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Studies of Issues on Hybrid Neural Networks for Pricing Financial Options

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Abstract: Neural networks (NN) in combination with parametric models (i.e., Hybrid models) are increasingly employed for option pricing. However, a fundamental question needs to be addressed within the current research domain of financial option pricing utilizing hybrid NN: Does integrating a NN with a more advanced mathematical option model enhance its pricing capabilities compared to integrating with a robust mathematical model? In this paper, we conducted a novel ANN-Heston and ANN-CS option pricing research based on the 50ETF options obtained from the Shanghai Stock Exchange covering January 2018 to December 2021. Having compared the pricing accuracies between ANN-Heston and ANN-CS, we show that the hybrid ANN in combination with the CS model is adequately competent in pricing Chinese options. We also comment that the parametric model should be robust with only some parameters to be estimated. The CS model can capture Chinese option features, and its hybrid ANN model exhibits remarkable competence in pricing options. This research is useful for practitioners and researchers in the field of option trading.

Keywords: hybrid neural network, option pricing, Chinese options, parametric models

1. Introduction

Option pricing has been a prominent area of research for many years since Black, Scholes, and Merton introduced their Nobel Prize-winning option pricing model (OPM) in 1973 [1]. The Black-Scholes-Merton model is widely regarded as one of the greatest achievements in financial theory over the past several decades. However, empirical research has demonstrated that the formula is subject to systematic biases [2].

As is widely recognized, the bias in the Black-Scholes (BS) model arises from the numerous assumptions upon which it is based, including constant market volatility, geometric Brownian motion of the underlying asset prices, constant interest rates, a fixed drift term, and the assumption of efficient market conditions. Consequently, subsequent research following the development of the BS model has sought to relax some of these assumptions, exploring alternatives such as stochastic volatility, stochastic interest rates, jump-diffusion processes, and regime-switching in economics.

On the other hand, models using machine learning models, such as artificial neural networks (ANNs), present promising alternatives to traditional parametric OPMs [3–8]. ANNs do not require specific assumptions regarding the input variables of options or their transaction data. Option pricing functions are inherently multivariate and highly nonlinear, making ANNs effective tools for option pricing in fluctuating market conditions. In particular, ANNs can capture the nonlinear dependencies between input and output variables. Consequently, ANNs can potentially address the biases associated with the BS model [9].

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There are numerous applications of ANNs in pricing financial options. Meanwhile, scholars have consistently reviewed research on machine learning for options. In particular, Ruf and Wang [10] conducted a comprehensive review after analyzing approximately 150 papers and provided valuable suggestions for implementing ANNs as nonparametric estimation tools in option pricing and hedging.

Investigations into option problems have focused on the input features of ANNs [11] and the outputs of ANNs [12, 13]. Some researchers concentrate on pricing American options [14, 15]. Others have explored exotic options that involve more complex processes, including jumps and stochastic volatilities [16, 17]. In recent years, several papers have addressed issues concerning market efficiency conditions, such as no arbitrage and market frictions [18–20], and some others have adopted interpretable ALE method [21] and GA-BP neural network [22].

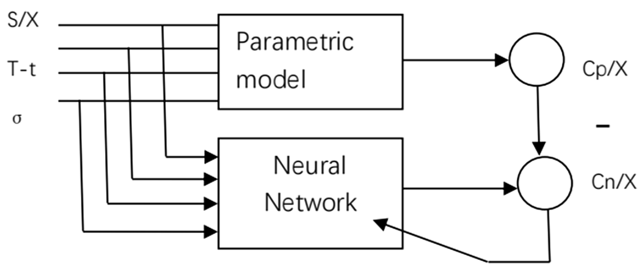
In this paper, we will utilize hybrid neural networks to price Chinese options. These networks differ from the previously mentioned models in that they integrate parametric models with ANN models, rather than relying solely on ANN models. We will introduce hybrid ANN models in the following section. More importantly, we will address several challenges in the field of option pricing. First, the literature indicates that hybrid ANNs are often combined with relatively robust parametric models, such as the BS model [23]. Therefore, we need to explore whether a more complex parametric model can enhance the option pricing performance of ANNs. Second, the Chinese options market is still emerging; notably, the Chinese SSE50 options exhibit complex features and are influenced by risk appetite [24]. Consequently, we are interested in applying hybrid ANN models to these options. In the following sections, we will first present the models used in our research and provide a detailed discussion on parametric models, including both the CS model and the Heston model, with a particular focus on hybrid neural networks for financial options. We will then present the results of pricing Chinese options using hybrid ANNs. The performance of the models will be illustrated, followed by our conclusions.

2. Models

2.1. Hybrid ANN model

As mentioned earlier, in most ANN models for option pricing, the option price is typically used as the output. In some instances, the output may be the option price normalized by the strike price. Notably, ANNs can be combined with certain parametric models to enhance their option pricing capabilities. In this approach, the ANN is trained to learn the so-called bias, which is the difference between the market price and the price estimated by a parametric model. Such an ANN is referred to as a hybrid ANN (see Figure 1), for example, the work of Andreou et al. [25]. In a hybrid ANN model, the ANN essentially calibrates the parametric models [23, 25].

Figure 1
Illustration of a hybrid ANN model



A recent surge in the application of hybrid ANNs has been observed in option pricing [10, 23, 26]. In this approach, option prices are initially mapped to a parametric model, which is subsequently utilized to determine the option prices [10]. This methodology can significantly enhance the accuracy of option pricing. More recently, models that are more complex than the BS model have been developed to improve ANNs. For example, Dimitroff et al. [27], and McGhee [28] have calibrated stochastic volatility models. Additionally, Hernandez [29] employed an ANN to calibrate a single-factor Hull-White model, while Bayer et al. [30] focused on calibrating rough volatility models.

The hybrid neural network depicted in Figure 1 employs a backpropagation (B-P) neural network in combination with a parametric model. The B-P neural network is a multilayer feedforward architecture that utilizes the error B-P algorithm. This type of neural network exhibits strong nonlinear fitting capabilities and robust generalization performance in information processing. In a B-P neural network with hidden layers, the model can effectively map complex nonlinear relationships from the input layer to the output layer, thereby revealing the underlying patterns and characteristics inherent in these mappings. We will also consider various parametric models, which will be discussed in detail below.

2.2. Parametric models

Parametric models describe stationary nonlinear relationships between theoretical option prices and related variables. However, it is well-documented that these models often produce significant discrepancies in their predictions of market option prices. Consequently, it is advantageous for such models to collaborate with ANNs.

When selecting a parametric model, we prefer to adhere to the recent guidance provided by Ruf and Wang [10], which suggests that stationary features should be utilized as inputs. While there are significant research efforts aimed at integrating ANNs with increasingly complex parametric models, such as jump-diffusion models, we believe that parametric models need not be overly complicated or involve numerous parameters that must be estimated.

As the benchmark in this paper, we first consider the semi-parametric Corrado and Su (CS) model for Chinese options, which accommodates excess skewness and kurtosis. This model can serve as a proxy for more complex parametric models. Developed by Corrado and Su [31], the CS model explicitly accounts for excess skewness and kurtosis, thereby providing a correction to the BS model to some extent. The CS model is classified as semi-parametric because it does not depend on specific assumptions regarding the underlying stochastic process. CS define their model for option price as follows:

$$C^{CS} = C^{BS} + \mu_3 Q_3 + (\mu_4 - 3)Q_4 \tag{1}$$

Where:

$$\mathrm{C^{BS}} = \mathrm{S_0 N(d_1) - Ke^{-rt}N(d_2)} \tag{2}$$

$$\mathrm{Q_3} = \frac{1}{3!}\mathrm{S_0}\sigma\sqrt{\mathrm{T}}\left[\left(2\sigma\sqrt{\mathrm{T}} - \mathrm{d_1}\right)\mathrm{n(d_1)} + \sigma^2\mathrm{TN(d_1)}\right] \tag{3}$$

$$\mathrm{Q_4} = \frac{1}{4!}\mathrm{S_0}\sigma\sqrt{\mathrm{T}}\left[\left(\mathrm{d_1^2} - 1 - 3\sigma\sqrt{\mathrm{T}}\mathrm{d_2}\right)\mathrm{n(d_1)} + \sigma^3\mathrm{T^{3/2}N(d_1)}\right] \tag{4}$$

$n(z)$ is the standard normal probability density function, N(z) is the standard normal cumulative distribution, S_t is a random stock price at time t, μ_3 is the skewness, and μ_4 is the kurtosis.

To address the primary challenge outlined in the introduction, we need to design an innovative hybrid ANN model that integrates the neural network with a more sophisticated parametric model than the classical model. For this purpose, we will consider a moderately complex OPM: Heston's volatility model.

Heston's model [32] is a bivariate composite model that consists of two coupled univariate models. First, the underlying asset follows:

$$dS_t = \mu S_t dt + \sqrt{V_t} S_t dW_t^{(S)} \tag{5}$$

S_t is the price of the underlying at time *t*, μ is the drift term, V_t is the volatility at t, $W_t^{(S)}$ represents Brownian motion. Secondly, Heston's model assumes that the volatility also follows a CIR process:

$$dV_t = \kappa(\theta - V_t)dt + \sigma_V \sqrt{V_t} dW_t^{(V)} \tag{6}$$

where $W_t^{(V)}$ is another Brownian motion。 $W_t^{(S)}$ and $W_t^{(V)}$ satisfy:

$$dW_t^{(S)} dW_t^{(V)} = \rho dt \tag{7}$$

where ρ *is* the instantaneous correlation.

This model usually corresponds to a price process whose volatility (variance rate) is governed by the second univariate model.

The main parameters of interest in the Heston Model are v, κ, θ, σ, and ρ. $v(t)$ is the instantaneous variance at time t, r is the risk-neutral rate of return, θ is the long-run average variance (as t tends to infinity, the expected value of $v(t)$ tends to θ), κ is the rate at which $v(t)$ reverts to θ and σ is the volatility of the variance. In the work, we shall calibrate the Heston's OPM by using simulated annealing algorithm (SAA).

3. Shanghai 50ETF Option

The data for 50ETF options were obtained from the Shanghai Stock Exchange, covering the period from January 2018 to December 2021. The underlying asset prices are illustrated in Figure 1 for the duration of the study. The options are classified as European options. Table 1 presents several typical option contracts along with their trading details.

There are 43,900 call option data points. The first 30,730 option contracts, representing 70% of the entire dataset, are designated as the training set, while the remaining 13,170 option contracts, or 30% of the dataset, are allocated as the test set. The factors that are theoretically considered to influence the option price include the underlying share price, the time remaining until the option's expiration date, the exercise price, the risk-free interest rate, and the volatility of the underlying share price returns.

When selecting the input variables, five key factors are considered in the model: the price of the underlying asset (S), the option strike price (K), the risk-free interest rate, the time to expiration, and the volatility.

It is important to note that, for the time to expiration (T) in each option contract, trading days are calculated based on the assumption of 252 days in a year. In terms of duration, an option contract is classified as having a short-term expiration if it has an expiration date of less than 60 days, a medium-term expiration if it has an expiration date between 60 and 180 days, and a long-term expiration if it has an expiration date of 180 days or more. To calculate the risk-free rate (r), we match each option contract to a continuous rate r using nonlinear cubic spline interpolation, utilizing 3-month, 6-month, and 1-year savings rates collected from the Bank of China.

4. Model Techniques and Pricing Performance

The ANN developed in this study is a standard densely connected neural network featuring four hidden layers, each containing 120 neurons. In this research, we implemented two technical steps to train and evaluate the ANN, which are outlined below:

1) We first use Moneyness Ratio, C^{mrk}/K (i.e., the call market price C^{mrk} normalized by its strike price K) as the objective function for approximation.
2) We then implemented the hybrid structure ($(C^{mrk} - C_p)/K$), where the objective function is represented by the pricing error between the option market price and the parameter model estimate C_p, and normalized by the strike price.

We employed a minimization process similar to that proposed by Andreou et al. [25] to derive four distinct sets of implicit parameters from the parametric CS model, utilizing the sum of squared errors (SSE).

1) the first optimization approach involves obtaining the daily average implied structural parameters, which we will denote as *implied volatility1 or imvol1*, *implied skewness1 or imsk1, implied kurtosis1 or imku1*, with all available option trading data.
2) In the second approach, we fit the CS model for options with the same maturity by minimizing the SSE to obtain the implied parameters, which we will denote as *implied volatility2 or imvol2*, *implied skewness2 or imsk2, implied kurtosis2 or imku2*.
3) In the third approach, for each maturity, the four nearest options are grouped based on their moneyness ratios to minimize the aforementioned SSE function for the implied parameters. We will refer to these parameters as *implied volatility3 or imvol3, implied skewness3 or imsk3, implied kurtosis3 or imku3*;
4) We finally calibrate the implied structural parameters, which we will denote as implied *volatility4 or imvol4*, *implied skewness4 or imsk4, implied kurtosis4 or imku4*, by focusing on the Brownian volatility for each contract, aiming to reduce the residual error to zero or a negligible value after fixing the skewness and kurtosis coefficients to the values obtained in the previous process.

Table 1
Examples of 50 ETF options on Shanghai stock exchange

Code	Open	High	Low	Close	K	t
510050C1803M02650	0.4831	0.4991	0.4507	0.455	2.65	0.230159
510050C1803M03100	0.1042	0.1131	0.0803	0.0845	3.1	0.230159
510050C1803M03200	0.0594	0.066	0.044	0.0469	3.2	0.230159
510050C1803M03300	0.0329	0.037	0.023	0.025	3.3	0.230159
510050C1803M03400	0.0184	0.0203	0.0121	0.0136	3.4	0.230159
510050C1803M03500	0.0106	0.0116	0.0068	0.0075	3.5	0.230159
510050C1806M03400	0.0675	0.0725	0.0562	0.0567	3.4	0.59127
510050C1806M03500	0.0502	0.0532	0.0409	0.0421	3.5	0.59127
510050C1806M03600	0.0381	0.0402	0.0305	0.031	3.6	0.59127
510050C1809M03000	0.3021	0.3104	0.2741	0.2763	3.0	0.952381
510050C1809M03100	0.2459	0.2509	0.2162	0.2184	3.1	0.952381
510050C1809M03200	0.1903	0.1993	0.1697	0.1721	3.2	0.952381
510050C1809M03300	0.1503	0.1579	0.1325	0.1328	3.3	0.952381
510050C1809M03400	0.1207	0.1232	0.1029	0.1042	3.4	0.952381
…	…	…	…	…	…	…

Table 2
Examples of Heston parameters obtained via optimizations

Settlement	*V0*	*ThetaV*	*Kappa*	*SigmaV*	*RhoSV*
28/03/2018	0.343461	0.400203	0.200249	0.595387	−0.09803
29/03/2018	0.255192	0.400172	0.200222	0.598773	−0.09464
17/12/2018	0.070361	0.400083	0.200366	0.574782	−0.11257
24/12/2018	0.149299	0.400098	0.200145	0.598538	−0.09919
16/04/2019	0.090128	0.399576	0.199421	0.599631	−0.09932
17/04/2019	0.116268	0.400697	0.200945	0.600602	−0.10112
21/06/2019	0.099597	0.399999	0.199999	0.599971	−0.09981
09/12/2019	0.045968	0.403208	0.208038	0.394389	−0.16461
12/12/2019	0.055748	0.402818	0.207313	0.341996	−0.19896
16/12/2019	0.070171	0.401575	0.204417	0.367826	−0.22405
20/03/2020	0.051153	0.399844	0.199753	0.601596	−0.10212
23/03/2020	0.040605	0.399866	0.199785	0.600689	−0.10323
24/03/2020	0.078926	0.399959	0.199936	0.600233	−0.10099
11/09/2020	0.035021	0.392188	0.183211	0.758964	−0.55937
08/12/2020	0.945955	0.405281	0.205389	0.614998	−0.14105
14/01/2021	0.198294	0.401991	0.202836	0.601829	−0.10459
15/01/2021	0.055365	0.397987	0.197099	0.599696	−0.09787
03/02/2021	0.251255	0.402254	0.203123	0.603121	−0.10702
08/02/2021	0.001737	0.393038	0.186911	0.598709	−0.24854
16/09/2021	0.074861	0.401378	0.203953	0.365974	−0.23529
17/09/2021	0.079451	0.400729	0.202154	0.465834	−0.17242
…	…	…	…	…	…

As a benchmark, we also use *rv60 (the 60-day realized volatility)*, *sk60 (the 60-day skewness), and ku60 (the 60-day kurtosis)* as an alternative input for hybrid ANN training.

For Heston's model, calibrating the model parameters involves high-dimensional optimization; therefore, it is not suitable to consider a large number of option contracts during each calibration. In this process, we calibrate the Heston model using the SAA to obtain the model parameters. For instance, in each calibration, we first fix the settlement date (e.g., March 28, 2018), and there will be many different option contracts (23 contracts in the case of March 28, 2018) with various strike prices, time to maturity, and market prices. We then conduct the optimization process to derive the model parameters. In this manner, the Heston model parameters are retained for multiple settlement dates based on our selections. Table 2 presents some examples of Heston parameters obtained through optimization (note: only a portion of the data is displayed here). Clearly, Heston model parameters are not unique; The five parameters in Table 2 depend on various factors, including option characteristics and the optimization processes employed.

After obtaining the parameters of the Heston model for a sufficient number of settlement days, we calculate the implied volatilities and model the option prices. These will be used for training and testing ANNs.

5. Neural Network Predictions

5.1. Neural network predictions: the output is C_n/K

First, we train the neural network by providing various groups of input variables using the training sets of option data. The initial group of inputs serves as our benchmark data, which includes moneyness, the risk-free rate, rv60, sk60, and ku60. This group does not consider the implied volatilities generated by the previously mentioned parametric CS model. Subsequently, we train the neural networks by incorporating four types of implied parameters as described in the preceding section. In total, there are five groups of input variables. Upon completion of the training, the networks are utilized to predict option prices using the test set of option data. For comparative purposes, in this section, the output value will be defined as the option price divided by the strike price.

It can be observed from the error table above that we have calculated the mean square error (MSE), root mean square error (RMSE), mean absolute error, and mean absolute percentage error.

The results presented in Table 3 indicate that the best-performing group is labeled CSimvol4, in comparison to the benchmark group labeled CSRV60, as the group yields the smallest pricing errors for options. For instance, when considering the error measure RMSE, the CSimvol4 group yields a low value of 0.03454, while the CSRV60 group produces a slightly higher value of 0.03543. Overall, the CSimvol3 group corresponds to the worst-performing set.

We need to emphasize that, in all the predictions of option prices made by the CS-NN model, the errors are relatively low when compared to the actual market prices. Even the "worst" group CSimvol3 has provided very accurate predictions of option prices. For example, Figure 2 illustrates a positive correlation between the actual prices and the predicted prices for all 29,050 option contracts, with the trendline closely resembling a positive linear correlation. Furthermore, the scatter points are concentrated and aligned within a narrow band, indicating that the spread of error is relatively small. Additionally, the error distribution tends to

Table 3
Prediction accuracy of option prices when the output is C_n/K

	$_{CS}{}^{RV}{}_{60}$ sk60 ku60	^{CS}imvol1 imsk1 imku1	^{CS}imvol2 imsk2 imku2	^{CS}imvol3 imsk3 imku3	^{CS}imvol4 imsk4 imku4	Heston imvol
MSE	0.00125	0.00137	0.00217	0.00439	0.00119	0.00314
RMSE	0.03543	0.03714	0.04658	0.06630	0.03454	0.05605
MAE	0.02823	0.02697	0.03660	0.05491	0.02613	0.03998
MPE	0.16140	0.16915	0.21218	0.30196	0.15731	0.33208

Figure 2
The underlying prices of 50 ETF options from January 2018 to December 2021

exhibit a normal shape with a mean error of −0.025, as shown in Figure 3. It is also worth noting that the range of the errors lies within a relatively wide domain (−0.15, 0.15).

In contrast, the ANN combined with the Heston model does not yield better prediction accuracy. For instance, MSE = 0.00314 and RSME = 0.05605. This model performs the worst in all cases. We will revisit this point later.

5.2. Hybrid neural network predictions: the output is $(C_n - C_p)/K$

In the second round of tests, the input variables are similarly divided into five groups, as described in Section 4. A key distinguishing feature is that the output value of the hybrid neural networks will be *(Cn − Cp)/K,* i.e., the pricing error between the option price and the parameter model estimate, normalized by the strike price. Therefore, the ANN effectively calibrates the CS model.

The results presented in Table 4 are consistent with those in Table 3, indicating a general improvement in prediction accuracy, except for the CSRV60 group, which utilizes realized volatility instead of implied volatility. Hybrid ANNs that incorporate CS-implied parameters as inputs significantly outperform the benchmark group. Furthermore, when considering the error measure RMSE, the CSimvol4 group achieves a low error value of 0.01495, compared to 0.03454 in Table 3, representing a 56% improvement in accuracy. In contrast, the CSRV60 group reports a slightly higher error value of 0.0403628. Overall, all hybrid neural networks demonstrate enhanced accuracy in their performance.

In the scatter plot presented in Figure 4, we illustrate the relationship between the actual prices and the predicted prices for the group CSimvol4. The data points align closely along a highly linear trend, indicating a strong positive correlation. In this scenario, the scatter of the points is minimal, suggesting that the results are approaching an ideal state of prediction accuracy.

Figure 5 illustrates that the error distribution is centered around a mean of zero when comparing the predicted values to the actual values. In comparison to Figure 3, the range of errors is much smaller roughly within (−0.03, 0.08). This further confirms that our hybrid neural network has performed satisfactorily.

It is important to note that the hybrid ANN utilizing Heston's model does not outperform any of the CS groups, although it performs slightly better than the CSRV60 group. For example, in terms of the RMSE, the hybrid ANN with the Heston model yields a value of 0.036518, which indicates a marginally better accuracy compared to the CSRV60 group, which has an RMSE of 0.0403628. In contrast, the hybrid CS models give relatively smaller errors, ranging from 0.010613 to 0.02663. The underlying reasons for this discrepancy can be attributed to the calibration of

Figure 3
Predicted option price vs actual market price for the group CS_{imvol3}

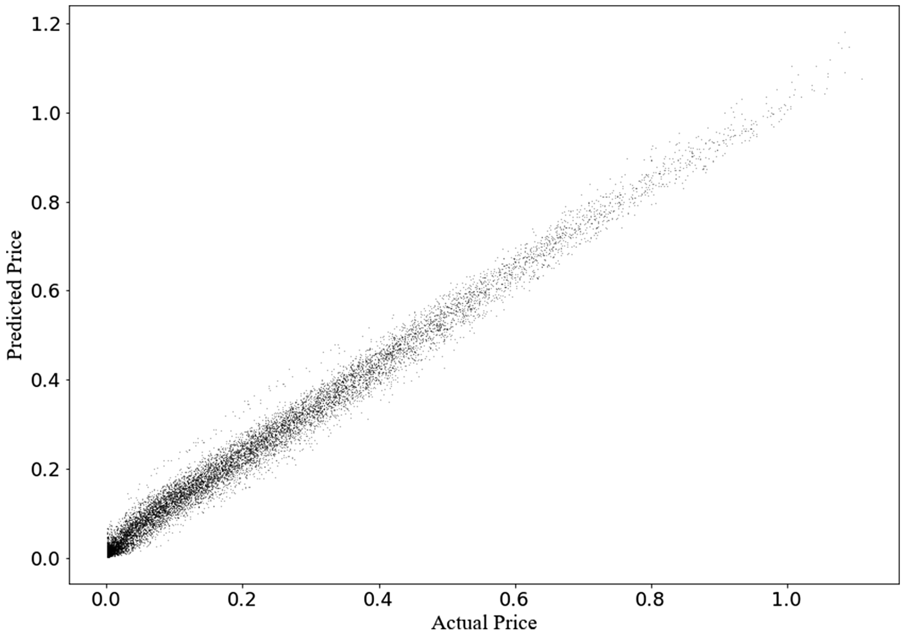


Table 4
Prediction accuracy of option prices when the output is *(Cn − Cp)/K*

	$CS^{RV}_{60\ sk60}$ ku60	^{CS}imvol1 imsk1 imku1	^{CS}imvol2 imsk2 imku2	^{CS}imvol3 imsk3 imku3	^{CS}imvol4 imsk4 imku4	Heston imvol
MSE	0.0016291	0.0001448	0.000112	0.000329	0.000223	0.001333
RMSE	0.0403628	0.012037	0.010613	0.02663	0.01495	0.036518
MAE	0.0292693	0.008047	0.00677	0.008491	0.00290	0.013714
MPE	0.1838273	0.054821	0.04833	0.09016	0.06811	0.216339

the Heston model, which presents a complex optimization problem for determining the model parameters numerically. In the context of option pricing, one may obtain the "optimized" set of parameters; however, this set likely corresponds to a local minimum rather than the global minimum. This is why the model parameters are not unique in our practice of option pricing. Consequently, such model uncertainty can propagate into the implied volatilities, resulting in larger errors when training ANNs to predict option prices.

5.3. Comparisons between hybrid neural networks and recent models

In this section, we compare the results obtained from the hybrid neural network used in this study with those from several recent neural network models for option pricing.

One of the significant models recently employed for option pricing is the Long Short-Term Memory (LSTM) neural network [33]. We have utilized this method to price Chinese options [33]. Another noteworthy approach is the combined use of the Black-Scholes model and neural networks (BS-NN), which was recently applied by Shvimer and Zhu [23]. In this research, we adopt these two methods to price Chinese exchange-traded fund (ETF) options and compare their pricing performance with the hybrid neural networks discussed in the preceding sections. It is important to note that, for the BS-NN model, we have chosen to use the 60-day realized volatility as one of the input parameters for the BS model.

Table 5 shows the prediction accuracies produced by using the aforementioned models. Typically, we have examined the output $(C_n - C_p)/K$ under the hybrid neural network.

The results presented in Table 5 and Figure 6 are intriguing, as the comparisons indicate that both the LSTM and CS hybrid models have outperformed the Heston hybrid model, while the BS-NN demonstrates a pricing performance comparable to that of the Heston hybrid model. As shown in the table, the CS hybrid model yields a RMSE of 0.01495, whereas the RMSE for the BS-NN is 0.0369105, and for the Heston hybrid model, it is 0.036518. Therefore, our research findings align with recent studies in which ANN is integrated with the parametric BS model. Furthermore, the results indicate that the hybrid model utilizing a complex parametric method, such as Heston, does not significantly outperform the other models; no superior accuracy is observed in this case.

Figure 4
Error probability distribution in the group CS_{imvol3}

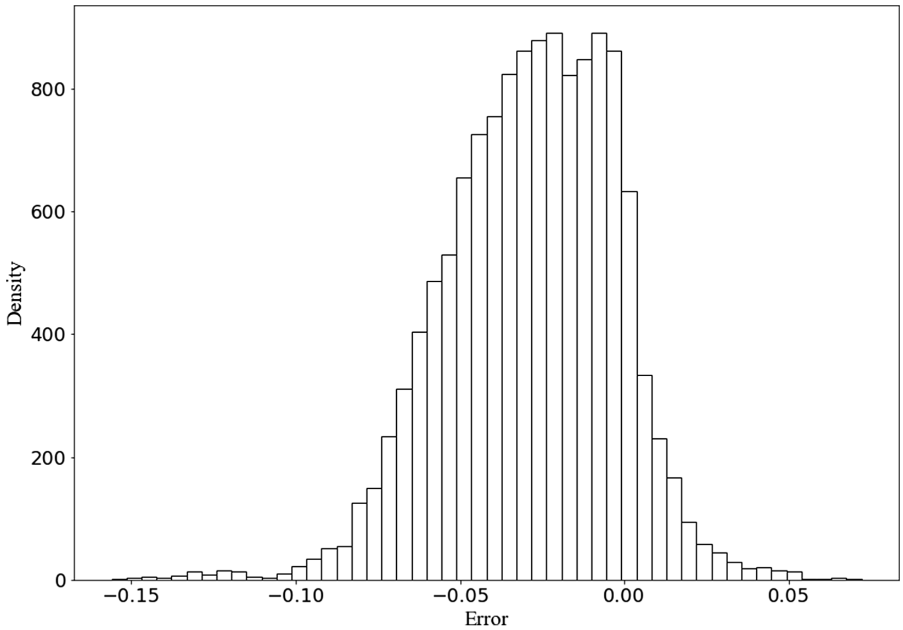


Figure 5
Predicted option price vs actual market price for the group CS_{imvol4}

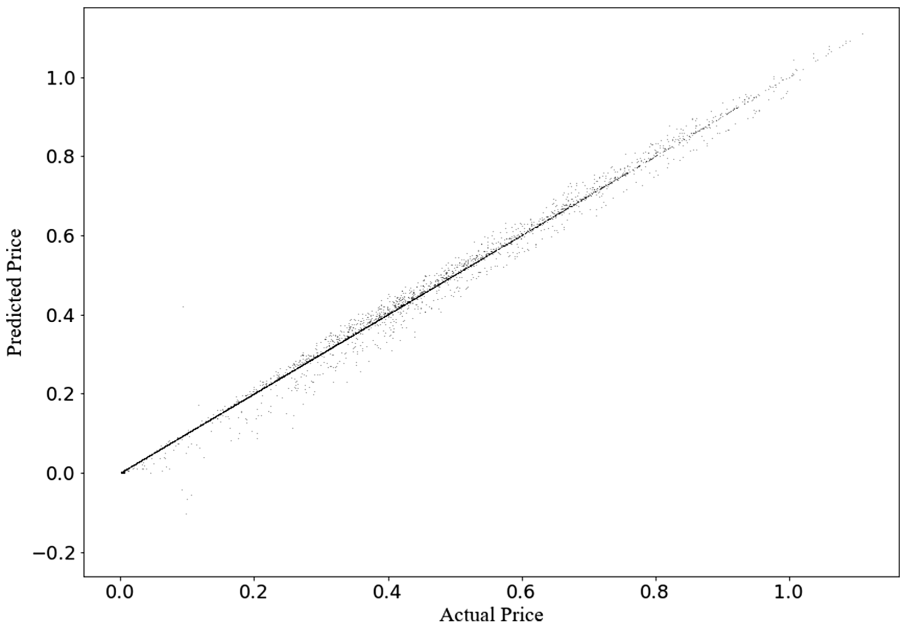
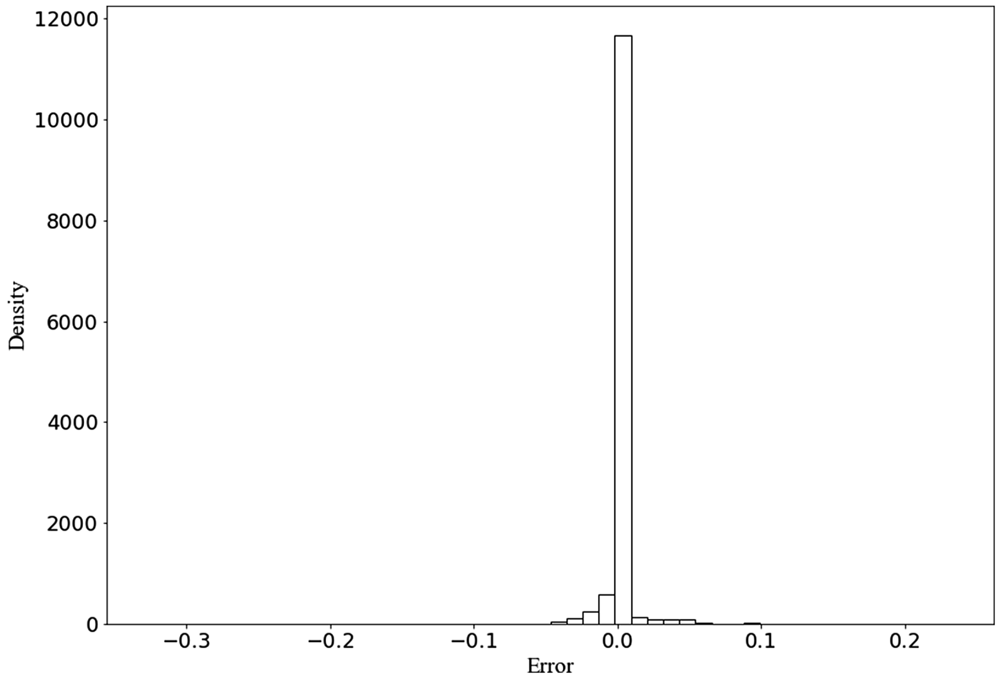


Table 5
Prediction accuracy of option prices

	LSTM	BS-NN	^{CS}imvol4 imsk4 imku4	Heston imvol
MSE	0.000663	0.0013624	0.000223	0.001333
RMSE	0.025756	0.0369105	0.01495	0.036518
MAE	0.018401	0.0159296	0.00290	0.013714
MPE	0.119731	0.2181041	0.06811	0.216339

Figure 6
Error probability distribution in the group CS_{imvol4}



6. Conclusion

In this study, hybrid ANN models are developed to price Chinese options by integrating parametric models, specifically the CS model and Heston's model. Typically, a hybrid objective function is employed, contrasting with the conventional objective function that focuses solely on the option price. By comparing the pricing predictive capabilities of the two approaches for 50 ETF call options, we demonstrate that the hybrid ANN models exhibit significantly enhanced pricing accuracy.

Further, the hybrid ANN approach is compared to recent models, such as LSTM networks and the hybrid BS model, for pricing Chinese options. The comparison of results indicates that the ANN, when combined with the CS formulas, is equally competent as the LSTM model and outperforms both the BS-ANN model and the Heston hybrid model.

Nevertheless, there is a growing trend of combining neural networks with increasingly complex parametric models, which requires significant offline effort to estimate model parameters. This approach is highly beneficial for scientific research. However, this paper demonstrates that hybrid ANNs are highly effective in pricing Chinese options, while the performance of parametric models does not justify their complexity. In contrast, a relatively robust model can outperform more complex parametric models, provided that the latter can accurately capture essential implied structural parameters. We hope this conclusion will be useful for practitioners analyzing option prices in the Chinese market.

This paper presents significant findings regarding the application of hybrid neural networks in the Chinese options market. However, several areas warrant further investigation. Our training and testing datasets encompass all types of options based on their moneyness, covering the period from 2018 to 2021. Nonetheless, it is essential to distinguish between three specific cases: real (in-the-money), imaginary (out-of-the-money), and two-even (at-the-money) options, whether they are call or put options with the same expiration date. In this study, all cases were analyzed collectively, but it may be beneficial to examine these cases separately. Consequently, more comprehensive research will be conducted in the future.

Ethical Statement

This study does not contain any studies with human or animal subjects performed by the author.

Conflicts of Interest

The author declares that he has no conflicts of interest to this work.

Data Availability Statement

Data sharing is not applicable to this article as no new data were created or analyzed in this study.

Author Contribution Statement

David Liu: Conceptualization, Methodology, Formal analysis, Resources, Writing – original draft, Writing – review & editing, Visualization, Validation, Investigation.

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