## RESEARCH ARTICLE

# Synthesis of Parameter Recognition Algorithm and State Evaluation for Separating Target 

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#### Abstract

This paper presents the results of the building of a parameter recognition algorithm and state evaluation for separate targets, to improve the effectiveness of target tracking for interceptor missiles. The assessment maneuver of the separating target is of great significance for the interceptor missile to recognize the maneuverability and assess the danger level of targets of the target in the group. With applied to the optimal Kalman filter algorithm, the structure of the filter is simple, and the evaluation error is small therefore the state evaluation accuracy of the algorithm is advanced, and the results of evaluating the system are verified through simulation on Matlab. After evaluation, the maneuvering parameters of the separating target will be included in the algorithm to evaluate the target's maneuvering direction later to the decision to track (or not to track) the initially selected target and destroy (or not destroy) the new separating target that appeared because of its danger.


Keywords: separating target, Kalman filter, parameter evaluation, state evaluation, supermaneuverable target

## 1. Introduction

During the guidance, interceptor missiles are launched from the aircraft toward the group of enemy aircraft. But maybe at some point, the enemy aircraft detects that there is an opposing target (it can be an aircraft or missile of the interceptor group or both at the same time) and responds by launching missiles at the enemy one of the selected antagonistic targets. During the journey to the initial target of the interceptor missile, by means of onboard self-guidance equipment, it is necessary to detect that the enemy missile is separated from the initial target group. Because enemy missiles are often much smaller in size and more maneuverable than the original target aircraft, it is classified as a particularly dangerous separating target group that needs to be destroyed priority. This target is called a supermaneuverable target (Pons, 2019; Zhu et al., 2018).

Currently, there are many works in the world that have used modern filtering algorithms to evaluate flight parameters (Gao et al., 2016; Kulikov \& Kulikova, 2015; Sun et al., 2018; Yin et al., 2018; Zhang et al., 2019). However, the works only evaluate the parameters of a single target, not being used to guide a missile to a group of $n$ targets. Besides, the works (Chen et al., 2014; Hosseini et al., 2020; Uhrmeister, 1994; Zhang et al., 2018) do not have the ability to distinguish and identify maneuvers for each target in the group. So, if there is a missile that separates from the target group, that algorithm does not have the ability to identify, follow, and destroy.

The problem that needs to be posed for the control system on the interceptor missile compartment is to detect some maneuver characteristics of the target separating the group. The purpose is to

[^0]enhance the efficiency of distinguishing and selecting dangerous targets in the group and to evaluate the parameters of the target in the group. On the basis of the formation of filters, it is necessary to determine the structure of the relative flight dynamics model between interceptors and targets. That model provides priori information, used when synthesizing optimal Kalman filters, as the basis for the state observation algorithm, which means identification and maneuvering detection of the target being tracked by the radar-homing head on the interceptor missile.

## 2. Solve the Problem

### 2.1. Model of separating target and interceptor missile

Based on the formation of filters (Ярлыков et al., 2012), we will determine the geometrical dynamic structure of the interceptor missile and the separating missile from the enemy aircraft as shown in the following form of the equation:
$\left\{\begin{array}{l}V_{\mathrm{t}}(\mathrm{t})=V_{0}+\Delta V(\mathrm{t}), V_{t}(0)=V_{0 \mathrm{t}} \\ \dot{V}_{\mathrm{t}}(\mathrm{t})=\Delta V(\mathrm{t})=a_{\mathrm{t}}(\mathrm{t}), \Delta V_{\mathrm{t}}(0)=\Delta V_{0 \mathrm{t}} \\ \dot{\mathrm{t}}_{\mathrm{t}}(\mathrm{t})=-2 \alpha_{\mathrm{t}} a_{\mathrm{t}}(\mathrm{t})-\left(\alpha_{\mathrm{t}}^{2}+\beta_{\mathrm{t}}^{2}\right) V_{\mathrm{t}}(\mathrm{t})-\sqrt{2 \alpha_{\mathrm{t}} \sigma_{\mathrm{t}}^{2}}\left(2 \alpha_{\mathrm{t}}+\sqrt{\alpha_{\mathrm{t}}^{2}+\beta_{\mathrm{t}}^{2}}\right) \xi(\mathrm{t})\end{array}\right.$
where $V_{t}(t), \Delta V_{t}(t), a_{t}(t)$ are the velocity, velocity variation, and radial acceleration of the separating target, respectively.
$V_{0}$ is the velocity of the carrier aircraft. $\alpha_{t}=\frac{1}{\tau}, \beta_{t}=\left(2 \pi f_{t}\right)^{2}, \tau, f_{t}$ are the correlation time and the oscillation frequency of the target carrier aircraft, respectively. $\sigma_{t}$ is the oscillating dispersion of the normal acceleration of the target carrier aircraft and $\xi(t)$ is the central white noise with a known one-sided spectral density.

The model of the interceptor missile is expressed in the form of differential equations:

$$
\left\{\begin{array}{l}
\dot{V}_{\mathrm{m}}(\mathrm{t})=0, V_{\mathrm{m}}(0)=V_{0 \mathrm{~m}} \\
\Delta \dot{V}_{\mathrm{m}}(\mathrm{t})=a_{\mathrm{m}}(\mathrm{t}), \Delta V_{\mathrm{m}}(0)=\Delta V_{0 \mathrm{~m}}  \tag{2}\\
\dot{a}_{\mathrm{m}}(\mathrm{t})=-\alpha_{\mathrm{m}} a_{\mathrm{m}}(\mathrm{t})-\beta_{\mathrm{m}} \Delta V_{\mathrm{t}}(\mathrm{t})+\sqrt{2 \alpha_{\mathrm{m}} \sigma_{\mathrm{m}}^{2}} \xi_{1}(\mathrm{t}), a_{\mathrm{m}}(0)=a_{0 \mathrm{~m}}
\end{array}\right.
$$

where $V_{\mathrm{m}}(t), \Delta V_{\mathrm{m}}(t), a_{\mathrm{m}}(t)$ are the velocity, velocity variation, and radial acceleration of the interceptor missile, respectively. $V_{0}$ is the speed of the interceptor carrier aircraft. $\sigma_{m}$ is the oscillating dispersion of the normal acceleration of the interceptor and $\xi(t)$ is the central white noise with a known one-sided spectral density.

### 2.2. Synthesis of the algorithm to parameter recognition of the separating target

The model of the interceptor missile and separating target of the enemy according to equations (1) and (2) is quite complete. According to Васин et al. (1970), the degree of incompleteness compared to the actual model is less than $15 \%$. Furthermore, the model ( $1-2$ ) is linear, so a discrete Kalman filter is applied to form a dynamic target detection filter. The synthesis of the component algorithms and the evaluation of the input parameters of the maneuverability detector will be carried out according to the principle of the discrete optimal Kalman filter with the mean squared error (MSE) of the evaluation standard. It will be used for evaluation of the quality of the filtering algorithm, applied simultaneously to two objects: interceptor missiles and separating targets. With the above characteristics, combined with the flight kinematics and homing kinematics model of intercepting missiles and separating targets. We can identify a reduced state model in the form of a system of differential equations describing the changing properties of phase coordinates related to the simultaneous flight motion of two interested objects, which are intercept missiles and the missiles of the enemy (Васин et al., 1970)

$$
\begin{equation*}
\dot{D}(t)=-V_{c}(t)=-\left[V_{m}(t)+V_{t}(t)+\Delta V_{m}(t)+\Delta V_{t}(t)\right] ; D(0)=D_{0} \tag{3}
\end{equation*}
$$

$$
\begin{gather*}
\dot{\omega}(\mathrm{t})=-\frac{2 \dot{\mathrm{D}}(\mathrm{t})}{\mathrm{D}} \omega(\mathrm{t})+\frac{1}{\mathrm{D}}\left(\mathrm{~W}_{\mathrm{T}}-\mathrm{W}_{\mathrm{M}}\right) ; \omega(0)=\omega_{0}  \tag{4}\\
\dot{\varepsilon}(t)=\omega(t), \varepsilon(0)=\varepsilon_{0}  \tag{5}\\
\dot{W}_{m}(t)=-k_{m} W_{m}(t)+\sqrt{2 k_{m} \sigma_{m}^{2}} \xi_{3}(t), W_{m}(0)=W_{0 m}  \tag{6}\\
\dot{W}_{t}(t)=-k_{t} W_{t}(t)+\sqrt{2 k_{t} \sigma_{t}^{2}} \xi_{4}(t), W_{t}(0)=W_{0 t} \tag{7}
\end{gather*}
$$

where $k_{\mathrm{m}}, k_{\mathrm{t}}$ are the corresponding coefficients for the maneuverability of interceptor and target; $\xi_{1}(\mathrm{t}), \xi_{2}(\mathrm{t}), \xi_{3}(\mathrm{t}), \xi_{4}(\mathrm{t})$ is the white noise. Thus, the state vector will have the form:
$x(t)=\left[D(t), V_{m}(t), \Delta V_{m}(t), a_{m}(t), V_{t}(t), \Delta V_{t}(t), a_{t}(t), \varepsilon(t), \omega(t), W_{m}(t), W_{t}(t)\right]$

The system of differential Equations (1)-(7) in a vector matrix has the form:

$$
\begin{equation*}
\dot{X}=A X+G E, X(0)=X_{0} \tag{9}
\end{equation*}
$$

where $X, X_{0}$ are the real state vector and initial state vector of the matrix with size [ $11 \times 1$ ] of the following form:

$$
\begin{gather*}
\dot{X}=\left[\begin{array}{llllll}
D V_{m} & \Delta V_{m} a_{m} V_{t} \Delta V_{t} & a_{t} \varepsilon & \omega & W_{m} W_{t}
\end{array}\right]^{T} \\
X_{0}=\left[\begin{array}{llllll}
D_{0} V_{0 m} & \Delta V_{0 m} a_{0 m} V_{0 t} \Delta V_{0 t} & a_{0 t} \varepsilon_{0} & \omega_{0} & W_{0 m} W_{0 t}
\end{array}\right]^{T} \tag{10}
\end{gather*}
$$

A -The size of a known coefficient matrix $[11 \times 11]$ has non-zero elements is determined according to equation (11) as follows:

$$
\begin{align*}
& a_{1,2}=a_{1,3}=a_{1,5}=a_{1,6}=-1 ; \quad a_{3,4}=a_{6,7}=a_{8,9}=1 ; \quad a_{4,3}=-\beta_{\mathrm{m}} \\
& a_{4,4}=-a_{\mathrm{m}} ; \quad a_{7,6}=-\left(\alpha_{\mathrm{t}}^{2}+\beta_{\mathrm{t}}^{2}\right) ; \quad a_{7,7}=-2 a_{\mathrm{t}} \tag{11}
\end{align*}
$$

$a_{9,9}=-\frac{2 V_{c}}{D} ; \quad a_{9,10}=-\frac{1}{D} ; \quad a_{9,11}=\frac{1}{D} ; \quad a_{10,10}=-k_{m} ; \quad a_{11,11}=-k_{t}$
$G-$ The factor matrix with the size $[11 \times 11]$ has the elements other than zero can be determined according to equation (12) below:

$$
\begin{gather*}
g_{4,4}=g_{10,10}=\sqrt{2 \alpha_{\mathrm{m}} \sigma_{\mathrm{m}}^{2}} ; g_{7,7}=\sqrt{2 \alpha_{\mathrm{t}} \sigma_{\mathrm{t}}^{2}}\left(2 \alpha_{\mathrm{t}}+\sqrt{\alpha_{\mathrm{t}}^{2}+\beta_{\mathrm{t}}^{2}}\right) \\
g_{11,11}=\sqrt{2 \alpha_{\mathrm{t}} \sigma_{\mathrm{t}}^{2}} \tag{12}
\end{gather*}
$$

The displacement vector for independently Gauss white noise with a mathematical expectation of 0 and variance of 1 has a size [ $11 \times 1$ ] of the form equation (13):

$$
E=\left|\begin{array}{lllllllllll}
0 & 0 & 0 & \xi_{1} & 0 & 0 & \xi_{2} & 0 & 0 & \xi_{3} & \xi_{4} \tag{13}
\end{array}\right|^{T}
$$

Then, the elements of the observed noise vector $\gamma(k)$ of the model (20) will not be correlated with the elements of the noise vector formed in expression (9).

The transformation of the continuous state equations (1)-(7) to the discrete model starts from choosing the time interval $[0, T]$ and then dividing it into $n$ points equally spaced $t_{k}=k \Delta t$, which we call the time discretization step $\Delta t=T / n$, where $k=\overline{0, n}$. Then, based on equation (14), the matrix $X$ will be determined after every step $k$, i.e.:
$x(k)=\left[D(k), V_{m}(k), \Delta V_{m}(k), a_{m}(t k), V_{t}(k), \Delta V_{t}(k), a_{t}(k), \varepsilon(k), \omega(k), W_{m}(k), W_{t}(k)\right]$

When synthesizing the component algorithm of the evaluation block, we use the multi-state linear discrete Kalman filter procedure (Kim et al., 2018; Тихонов \& Харисов, 1991; Фарина \& Студер, 1993; Ярлыков et al., 2012) which is described by the following system of equations:

$$
\begin{gather*}
P^{n s}(k+1) \Phi(k) P(k) \Phi^{T}(k)+Q(k)  \tag{15}\\
\psi(k+1)=H P^{n s}(k+1) H^{T}+R(k)  \tag{16}\\
Z(k+1)=Y(k+1)-H \Phi(k) \hat{X}(k)  \tag{17}\\
P(k+1)=[I-K(k+1) H] P^{n s}(k+1)  \tag{18}\\
\hat{X}(k+1)=\Phi(k) \hat{X}(k)+K(k+1) Z(k+1)  \tag{19}\\
Y(k)=H(k) X(k)+R(k) \gamma(k) \tag{20}
\end{gather*}
$$

where
$H(k)$ - observation matrix.
$X(k)$ - state vector.
$R(k)$ - observed noise matrix.
$P^{n s}(k), P(k)$ - extrapolated and filtered error covariance matrix.
$\Phi(k)$ - transition matrix.
$Q(k)$ - excitation noise covariance matrix.
$K(k)$ - matrix of weights.
$I$ - unit matrix.
$\hat{X}(k)$ - the state vector is evaluated.
$Y(k)$ - observation vector.
$\gamma(k)$ - Gaussian white noise column vector with zero mathematical expectation and unit variance (equal to 1 ).

In equation (21), the elements of the observation matrix $H$ will be determined by the element to be evaluated and have the size $[7 \times 11]$ with the following non-zero elements:

$$
\begin{gather*}
h_{1,1}=h_{2,2}=h_{2,3}=h_{3,4}=h_{4,2}=h_{4,3}=1 \\
h_{4,5}=h_{4,6}=h_{5,8}=h_{6,0}=h_{7,10}=1 \tag{21}
\end{gather*}
$$

The elements of matrices $\Phi$ and $Q$ used for the Kalman filter are calculated according to the following equations (22)-(23) (Васин et al., 1970; Ярлыков et al., 2012).

$$
\begin{gather*}
\Phi(\mathrm{k})=\mathrm{e}^{\mathrm{A} \Delta \mathrm{t}} \approx \mathrm{I}+\mathrm{A} \Delta \mathrm{t}  \tag{22}\\
Q(k)=M\left[j(k) j^{T}(k)\right] \tag{23}
\end{gather*}
$$

where $j(k)$ is the criterion function (penalty function), obtained from the following equation (24):

$$
\begin{equation*}
\mathrm{j}(\mathrm{k})=\int_{\mathrm{k} \Delta \mathrm{t}}^{(\mathrm{k}+1) \Delta \mathrm{t}} \Phi[(\mathrm{k}+1) \Delta \mathrm{t}-\tau] \mathrm{G}(\tau) \mathrm{E}(\tau) \mathrm{d} \tau \tag{24}
\end{equation*}
$$

As a result, the matrix $\Phi(k)$ in equation (25) below will have size [ $11 \times 11$ ] with non-zero elements:
$\Phi_{1,1}=\Phi_{2,2}=\Phi_{3,3}=\Phi_{5,5}=\Phi_{6,6}=\Phi_{8,8}=1 ; \quad \Phi_{1,2}=\Phi_{1,3}=\Phi_{1,5}=\Phi_{1,6}=-\Delta \mathrm{t} ;$
$\Phi_{3,4}=\Phi_{6,7}=\Phi_{8,9}=\Delta \mathrm{t} ; \quad \Phi_{4,3}=-\beta_{\mathrm{m}} \Delta \mathrm{t} ; \quad \Phi_{4,4}=\Phi_{10,10}=1-\alpha_{\mathrm{m}} \Delta \mathrm{t} ;$

$$
\begin{gather*}
\Phi_{7,6}=-\left(\alpha_{\mathrm{t}}^{2}+\beta_{\mathrm{t}}^{2}\right) \Delta \mathrm{t} ; \quad \Phi_{7,7}=1-2 \alpha_{\mathrm{t}} \Delta \mathrm{t} ; \quad \Phi_{9,9}=1-\frac{2 \mathrm{~V}_{\mathrm{c}}}{\mathrm{D}} \Delta \mathrm{t} \\
\Phi_{9,10}=-\frac{\Delta \mathrm{t}}{\mathrm{D}} ; \quad \Phi_{9,11}=\frac{\Delta \mathrm{t}}{\mathrm{D}} ; \quad \Phi_{11,11}=1-\alpha_{\mathrm{T}} \Delta \mathrm{t} \tag{25}
\end{gather*}
$$

The matrix $Q(k)$ also has size $[11 \times 11]$ with the following non-zero elements:

$$
\begin{gathered}
q_{4,4}=2 \alpha_{m} \sigma_{m}^{2}\left(\Delta t-\alpha_{m} \Delta t^{2}+\alpha_{m}^{2} \frac{\Delta t^{3}}{3}\right) ; \\
q_{4,7}=q_{7,4} \\
=2 \sqrt{\alpha_{m} \sigma_{m}^{2} \alpha_{t} \sigma_{t}^{2}}\left(\alpha_{t} \Delta t^{2}-2 \alpha_{m} \alpha_{t} \frac{\Delta t^{3}}{3}-\Delta t+\alpha_{m} \frac{\Delta t^{2}}{2}\right) ; \\
q_{4,10}=q_{10,4}=2 \sigma_{m}^{2} \sqrt{\alpha_{m} k_{m}}\left(1-\left(\alpha_{m}-k_{m}\right) \frac{\Delta t^{2}}{2}+\alpha_{m} k_{m} \frac{\Delta t^{3}}{3}\right) ; \\
q_{4,11}=q_{11,4}=2 \sqrt{\alpha_{\mathrm{m}} \alpha_{\mathrm{m}}^{2} \alpha_{\mathrm{t}} \sigma_{\mathrm{t}}^{2}}\left(1-\left(\alpha_{\mathrm{m}}-\alpha_{\mathrm{t}}\right) \frac{\Delta \mathrm{t}^{2}}{2}+\alpha_{\mathrm{t}} \alpha_{\mathrm{t}} \frac{\Delta t^{3}}{3}\right) ; \\
q_{7,10}=q_{10,7}=2 \sqrt{\alpha_{\mathrm{t}} \sigma_{\mathrm{t}}^{2} \mathrm{k}_{\mathrm{m}} \sigma_{\mathrm{m}}^{2}}\left(\alpha_{\mathrm{t}} \Delta \mathrm{t}^{2}-2 \mathrm{k}_{\mathrm{m}} \alpha_{\mathrm{t}} \frac{\Delta \mathrm{t}^{3}}{3}-\Delta \mathrm{t}+\mathrm{k}_{\mathrm{m}} \frac{\Delta \mathrm{t}^{2}}{2}\right) ; \\
q_{7,11}=q_{11,7}=\sigma_{\mathrm{t}}^{2} \sqrt{\alpha_{\mathrm{t}} k_{\mathrm{t}}}\left(1-\left(\alpha_{\mathrm{t}}-k_{\mathrm{t}}\right) \frac{\Delta t^{2}}{2}+\alpha_{\mathrm{t}} k_{\mathrm{t}} \frac{\Delta t^{3}}{3}\right) ;
\end{gathered}
$$

$$
\begin{gather*}
q_{10,10}=2 \mathrm{k}_{\mathrm{m}} \sigma_{\mathrm{m}}^{2}\left(\Delta \mathrm{t}-\alpha_{\mathrm{m}} \Delta \mathrm{t}^{2}+\alpha_{\mathrm{m}}^{2} \frac{\Delta \mathrm{t}^{3}}{3}\right) \\
q_{10,11}=q_{11,10}=2 \sqrt{\mathrm{k}_{\mathrm{m}} \sigma_{\mathrm{m}}^{2} \mathrm{k}_{\mathrm{t}} \sigma_{\mathrm{t}}^{2}}\left(1-\left(\mathrm{k}_{\mathrm{m}}-\mathrm{k}_{\mathrm{t}}\right) \frac{\Delta \mathrm{t}^{2}}{2}+\mathrm{k}_{\mathrm{m}} \mathrm{k}_{\mathrm{t}} \frac{\Delta \mathrm{t}^{3}}{3}\right) \\
q_{11,11}=2 \mathrm{k}_{\mathrm{t}} \sigma_{\mathrm{t}}^{2}\left(\Delta \mathrm{t}-\mathrm{k}_{\mathrm{t}} \Delta \mathrm{t}^{2}+\mathrm{k}_{\mathrm{t}}^{2} \frac{\Delta \mathrm{t}^{3}}{3}\right) \tag{26}
\end{gather*}
$$

According to Богданов et al. (1998) when calculating the elements of the noise covariance matrix $Q(k)$ in equation (26), as a rule, it is limited to taking into account only first-order linear elements to satisfy the error characteristics of the filter, but when the resource requirement of the on-board computing device MTS is reduced. When the second- and third-order elements are removed from the matrix $Q$, the filter will probably diverge. Therefore, it is necessary to use a full noise covariance matrix with elements of order 1,2 , and 3 .

### 2.3 Determine the boundary matrix values for the filter

Next, we determine the elements of the matrices $R, X 0$, and $P$, which are the input boundary conditions of the optimal Kalman filter described by the system of Equations (1)-(9).

The elements of the observed (measured) noise covariance matrix $R$ of size [ $7 \times 7$ ] are determined by the following formulas:

$$
\begin{equation*}
\sigma^{2}=\left(\frac{\delta}{\sqrt{\mathrm{q}}}\right)^{2} \tag{27}
\end{equation*}
$$

where $\delta$ is the allowable ability of the measured coordinate system; $q$ is the signal/noise correlation at the filter input.

To calculate the measurement error variances of the parameters to be observed, we take the signal/noise ratio to be 20 dB .

Allowability according to distance $(D)$ when pulse signal simple probe, width $1 \mu$ s equals 150 m , according to equation (27) then variance of distance measurement error will be equal to $225 \mathrm{~m}^{2}$.

The ability to distinguish the specific velocity of the intercept missile $\left(V_{M}\right)$ can be taken as $0.16 \mathrm{~m} / \mathrm{s}$, then according to equation (27) the variance of the missile velocity measurement error will be $256 \cdot 10^{-4}\left(\mathrm{~m} / \mathrm{s}^{2}\right)$.

The ability to distinguish close velocity $\left(V_{c}\right)$ by onboard radar is determined by the formula:

$$
\begin{equation*}
\delta\left(V_{\mathrm{c}}\right)=\frac{\lambda}{2} \Delta f_{\mathrm{D}} \tag{28}
\end{equation*}
$$

where $\Delta f_{\mathrm{D}}$ is the passband of the Doppler frequency filter of the onboard radar, $H z ; \lambda$ is the wavelength of the radar's spatially detected pulse generator frequency.

The Doppler frequency filter passband is calculated as follows (Черных et al., 2000):

$$
\begin{equation*}
\Delta f_{\mathrm{D}}=\sqrt{\frac{2\left(a_{\mathrm{m}}+a_{\mathrm{t}}\right)}{\lambda}} \tag{29}
\end{equation*}
$$

where $\Delta f_{\mathrm{D}}, a_{\mathrm{t}}$ corresponds to the centripetal acceleration of the intercept missile and the target.

The passband $\Delta f_{\mathrm{D}}$ of a narrow band Doppler frequency filter with good impedance matching can be taken as equal to the signal spectral width reflected from the target according to Дудник and Чересов (1986), for example.

$$
\begin{equation*}
\Delta f_{\mathrm{D}}=\Delta f_{\mathrm{C}} \tag{30}
\end{equation*}
$$

From equations (29) and (30), in theory, when the wavelength of the probing signal is known in advance, the spectral width of the signal reflected from the target will be determined by the centripetal acceleration of both missile intercept and the target. We estimate the necessary value of the Doppler frequency filter passband $\Delta f_{D}$ in case we assume that the target missile decelerates with a negative centripetal acceleration $\left(a_{t}=-16 \mathrm{~m} / \mathrm{s}^{2}\right)$ and intercept missile acceleration is $0\left(a_{m}=0\right)$, knowing in advance the onboard radar wavelength is " $\lambda=0,04 \mathrm{~m}$ " then we can estimate the necessary value of the Doppler frequency filter passband according to equation (27) is $\Delta f_{D} \cong 28 \mathrm{~Hz}$. From the analysis in Ярлыков (1985), the authors show that the required bandwidth of a narrowband Doppler filter of an airborne radar should be $26-31 \mathrm{~Hz}$, so we can choose $\Delta f_{D}=30 \mathrm{~Hz}$ to be suitable for calculation. Then discrimination ability of the measurement according to the close velocity is calculated according to equation (28) is $\sigma\left(V_{c}\right)=0.06 \mathrm{~m} / \mathrm{s}$, and the measurement error variance is $\sigma^{2}\left(V_{c}\right)=3.6(\mathrm{~m} / \mathrm{s})^{2}$. The ability to distinguish the line-of-sight angle determined by the corresponding receiving antenna wave width in each plane ( $\varepsilon$ and $\beta$ ) observing the target at 0.5 power level is expressed as the following equation (31):

$$
\begin{equation*}
\delta(\varepsilon)=\theta_{0.5 \varepsilon} \tag{31}
\end{equation*}
$$

Therefore, for modern homing heads using phased array antenna radar, the measurement variance is $\delta(\varepsilon)=\theta_{0.5 \varepsilon}=1.8^{\circ}$. In all cases, we can take the measurement error variance for the line-of-sight angle according to equation (32) as follows:

$$
\begin{equation*}
\delta(\varepsilon)=\theta_{0.5 \varepsilon}=2^{\circ} \tag{32}
\end{equation*}
$$

According to equation (27), if the signal/noise ratio is $q=100$, then the measurement variance of the line-of-sight angle will be $\sigma^{2}(\varepsilon)=0.04(0)^{2}$. The angular velocity measurement variance of the line-of-sight in terms of equation (27) will be $\sigma^{2}(\omega)=4.10^{-4}(0 / s)^{2}$.

The error measurement of normal acceleration along axes perpendicular to the longitudinal axis of the missile, including the measurement of axial acceleration by accelerometers in the inertial measurement unit (IMU), can be determined by randomly MSE of the accelerometers themselves. Then, the variance of the vertical and horizontal acceleration measurements of the missile will be equal to:

$$
\sigma^{2}\left(a_{\mathrm{m}}\right)=\sigma^{2}\left(W_{\mathrm{m}}\right)=3 \cdot 8 \cdot 10^{-4}\left(\mathrm{~m} / \mathrm{s}^{2}\right)^{2}
$$

Thus, the diagonal elements of the measured noise covariance matrix $R$ will be the measured error variance values of the observing coordinates defined above. All other elements in the matrix have a value of 0 . The values of the elements lying on the diagonal of the matrix $R$ are as follows:

$$
\begin{gathered}
r_{1,1}=225 ; \quad r_{2,2}=2.56 .10^{-4} ; \quad r_{3,3}=3.84 .10^{-4} ; \quad r_{4,4}=3.6 .10^{-3} ; \\
r_{5,5}=0.04 ; \quad r_{6,6}=4.10^{-4} ; \quad r_{7,7}=3.84 .10^{-4} ;
\end{gathered}
$$

Next, we determine the first elements (when $k=0$ ) lying on the diagonal of the filter error matrix $P$, i.e. $P_{i, i}(0)$ with, $i=\overline{1,11}$. Therefore, we can apply as equation (33) following to the calculation filter error matrix (Канащенков \& Меркулов, 2004):

$$
\begin{equation*}
p_{\mathrm{i}, \mathrm{i}}(0)=\frac{\left(X_{\mathrm{imax}}-X_{\mathrm{imin}}\right)^{2}}{36} \tag{33}
\end{equation*}
$$

The filter error matrix $P(k=0)$ whose diagonal elements are not zero: $p_{1,1}=6.25 .10^{6} \mathrm{~m}^{2} ; \quad p_{2,2}=1.1 .10^{3}\left(\mathrm{~m}^{2} / \mathrm{s}^{2}\right)^{2} ; \quad p_{3,3}=11.28(\mathrm{~m} / \mathrm{s})^{2} ;$

$$
\begin{gathered}
p_{4,4}=0.528\left(\mathrm{~m}^{2} / \mathrm{s}^{2}\right)^{2} ; \quad p_{5,5}=2.5 .10^{4}(\mathrm{~m} / \mathrm{s})^{2} ; \quad p_{6,6}=7.11(\mathrm{~m} / \mathrm{s})^{2} ; \\
p_{7,7}=0.278(\mathrm{~m} / \mathrm{s})^{2} ; \quad p_{8,8}=25\left({ }^{\circ}\right)^{2} ; \quad p_{9,9}=0.4\left({ }^{\mathrm{o}} / \mathrm{s}\right)^{2} ; \\
p_{10,10}=0.46(\mathrm{~m} / \mathrm{s})^{2} ; \quad p_{11,11}=0.13(\mathrm{~m} / \mathrm{s})^{2} .
\end{gathered}
$$

To determine the state vector initial values evaluated $\hat{X}(0)$, that is, $x_{i}(0), i=\overline{1,11}$, according to Канащенков and Меркулов (2004) we use the following expression:

$$
\begin{equation*}
x_{i}(0)=\frac{X_{\text {imax }}+X_{\text {imin }}}{2} \tag{34}
\end{equation*}
$$

In equation (34), the initial values of the elements $x_{i}(0)$ are not available; they will be very different when changing the initial conditions of filtering phase coordinates in self-guidance.

Thus, the evaluation filter has been synthesized according to the criterion of least squared error in the form of a discrete linear Kalman filter (15)-(20). The dimensions and properties of the matrices as well as their elements have been clearly defined.

## 3. Simulation and Evaluation of Results

To investigate the quality of the optimal Kalman filter algorithm applied for parameters estimating of the separating targets as well as of the target angular coordinate system. This article conducts a simulation consisting of the intercepting missile and the separating target within a horizontal plane and the target of one-sided maneuvering. Since the state error itself is a random process, to compare the quality criteria of the algorithms it is necessary to follow the Monte-Carlo test method (Angelova et al., 2001). According to this method, the same target maneuvering trajectory is tested, but each time the gauges (measurement models) get different values due to random effects of measurement noise. The mean squared error of evaluation (MSE) is used to evaluate the quality of the filtering algorithm. The major parameters used for the simulation are as follows:

## Missile interceptor:

Initial velocity: $1000(\mathrm{~m} / \mathrm{s})$
Horizontal distance: 0 (km)
Altitude: 0 (km)
The guidance law OPPN (Zarchan, 2013).
Initial orbital tilt angle: $30^{\circ}$.

## Target:

Initial velocity: $350(\mathrm{~m} / \mathrm{s})$
Horizontal distance: 25 (km)
Altitude: 15 (km)
Initial orbital tilt angle: $140^{\circ}$.
The two-target normal acceleration in the group generated from equation (10) is a one-side maneuver with the following control signal. Where $u$ is the control signal or maneuvering command for targets 1 and 2 shown in equations (35)-(36):

$$
\begin{align*}
& u_{1}= \begin{cases}0 & \text { when } t<2[\mathrm{~s}] \\
10\left[\mathrm{~m} / \mathrm{s}^{2}\right] & \text { when } t<14[\mathrm{~s}] \\
0 & \text { when } t \geq 14[\mathrm{~s}]\end{cases}  \tag{35}\\
& u_{2}= \begin{cases}0 & \text { when } t<2[\mathrm{~s}] \\
40\left[\mathrm{~m} / \mathrm{s}^{2}\right] & \text { when } t<14[\mathrm{~s}] \\
0 & \text { when } t \geq 14[\mathrm{~s}]\end{cases} \tag{36}
\end{align*}
$$

### 3.1. Model simulation results for the separating target

Figure 1 describes the trajectory of the intercepting missile and separating target. The velocity and normal acceleration of the target (1) and target (2) are illustrated in Figure 2. Initially, both targets fly at the same distance and the same speed, after a period of 2 s , target 2 appears as a separating target. Accurate evaluation of maneuver parameters is of great significance for building an algorithm to detect the direction of maneuver of separate targets later. Based on detecting the direction of maneuver, the interceptor can reselect the target in the group to destroy, ensuring the safety of the combat formation.

Figure 1
Trajectory of the missile intercepts the separating target


Figure 2
Change of velocity, normal acceleration target (1) and separating target (2)


### 3.2. Evaluate the parameters of the 2 targets in the group

Target (1)
In this case, the evaluation line-of-sight angle, and error evaluation of the line-of-sight angle after 100Monte Carlo are depicted in Figure 3 and Figure 4, while Figure 5 and Figure 6 describe the evaluation of the angular velocity of the line-of-sight Missile - Target and error evaluation after 100Monte Carlo.

Figure 3
Evaluation of the angle of line-of-sight missile-target


Figure 4
Evaluation of angle of line-of-sight missile-target after 100
Monte Carlo


Figure 5
Evaluation of the angular velocity of line-of-sight missile-target


Figure 6
Evaluation of the angular velocity of line-of-sight missile-target after $\mathbf{1 0 0}$ Monte Carlo


The normal acceleration of target-1 is also evaluated in Figure 7. Target (2)

The results for the evaluation line-of-sight angle, and error evaluation of the line-of-sight angle after 100Monte Carlo for target- 2 are shown in Figure 8 and Figure 9 as follows.

The angular velocity line of sight is evaluated according to the results as shown in Figure 10, and the evaluation error after 100Monte Carlo is also shown in Figure 11.

Figure 7
Evaluation of the normal acceleration target (1)


Figure 8
Evaluation of the angle of line-of-sight missile-target


Normal acceleration of target- 2 is evaluated according to the results of Figure 12.

Comment: Through the survey results and running 100 Monte Carlo, we see that the optimal filtering algorithm can evaluate the parameters for the 2 targets in the group. The result evaluation of the algorithm still ensures the determination of state variables and the evaluation of the target parameters.

Figure 9
Evaluation of the angle of line-of-sight missile-target after 100 Monte Carlo


Figure 10
Evaluation of the angular velocity of line-of-sight missile-target after 100 Monte Carlo


## 4. Conclusion

This paper presented the results of synthesizing the separation target model recognition algorithm. Basically, it is a synthetic algorithm that states evaluation and evaluates target parameters using an optimal Kalman filter. Simulation results using MATLAB software show that the synthetic filter can accurately estimate the parameters of separate targets. This is an important basis for building guidance laws for intercepting missiles and detecting the direction of maneuver of separate targets.

## Ethical Statement

This study does not contain any studies with human or animal subjects performed by any of the authors.

Figure 11
Evaluation of angular velocity of line-of-sight missile-target after 100 Monte Carlo


Figure 12
Evaluation of the normal acceleration target (2)


## Conflicts of Interest

The authors declare that they have no conflicts of interest to this work.

## Data Availability Statement

Data sharing is not applicable to this article as no new data were created or analyzed in this study.

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