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On the Efficient and Optimal Predictive Values for Economic Growth with Covariate Predictors: Ordinary Least Square and Ridge Regression Approach

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Abstract: This study examined an efficient and optimal estimate and predicted value for economic growth using the explanatory variables that include internal and external debts, interest and exchange rate, and trade openness, which were correlated to cause a multicollinearity problem. The ordinary least square and ridge regression (RR) technique were adopted to analyze data gathered from the Central Bank of Nigeria (1986–2022). The diagnostic test conducted using the variance inflation factor revealed that the ordinary least square technique established a multicollinearity problem, which was addressed when the RR method was used with an appropriate ridge constant. The predictive performance metrics such as root mean square error, the mean absolute error, the mean absolute percentage error, and the bias proportion for the fitted RR were smaller when compared with the values obtained using the ordinary least square regression technique. Consequently, an RR was chosen as the most efficient and optimal technique to predict stable and reliable values for the economic growth. Therefore, this study was beneficial to the policymakers because it provided the needed understanding that captured the contributions of the identified economic growth drivers. The negative influence of the external debt on the economic growth was not in doubt. In addition, this study was significantly beneficial to the researchers by strengthening their understanding of the appropriate estimation technique to be adopted for efficient, optimal, stable, and reliable predictive values in the presence of multicollinearity.

Keywords: debts, monetary policy, trade openness, economic growth, multicollinearity, ridge regression

1. Introduction

Economic growth is one of the fundamental macroeconomic policy objectives, which is pursued by countries all over the world, and Nigeria is not an exemption. Nigeria's economic growth is anchored on oil and gas resources [1]. However, the overall economic performance of Nigeria since independence in 1960 has been absolutely weak and unimpressive [2]. For Nigeria, economic growth is a key policy objective of the government to drive development. Reference [3] noted that sustainable economic growth relies on the nation's capacity to invest, make efficient, and productive utilization of the available resources. It can be emphasized that good governance can significantly influence economic growth. According to [4], good management of the national debt profile and feasible policy formation are keys to control external debt and as such enhance economic growth. Reference [5] stressed that the long-term growth is hampered despite the fact that in the short-term, growth enhancing was observed.

Reference [6] asserted that the government should allow expansionary monetary policy to stabilize economic growth. In [7], it was emphasized that the need for the government to ensure sustainable infrastructural development to enhance economic growth. Thus, [8] posited that a stable exchange rate through the implementation of monetary policies is required and must be pursued to propel diversification via export to boost economic growth. Reference [9] stated that a rigorous effort must be made by the government to ensure the growth of the economy via diversification led by export. Therefore, an accurate and reliable prediction of the economic growth assists in managing the investment risk. A suitable and sustainable upward trajectory of economic growth makes investments profitable to the investors and enhances the economic outlook and performance aimed of the policymakers. Therefore, this study focuses on the comparative study of the ordinary least square regression and ridge regression (RR) methods for the analysis, parameter estimation, and prediction of economic growth in Nigeria using the identified economic growth drivers that include internal and external debt, interest, and exchange rate, as well ase the trade openness.

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2. Literature Review

Thus, in order to predict economic growth using the identified economic growth drivers considered in this study, the ordinary least squares (OLS) estimation technique is often used [10–12] further emphasized that, if there is collinearity problem among the economic growth drivers under consideration, the OLS method will be unreliable and rugged to interpret, which will significantly influence the coefficients of the model, leading to unreliable results. Thus, an RR model is proposed to estimate and predict reliable economic growth (RGDP) in the presence of multicollinearity. This involves a deliberate use of a biased estimating method or procedure to improve the accuracy of the estimated economic growth parameters.

According to [13] as cited in [12], it can be stressed that if the mean square error (MSE) or root mean square error (RMSE) as a performance metric criterion is used to examine the accuracy and the efficiency of the model, an RR method will generate better and more accurate estimates than the unbiased OLS estimates. Also, [14] opined that the RR method generates a better and more promising results than deliberately deleting relevant variable(s) to avoid multicollinearity problem. Also, the aforementioned provides a smallest value for the adopted evaluation techniques than the unbiased OLS method [1] and [12]. Reference [15] carried out study to optimize linear regression models by considering Lasso and RR method to examine financial behavior in the United Arab Emirate during the COVID-19 pandemic period. In this study, it was found that RR was effective in correcting the existing collinear variables and missing values, by providing robust estimate into financial decision-making during crises.

Reference [16] analyzed factors that determine stable economic growth in the Czech Republic, Slovak Republic, Hungary, Poland, and Romania between 2003 and 2016. An RR technique was used for the investigation, and it was found from the results that, apart from Slovak Republic, foreign direct investment can be used to promote economic growth in the aforementioned countries. It was also found in Czech Republic that government spending on education significantly contributed to the economic growth, while in Romania, Hungary, and Czech Republic, the impact of government expenditure on human capital positively influenced economic growth.

Reference [17] analyzed various factors that affect the tertiary industry in achieving sustainable and stable national economic growth in Beijing (2000–2015). In the study, various factors that include the proportion of labor productivity, employed people, fixed-asset on investment, actual utilization of foreign capital, total energy consumption, and resident population. The RR method used for analysis showed that the identified various factors have a significant impact on the development of the tertiary industry. Thus, it can be stressed that energy, population, and investment were the most influential factors affecting national economic growth.

Reference [18] investigated the prediction of food prices using time-series data. In the study, a linear RR with the best curbing factor was employed to predict food prices. This technique was found capable to address a multicollinearity issue in time-series data through the optimization of the curbing factor, and as such, it leads to an enhanced computational efficiency and accuracy that is required for predicting reliable food prices. Reference [19] examined multicollinearity regularization on economic data. The focus of the study was to address multicollinearity among the economic factors used as the predictors of gross domestic product (GDP) in Nigeria. Thus, the Lasso and Ridge regression techniques adopted for the analysis revealed the significant economic variables among the predictors that influence GDP during the investigated period.

Reference [20] investigated the impact of technological standardization on the economic growth. The study examined technological innovation as a pathway to formulate corresponding hypotheses. The evolution of standardization in China was considered, and the study focused on encompassing standardized and innovative technology in economic growth. The economic data collected between the period 1989 and 2019 was used. An RR method was applied due to the collinearity problem among the explanatory and mediating variables. The findings showed a more stable estimated parameter value. Also, a comparative analysis revealed that technological standardization, as an intermediate variable, enhanced economic growth.

Reference [21] carried out a study on modern economic management analysis of commercial insurance by investigating various factors affecting households' readiness to subscribe to commercial insurance based on their household expenditure using the China Family Panel Study survey data. An RR method was employed for the study due to a strong correlation that showed evidence of a multicollinearity problem among the factors, such as the annual expenditure of a family on food, beauty, tourism, education, and training, and medical healthcare. Thus, it was revealed from the results that the aforementioned factors significantly affected households' willingness to purchase commercial insurance. Hence, there is a need to develop commercial insurance based on the use of big data to attract more customers, increase innovation, and face the development gap and rational control.

Reference [22] evaluated physical infrastructure and economic performance in Nigeria. In the study, an RR method was applied to examine the influence of physical infrastructure on the performance of the economy in Nigeria. The existing multicollinearity among the explanatory variables was effectively managed, and the findings show a positive and significant impact of physical infrastructure on economic performance during the period under consideration. Reference [23] used RR, random forest, and the mean-variance model in portfolio investment optimization. The study combined RR and random forest models within the context of a mean-variance to optimize investment portfolios. Thus, it was found that the aforementioned techniques improved the risk-adjusted returns and also minimized volatility.

Reference [24] examined the performance of RR and principal component regression in a comparative study of addressing multicollinearity. The economic data extracted from Central Bank of Nigeria statistical bulletin were used for this study. Thus, in the findings, it was revealed that an RR performed better than a principal component regression as a result of a smaller mean square of error obtained from the investigation. This implies the effectiveness of RR in handling collinear variables. However, in the various studies and literature reviewed for this study, the study that estimates the parameter's value for the identified economic growth drivers that include internal and external debt, interest, and exchange rate, as well as the trade openness to efficiently predict economic growth (RGDP) in Nigeria using the RR method is dearth. Thus, a gap worth filling to enhance the estimate, efficiency, and accuracy, and to predict stable and reliable values for the economic growth (RGDP) in Nigeria.

3. Research Methodology

In this study, time-series data that are secondary in nature are gathered on the explanatory variables such as internal debt, external debt, interest rate exchange rate and trade openness, and the economic growth, which serve as the dependent (response) variable. These are extracted from the Central Bank of Nigeria statistical bulletin spanning the period between 1986 and 2022. Also, in this section, the methodology based on ordinary least square and RR for handling multicollinearity is explained in detail. Thus, according to [25], [26] and [27], which are widely used in economic and statistical research, the general linear regression equation can be given as

$$Y = X'\beta + \varepsilon, \tag{1}$$

The parameters to be estimated (β's) can be expressed as

$$\hat{\beta} = (X'X)^{-1}X'Y, \tag{2}$$

Also, variance co-variance matrix for the estimated parameters (β's) can be computed by

$$\mathrm{Cov}(\hat{\beta}) = \sigma^2(X'X)^{-1}, \tag{3}$$

where Y is a vector of dependent variable with $n \times 1$ dimension, X is $n \times p$ matrix for the explanatory variables, β's are $p \times 1$ vector of the estimated parameters, and ε is an $n \times 1$ vector for the error term.

3.1. Ridge regression technique

Consider a general RR model [13] expressed as

$$\hat{\beta}_{RR} = (X'X + KI_P)^{-1} X'Y, \; k \geq 0, \tag{4}$$

In this case, $0 < K < 1$. This is a biased technique adopted to reduce the estimated variances of the parameters. According to [28], an RR technique for parameter estimation can be stated by

$$\hat{\beta}_{RR,b} = (X'X + KI_P)^{-1} X'Y + Kb, \; k \geq 0, \tag{5}$$

where b is the prior estimate of β and as K equal to one (1), the estimated value using RR becomes b. Furthermore, [29] proposed an unbiased RR (URR) estimation method given as

$$\hat{\beta}_{UR,J} = (X'X + KI_P)^{-1} X'Y + KJ, \; k \geq 0, \tag{6}$$

where $J \sim N(\beta, \frac{\sigma^2}{K} I_P)$. for $K > 0$.

To consider the spectral decomposition of $X'X$, the model stated in (4) can be transformed and expressed in (7) using $X'X = T\Lambda T'$, where $TT' = T'T = I$.

$$Y = XTT'\beta + \epsilon,$$

$$Y = Z\alpha + \epsilon, \tag{7}$$

with $Z = XT$, $\alpha = T'\beta$ where $Z'Z = T'X'XT = \Lambda = \mathrm{diag}(D_1, D_2, D_3, \ldots, D_P)$. The diagonal elements denoted by Λ are the eigenvalues obtained from the $X'X$ matrix of the explanatory variables and T are the corresponding eigenvectors generated from the $X'X$ matrix of the explanatory variables. Hence, the transformed OLS, ORR, and URR with parameter α based on the spectral decomposition of $X'X$ matrix of the explanatory variables are written as

$$\hat{\alpha}_{OLS} = \Lambda^{-1}Z'Y, \tag{8}$$

$$\hat{\alpha}_{RR} = (\Lambda + KI_P)^{-1}Z'Y, \tag{9}$$

$$\hat{\alpha}_{UR,J} = (\Lambda + KI_P)^{-1}(Z'Y + KJ). \tag{10}$$

According to [30], an improved and URR estimation technique can be derived from URR. Similarly, an ordinary ridge regression (ORR) technique can be derived by using OLS regression technique. This modification aims to reduce the overestimated standard error that arise when eliminating bias using URR. The underlying logic involves premultiplying the URR by the matrix $[I - K(X'X + KI_P)^{-1}]$ in order to obtain the estimator of β. Recalled (6) given as:

$$\hat{\beta}_J(K) = [I - K(X'X + KI_P)^{-1}]\, \hat{\beta}_{UR,J},$$
$$\hat{\beta}_J(K) = [I - K(X'X + KI_P)^{-1}]\, ((X'X + KI_P)^{-1}(X'Y + KJ), \tag{11}$$

where $J \sim N(\beta, \frac{\sigma^2}{K} I_P)$ for $K > 0$. This technique is known as the improved and the URR obtained using URR. The modified and the unbiased ridge estimation technique can be through spectral decomposition and transformation. Thus, it can be expressed as

$$\hat{\alpha}_J(K) = [I - K(X'X + KI_P)^{-1}]\, \hat{\alpha}_{UR,J}, \tag{12}$$

The following properties of techniques can be obtained:

bias of $\hat{\alpha}_J(K)$

$$\text{bias } (\hat{\alpha}_J(K)) = E(\hat{\alpha}_J(K)) - \alpha,$$

$$\text{bias } (\hat{\alpha}_J(K)) = -KG_K^{-1}\beta, \tag{13}$$

where $G = X'X$ and $G_K = (G + KI)$,

Variance-covariance matrix of $\hat{\alpha}_J(K)$

$$\mathrm{Var}(\hat{\alpha}_J(K)) = E\left[(\hat{\alpha}_J(K)) - E(\hat{\alpha}_J(K))\;(\hat{\alpha}_J(K) - E(\hat{\alpha}_J(K)))'\right],$$

$$\mathrm{Var}\;(\hat{\alpha}_J(K)) = \sigma^2 W G_K^{-1} W', \tag{14}$$

where $W = [I - KG_K^{-1}]$.

Matrix mean squared error (MMSE) of $\hat{\alpha}_J(K)$

$$\mathrm{MMSE}\;(\hat{\alpha}_J(K)) = \mathrm{Var}(\hat{\alpha}_J(K)) + [\mathrm{Bias}\;(\hat{\alpha}_J(K))]\,[\mathrm{Bias}\;(\hat{\alpha}_J(K))]'$$

$$\mathrm{MMSE}(\hat{\alpha}_J(K)) = \sigma^2 W G_K^{-1} W' + K^2 G_K^{-1} \alpha\alpha' G_K^{-1}, \tag{15}$$

Scalar mean squared error (SMSE) of $\hat{\alpha}_J(K)$

$$\mathrm{SMSE}(\hat{\alpha}_J(K)) = E(\hat{\alpha}_J(K) - \alpha)'\,(\hat{\alpha}_J(K)) - \alpha)$$

$$\mathrm{SMSE}(\hat{\alpha}_J(K)) = \mathrm{tr}(\mathrm{MMSE}(\hat{\alpha}_J(K))),$$

Then,

$$\mathrm{SSME}(\hat{\alpha}_J(K)) = \sigma^2 \sum_{i=1}^{P} \frac{D_i^2}{(D_i + K)^3} + K^2 \sum_{i=1}^{P} \frac{(D_i + K)\alpha_i^2}{(D_i + K)^3} \tag{16}$$

where $\{D_i\}$ are eigenvalues of $X'X$

$\hat{\alpha}_J(K = 0) = \hat{\alpha}_{LS} = (X'X)^{-1} X'Y$ is the OLS estimator $\lim_{K \Longrightarrow 0} \alpha(K) = \hat{\alpha}_{LS}$.

3.2. Estimation of ridge constant K

There are several methods for estimating the ridge constant. According to [13], [31] and [29], this can be obtained based on harmonic mean computation of the optimal ridge values. This depends on the unidentified parameters α and σ^2. Thus, the unidentified parameters can be obtained using ordinary least square estimated values, and as such, the operational ridge constant with the value $(\hat{K})$, which minimizes MSE can be expressed as

$$K = \frac{\hat{\sigma}^2}{\hat{\alpha}_{max}}, \tag{17}$$

where $\hat{\sigma}^2$ denotes the error or variance associated with the fitted model, $\hat{\alpha}_{max}$ is the highest value of the elements $\alpha = D'\hat{\alpha}$, where D is an orthogonal matrix. The estimated ridge constant called the (FG) ridge constant K can be expressed in the form given by

$$\hat{K}_{FG} = \frac{p\hat{\sigma}^2}{\sum_{i=1}^{p}\left[\hat{\alpha}_i^2/\left[\left(\frac{\hat{\alpha}_i^4 D_i^2}{4\hat{\sigma}^4} + \frac{D_i\hat{\alpha}_i^2}{\hat{\sigma}^2}\right)^{1/2} - \frac{D_i\hat{\alpha}_i^2}{2\hat{\sigma}^2}\right]\right]}, \tag{18}$$

Also, the HKB ridge constant by [31] is given as

$$\hat{K}_{HKB} = \frac{p\hat{\sigma}^2}{(\hat{\alpha}_{LS}')'(\hat{\alpha}_{LS})}, \tag{19}$$

Reference [29] ridge constant K denoted by CJH ridge constant is expressed in the form given by

$$\hat{K}_{CJH} = \begin{cases} \frac{p\hat{\sigma}^2}{(\hat{\alpha}_{LS}-J)'(\hat{\alpha}_{LS}-J) - \hat{\sigma}^2 tr(X'X)^{-1}} & \text{if } (\hat{\alpha}_{LS}-J)'(\hat{\alpha}_{LS}-J) > \\ \hat{\sigma}^2 tr(X'X)^{-1}, \frac{p\hat{\sigma}^2}{(\hat{\alpha}_{LS}-J)'(\hat{\alpha}_{LS}-J)} & \text{otherwise} \end{cases} \tag{20}$$

where $\hat{\sigma}^2 = \frac{(Y-X\hat{\alpha}_{LS})'(Y-X\hat{\alpha}_{LS})}{n-p}$ is an unbiased estimator of σ^2 and $\hat{K}_{CJH}$ is a generalization of [31] RR constant K given in (19).

3.3. Choosing of ridge parameter k

Reference [13] showed the appropriate ways of obtaining the value of ridge constant $k > 0$ in such a way that the MSE is smaller when compared with the MSE obtained from OLS. Thus, the ideal value of ridge constant k is estimated from the data using the traditional ridge trace plot proposed by Hoerl and Kennard. This is a graphical method that can be used to select ridge constant k. Estimated coefficients and VIFs are plotted against a range of specified values of k. Hence, from the plot, the value of ridge parameter k that stabilizes and efficiently shows the statistically independent explanatory variables leads to coefficients with reasonable values, ensures that coefficients with improper signs at ridge constant $k = 0$ changed to a proper sign, and ensures that the value of sum of squares of the regression residual is not unreasonably inflated is then chosen. However, since these approached or criteria are subjective, it is imperative to adopt a more objective method known as generalized cross validation. A cross validation thus implies examining parts or subsets of the data and computing the estimated coefficients for each part or subset of the data by using the same value of ridge constant k across subsets. This is then repeated for multiple number of times with different values of ridge constant k. The value of ridge constant k that minimizes the differences in coefficient estimates across these data subsets is then selected. According to [32], it had been proved that the model with the smallest predictive errors can be obtained by simply selecting the value of ridge constant k that minimizes the generalized cross-validation equation given by

$$Var(K) = \frac{\frac{1}{n} \| (I_P - A(K)y) \|^2}{\left[\frac{1}{n}\, Trace\ (I_P - A(K))\right]^2} \tag{21}$$

where

$$A(K = X(X'X + nKI_P)^{-1} X' \tag{22}$$

Thus, the value of k that minimizes this equation is computed using statistical software. Also, [33] emphasized that the process of splitting of the data into various groups and to train the RR model on different groups. This process is repeated for different values of k. Thus, through a performance evaluation on the remaining groups, the ridge parameter k that produces the results with minimum or lowest MSE is selected.

4. Empirical Results

In this section, the empirical results of the fitted RR model discussed in Section 2 are presented to capture multicollinearity among the explanatory variables. Table 1 shows the maximum likelihood parameter estimates of the OLS regression model. However, it must be noted that the preliminary analysis of the dataset such descriptive analysis, correlation analysis, and various diagnostic test established the presence of multicollinearity among the covariates or predictors of the economic growth (RGDP) as presented in [1],

Table 1
ML OLS regression results

	Parameter estimate	*p*-value
Constant term	7.4666	0.0001
INTD	0.2817	0.0001
EXTD	-0.1128	0.0001
INTR	0.3885	0.0001
EXCR	0.0998	0.0001
TROP	0.5078	0.0200
R-squared	0.9147	
Adjusted *R*-squared	0.9116	
F-statistic	289.6333	0.0001

Table 1 shows the results of the estimated parameters for the covariate economic growth drivers (internal debt (INTD), external debt (EXTD), interest rate (INTR), exchange rate (EXCR), and trade openness (TROP)) that are used to assess economic growth (RGDP) (dependent variable) in Nigeria. Findings from the OLS reveal that INTD, EXTD, INTR, EXCR, and TROP are positive in relation to the RGDP, while EXTD has negative relationship with the RGDP. To be specific, INTD, INTR, EXCR, and TROP contribute 28.17%, 38.85%, 9.98%, and 50.78%, respectively, to enhance RGDP, while EXTD causes 11.28% decline in RGDP. A p-value < 0.05 shows the statistical significance in the estimated coefficients of the INTD, EXTD, INTR, EXCR, and TROP. Furthermore, the adjusted R-square of 0.912 reveals that INTD, EXTD, INTR, EXCR, and TROP as predictors can explain 91% of variations in RGDP. Moreso, F-statistic value (289.633) with p-value < 0.05 measured the significance of the fitted model. Thus, the statistical significance of the method in examining the contribution of the INTD, EXTD, INTR, EXCR, and TROP to RGDP is established. However, according to [10], [11], and [12], the 91% variability caused by the predictors under consideration is another indication to the existence of collinearity among the predictors. Also, since exploratory data analysis has revealed the existence multicollinearity problem caused by the economic growth drivers identified for this study, an RR model is fitted to account for the multicollinearity with the appropriate ridge constant *k*. Thus, a ridge trace constant is plotted, as shown in Figure 1.

In Figure 1, a ridge constant *k* that produces a better and reliable estimate for an RR model is chosen for the parameter estimate. Thus, an RR model is fitted with a parameter *k*(0.29), as shown in Figure 1. It is observed from the ridge trace plot that each predictors contrast or expands after the chosen ridge parameter *k*(0.29) to indicate its reliability in fitting an appropriate RR model for the results in Table 2.

Table 2 shows the result of ridge cross validation that is been carried out in this study to select the optimal and stable ridge parameter k to fit an efficient RR model for the prediction of economic growth (RGDP) for Nigeria. Thus, it is found from the result presented in Table 2 that the suitable and appropriate ridge parameter k for this study is 0.3229 with the cross-validation (CV) error and MSE of 0.2146 and 0076 (1%). Therefore, the ridge parameter k (0.3229) with approximately1% MSE obtained through the ridge cross validation for the five (5) groups of the data set is selected to produce the fitted RR results presented in Table 3.

In Table 3, the results for the estimated parameters of the economic growth drivers that are INTD, EXTD, INTR, EXCR, and TROP that can be used to explore RGDP is presented. In this study, parameters are estimated through a ridge estimation method to address the problem multicollinearity earlier established. Findings from Table 3 reveal that INTD, INTR, EXCR, and TROP are positive and significantly related with the RGDP at 5% level of significance as earlier observed from the ordinary least square results presented in Table 1 except EXTD, which is negative and significant in relation to the RGDP. Specifically, the results indicate that INTD, INTR, EXCR, and TROP increase Nigeria's RGDP by 70.40%, 9.79%, 12.58%, and 35.86%, respectively. In comparison, EXTD causes 11.68% reduction in Nigeria's RGDP as considered in this study. Moreso, estimated *p*-values < 0.05 for INTD, INTR, EXCR, and TROP show the significant of the estimated coefficients for the fitted RR model, while, *p*-value < 0.10 shows the significant of the estimated coefficient for EXDT using RR method.

Figure 1
Ridge trace plot

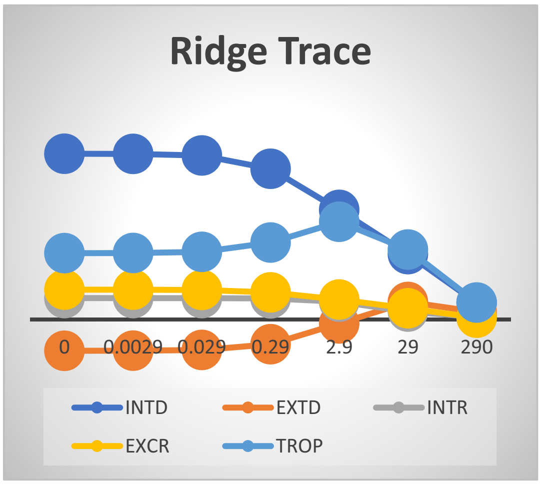

Table 3
Ridge regression model results

RGDP	Parameter estimate	*p*-value
INTD	0.7040	0.0000
EXTD	-0.1168	0.0621
INTR	0.0979	0.0301
EXCR	0.1258	0.0096
TROP	0.3586	0.0006
R-squared	0.8973	
Adjusted *R*-squared	0.8910	
F-statistic	147.7685	0.0000

However, an adjusted R-square value (0.891), a measure of explained variability in the RGDP, is revealed to be 89% as a result of variations in the INTD, EXTD, INTR, EXCR, and TROP. Also, F-statistic value (147.769) with the associated *p*-values < 0.05, which determines the appropriateness of the fitted model, reveals the reliability of a ridge estimation technique in estimating and predicting the stable value for RGDP using INTD, EXTD, INTR, EXCR, and TROP as predictors or covariates. However, in order to examine the presence of collinearity among the predictors, the variance inflation factor (VIF) using ordinary least square method in comparison with the RR method that addressed the collinearity among the identified economic growth drivers for this study is conducted and the results are presented in Table 4.

Table 4
Variance Inflation Factor (VIF) for OLS and ridge estimation technique

Variable	OLS estimate	Ridge estimate
INTD	14.2700	9.1430
EXTD	3.5300	2.7334
INTR	1.4400	1.3858
EXCR	1.7600	1.5347
TROP	9.5600	6.2876

In Table 4, the variance inflation factor results are presented to examine the presence of multicollinearity. The results reveal that the VIF for the OLS regression for the INTD, EXTD, INTR,

Table 2
Ridge cross-validation results

Number of Group	Ridge Parameter *k*	CV Error	MSE
5	0.3229	0.2146	0.0076

EXCR, and TROP to be 14.27, 3.53, 1.44, 1.76, and 9.56, respectively. The results reveal that the VIF obtained by using RR for the INTD, EXTD, INTR, EXCR, and TROP are 9.14, 2.73, 1.39, 1.54, and 6.29, respectively. Thus, it can be stressed that the collinearity problem caused by the predictors has been addressed considering a ridge estimate, where the VIF statistics < 10.00 for each identified economic growth drivers in comparison with OLS estimation method. Therefore, a ridge estimation technique and the selected ridge selected ridge parameter k (0.3229) after cross validation with MSE of 0.0076 or approximately (1%) as indicated in Table 2 and reveal in Table 4. This efficiently addressed the collinearity problem caused by the predictors (INTD, EXTD, INTR, EXCR, and TROP) considered for this study. It is also important to emphasize that these models or techniques will be considered to efficiently predict or forecast stable and reliable RGDP for the next quarters based on the parameter estimates obtained. Thus, the need to evaluate the predictive performance metrics of the methods in order to choose appropriate technique for the prediction. In this case, the results are presented in Table 5.

Table 5
Predictive evaluation metrics of a fitted OLS and ridge models

Model evaluation metrics	ML OLS estimate	Ridge estimates
RMSE	0.2927	0.2910
MAE	0.2319	0.2093
MAPE	1.9672	1.9538
TIC	0.1693	0.1493
BiasP	0.0103	0.0000
VarP	0.0284	0.0223

Table 5 presents the criterion to be used in selecting an optimal and efficient estimate for the prediction of economic growth using OLS regression and RR technique employed in this study. This is determined by RMSE, MAE, and MAPE. Findings from Table 4 show that a fitted RR model with RMSE, MAE, and MAPE obtained as 0.2910, 0.2093, and 1.9538, respectively, are the smallest when compared with the values obtained using OLS regression model. Hence, based on the estimated predictive evaluation criterion, a ridge estimation method is chosen as an efficient technique to predict economic growth in this study. Thus, a predictive plot showing the predicted values of RGDP for the next ten (10) quarters and are shown in Figure 2.

Predicted Values for RGDP using Ridge Estimation Model

Figure 2
Forecast plot for RGDP using ridge regression technique

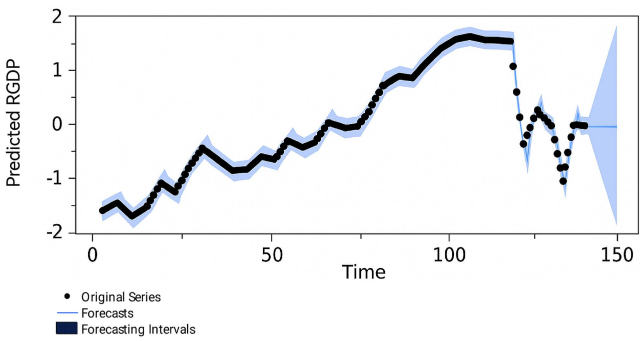

Figure 2 shows a predictive RR model's plot that is efficient in generating stable and reliable values for the economic growth rate (RGDP) in the study under consideration.

5. Conclusion

An examination of an efficient method of estimating and predicting the economic growth (RGDP) using INTD, EXTD, INTR, EXCR, and TROP as an economic growth drivers or predictors that are collinearly related. This investigation was done using an ordinary least square estimation method and a ridge estimation technique. In the study, findings from an ordinary least square estimation method using variance inflation factor show a collinearity problem among the predictors due to the influence of internal debt. Also, an ordinary least square estimation method reveals an underestimated standard error, leading to the statistical significance of all the estimated parameters of the fitted model at 5% level due to the existing strong correlation among the identified economic growth drivers (predictors) for this study. However, it is evident from the results obtained using variance inflation factor results for a ridge estimation technique with appropriate ridge parameter k (0.3229) after cross validation with MSE of 0.0076 or approximately (1%) to reduce the influence of the internal debt to address collinearity among the economic growth drivers (predictors). Moreso, the results indicate that the estimated standard errors of the parameters are efficient and better estimated with a ridge estimation technique at appropriate ridge parameter k (0.3229) after cross validation of the ridge trace plot. Thus, it can be posited according to 13 that a fitted ridge model with an efficient ridge parameter chosen from the ridge trace results will produce a minimum mean square of error and other model performance evaluation metrics. It can be emphasized that a ridge estimation method is more efficient than an ordinary least square technique in estimating the parameters and predicting economic growth using covariate predictors.

Recommendations

Therefore, based on the findings, this study is beneficial to the policymakers because it provides an understanding of the extent to which each of the identified economic growth drivers contributed to the economic growth (RGDP). Also, the negative influence of the external debt on the economic growth is not a doubt. Furthermore, to the researchers, it has significant benefit by strengthening the understanding of an appropriate estimation technique to be adopted when dealing with the covariate predictors as established in the data under consideration for efficient, optimal, stable, and reliable predictive values.

Ethical Statement

This study does not contain any studies with human or animal subjects performed by the author.

Conflicts of Interest

The author declares that he has no conflicts of interest to this work.

Data Availability Statement

The data that support the findings of this study are openly available in the Central Bank of Nigeria Statistical Bulletin via the link: https://www.cbn.gov.ng

Author Contribution Statement

Ebiwonjumi Ayooluwade: Conceptualization, methodology, software, validation, formal analysis, Iinvestigation, resources, data curation, writing – original draft, writing – review & editing, vsualization, supervision, project administration.

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