, which are measures of profitability. [18] investigated whether augmenting models with the variance risk premium and Google search data improved the quality of the forecasts for real oil prices (2007–2019). Based on the high volatility in the oil market, evidence showed that adopted penalized regression methods provided the best forecasting performances across most of the forecasting horizons. Moreover, it was found that a hierarchical vector auto-regressive model estimated with the LASSO provided more robust forecasts from the Google trend data.

[19] investigated the application and comparative performance of multifactorial machine learning models in forecasting stock prices, following a rigorous screening of influential determinants. The analysis incorporated four linear approaches: Ordinary Least Square (OLS) regression, Lasso regression, ridge regression, and elastic net regression alongside a nonlinear ensemble method, XGBoost. Empirical findings indicated that the nonlinear model consistently outperformed its linear counterparts, yielding predicted returns with stronger correlations to observed market returns. Overall, the results provide evidence that integrating machine learning techniques with multifactor models holds significant potential for advancing quantitative investment strategies.

[20] investigated an accurate prediction of stock price movement as it remained a critical challenge facing financial markets. In this study, high dimensionality, nonstationarity, and nonlinear characteristics of financial data were identified as the major problems in the prediction of stock price. To address these, an integrated method that leverages both technical and sentiment indicators to ensure efficient predictive accuracy was proposed. Multicollinearity among explanatory variables was addressed using an enhanced long short-term memory (LSTM) model constructed based on LASSO. Thus, the proposed LASSO-LSTM model used the selected variables as input features and was evaluated against baseline models in terms of accuracy and robustness. Hence, results indicated that the predictive accuracy of LASSO-LSTM improved with an average of 8.53% confirming the effectiveness of combining sentiment and technical information with variable selection strategies. [21] examined the forecast values for stock market out-of-sample in finance and emerging markets using regularized regression training techniques. The regularized regression training models adopted to forecast the equity premium out-of-sample in the study were ridge, forward-backward ridge, LASSO, relaxed LASSO, elastic net, and least angle regression. Thus, evaluation results revealed that the ridge performed better than other identified methods, while the LASSO appeared to most economically meaningful because it provided timely information on mean-variance portfolio investment for the investors operating in the market to make a profit with lower risk.

[22] identified financial risk in the banking sector by presenting a novel approach that used historical financial ratios data sourced from the FDIC database to predict bank failures in the United States. In the study, a novel hybrid approach to loss estimation in the context of bank failures was employed. Elastic net regression and relevant data extraction techniques are combined as a method to improve the predictive accuracy. The performance of the method was evaluated in comparison with the conventional regression techniques. Thus, the mean squared error, mean absolute error, a significantly high R-square value, and a

high value for the explained variance score revealed the superior performance of the hybrid approach when compared with the other existing method. [23] employed principal component analysis (PCA) and hierarchical regression to examine the influence of macroeconomic variables on Kenya's economic performance. In the investigation, 18 macroeconomic indicators spanning the period 1970–2019 were obtained from the Kenya National Bureau of Statistics and the World Bank. PCA was applied to reduce dimensionality and extract components. Subsequently, a hierarchical regression was used to fit a model based on the extracted components for the prediction of growth trajectory.

[24] examined the model to fit and predict the potential future value for cryptocurrencies. The study used multiple linear, ridge, LASSO, and elastic net regressions as the variable selection and regularization methods. The weekly Bitcoin and Ethereum along with other variables such as stock market indices, gold and oil prices, interest rates, exchange rates, and policy uncertainty were used. Thus, from the results, it was found that the elastic net approach performed better than other methods in fitting a predictive model for the Bitcoin and Ethereum. [25] explored factors that influenced Anhui Province's economic growth. The data gathered from the Anhui Provincial Bureau of Statistics on total import and export volume, number of colleges and universities, per capita consumption expenditure of urban residents, general budget revenue, and highway mileage were used for the study. The preliminary analysis carried out on the data revealed a strong correlation between different identified factors. The factor analysis was used to select social finance and social construction as the main factors to determine the economic growth. Thus, a fitted linear regression model based on two factors showed significant influence on the economic growth.

[26] examined heterogeneous autoregressive (HAR) model as a benchmark in predicting volatility, by imposing a structured lag specification with three parameters that approximate an AR(22) process. The study employed LASSO penalized technique to facilitate the selection of a parsimonious and flexible lag structure, while also permitting additional lags beyond the stated 22 lags. In the results, in-sample analysis revealed that the most influential predictors were concentrated within the first 22 lags, which aligning with the HAR specification. Moreover, the results showed that additional predictive power was captured by incorporating more lags, as evidenced by the LASSO technique to show better or superior fit. [27] investigated the limitations and potential of LASSO-based methods when predictors showed stationarity in the short and long run. The inability of the adaptive LASSO to asymptotically remove cointegrated variables associated with zero estimated coefficients of the regression model led to the introduction of twin adaptive LASSO. This served as a post-selection modification method that restored the consistency in the variable selection even with the existence of heterogeneous regressor dynamics. The results revealed that twin adaptive LASSO satisfactorily generated consistent and estimated coefficient and variable screening that demonstrated empirical evaluation and practical relevance method for short- and long-run prediction.

[28] investigated factors influencing poverty in ECOWAS through the combination of statistical methods with economic theory to validate variable selection. In the study, the analysis was done by incorporating the Fraser Institute's Economic Freedom Index, an often-overlooked dimension in West African poverty study. LASSO and elastic net penalized regression methods employed for the study to identify relevant explanatory variables that addressed the high-dimensional data problems. The techniques were tolerated sparse solutions that helped in obtaining lower mean squared error (MSE) and providing a robust understanding of an existing relationship between economic freedom, growth, and poverty dynamics in ECOWAS. [29] examined high-dimensional data that were on the increase and prevalent in medical research due to the incorporation of electronic health records, high-throughput screening technologies, and large-scale genomic profiling. In the study, penalized estimation techniques were employed for survival analysis due to numerous predictors mostly involved to mitigate overfitting. Also, when the survival data were frequently subjected to left truncation, a penalized Cox proportional hazards framework with right-censored data was applied. The simulation experiments and a large-scale clinico-genomic dataset used for the evaluation of the method revealed the impact of left truncation adjustment on bias and inference.

[30] investigated the determinants of mortality among hospitalized patients diagnosed with COVID-19 using a machine learning-based predictive model to determine the mortality risk. In the research, patients admitted to Imam Reza Hospital, Tabriz, Iran, during the period from March 2020 to November 2021 were examined. Elastic net penalized least squares technique was applied to identify and arrange variables associated with mortality. Consequently, an artificial neural network (ANN) model was employed to predict patient risk of mortality through a predictive evaluation performance assessed using receiver operating characteristic curve analysis. Findings revealed that the area under the ROC curve obtained was 0.988, and it indicated a high discriminatory power in stratifying patients based on mortality risk. [31] research focused on developing and validating predictive models for estimating the risk of mortality and intensive care unit (ICU) admission among hospitalized COVID-19 patients. The study included all RT-PCR-confirmed adult inpatients admitted to Fujian Provincial Hospital between October 2022 and April 2023. Elastic net regression method was employed for analysis based on demographic data, clinical symptoms, comorbidities, laboratory findings, treatment details, and patient outcomes. The performance of the model was evaluated through discrimination, calibration, and overall accuracy, with AUROC values of 0.858 for ICU admission and 0.906 for mortality, thus demonstrating strong predictive model validity on the application of clinical trials.

[32] examined county-level sociodemographic factors associated with COVID-19 case rates using the social vulnerability index (SVI) and democratic voting percentages. Elastic net regression method was employed, stratifying analyses across the ten Health and Human Services regions and two periods to address collinearity and reduce overfitting. Thus, elastic net reliably outperformed multiple regression, yielding lower root mean square error and satisfactory $R^2$ value. [33] examined the integration of high-dimensional omics data, which had increased the advancement of predictive modeling in healthcare. Despite its promise, the study emphasized that effective integration was hindered by heterogeneity across data types, sequencing of information flow between omics layers, multicollinearity, and prioritization of data blocks required for prediction. Priority-elastic net, a hierarchical regression procedure that extended the Priority-LASSO technique for logistic regression, was introduced to address the problem. The technique sequentially applied elastic net methods to ordered variable blocks, incorporating fitted values as offsets, with well-determined penalty variant. The Cancer Genome Atlas glioma dataset (Lower Grade-Glioma (LGG) and Glioblastoma Multiforme (GBM)) was used for the analysis, and the results indicated a better, stable, accurate, and efficient computation of the predictive values in comparison with the competing methods.

[34] examined infant mortality rate as a widely recognized key indicator of public health status and the quality of maternal and child care services within a population. In efforts to reduce infant mortality and to offer valuable insights into identifying determinants, the study employed regularization methods comprising of ridge regression, LASSO, and elastic net to mitigate multicollinearity, arising from strong correlations among predictors. In the study, it was found that elastic net performed better when compared with the ridge and LASSO, thus emphasizing its effectiveness in addressing multicollinearity. [35] examined the dynamic interconnections of housing price shocks across the 50 U.S. states and the District of Columbia. A conventional vector autoregressive (VAR) model alongside shrinkage-based approaches comprising of elastic net, LASSO, and ridge was employed for the analysis. The techniques were used to explore the regional transmission of real housing return shocks. In the results, it was found that shocks from the other three regions were originated from the Southern states, with the Northeast primarily acting as a recipient of the shocks. The West served as a receiver from the South and as a transmitter to the Midwest and Northeast, while the Midwest transmitted to the Northeast and absorbed shocks from the South and West, thus providing insights into cyclical housing market, regional risk exposure, and strategic asset allocation for policymakers and investors.

[36] evaluated extensive analysis of stock market volatility considering more than one hundred monthly macroeconomic indicators and applying machine learning methods. The study specifically used the integration of the random forest (RF) algorithm with the LASSO technique. Findings revealed that the technique consistently demonstrated better predictive performance across diverse market conditions. [37] conducted a study to fit predictive models for a typical dataset using the ordinary least squares method and three other methods: ridge regression, LASSO regression, and principal component regression. Their study compared the results using the estimated mean square error of the aforementioned methods and found that principal component regression outperformed the others with minimal mean square error. Therefore, it was suggested that principal component regression would be optimal for building predictive models for related datasets.

Therefore, in this study, variable(s) or model selection method known as regularized or shrinkage technique is proposed to explore and fit a stable and reliable model via continuous process selection methods due to the presence of multicollinearity and outliers among the variables under consideration to predict economic growth (RGDP) for Nigeria using the identified drivers that include internal debt (INDT), external debt (EXDT), interest rate (RINR), exchange rate (REXR), and trade openness (OPEN). Thus, this approach is said to enrich the body of knowledge on various works and studies on the aforementioned macroeconomic variables in the literature.

## 3. Research Methodology

This study employs time series data of a secondary nature, comprising explanatory variables such as internal debt, external debt, interest rate, exchange rate, and trade openness, with economic growth serving as the dependent (response) variable. The data were obtained from the Central Bank of Nigeria Statistical Bulletin (1986–2023). Furthermore, in this section, a detailed explanation of the research method framework is provided, emphasizing the application of penalized least squares regression methods comprising of LASSO, ridge regression, and elastic net method. These are the extension of OLS in addition to the penalty term, and it shall be used to explore and predict economic growth using the identified colinear predictors with extreme values.

### 3.1. Panelized least squares regression model

According to [14, 38], it can be argued that response variables such as RGDP used in this study are influenced by predictors, and when there is dependency among predictors, using OLS method can lead to estimation errors and inefficient predictive value(s). Therefore, ridge regression is proposed as an alternative method for parameter estimation, introducing a penalty ($\ell$) that reduces the variance of the estimates [38, 39]. This technique effectively shrinks the correlation coefficients of the explanatory variables [13]. Without restrictions on the parameter $\beta$(s), extreme values with wide variances are obtained. To mitigate this, it is necessary to regularize the estimated parameters $\beta$(s) of the model [12].

LASSO is a penalized technique that restricts the model's parameters by restricting all the absolute values to be less than a given value. This can be done by regulating the process where the estimated parameters of the model shrink or part of the parameters are reduced to zero. In the process of selecting the parameters, variables associated with the estimated parameters that are not zero after regularization are made to be part of the variables for model specification. In this case, the forecast errors are reduced [11].

According to [40], elastic net is used jointly to regulate and select correlated variables and a group of correlated variables. Elastic net as a penalized regression technique is a single linear function that combines ridge and LASSO features. The linear equation can be estimated using a two-step method that involves determining parameter estimates for a given regression model as the first step and using the LASSO technique for regularization as a second step to reduce the unnecessary estimated values. This process may be more efficient than the LASSO or ridge as an individual penalized technique. Thus, the performance of the penalized regression techniques such as ridge, LASSO, and elastic net is to be considered in this study.

### 3.2. Shrinkage model

The specified model requires to examine the efficiency of the aforementioned penalized regression technique is expressed as:

$$Y = X\beta + \mu, \tag{1}$$

$$\mu \sim N\left(0, \sigma^2\right).$$

In this case, $Y = (y_1, y_2, y_3, \ldots, y_n)'$ is a vector of response variable observations, X denoted an nxr matrix for the predictors, $\beta = (\beta_1, \beta_2, \ldots, \beta_r)$ is a vector of estimated parameters of the model, and $\mu$ denoted the error terms of the model and its variance denoted by $var(\mu) = \sigma^2 I$ [41]. The coefficients of the regression model can be estimated using ordinary least squares through the minimization of the expression given by:

$$\underset{\beta_0, \beta}{argmin} \left\{\sum_{i=1}^{r} (y_i - \beta_0 - X_{i1}\beta_1 - \ldots - X_{ir}\beta_r)^2\right\}, \tag{2}$$

It must be noted that the estimated parameters $\beta_1, \beta_2, \ldots, \beta_r$ from the expression given in (2) are not zeros; thus, a large $r$ makes the interpretation of the estimated parameters to become a challenge. That is, $n < r$ leads to inefficient estimate of the parameters using OLS. Thus, the need for alternative estimation techniques that equate the expression under consideration to be zero. Therefore, the estimated parameters are restricted or regulated (penalty).

### 3.3. Ridge penalized regression

This method is appropriate when many endogenous variables are independent. Specifically, ridge regression is an appropriate technique when there are many explanatory variables with small estimated parameter values and regulates the estimated variance of the correlated independent variables from obtaining a large variance that can lead to the insignificance of the estimated parameters. Hence, the estimated parameters of the linear model using the ridge penalized regression method are given as follows:

$$\hat{\beta}_{ridge} = \underset{\beta}{argmin} \parallel y - \beta' X \parallel^2 + \lambda \parallel \beta \parallel_2^2, \tag{3}$$

where $\parallel y - \beta' X \parallel^2 = \sum_{i=1}^{r} (y_i - \beta' X)^2$ denoted by $\ell_2$, an average loss function, $X_i{}'$ is the ith value of vector $X$, $\parallel \beta \parallel_2^2 = \sum_{j=1}^{r} \beta_j{}^2$ the $\ell_2$ model penalty on $\beta$. Then, $\lambda$ , which is greater than or equal to zero, is a parameter that normalizes penalty in the model. Thus, a comparative influence of data that are not error independent can be examined by $\lambda$ using cross-validated approach that allows the substitution of $\lambda$ in equation (3);then, the next expression given can be obtained:

$$\underset{\beta_0, \beta}{argmin} \left\{ \sum_{i=1}^{r} (y_i - \beta_0 - X_{i1}\beta_1 \ldots - X_{ir}\beta_r)^2 \right\} \quad s.t\ \lambda \sum_{j=1}^{r} \beta_j^2 \leq t, \tag{4}$$

where parameter $t$ can be defined by the user.

Least Absolute Shrinkage and Selection Operator (LASSO)

This technique regulates the sum of squares for the loss of absolute error and constraints of the sum of the absolute value of the coefficients. According to [42] as cited in [43], the constraint has a regularizing effect on the parameters estimated. Also, it sets some to zero to provide a proper interpretation for the fitted regression model. In LASSO, the penalized technique is obtained as a sparse solution using the optimization problem given by:

$$\hat{\beta}_{lasso} = \underset{\beta}{argmin} \parallel y - \beta' X \parallel^2 + \lambda \parallel \beta \parallel_1, \tag{5}$$

where $\parallel \beta \parallel_1 = \sum_{j=1}^{r} \beta_j$, the norm penalty of $\ell_2$ under $\beta$ that represented the sparsity level and $\lambda \geq 0$ serve as shrinking parameter for the estimated model. Thus, expression (5) can be written as:

$$\underset{\beta_0, \beta}{argmin} \left\{ \sum_{i=1}^{r} (y_i - \beta_0 - \beta_{i1}\beta_1 - X_{I2}\beta_2 - \ldots - X_{ir}\beta_r)^2 \right\} \quad s.t\ \lambda \sum_{j=1}^{r} |\beta_j| \leq t, \tag{6}$$

$\ell_1$ is the penalty on $\beta$ that helps in reducing the number of components to zero, with excellent selection that can be carried out simultaneously, and as such, it is a good technique used to shrink the estimates of linear model.

### 3.4. Elastic net method

This is a penalized least squares and variable selection technique with turning parameter $\alpha \geq 0$. It is a technique that combined the features of LASSO and ridge models. Elastic net is designed to address the correlation and selection problem from ridge and LASSO penalized least squares methods and penalties value denoted by $\ell_1$ and $\ell_2$, respectively. Thus, elastic net penalized model reduced to ridge model if $\alpha = 1$. According to [41, 44], it can be emphasized that elastic net penalized technique consists of LASSO and ridge penalized techniques properties, respectively. Thus, if $\alpha = 0$, the penalty function is undefined, and if $\alpha > 0$, the penalty function will be convex. Elastic net regression technique with penalized parameters $\ell_1$ and $\ell_2$ can be expressed in the form given as:

$$\hat{\beta}_{elastic} = \left(1 + \frac{\lambda_1}{n}\right)\left\{argmin \parallel G \parallel^2 + \lambda_2 \parallel \beta \parallel^2 + \lambda_1 \parallel \beta \parallel_1\right\} \tag{7}$$

setting $\alpha = \frac{\lambda_2}{\lambda_1 + \lambda_2}$, the function expressed in (7.7) is similar to the minimization of expression given as:

$$\hat{\beta}_{elastic} = \left(1 + \frac{\lambda_1}{n}\right)\{argmin \parallel G \parallel^2 + \lambda_2 \parallel \beta \parallel^2 + \lambda_1 \parallel \beta \parallel_1 \text{ s.t } (1 - \alpha) \parallel \beta_1 \parallel + \alpha \parallel \beta \parallel^2 \leq t, \tag{8}$$

where $G = y - \beta' X$ and the elastic net penalty parameters are given as $(1 - \alpha) \parallel \beta_1 \parallel + \alpha \parallel \beta \parallel^2$.

### 3.5. Evaluation techniques

It is important to select penalized regression models by adopting suitable evaluation techniques. Thus, in this study, the evaluation techniques to be adopted are root mean square error (RMSE), mean absolute error (MAE), mean absolute percentage error (MAPE), and lambda penalty ($\lambda$), with the least adopted evaluation techniques selected.

## 4. Empirical Results

This section presents the empirical results of the fitted average penalized least squares regression models discussed under the research method. Thus, Table 1 presents the maximum likelihood estimate for LASSO, ridge, and elastic net as a penalized least squares regression technique.

In Table 1, the results of a penalized least squares regression parameter estimate for each identified economic growth drivers are presented. The economic growth drivers include internal debt (INDT), external debt (EXDT), interest rate (RINR), exchange rate (REXR), and trade openness (OPEN). These are used as drivers to determine and predict economic growth RGDP [1, 2]. This study employs the penalized regression methods comprising of LASSO, ridge, and elastic net techniques. In Table 1, the results of the LASSO technique show that the identified economic growth drivers or determinants are linearly related to RGDP. The result further shows that INDT, RINR, REXR, and OPEN are positive, and their contributions to RGDP are 4.27%, 0.40%, 0.49%, and 0.52%, respectively. EXDT is negative and causes a decline of 0.97% in RGDP during the period under study. The standard error values of 0.6663, 0.3826, 0.2305, and 0.1548 with $p$-value < 0.05 for the estimated parameters of the economic growth drivers consisting of INDT, EXDT, RINR, and REXR show the

**Table 1**
**ML estimates for the penalized least squares techniques**

| Variables | Estimate | Std error | Wald Chi-Sqr. | Prob >Chi-Sqr |
|---|---|---|---|---|
| | | LASSO technique | | |
| Constant | 10.3297 | 0.0131 | 614660 | 0.0001* |
| INDT | 4.2717 | 0.5561 | 59.0021 | 0.0001* |
| EXDT | –0.9760 | 0.2818 | 11.9886 | 0.0005* |
| RINR | 0.4007 | 0.1721 | 5.4163 | 0.0199* |
| REXR | 0.4948 | 0.1852 | 7.1348 | 0.0076* |
| OPEN | 0.5262 | 0.4691 | 1.2581 | 0.2620 |
| | | Ridge technique | | |
| Constant | 10.2914 | 0.0135 | 574342 | 0.0001* |
| INDT | 4.0771 | 0.6663 | 37.4351 | 0.0001* |
| EXDT | –1.1178 | 0.3826 | 8.5316 | 0.0035* |
| RINR | 0.7247 | 0.2305 | 9.8784 | 0.0017* |
| REXR | 0.6629 | 0.1548 | 18.3298 | 0.0001* |
| OPEN | 0.7843 | 0.5565 | 1.9863 | 0.1587 |
| | | Elastic net technique | | |
| Constant | 10.2857 | 0.0123 | 694138 | 0.0001* |
| INDT | 3.4575 | 0.5829 | 35.1736 | 0.0001* |
| EXDT | –0.9222 | 0.2935 | 9.8721 | 0.0017* |
| RINR | 0.6414 | 0.2114 | 9.2038 | 0.0024* |
| REXR | 0.5809 | 0.1335 | 18.9355 | < 0.0001* |
| OPEN | 1.1485 | 0.4905 | 5.4811 | 0.0192* |

**Table 2**
**Distribution and scaled parameter for penalized least squares techniques**

| Normal distribution parameters | Estimate | Std error | Wald Chi-Sqr. | Prob > Chi-Sqr. |
|---|---|---|---|---|
| LASSO scale | 0.1304 | 0.0149 | 76.6142 | 0.0001* |
| Ridge scale | 0.1344 | 0.0160 | 70.4006 | 0.0001* |
| Elastic net scale | 0.1222 | 0.0119 | 104.0088 | 0.0001* |

statistical significance of the identified economic growth drivers in determining and predicting economic growth (RGDP) in Nigeria.

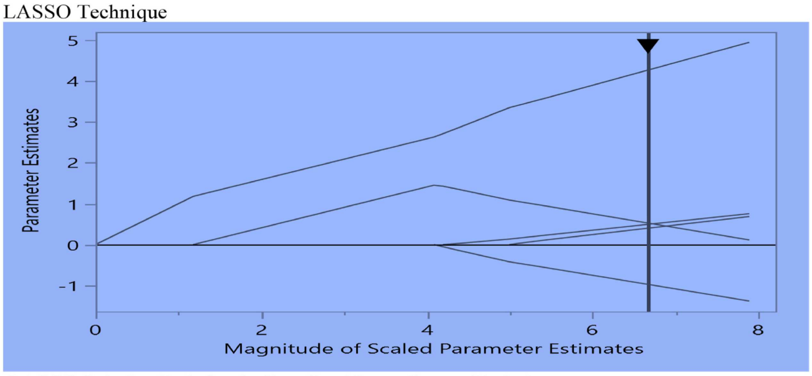
Also, in Table 1, the results of the ridge estimation technique presented reveal that the identified economic growth drivers have a linear relationship with RGDP. Specifically, the results indicate that INDT, RINR, REXR, and OPEN are positive, which contribute 4.07%, 0.72%, 0.66%, and 0.78%, respectively, to the economic growth rate (RGDP). The results also reveal that EXDT is negative and as such causes a decline of 1.11% in economic growth (RGDP) in the period under investigation. The standard error values of 0.6663, 0.3826, 0.2305, and 0.1548 and the $p$-value < 0.05 for the estimated parameters such as INDT, EXDT, RINR, and REXR evidently establish the statistical significance of the identified economic growth drivers in determining and predicting economic growth (RGDP) in Nigeria.

The elastic net estimation method reveals that economic growth drivers are linearly related to economic growth (RGDP). Thus, in Table 1, the results show that INDT, RINR, REXR, and OPEN contribute positively to the RGDP in Nigeria to the turn of 3.45%. 0.64%, 0.58%, and 1.14%, respectively. It also reveals that the contribution of EXDT is negative and, as such, leads to a decline in the economic growth rate (RGDP) by 0.92%. The standard error values of 0.5829, 0.2935, 0.2114, 0.1335, and 0.4905 and $p$-value < 0.05 for the estimated parameters of the aforementioned economic growth drivers (INDT, EXDT, RINR, and REXR) evidently show the statistical significance of the identified economic growth drivers in examining and predicting economic growth rate (RGDP) in Nigeria. In Table 2, the distribution results for the penalized least squares methods are presented.

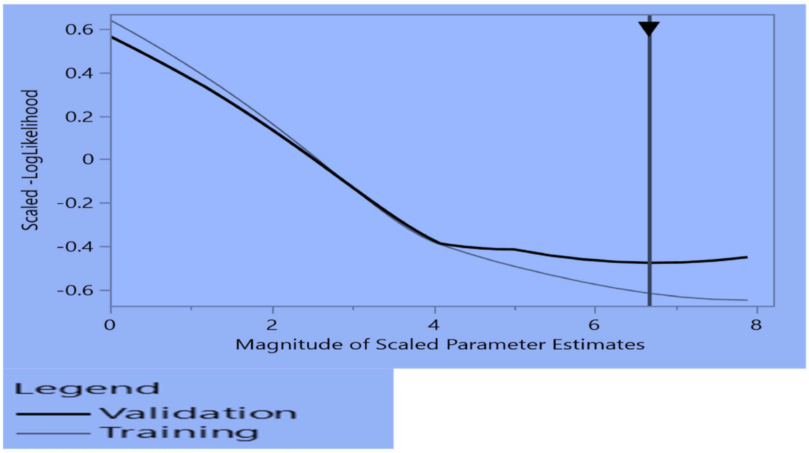
Table 2 presents the results of penalty parameter lambda for the penalized regression models such as LASSO, ridge, and elastic net technique been employed by this study. Thus, from the results, the distribution parameter lambda or estimated scale for the LASSO, ridge, and elastic net are 0.1304, 0.1344, and 0.1222, respectively. The standard error values of the techniques are 0.0149, 0.0160, and 0.0119 with $p$-value < 0.05. As such, the significance of the scale parameter is established and the need to includes penalty in modeling the identified economic growth drivers and the RGDP for the purpose of prediction. Thus, in Figure 1, the solution path for the employed penalized least squares regression techniques is shown, and it reveals the trends for the 98 training and 43 validated data used for this study. The

**Figure 1**
**Solution path for the penalized least squares techniques. a LASSO solution path for the penalized least squares techniques and cross-validation error curve.**

(a) LASSO Technique




LASSO Solution Path for the Penalized Least Square Techniques




LASSO Cross-Validation Error Curve

solution path describes the magnitude of the scaled parameters and efficiently maximizes the likelihood function of the estimates. The vertical line in the plot presented in Figure 1, indicates the optimal value for lambda. The following two plots show the display of the coefficient "path" and the sorted magnitude of the coefficients at the optimal lambda.

From Figure 1a, the plot on the above is the LASSO solution path, while the one below is the LASSO cross-validation error curve. In Figure 1b, the plot on the above is the ridge solution path, while the one below is the ridge cross-validation error curve. In Figure 1c, the plot on the above is the elastic net solution path, while the one below is the elastic net cross-validation error curve. These are presented for the three penalized least squares considered for this study. The solution (coefficient or parameter estimate) path for the LASSO, ridge, and elastic net penalized least squares models, and it reveals how the parameter estimates of the models changes as the magnitude of the scale parameter (penalty) increases. In the solution path, the penalty is low, which indicates weak penalty, and, as such, the parameter estimates assume estimates from OLS method. In a situation where some of the parameter estimates shrink faster than the others, the penalty is said to be moderate such that predictors (explanatory variables) with less explanatory power or have low contribution to the response or dependent variable are tend or lean toward zero. However, in a situation where almost all the parameter estimates shrink to zero, the penalty is high and as such the model becomes sparse. Thus, it can be emphasized from Figure 1a that LASSO penalized least squares technique produce moderate penalty as some of the parameter estimates shrink faster than others when compared with the parameter estimates from ridge in and elastic net penalized least squares methods in Figure 1b and c, respectively. In Figure 1a–c, the plot on below shows the cross-validation error plot for each penalized least methods under consideration. These are used to select the optimal penalty (magnitude of the scale parameter) and the model that generates the optimal penalty. In the cross-validation plot for the three penalized least squares, it is revealed that LASSO produces the smallest cross-validation error with minimum penalty value of 6.40 compared to the ridge and elastic net with value of 7.80 and 6.80, respectively. Hence, it can be stressed that LASSO penalized least squares method is more appropriate and efficient than the other two techniques in predicting economic growth in this study.

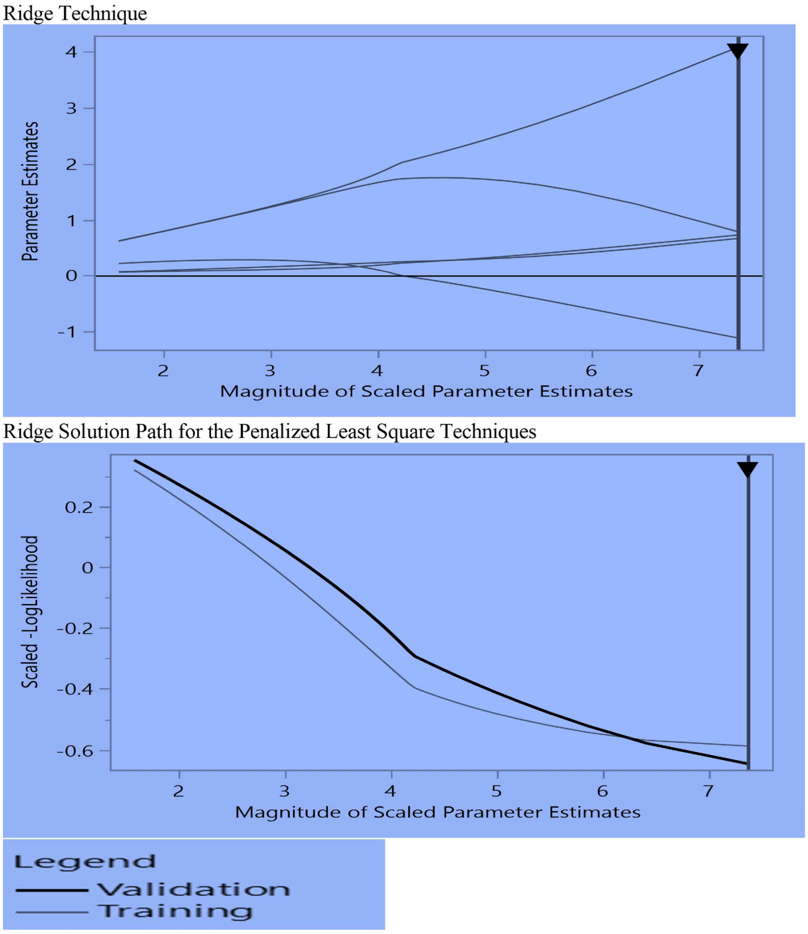
## Quantile prediction for penalized least squares techniques

The quantile prediction plot is shown in Figure 2 for the penalized regression techniques used for this study that include LASSO, ridge, and elastic net methods presented in Figure 2a–c, respectively.

**Figure 1b**

**Ridge solution path for the penalized least squares techniques and cross-validation error curve.**

(b) Ridge Technique



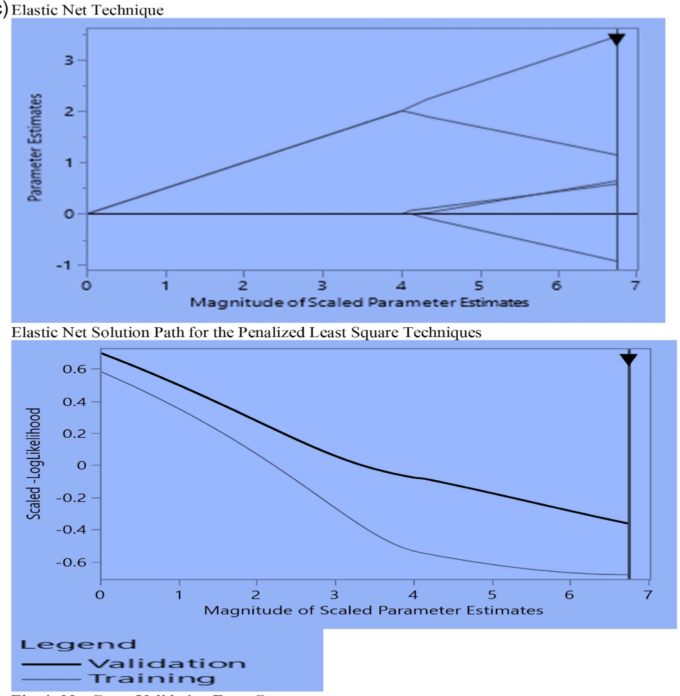

Ridge Solution Path for the Penalized Least Square Techniques

Ridge Cross-Validation Error Curve

**Figure 1c**

**Elastic net solution path for the penalized least squares techniques and cross-validation error curve**

(c) Elastic Net Technique




Elastic Net Solution Path for the Penalized Least Square Techniques

Elastic Net Cross-Validation Error Curve

**Figure 2**
**Quantile prediction plot for penalized techniques. (a) LASSO quantile prediction plot for penalized techniques. (b) Ridge quantile prediction plot for penalized techniques. (c)Elastic net quantile prediction plot for penalized techniques**

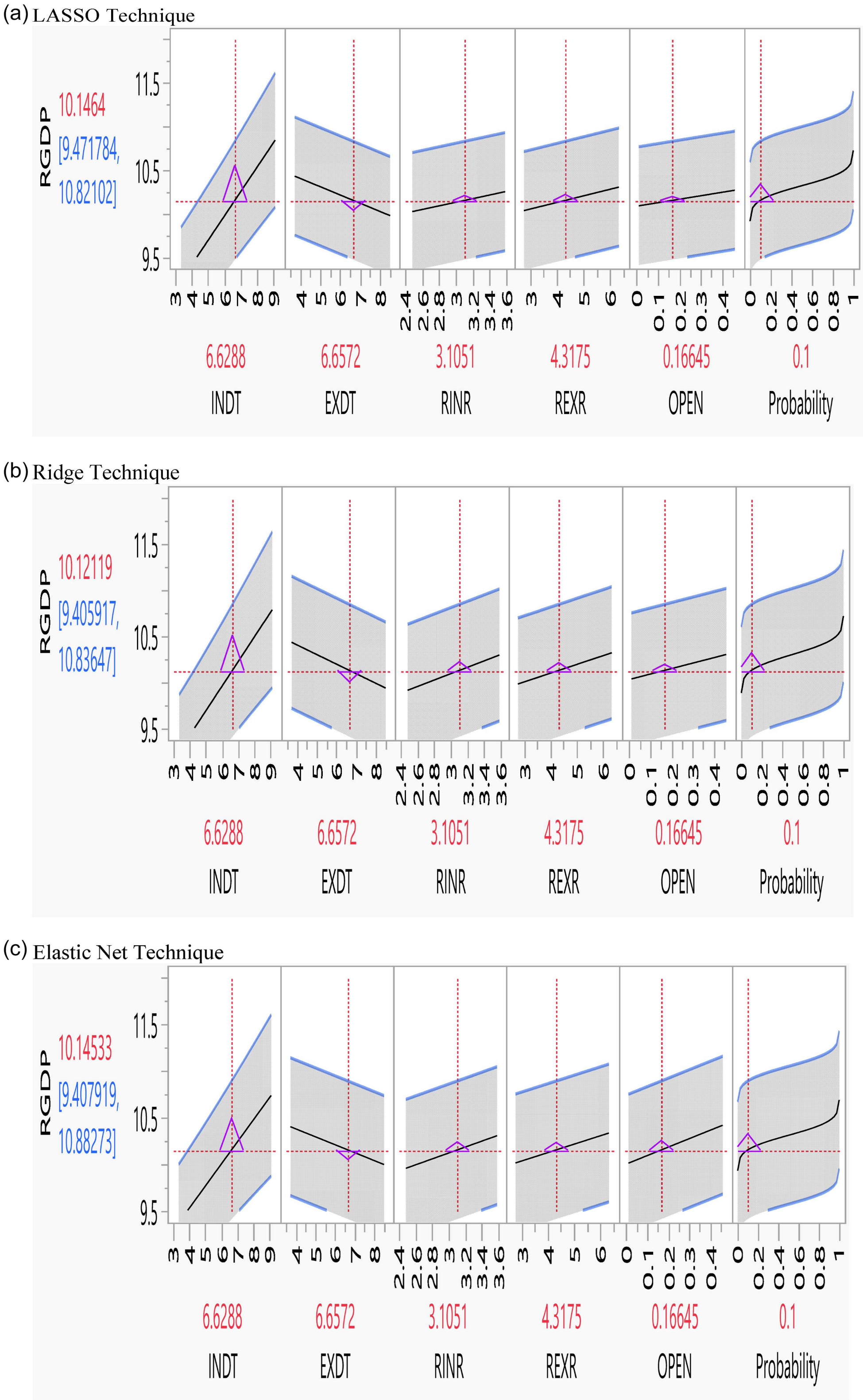
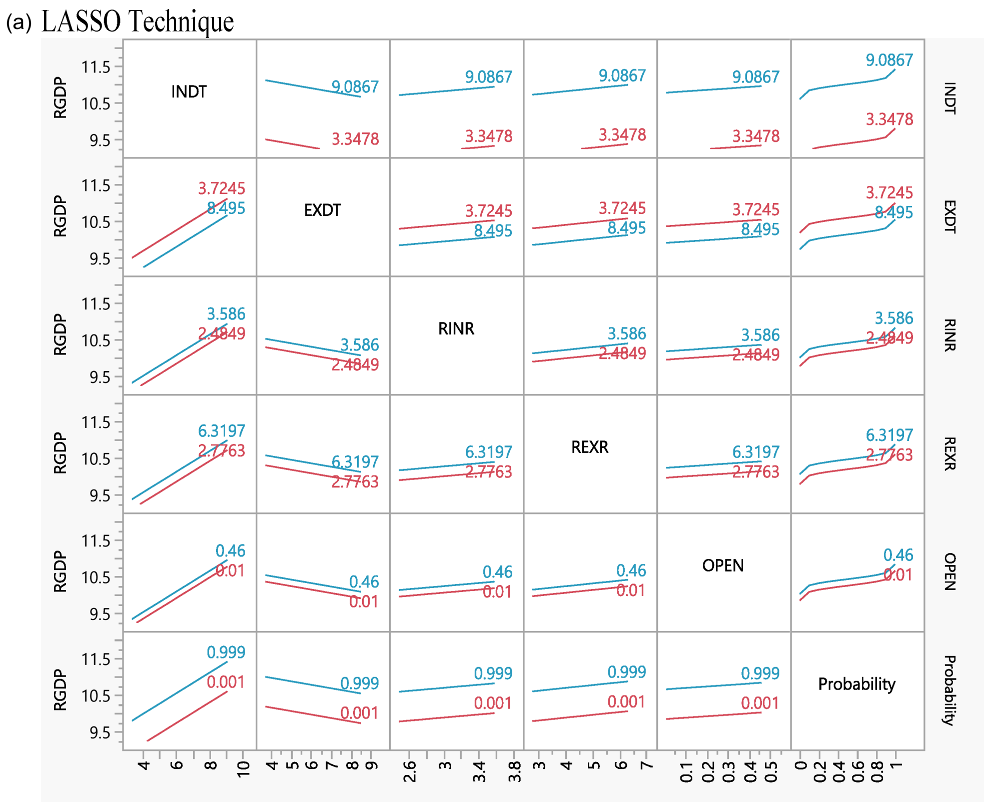


Figure 2a–c, respectively, presents plot that reveals the sensitivity of the LASSO, ridge, and elastic net as the penalized regression technique employed for predicting economic growth (RGDP). In the plots, the dotted red lines represent expected distribution of the *p*-value, while the blue lines represent the observed distribution of the *p*-value. Thus, it indicates that the values written in red for INDT (6.6288%), EXDT (6.6572%), RINR (3.1051%), REXR (4.3175%), and OPEN (0.16645%), respectively, predicted 10.1464% of the economic growth (RGDP) at the error margin of 0.1 or 10% error margin, when LASSO penalized regression technique is employed. The 95% confidence interval reveals that the predicted economic growth (RGDP) value is 10.1464%. This is good precision because the value is within the confidence interval, as shown in the LASSO plot. In Figure 2b, the plot reveals that INDT (6.6288%), EXDT (6.6572%), RINR (3.1051%), REXR (4.3175%), and OPEN (0.16645%), respectively, efficiently predict 10.1212% of economic growth rate (RGDP) at 10% error margin when ridge penalized regression technique is used. The 95% confidence interval reveals that the predicted value for economic growth (RGDP) is 10.1212%. This is good because the value is within the confidence interval, as shown in the ridge plot. Also, in Figure 2c, the plot shows that INDT (6.6288%), EXDT (6.6572%), RINR (3.1051%), REXR (4.3175%), and OPEN (0.16645%), respectively, predict economic growth RGDP to be 10.1453% at 10% error margin, when elastic net penalized regression technique is adopted. The 95% confidence interval reveals that the predicted economic growth (RGDP) value is 10.1453%. This is good, and it signifies precision because the value is within the confidence interval, as shown in the elastic net plot. Thus, it can be emphasized that the LASSO penalized regression technique has the highest predictive value of 10.1464% for economic growth (RGDP) in Nigeria compared to the predicted ridge and elastic net values. This is also evidence in the precision of the solution path plots for the penalized techniques as shown in Figure 1a–c. The interaction plots among the economic growth drivers are considered for each of the penalized least squares techniques and are shown in Figure 3a–c.

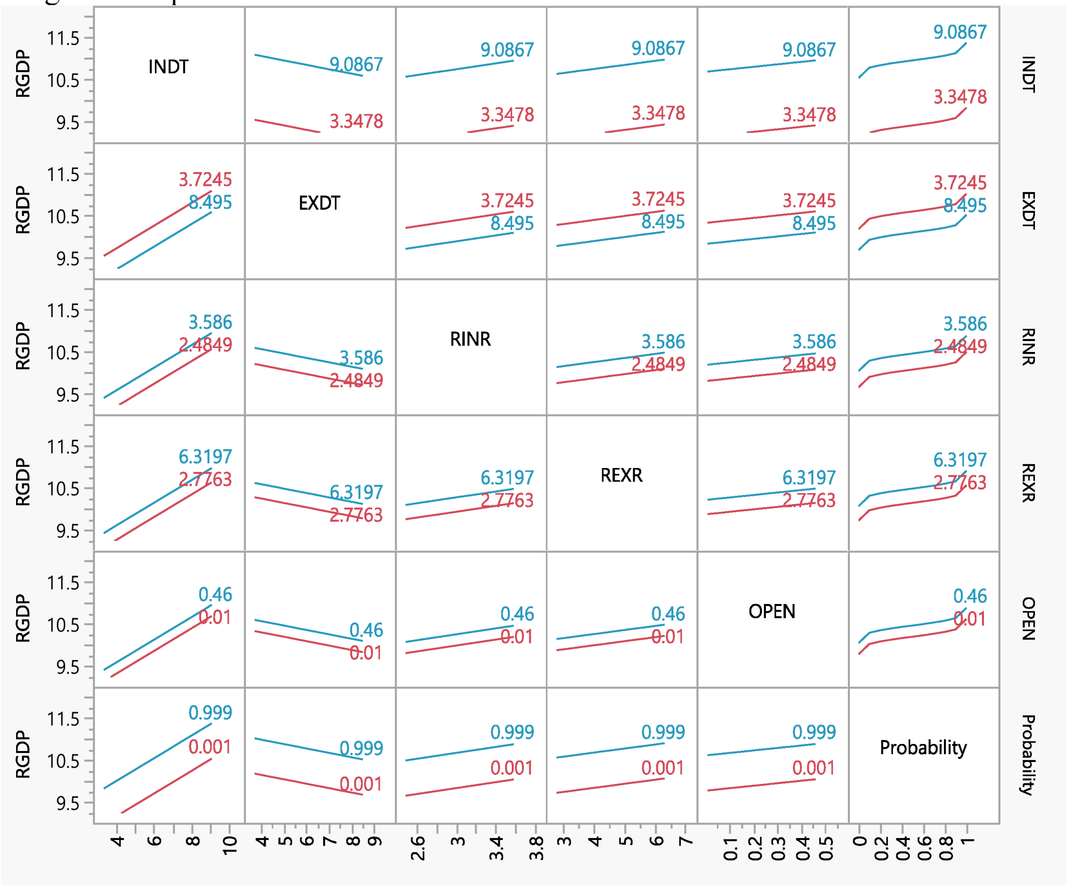
Figure 3a–c shows the plots for the interaction profile of the penalized least squares regression method comprising of lasso, ridge, and elastic net employed in this study. In Figure 3a–c, the blue and the red lines show the relationship between two predictors such as INDT and EXDT, INDT and RINR, INDT and REXR, and INDT and OPEN, and so on in determining their joint influence on RGDP. Thus, the interaction plots show evidence of parallel lines between INDT and EXDT, INDT and RINR, INDT and REXR, and INDT and OPEN for all the three penalized least squares methods employed for this study. Thus, it indicates the independence of the identified economic growth drivers and do not influence each other in determining and predicting economic growth rate (RGDP). Following the interaction plots, the normal quantile plots for the panelized least squares methods for this study are considered, and these are shown in Figure 4a–c for both training and validation.

**Figure 3**

**Interaction plot for the penalized least squares techniques. (a) Interaction plot for the penalized least squares techniques.(b) Ridge iInteraction plot for the penalized least square techniques. (c) Elastic net Interaction plot for the penalized least square techniques**



(b) Ridge Technique




(c) Elastic Net Technique

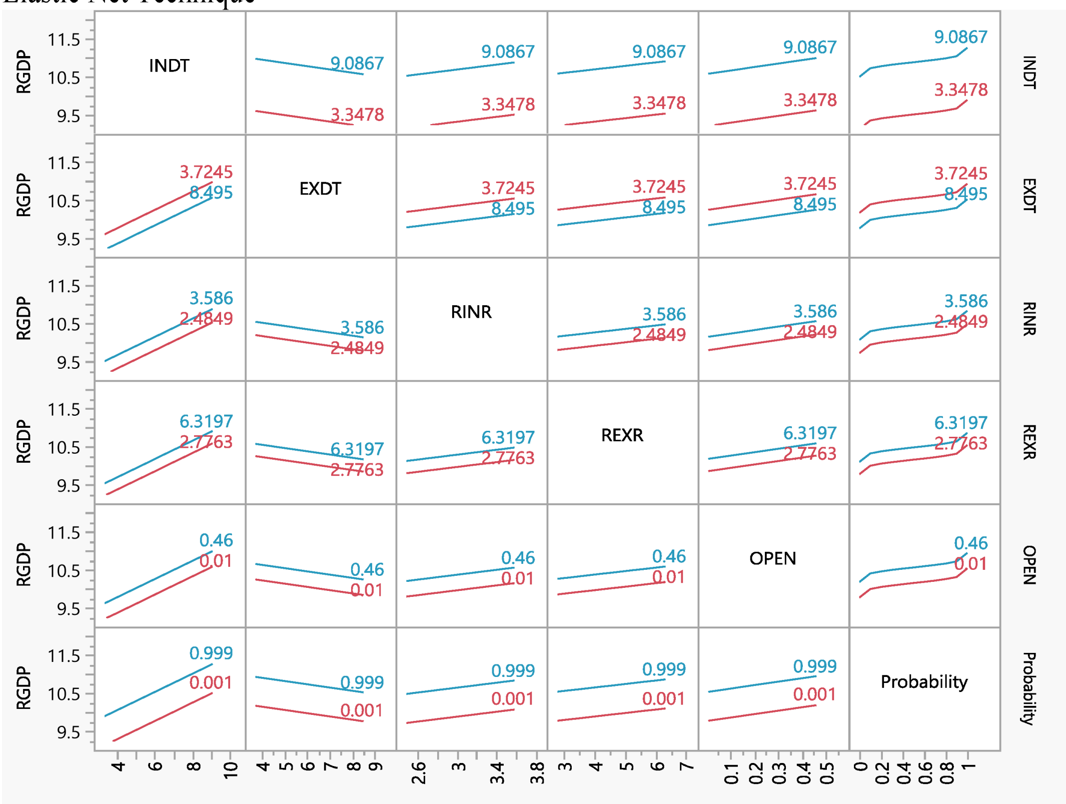


**Figure 4**
**Normal quantile plot for the penalized techniques. (a) LASSO normal quantile plot for the penalized techniques.(b) Ridge normal quantileplot for the penalized techniques.(c) Elastic net normal quantile plot for the penalized techniques**

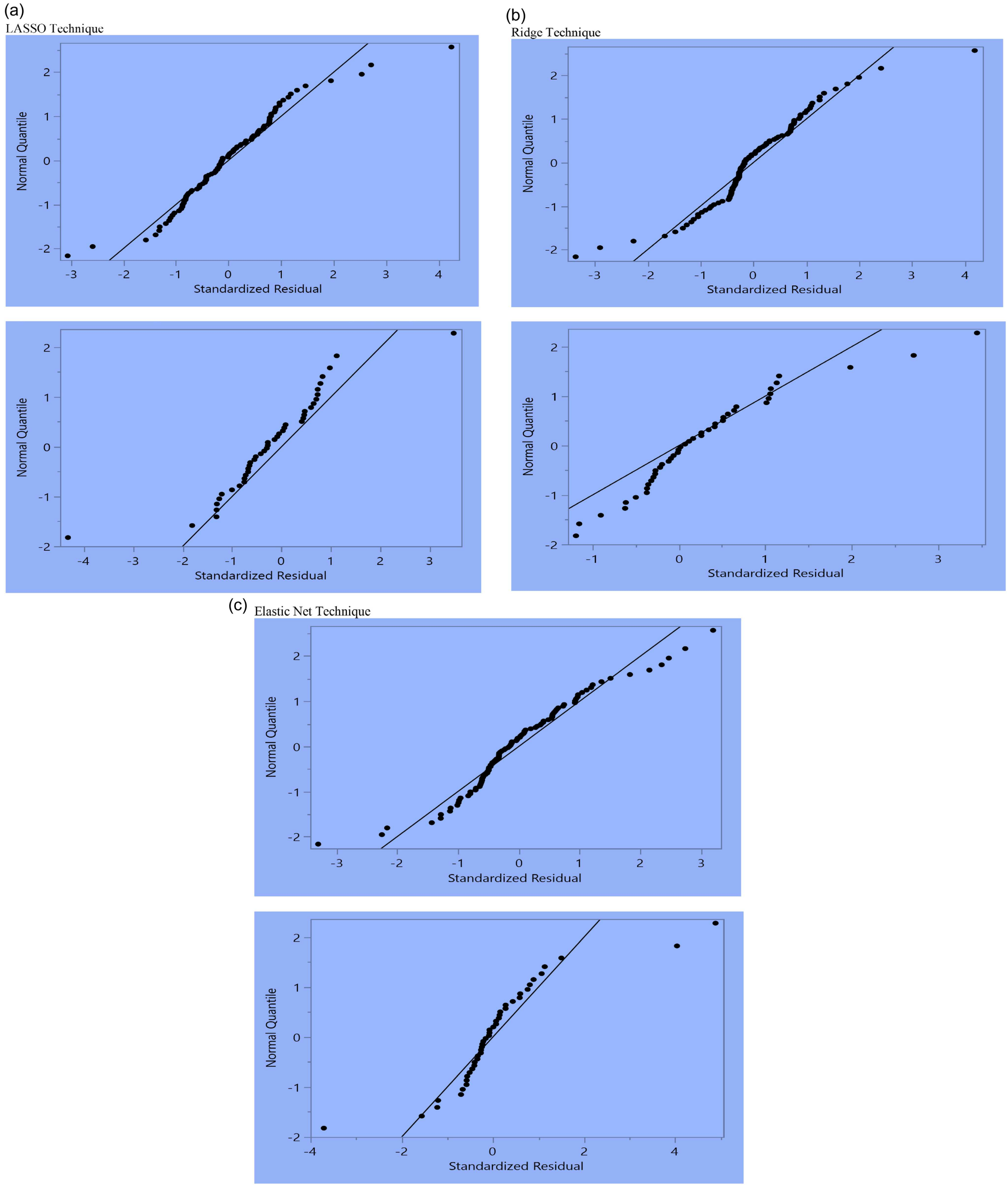


**Figure 5**
**Predicted plot for the penalized techniques**

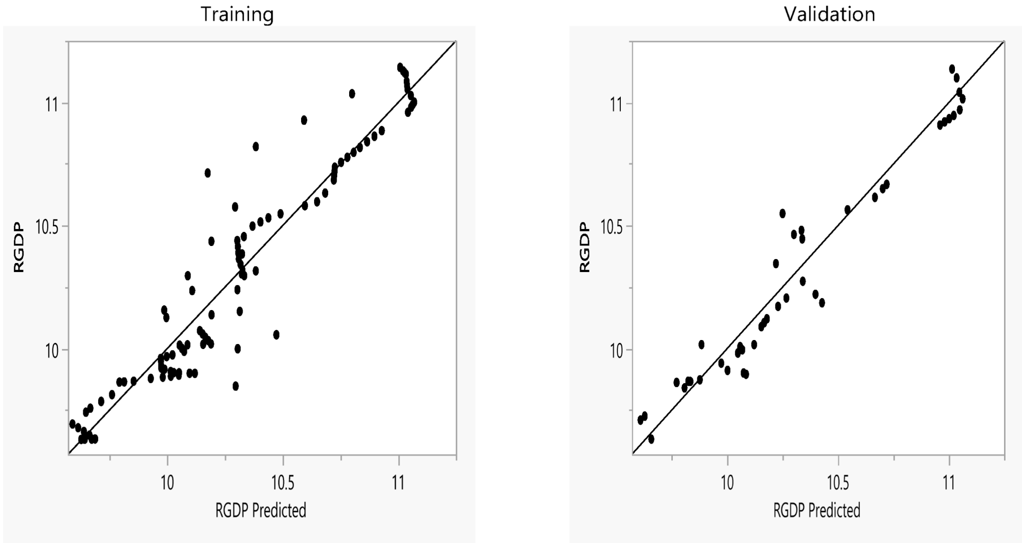
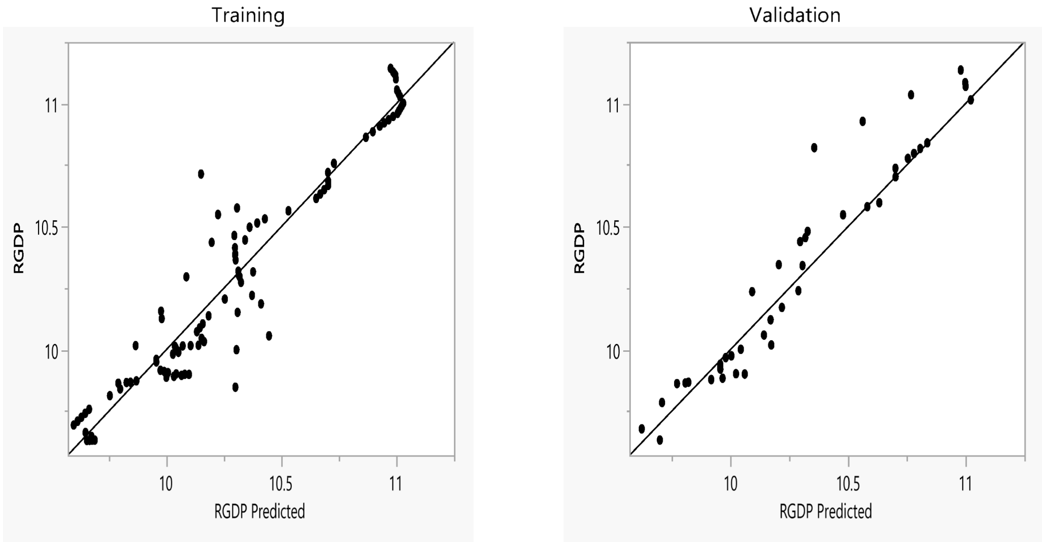



**Figure 6**
**Actual by predicted plot using ridge technique**




The actual predicted plot for the penalized techniques used in this study is shown in Figures 5, 6, 7.

Figures 4a–c, 5, 6, 7 present the normal quantile plots and the predicted plots for the penalized least squares techniques comprising of LASSO, ridge, and elastic net. The plots show that the dataset under consideration is normally distributed, as revealed by all the techniques. Thus, the dotted lines represent the individual predicted observations during the period under consideration using the penalized least squares methods. Having determined the contributions of the identified economic growth drivers and established their independence using various fitted penalized least squares techniques, it is also imperative to investigate the predictive power of these methods. Evaluating their ability to predict stable and reliable values for the economic growth rate (RGDP) is crucial. Therefore, there is a need to examine the performance metrics in other to assess the predictive strength of the methods. The results are presented in Table 3.

Table 3 presents the results of various performance metric evaluations to be considered for selecting the best-fit model among the average-centered penalized least squares techniques employed for this study. Here, we used root mean square error (RMSE), mean absolute error (MAE), mean absolute percentage error, and lambda penalty. Thus, based on the results, the RMSE for the fitted LASSO, ridge, and elastic net penalized least squares techniques are 0.2895, 0.3197, and 0.2979, respectively. The MAE for the fitted LASSO, ridge, and elastic net penalized least squares techniques are 0.2174, 0.2400, and 0.2237, respectively. The MAPE for the fitted LASSO, ridge, and elastic net penalized least squares techniques are 2.1298, 2.3489, and 2.1985, respectively. The R-square for the fitted LASSO, ridge, and elastic

**Figure 7**
**Actual by predicted plot using elastic net technique**

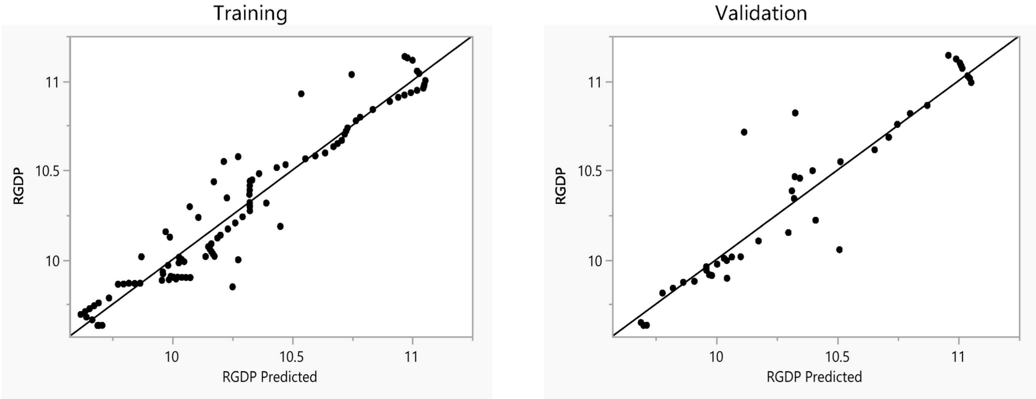



**Table 3**
**Performance metrics of the penalized least squares regression models**

| | Lasso technique | | Ridge technique | | Elastic net technique | |
|---|---|---|---|---|---|---|
| Evaluation | Training | Validation | Training | Validation | Training | Validation |
| Rows | 98 | 43 | 98 | 43 | 98 | 43 |
| Sum of freq | 98 | 43 | 98 | 43 | 98 | 43 |
| –LogLikelihood | –60.56 | –20.53 | –57.60 | –27.77 | –66.94 | –15.66 |
| BIC | –89.03 | –14.73 | –83.10 | –29.22 | –101.7 | –4.99 |
| AICc | –105.8 | –23.85 | –99.95 | –38.34 | –118.6 | –14.13 |
| R-square | 0.919 | 0.873 | 0.910 | 0.919 | 0.920 | 0.892 |
| Lambda penalty | 0.0840 | | - | | 0.0000 | |
| RMSE | 0.2895 | | 0.3197 | | 0.2979 | |
| MAE ; | 0.2174 | | 0.2400 | | 0.2237 | |
| MAPE | 2.1298 | | 2.3489 | | 2.1985 | |

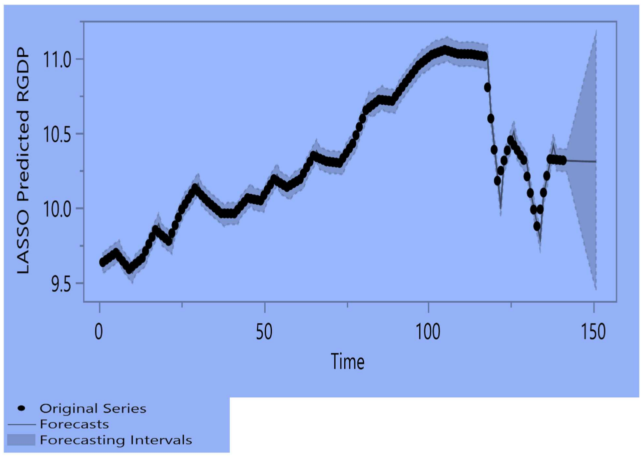
net penalized least squares techniques were 0.919, 0.910, and 0.920, respectively, indicating that identified economic growth drivers explained 91.9%, 91.0%, and 92.0% variations or changes in the economic growth in Nigeria using LASSO, ridge, and elastic net as penalized least squares regression techniques, respectively. The lambda penalty for the fitted model using the LASSO technique is 0.0840, which is higher than that of the elastic net technique value of 0.0000. Thus, it can be emphasized that based on these performance metrics, LASSO technique is the most efficient penalized regression model for this study.

## Predicted plot for RGDP using LASSO penalized method

Based on the RMSE, MAE, MAPE, and lambda penalty, LASSO is selected as the most efficient average-centered penalized least regression method to be used in predicting a stable and reliable economic growth rate (RGDP) for this study. Therefore, in the next ten (10) quarters, the economic growth rate (RGDP) is predicted using the LASSO penalized regression model. The result is shown as forecast in the plot presented in Figure 8.

In Figure 8, the original data for the economic growth during the period under consideration are denoted by the dark dotted color line. The forecasting interval or the region is denoted by light blue color, and this ranges between 9.50 and 11.20. The stable and reliable predicted value of the economic growth for Nigeria is the line denoted by the forecast, which stood at 10.25 for the ten (10) quarters.

**Figure 8**
**Forecast plot for the adopted LASSO penalized least squares technique**



## 5. Conclusion

This study examined the impact of some important macroeconomic indicators: internal debt (INDT), external debt (EXDT), interest rate (RINR), exchange rate (REXR), and economy openness (OPEN) in predicting Nigeria's economic growth rate (RGDP) using penalized regression techniques. To address multicollinearity and outlier effects previously established in the study by [1, 2], three penalized least squares approaches including LASSO, ridge, and elastic net were applied. The results obtained reliably establish that internal debt, interest rate, exchange rate, and economy openness have a positive impact on economic growth, while external debt impact is negative. Explicitly, the LASSO penalized model indicates that INDT, RINR, REXR, and OPEN contribute positively to RGDP by 4.27%, 0.40%, 0.49%, and 0.52%, respectively, while EXDT leads to decline of growth by 0.97%. These findings support previous studies [4, 5, 8, 9], which argue that openness of the economy and monetary policy transformations stimulate international market competition and enhance growth. On the contrary, the negative influence of EXDT aligns with [6, 7], by cautioning on the poor management of external borrowing and more reliance on debt-financed practice that can hinder the growth and development of the economy. Ridge penalized least squares regression results equally reveal positive contributions of INDT, RINR, REXR, and OPEN (4.07%, 0.72%, 0.66%, and 0.78%, respectively), while EXDT again reveals a negative impact (−1.11%). The elastic net penalized least squares technique also affirms the same relationships, with INDT, RINR, REXR, and OPEN having positive contributions (3.45%, 0.64%, 0.58%, and 1.14%, respectively) and EXDT with negative contribution (−0.92%). The significance of the estimated parameter for OPEN as revealed by the elastic net is due to its efficiency in combining the estimating power of both LASSO and ridge penalized methods in addressing the problem associated with the correlated predictors and high-dimensional data, which is the situation experienced in this study. This technique ensures accuracy and stable estimate without over-shrinking or dropping important correlated variable. The consistency that transversed the penalized least squares methods underlines the robustness of the identified economic growth predictors.

The evaluation of the penalized least squares models using various predictive performance criterions further emphasizes the better performance of the LASSO method over other penalized techniques employed for this study. Thus, it generated the minimum root mean square error (RMSE = 0.2895), mean absolute error (MAE = 0.2174), and mean absolute percentage error (MAPE = 2.13%), indicating better performance over ridge (RMSE = 0.3197; MAE = 0.2400; MAPE = 2.35%) and elastic net (RMSE = 0.2979; MAE = 0.2237; MAPE = 2.20%). The explanatory power of all three penalized least squares models shows the robustness of these methods, with $R^2$ values greater than 91%, demonstrating that the selected predictors (explanatory variables) jointly explain over 90% of the variation in RGDP in Nigeria. Moreso, the estimated penalty parameter in the LASSO model further validates its predictive efficiency, consistent with previous findings on its optimal variable selection capabilities [16, 20].

On the policy implications of findings, it is empirically evident that the strong positive effect of internal debt indicates the need for policies that enhance the effective uses and the management of domestic borrowing to enhance socio-economics and infrastructural development, as well as industrial growth for better economic performance. Also, considering the positive influence of economy openness, there is a need for the formulation and implementation of policies to enhance export promotion strategies, reforms to facilitate and strengthen international trade participation, and regional integration to support Nigeria's economy in global market competition. The positive but uncertain impact of RINR and REXR emphasizes the relevance of maintaining stable exchange rate and adoption of suitable interest rate policies rather than speculative flows to attract investors and boost production in the economy. The persistent negative impact of EXDT posits that external borrowing in Nigeria has been destabilized due to the inefficient management, uncontrollable desire for debt accumulation, and thus the need to enhance management and administrative strategies to reduce wastage and elephant projects that add no value to economy. On the methodology employed, the study establishes the importance of applying advanced and robust econometric methods such as penalized least squares regression techniques in policy analysis, particularly in economies where available data are highly correlated and contain a lot of extreme values or anomalies, which may lead to biased estimates of the classical linear models.

Therefore, this study contributes to the knowledge and existing literature by empirically corroborating the sensitivity and predictive performance of penalized least squares regression techniques in modeling Nigeria's economic growth dynamics. The results consistently identify internal debt, economy openness, interest rates, and exchange rates as good predictors with a positive impact on growth, while external debt has a negative effect on economic growth. Among the econometrics methods, LASSO penalized least squares least is found to be the most efficient, stable, and reliable method, which provides suitable variable(s) selection and higher and better predictive accuracy.

## 6. Recommendations

Based on the study's findings, it is recommended that government should ensure proper management of the internal borrowed funds and channel such funds by investing heavily in infrastructure, technology advancement, and innovation systems to maximize the growth benefits of the loans; policymakers should put measures in place to increase free market entry, restructure export processes, and negotiate for satisfactory international trade agreements that are indispensable to consolidate the advantages of economy openness; an optimal balance between growth-based interest rate policies and stable exchange rate to encourage investment and sustain investor confidence must be implemented; government and policymakers must change from quantity-driven to quality-driven approaches by focusing on value addition financing and to reduce heavy reliance on foreign loans; application of penalized regression methods, particularly LASSO, should be mainstreamed into economic forecasting units within government and research institutions to improve evidence-based policy formulation. However, this technique is limited to the use of a single-country (Nigeria) time series data without putting into consideration the influence of structural break(s), global crises, pandemics, and other financial instability in the fitted models, and as such, a good potential for further study. Also, it is another opportunity to extend this research through the application of panel data or comparative analysis across other developing economies.

## Acknowledgment

I want to use this opportunity to acknowledge an earlier version of this manuscript was posted as a preprint on https://doi.org/

10.21203/rs.3.rs-4623084/v1. The preprint does not impose any restrictions on the author copyright or re-use rights. Upon publication in this journal, the preprint has been updated to include a link to the final published version. I also acknowledge Prof. Chifurira and Prof. Chinhamu for their corrections and useful suggestions to ensure that this manuscript looks good and acceptable for publication in a reputable journal.

## Ethical Statement

This study does not contain any studies with human or animal subjects performed by the author.

## Conflicts of Interest

The author declares that he has no conflicts of interest in this work.

## Data Availability Statement

The data that support the findings of this study are openly available in the central bank of Nigeria statistical bulletin at https://www.cbn.gov.ng.

## Author Contribution Statement

**Ayooluwade Ebiwonjumi:** Conceptualization, Methodology, Software, Validation, Formal analysis, Investigation, Resources, Data curation, Writing – original draft, Writing – review & editing, Visualization, Supervision, Project administration.

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