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RESEARCH ARTICLE

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Spectral Graph Theory-Based Knowledge Representation for Analyzing Wireless Mesh Networks

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Abstract: The analysis of Wireless Mesh Networks (WMNs) has traditionally focused on key performance metrics such as throughput and connectivity. However, this paper introduces a novel and advanced approach that utilizes spectral graph theory-based knowledge representation to more effectively analyze the topological and functional characteristics of WMNs. By carefully examining the eigenvalues and eigenvectors of the network's graph Laplacian, this method uncovers deep underlying patterns in connectivity, resilience, and overall efficiency. These insights provide a more comprehensive understanding of network performance and reliability, moving beyond traditional evaluation techniques. The method offers a systematic and effective way to visualize and optimize the structures of WMNs, leading to the potential for more efficient network design. Ultimately, this approach contributes valuable knowledge that can enhance both the theoretical and practical understanding of WMNs, offering new avenues for improving network performance and resilience.

Keywords: spectral analysis, mesh networks, routing, eigenvalues

1. Overview of Wireless Mesh Networks and Spectral Graph Theory

A wireless mesh network [1] (WMN) is a mesh network consisting of nodes (wireless access points) that are installed locally, which is a decentralized type of wireless network. Such a network can be represented in the form of a graph, which can be analyzed by spectral graph theory.

Spectral graph theory is closely related to artificial intelligence (AI) in several ways.

One important connection between spectral graph theory and AI is the use of graph convolutional neural networks [2] (GCNNs) for graph classification and representation learning tasks. GCNNs are a type of neural network that operate directly on graphs, and they make use of spectral graph theory concepts such as graph Laplacians and graph Fourier transforms.

Another connection between spectral graph theory and AI is the use of graph embeddings for representation learning. Graph embeddings are low-dimensional vector representations of graph structures that can be used as input to machine learning algorithms. Spectral graph theory methods, such as the graph Laplacian and its eigenvectors, are often used to generate graph embeddings that capture the structure and properties of the graph.

In addition, spectral graph theory is used in AI applications such as graph partitioning and graph clustering [3], which are important for tasks such as community detection in social networks and network analysis. It is also used in AI applications that involve optimization on graphs, such as graph-based semi-supervised learning and graph-based reinforcement learning.

Overall, spectral graph theory is a powerful tool that is widely used in AI and machine learning for tasks involving graph data.

Many spectral methods are developed in the past period of time which can be applied on area of theory of graph, virtualization, machine learning, computer graphic, social networks, communication networks, etc. Generally, spectral methods resolve the problems using or manipulating their eigenvalues, eigenvectors, eigenspace projection, or the combination of this parameters.

The article [4] discusses two different graph-based methods for Vehicular ad hoc networks (VANETs) connectivity analysis showing that they capture the same behavior as estimated using probabilistic models. The study is, then, extended to include the case of directed Vehicular ad hoc networks (VANETs), resulting from the utilization of different communication ranges by different vehicles.

The characteristic of mesh technology networking is quicker and easier access to the computer network. Benefits of this technology of networking are easier expansion, development, upgrade, and reliability with very few interruptions. Mesh network are made of clients (end devices) and routers, respectively nodes for forwarding packages.

Mesh networks are often used not only in wireless networks (WMN) but also in all other types of networks [5].

A WMN is a very popular technology that can provide broadband Internet access, wireless local area network coverage, and network connectivity for network operators and users. Due to the low cost, as well as the rapid development and popularity of wireless technologies, wireless networks (WMN) is increasingly attractive to Internet service providers [6].

Routing mesh network consists of two stages. The first step involves determining the cost of communications, paths, and the

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second stage routing information obtained in the distribution network. In general, we can say that the routing of mesh routers is formed from the mesh clients and the mesh received services. Mesh routers are divided into two categories: gateways and backbone. Gateway routers are connected to a wired network.

Today, there are a number of different algorithms used to route packets through a mesh network, and some of the most well-known are Destination Sequenced Distance Vector (DSDV), Optimized Link State Routing (OLSR), Dynamic Source Routing (DSR), and Link Quality Source Routing Algorithm (LQSR) [7].

In this paper, we have analyzed WMN routing protocols. A software system for calculating the parameters of spectral graph theory has been implemented [8]. The observed mesh network is represented by a suitable graph, and then the resulting graph is analyzed using spectral graph theory techniques.

2. Spectral Graph Theory for Analyzing Wireless Mesh Networks

The basis of the spectral graph theory is to find the appropriate matrices associated with a given graph, especially the adjacency matrix and the Laplacian matrix. After that, it is necessary to determine the eigenvalues and eigenvectors of those matrices and connect the obtained values with the topological properties of the observed graph [9].

In spectral graph theory, each graph is analyzed using the eigenvalues of the corresponding matrix that describes that graph. The matrices used in that analysis are adjacency matrix A, the Laplace matrix L, and the distance matrix D. Also, normalized matrices are used.

This method has an important role in the study of complex networks such as Internet search, image processing, shape recognition, and clustering. The application gives significant results in the interconnection of networks, social networks, mathematical chemistry, economics, and other sciences [10, 11].

Spectral theory of graphs is based on their eigenvalues and eigenvectors.

An eigenvalue is a scalar value that is associated with a linear transformation. Given a linear transformation represented by a matrix A, an eigenvalue of A is a scalar λ [12] that satisfies the equation:

$$Ax = \lambda x \tag{1}$$

where x is a non-zero vector, called the eigenvector. The equation says that when the matrix A is applied to the eigenvector x, the result is a scalar multiple of the original vector x. Matrix A is the adjacency matrix.

Eigenvalues and eigenvectors have many important applications in mathematics and physics. They are used, for example, to study the stability of equilibrium points in dynamical systems, to diagonalize matrices, and to understand the structure of certain types of graphs.

Spectrum of the graph G is defined by the eigenvalues of the matrix A for given graph.

To obtain certain information about the graph, spectral graph theory uses the following matrices:

- Adjacency matrix,
- Laplacian matrix, and
- Normalized Laplacian matrix.

For a given graph, the adjacency matrix is calculated as follows:

$$A_{i,j} = \begin{cases} 1, & \textit{if there is a branch from i to j} \\ & 0, \ \textit{other} \end{cases} \tag{2}$$

If the graph is weighted, then adjacency matrix is determined in the following way:

$$A_{i,j} = \begin{cases} w(i,j), & \textit{if there is a branch from i to j} \\ & 0, \ \textit{other} \end{cases} \tag{3}$$

Normalized adjacency matrix is calculated as follows:

$$\hat{A} = \sqrt{D^{-1}}\, A\, \sqrt{D^{-1}} \tag{4}$$

The eigenvalues of the adjacency matrix A are denoted by $\lambda_1, \lambda_2, \ldots, \lambda_n$ and represent the spectrum of the matrix A.

$$\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_{n-1} \geq \lambda_n = 0 \tag{5}$$

In graph theory, the degree matrix of a graph D is a diagonal matrix that represents the degree of each vertex in the graph. The degree of a vertex is the number of edges incident to it, and the degree matrix is a diagonal matrix with the degrees of the vertices on the main diagonal.

The degree matrix is often used in the definition of the graph Laplacian matrix, which is a matrix that encodes the structure of the graph. The Laplacian matrix is defined as the difference between the degree matrix and the adjacency matrix of the graph, where the adjacency matrix is a matrix that represents the connections between the vertices. The Laplacian matrix is a useful tool for analyzing the structure and properties of a graph.

$$L = D - A \tag{6}$$

Eigenvalues of the matrix L are called Laplacian eigenvalues:

$$\mu_1 \geq \mu_2 \geq \cdots \geq \mu_{n-1} \geq \mu_n \tag{7}$$

For graphs without isolated vertices, the normalized Laplacian has the following relationship to L, A, and D [13]:
Laplacian matrix is calculated as follows:

$$Lnorm = D - 1/2AD - 1/2 \tag{8}$$

3. Performance Evaluation Using Spectral Graph Metrics

Based on the created topology, software for spectral analysis of graph is starting and compatible graph is generated (picture 3). Graph branches correspond to characteristics of the links given in the topology. Created topology has characteristics of real WMN.

The structural features of graphs can be used to study connectivity, and they have a significant impact on various processes in complex networks, so the analysis of these networks is based on the use of metrics that can be expressed using observed topological features [14].

Graph characteristics can be studied through graph topology. The topology of the graph defines the connection, as well as the relationships between the nodes.

Each node in the graph can be represented by some characteristics. Each branch of the graph can be specified as a set of weight functions. A characteristic can be, for example, processing time. A branch can represent, for example, delay, bandwidth, packet loss, etc.

Metrics is a topological, if it's possible to calculate only with adjacency matrix. Topological metrics can be classified on matrix based on distance, connectivity, and graph spectrum [14].

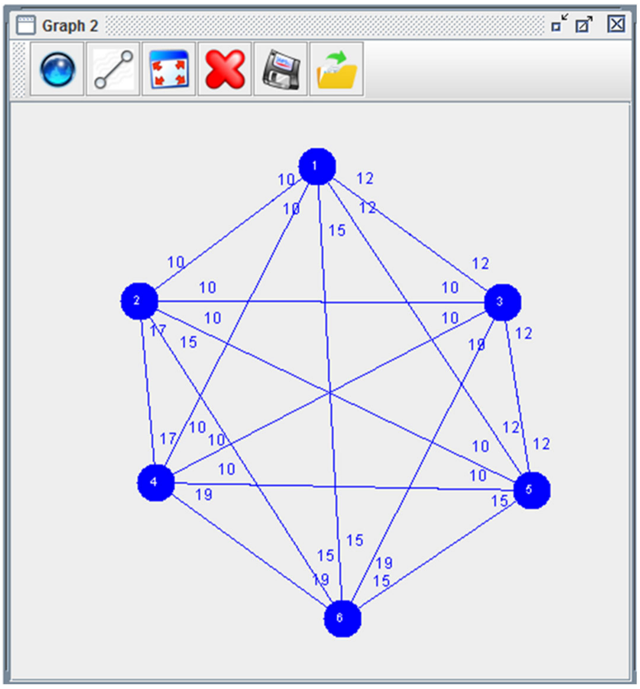
The following topological metrics are used in the analysis of complex networks using spectral graph theory:

- Fidler vector
- Algebraic connectivity
- Spectral radius
- Principal eigenvector.

3.1. Fiedler's vector

In graph theory, a Fiedler's vector is a special kind of eigenvector of the graph Laplacian matrix. The Fiedler's vector of a graph is defined as the eigenvector corresponding to the second smallest eigenvalue (also known as the Fiedler's eigenvalue) of the graph Laplacian matrix [10]. This eigenvalue is known as the algebraic connectivity of the graph, and the corresponding eigenvector is known as the Fiedler's vector.

Figure 1
Topology graph



The Fiedler's vector can be used to partition a graph into two clusters by finding a "cut" through the graph such that the vertices on one side of the cut have relatively small values in the Fiedler's vector, and the vertices on the other side have relatively large values [14]. The idea is that this cut will divide the graph into two connected components with a relatively small number of edges between them, resulting in a "good" partition of the graph.

If the Fielder's vector is x_{n-1}, then the clusterization starts between nodes that correspond to positive values of vector x_{n-1} and they are joined to the first cluster, then nodes that correspond to negative values of vector x_{n-1} are joined to second cluster [15, 16].

3.2. Algebraic connectivity

Algebraic connectivity is a measure of the connectedness of a graph, which is a mathematical structure used to represent pairwise relationships between objects. It is defined as the second smallest eigenvalue [10] of the Laplacian matrix of the graph. The algebraic connectivity of a graph is a measure of how well connected the graph is, and it is closely related to the number of paths between pairs of nodes in the graph. A graph with a high algebraic connectivity is said to be well connected, while a graph with a low algebraic connectivity is said to be poorly connected. Algebraic connectivity is an important concept in graph theory and has numerous applications in fields such as computer science, engineering, and physics. It is used for analysis of the robustness and synchronizability of the networks [15].

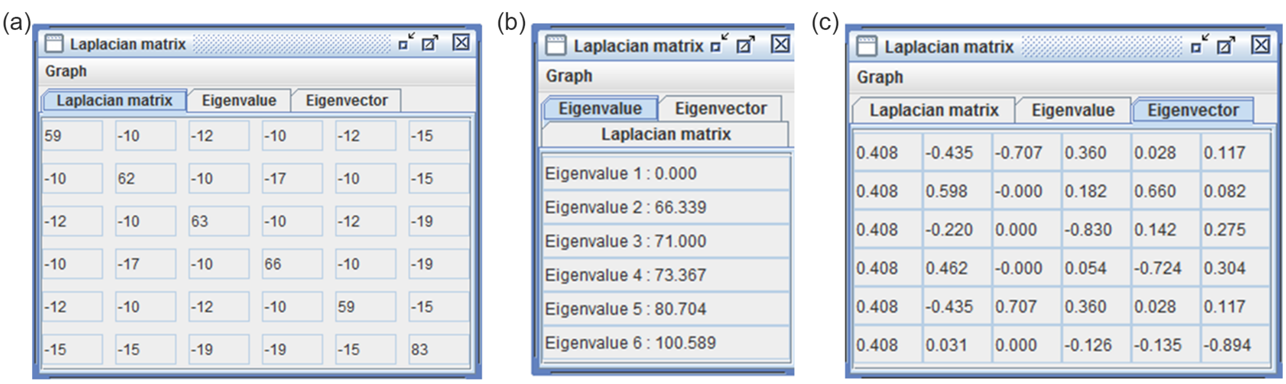
3.3. Spectral radius

The spectral radius of a matrix is the maximum absolute value of its eigenvalues.

$$\rho = \max_{1 \le i \le n} |\lambda_i| \tag{9}$$

It is a measure of how large the eigenvalues of a matrix are in magnitude. The spectral radius is also known as the matrix norm, and it is closely related to the operator norm. The spectral radius is often used as a measure of the stability of a system, and it plays an important role in the analysis of dynamical systems. In particular, the spectral radius of the matrix representing the linear part of a system's dynamics determines the stability of the system. If the spectral radius is less than 1, the system is stable, while if the spectral radius is greater than 1, the system is unstable. The spectral radius is also used in the analysis of

Figure 2
Eigenvalues and eigenvectors of the Laplacian matrix for the graph in Figure 1



(a)

59	-10	-12	-10	-12	-15
-10	62	-10	-17	-10	-15
-12	-10	63	-10	-12	-19
-10	-17	-10	66	-10	-19
-12	-10	-12	-10	59	-15
-15	-15	-19	-19	-15	83

(c)

0.408	-0.435	-0.707	0.360	0.028	0.117
0.408	0.598	-0.000	0.182	0.660	0.082
0.408	-0.220	0.000	-0.830	0.142	0.275
0.408	0.462	-0.000	0.054	-0.724	0.304
0.408	-0.435	0.707	0.360	0.028	0.117
0.408	0.031	0.000	-0.126	-0.135	-0.894

Table 1
Changes on algebraic connectivity by changing the signal strength in WMN

Link	Value	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
L_{1-2}	**10**	**15**	10	10	10	10	10	10	10	10	10	10	10	10	10	10
L_{1-3}	**12**	12	**17**	12	12	12	12	12	12	12	12	12	12	12	12	12
L_{1-4}	**10**	10	10	**15**	10	10	10	10	10	10	10	10	10	10	10	10
L_{1-5}	**12**	12	12	12	**17**	12	12	12	12	12	12	12	12	12	12	12
L_{1-6}	**15**	15	15	15	15	**20**	15	15	15	15	15	15	15	15	15	15
L_{2-3}	**10**	10	10	10	10	10	**15**	10	10	10	10	10	10	10	10	10
L_{2-4}	**17**	17	17	17	17	17	17	**22**	17	17	17	17	17	17	17	17
L_{2-5}	**10**	10	10	10	10	10	10	10	**15**	10	10	10	10	10	10	10
L_{2-6}	**15**	15	15	15	15	15	15	15	15	**20**	15	15	15	15	15	15
L_{3-4}	**10**	10	10	10	10	10	10	10	10	10	**15**	10	10	10	10	10
L_{3-5}	**12**	12	12	12	12	12	12	12	12	12	12	**17**	12	12	12	12
L_{3-6}	**19**	19	19	19	19	19	19	19	19	19	19	19	**24**	19	19	19
L_{4-5}	**10**	10	10	10	10	10	10	10	10	10	10	10	10	**15**	10	10
L_{4-6}	**19**	19	19	19	19	19	19	19	19	19	19	19	19	19	**24**	19
L_{5-6}	**15**	15	15	15	15	15	15	15	15	15	15	15	15	15	15	**20**
μ_{n-1}	66,339	**68,684**	66,429	68,167	66,339	66,895	67,966	66,394	**68,684**	67,442	67,530	66,429	66,537	**68,167**	67,021	66,895

Table 2
Change of the spectral radius by changing the signal strength in WMN

Link	Value	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
L_{1-2}	**10**	5	10	10	10	10	10	10	10	10	10	10	10	10	10	10
L_{1-3}	**12**	12	7	12	12	12	12	12	12	12	12	12	12	12	12	12
L_{1-4}	**10**	10	10	5	10	10	10	10	10	10	10	10	10	10	10	10
L_{1-5}	**12**	12	12	12	7	12	12	12	12	12	12	12	12	12	12	12
L_{1-6}	**15**	15	15	15	15	10	15	15	15	15	15	15	15	15	15	15
L_{2-3}	**10**	10	10	10	10	10	5	10	10	10	10	10	10	10	10	10
L_{2-4}	**17**	17	17	17	17	17	17	12	17	17	17	17	17	17	17	17
L_{2-5}	**10**	10	10	10	10	10	10	10	5	10	10	10	10	10	10	10
L_{2-6}	**15**	15	15	15	15	15	15	15	15	10	15	15	15	15	15	15
L_{3-4}	**10**	10	10	10	10	10	10	10	10	10	5	10	10	10	10	10
L_{3-5}	**12**	12	12	12	12	12	12	12	12	12	12	7	12	12	12	12
L_{3-6}	**19**	19	19	19	19	19	19	19	19	19	19	19	14	19	19	19
L_{4-5}	**10**	10	10	10	10	10	10	10	10	10	10	10	10	5	10	10
L_{4-6}	**19**	19	19	19	19	19	19	19	19	19	19	19	19	19	14	19
L_{5-6}	**15**	15	15	15	15	15	15	15	15	15	15	15	15	15	15	10
λ_1	**66.152**	**64.761**	64.737	64.675	64.829	64.401	64.664	64.610	**64.761**	64.312	64.571	64.737	64.281	**64.675**	64.199	64.401

Figure 3
Fiedler vector chart

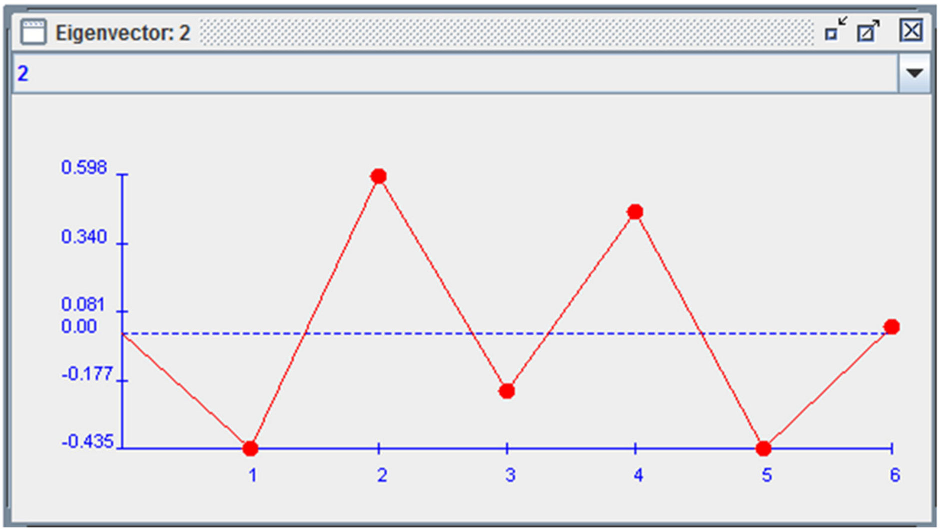
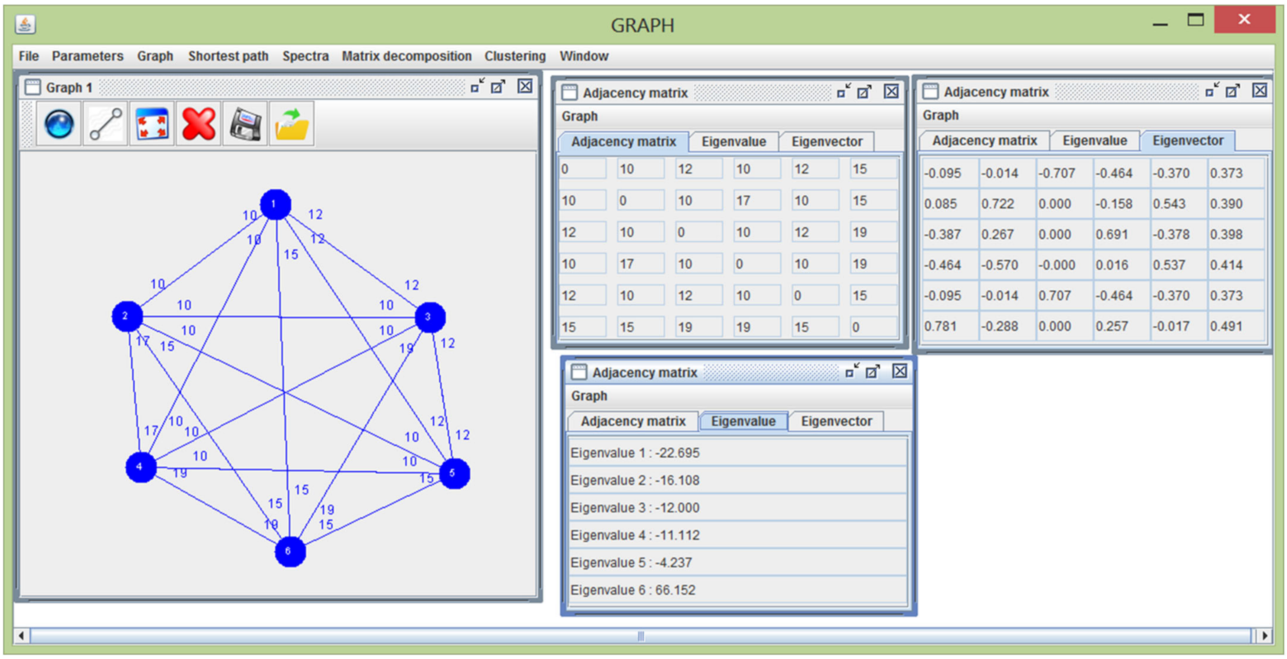


Figure 4
Adjacency matrix, eigenvalues, and eigenvectors



network systems, where it is related to the connectivity and robustness of the network.

Smaller spectral radius corresponds to greater robustness in the network regarding the spreading of the viruses, also the greater protection from viruses can be achieved with the minimization of spectral radius [11, 15, 17].

3.4. Principal eigenvector

The principal eigenvector of a matrix is the eigenvector corresponding to the largest eigenvalue of the matrix [18]. The principal eigenvector is often of particular interest because it corresponds to the direction in which the matrix has the greatest effect. It is also known as the dominant eigenvector or leading eigenvector. The principal eigenvector is often used in the analysis of networks, where it can provide insight into the centrality or importance of different nodes in the network [11]. It is also used in machine learning and data analysis, where it can be used to identify patterns in data.

Google's PageRank algorithm is using the variation of principal eigenvector in order to indicate the importance of the web page [19]. Analyzing of the coefficient of Laplacian characteristic polynomial and biggest eigenvalue of distance matrix, and also two invariants which are based on the graph spectrum – energy and Estrada index.

Table 3
Dependence of change of the spectral radius from ratio ($x1(i)$ * $x1(j)$)

	$L_{1\text{-}5}$	$L_{1\text{-}2}$	$L_{2\text{-}5}$	$L_{1\text{-}3}$	$L_{3\text{-}5}$	$L_{1\text{-}4}$	$L_{4\text{-}5}$	$L_{2\text{-}3}$	$L_{2\text{-}4}$	$L_{3\text{-}4}$	$L_{1\text{-}6}$	$L_{5\text{-}6}$	$L_{2\text{-}6}$	$L_{3\text{-}6}$	$L_{4\text{-}6}$
λ_1	64.829	**64.761**	**64.761**	64.737	64.737	64.675	**64.675**	64.664	64.61	64.571	64.401	64.401	64.312	64.281	64.199
$x_1(i) * x_1(j)$	0.139	0.145	0.145	0.148	0.148	0.154	0.154	0.155	0.161	0.164	0.183	0.183	0.191	0.195	0.203

3.5. Analysis: Signal strength influence on the algebraic connectivity WMN

Mesh nodes are shown like graph nodes; OLSR matrix value which represents signal strength between WMN nodes is shown with branches of graph (Figure 1).

Possibility of modification of graph is considered, respectively WMNs, in relation to changes in signal strength between some nodes so optimization of robustness inside network.

The method of eigenvalues is described, which determines whether the entities are connected as one network, as well as the adjacency exponent method, which determines whether there is a path between two entities [4].

Robustness of the network is shown with spectral parameters, such as algebraic connectivity.

Here, it will be shown how to establish the connection between the algebraic connectivity and the value of the Fiedler's vector.

For the mesh network, which is represented by the graph in Figure 1, the corresponding adjacency matrices, eigenvalues, and eigenvectors, as well as the Laplacian matrix are shown in Figure 2.

The idea is to analyze changes in the algebraic connectivity of mesh networks depending on the change in signal strength between the corresponding nodes.

Special software has been developed for this purpose. It can be used to analyze complex networks using spectral graph theory. Specific topological features of graphs are used to characterize connectivity and have a significant impact on dynamic processes in complex networks, and spectral graph theory studies the relationship between graphs and eigenvalues and eigenvectors [20].

Table 1 shows the dependence of algebraic connectivity depending on the change in signal strength between different nodes for a constant value.

Value of algebraic connectivity for graph from the picture 1 is $\mu5 = 66{,}339$. By changing the signal strength for value 5, algebraic connectivity is changed from 66,339, when $L_{1\text{-}5}$ is changed from 12 to 17, until 68,684, when $_{L2\text{-}5}$ is changed from 10 to 15 or $L_{1\text{-}2}$ is changed from 10 to 15 (Figure 1).

The values of the algebraic connectivity correspond to the second eigenvector, so it can be seen from Figure 2 that the corresponding values for the algebraic connectivity are −0.435, 0.598, −0.220, 0.462, −0.435, and 0.031.

Based on the experimental results, the conclusion is:

1) If the signal strength in network is increased, the value of algebraic connectivity is also increased or stays the same.
2) Algebraic connectivity will not be changed if the signal strength is changed between the nodes which have the same Fiedler's vector values (Figure 3).
3) If the signal strength is changed between the nodes with the min or max values of Fiedler's vector, maximum increase of algebraic connectivity can be seen.

3.6. Analysis: Signal strength influence on spectral radius of WMN

Eigenvalues of vectors that correspond to the spectral radius are (0.373, 0.390, 0.398, 0.414, 0.373, 0.491) (Figure 4).

Table 2 shows dependence of spectral radius from the signal strength between different nodes for constant value.

Based on the experimental results, the conclusion is:

1) Spectral radius is changed if the signal strength inside the network is changed in accordance with the change of value $x_1(i) * x_1(j)$, where x_1 represents principal eigenvector which corresponds to the biggest eigenvalue (spectral radius) (Table 3).

2) If the signal strength is decreasing, the value of spectral radius is reduced also.

The main goal is that algebraic connectivity be as big as possible and spectral radius as low as possible. This opposed demand is possible to accomplish by reducing the signal strength between nodes for which the value of Fiedler's vector remains the same.

4. Conclusions

In conclusion, this study demonstrated the utility of spectral graph theory for analyzing WMNs.

The algebraic connectivity and spectral radius of the mesh network are examined in relation to changes in signal strength between nodes.

It was found that when the signal strength increased between nodes with minimum or maximum values of the Fiedler's vector, the algebraic connectivity reached its maximum. On the other hand, no change in algebraic connectivity was observed when the signal strength between nodes with the same values as the Fiedler vector changed.

Also, it has been shown that the spectral radius is changed if the signal strength in the network is changed in accordance with the change of value of the principal eigenvector which corresponds to the largest eigenvalue.

By increasing or decreasing the signal strength between the corresponding nodes in the network, it is possible to directly influence the values of the spectral radius, as well as the algebraic connections, and thus influence the specific topological characteristics of the graph, that is, the dynamic processes in WMN.

Our simulations and experiments validated the usefulness of spectral graph theory as a tool for understanding and optimizing the performance of WMNs. Our results can inform the design and deployment of WMNs for various applications, such as Internet access, disaster response, and military communication.

Ethical Statement

This study does not contain any studies with human or animal subjects performed by the author.

Conflicts of Interest

The author declares that he has no conflicts of interest to this work.

Data Availability Statement

Data sharing is not applicable to this article as no new data were created or analyzed in this study.

Author Contribution Statement

Nenad M. Jovanovic: Conceptualization, Methodology, Software, Validation, Formal analysis, Investigation, Resources, Data curation, Writing – original draft, Writing – review & editing, Visualization, Supervision, Project administration, Funding acquisition.

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