Synthesis of Parameter Recognition Algorithm and State Evaluation for Separating Target.

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Abstract: This paper presents the results of the building of a parameter recognition algorithm and state evaluation for separate targets, to improve the effectiveness of target tracking for interceptor missiles. The assessment maneuver of the separating target is of great significance for the interceptor missile to recognize the maneuverability of the target in the group. The decision to abandon (or not abandon) the original target and destroy (or not destroy) the new separating target that appeared because of its danger.

Keywords: separating target, Kalman filter, parameter evaluation, state evaluation, supermaneuverable target

1. Introduction

During the guidance, interceptor missiles are launched from the aircraft toward the group of enemy aircraft. But maybe at some point, the enemy aircraft detects that there is an opposing target (it can be an aircraft or missile of the interceptor group or both at the same time) and responds by launching missiles at the enemy one of the selected antagonistic targets. During the journey to the initial target of the interceptor missile, by means of onboard self-guidance equipment, it is necessary to detect that the enemy missile is separated from the initial target group. Because enemy missiles are often much smaller in size and more maneuverable than the original target aircraft, it is classified as a particularly dangerous separating target group that needs to be destroyed priority. This target is called a supermaneuverable target (Pons, 2019; Zhu, Huang, Gou, Tang, & Chen, 2018).

Currently, there are many works in the world that have used modern filtering algorithms to evaluate flight parameters (Gao, Zhang, Zhou, & Zhang, 2016; Kulikov & Kulikova, 2015; Sun, Zhang, Wang, & Gao, 2018; Yin, Li, Zhang, & Liu, 2018; A. Zhang, Shuida, Fei, & Wenhao, 2019). However, the works only evaluate the parameters of a single target, not being used to guide a missile to a group of n targets. Besides, the works (Chen, Gao, & Han, 2014; Hosseini, Haeri, & Khloozadeh, 2020; Uhrmeister & Dynamics, 1994; H. Zhang, Li, & Xie, 2018) do not have the ability to distinguish and identify maneuvers for each target in the group. So, if there is a missile that separates from the target group, that algorithm does not have the ability to identify, follow, and destroy.

The problem that needs to be posed for the control system on the interceptor missile compartment is to detect some maneuver characteristics of the target separating the group. The purpose is to enhance the efficiency of distinguishing and selecting dangerous targets in the group. To evaluate the parameters of the target in the group. On the basis of the formation of filters, it is necessary to determine the structure of the relative flight dynamics model between interceptors and targets. That model provides priori information, used when synthesizing optimal Kalman filters, as the basis for the state observation algorithm. That means, identification and maneuvering detection of the target being tracked by the radar-homing head on the interceptor missile.

2. Solve the Problem

2.1. Model of separating target and interceptor missile.

Based on the formation of filters (M. Ярлыков, Богачев, Меркулов, & Дрогалин, 2012), we will determine the geometrical dynamic structure of the interceptor missile and the separating missile from the enemy aircraft as shown in the following form of the equation:

\[
\begin{align*}
V_1(t) &= V_0 + \Delta V_1(t), V_1(0) = V_{\text{ist}} \\
V_2(t) &= \Delta V_2(t) = a_1(t), \Delta V_2(0) = \Delta V_{\text{ist}} \\
a_1(t) &= -2\alpha_1\alpha_2 - \beta_1\Delta \xi(t) - \sqrt{2\alpha_1\alpha_2 - 2\alpha_1 + \alpha_1^2 + \beta_1^2}\xi(t)
\end{align*}
\]

(1)

Where: \(V_0(t), \Delta V_1(t), a_1(t)\) are the velocity, velocity variation, and radial acceleration of the separating target, respectively. \(V_0\) is the velocity of the carrier aircraft. \(a_1 = \frac{1}{2} \beta_1 = (2\pi f)^2, \tau f\) are the correlation time, and the oscillation frequency of the target carrier aircraft, respectively. \(\alpha_1\) is the oscillating dispersion of the normal acceleration of the carrier aircraft. \(\xi(t)\) is the central white noise with a known one-sided spectral density.

The model of the interceptor missile is expressed in the form of differential equations:
\[
\begin{align*}
V_m(t) = 0, V_m(0) = V_{0m} \\
\Delta V_m(t) = a_m(t), \Delta V_m(0) = \Delta V_{0m} \\
\dot{a}_m(t) = -a_m a_m(t) - \beta_m V_m(t) + \sqrt{2a_m \sigma_2} Z(t), a_m(0) = a_{0m}
\end{align*}
\] (2)

Where: \(V_m(t), \Delta V_m(t), a_m(t)\) are the velocity, velocity variation, and radial acceleration of the interceptor missile, respectively. \(V_0\) is the speed of the interceptor carrier aircraft. \(\sigma_m\) the oscillating dispersion of the normal acceleration of the interceptor and \(Z(t)\) the central white noise with a known one-sided spectral density.

2.2. Synthesis of the algorithm to parameter recognition of the separating target.

The model of the interceptor missile and separating target of the enemy according to (1) and (2) is quite complete. According to (Васиц, Власов, Григорин, Дудник, & Степанов, 1970) the degree of incompleteness compared to the actual model is less than 15%. Furthermore, the model (1-2) is linear, so a discrete Kalman filter is applied to form a dynamic target detection filter. The synthesis of the component algorithms and the evaluation of the input parameters of the maneuverability of the oscillating dispersion of the normal acceleration of the interceptor and \(Z(t)\) the central white noise with a known one-sided spectral density.

The displacement vector for independently Gauss white noise with a mathematical expectation of 0 and variance of 1 has a size \([1x1]\) of the form:

\[
E = [0 \ 0 \ 0 \ \xi_1 \ 0 \ 0 \ \xi_2 \ 0 \ 0 \ \xi_3 \ \xi_4]^T
\] (13)

Then, the elements of the observed noise vector \(\gamma(k)\) of the model (20) will not be correlated with the elements of the noise vector formed in expression (9). The transformation of the continuous state model (1) - (7) to the discrete model starts from choosing the time interval \([0, T]\) and then dividing it into \(n\) points equally spaced \(t_k = k \Delta t\), which we call the time discretization step \(\Delta t = T/n\), where \(k = 0, n\). Then the matrix \(X\) will be determined after every step \(k\), i.e.:

\[
x(k) = [b(k), V_m(k), \Delta V_m(k), a_m(0), V_0(k), \Delta V_0(k), E(k), \omega(k), W_m(k), W_0(k)]
\] (14)

When synthesis the component algorithm of the evaluation block, we use the multi-state linear discrete Kalman filter procedure (Kim, Bang, & Filter, 2018; Тихонов & Харисов, 1991; Фарина & Студен, 1993; М. Ярлыков et al., 2012) which is described by the following system of equations:

\[
P^{ns}(k+1) = \Phi(k) P(k) \Phi^T(k) + Q(k)
\] (15)

\[
\psi(k+1) = H P^{ns}(k+1) H^T + R(k)
\] (16)

\[
Z(k+1) = Y(k+1) - H \hat{\Phi}(k) \tilde{X}(k)
\] (17)

\[
P(k+1) = [I - K(k+1) H] P^{ns}(k+1)
\] (18)

\[
\tilde{X}(k+1) = \Phi(k) \tilde{X}(k) + K(k+1) Z(k+1)
\] (19)

\[
Y(k) = H(k) X(k) + R(k) Y(k)
\] (20)

In there:

- \(H(k)\) - observation matrix.
- \(X(k)\) - state vector.
- \(R(k)\) - observed noise matrix.
- \(P^{ns}(k), P(k)\) - extrapolated and filtered error covariance matrix.
- \(\Phi(k)\) - transition matrix.
- \(Q(k)\) - excitation noise covariance matrix.
- \(K(k)\) - matrix of weights.
- \(I\) - unit matrix.
- \(\tilde{X}(k)\) - the state vector is evaluated.
- \(Y(k)\) - observation vector.
- \(Y(k)\) - Gaussian white noise column vector with zero mathematical expectation and unit variance (equal to 1).
The elements of the observation matrix H will be determined by the element to be evaluated and have the size [7x11] with the following non-zero elements:

\[
\begin{align*}
 h_{1,1} &= h_{2,2} = h_{3,3} = h_{4,2} = h_{4,3} = 1 \\
 h_{4,4} &= h_{4,5} = h_{5,5} = h_{6,0} = h_{10,10} = 1
\end{align*}
\]  

(21)

Determine the elements of the matrix Φ and Q correspond to the following expressions (Bacun et al., 1970; М. Ярлыков et al., 2012).

Φ(k) = \text{Adt} ≈ 1 + αΔt

(22)

Q(k) = M(=[[k+1]])^2(k)

(23)

Where j(k) is the criterion function (penalty function):

\[
j(k)=j_{\text{kat}}^2(k+1)\Phi[(k+1)\Delta t]G(\tau)E(\tau)d\tau
\]

(24)

As a result, the matrix Φ(k) will have size [11x11] with non-zero elements:

\[
\begin{align*}
 Φ_{1,1} &= Φ_{2,2} = Φ_{3,3} = Φ_{4,4} = Φ_{6,6} = Φ_{8,8} = 1; \\
 Φ_{1,2} = Φ_{1,3} = Φ_{1,5} = Φ_{4,6} = Δt; \\
 Φ_{4,4} &= Φ_{6,6} = Φ_{8,8} = Δt; \\
 Φ_{4,3} &= αΔt; \\
 Φ_{4,10} &= 1 - αΔt;
\end{align*}
\]

(25)

The matrix Q(k) also has size [11x11] with the following non-zero elements:

\[
\begin{align*}
 q_{4,4} &= 2αm_σ^2 + α_σ^2 + 2α_σ^2 Δt^2/3; \\
 q_{4,7} &= q_{7,4} = 2αm_σ^2α_σ^2 Δt^2/3; \\
 q_{4,10} &= 2αm_σ^2α_σ^2 Δt^2/3; \\
 q_{4,11} &= 2αm_σ^2α_σ^2 Δt^2/3; \\
 q_{7,10} &= 2αm_σ^2α_σ^2 Δt^2/3; \\
 q_{7,11} &= 2αm_σ^2α_σ^2 Δt^2/3; \\
 q_{10,10} &= 2αm_σ^2α_σ^2 Δt^2/3; \\
 q_{10,11} &= 2αm_σ^2α_σ^2 Δt^2/3;
\end{align*}
\]

(26)

The elements of the observed (measured) noise covariance matrix R of size [7x7] are determined by the following formulas:

\[
σ^2 = \left(\frac{\delta}{\sqrt{q}}\right)^2
\]

(27)

With: δ – the allowable ability of the measured coordinate system; q – signal/noise correlation at the filter input.

To calculate the measurement error variances of the parameters to be observed, we take the signal/noise ratio to be 20dB.

Allowability according to distance (D) when pulse signal simple probe, width 1μs equals 150m, according to (27) then variance of distance measurement error will be equal to 225m².

The ability to distinguish the specific velocity of the intercept missile (V_m) can be taken as 0.16m/s, then according to (27) the variance of the missile velocity measurement error will be 256.10⁻³(m²/s²).

The ability to distinguish close velocity (V_c) by onboard radar is determined by the formula:

\[
δ(V_c) = \frac{1}{2} Δf_0
\]

(28)

Where: Δf_0 - the passband of the Doppler frequency filter of the onboard radar, Hz; λ is the wavelength of the radar's spatially detected pulse generator frequency.

The Doppler frequency filter passband is calculated as follows (Черных, Васильев, Богданов, Савельев, & Макаев, 2000):

\[
Δf_0 = \sqrt{\frac{2(\lambda m_3)}{λ}}
\]

(29)

With: Δf_0 is the corresponds to the centripetal acceleration of the intercept missile and the target.

The passband Δf_0 of a narrow band Doppler frequency filter with good impedance matching can be taken as equal to the signal spectral width reflected from the target according to (Дудник & Чересов, 1986), for example.

\[
Δf_0 = Δf_0
\]

(30)

From expressions (29) and (30), in theory, when the wavelength of the probing signal is known in advance the spectral width of the signal reflected from the target will be determined by the centripetal acceleration of both missile intercept and the target. We estimate the necessary value of the Doppler frequency filter passband Δf_0 in case we assume that the target missile decelerates with a negative centripetal acceleration (a_σ = -16m/s²) and intercept missile acceleration is 0 (a_m = 0), knowing in advance the onboard radar wavelength is λ=0.04 m then Δf_0 ≈ 28Hz. Based on the experimental results (М. Ярлыков, 1985), we see that the bandwidth of the Doppler frequency filter changes in the narrow band of 26Hz-31Hz, so for calculating, we can choose Δf_0 = 30 Hz. Then, the ability to distinguish the close velocity measurement σ(V_c) = 0.06 m/s , and the close velocity measurement error variance will be σ^2(V_c) = 3.6 (m/s)².

The ability to distinguish the line-of-sight angle (the angle of deviation between the velocity vector and the line-of-sight missile – target determined by the corresponding receiving antenna swing widths in each target observation plane (ε and β) at 0.5 power, ie:
δ(ε) = θ_{0.5ε} \quad (31)

For the modern homing head using phased antenna radar, θ_{0.5ε}=1.8^o, in general, it can be taken.

δ(ε) = θ_{0.5ε} = 2^o \quad (32)

According to (27), if the signal/noise ratio q=100, then the measurement variance of the line-of-sight angle will be σ^2(ε) = 0.04 (0^o)^2. The angular velocity measurement variance of the line-of-sight in terms of (27) will be σ^2(ω) = 4.10^{-4} (0/s)^2.

The error measurement of normal acceleration along axes perpendicular to the longitudinal axis of the missile, including the measurement of axial acceleration by accelerometers in the inertial measurement unit (IMU). Can be determined by randomly mean squared error of the accelerometers themselves. Then the variance of the vertical and horizontal acceleration measurements of the missile will be equal to:

σ^2(a_o) = σ^2(W_m) = 3.8.10^{-4} (m/s^2)^2

Thus, the diagonal elements of the measured noise covariance matrix R will be the measured error variance values of the observing coordinates defined above. All other elements in the matrix have a value of 0. The values of the elements lying on the diagonal of the matrix R are as follows:

r_{1,1} = 2.25; \quad r_{2,2} = 2.56.10^{-4}; \quad r_{3,3} = 3.84.10^{-4}; \quad r_{4,4} = 3.6.10^{-3};
\quad r_{5,5} = 0.04; \quad r_{6,6} = 4.10^{-4}; \quad r_{7,7} = 3.84.10^{-4};

Next, we determine the first elements (when k = 0) lying on the diagonal of the filter error matrix P, i.e. P_{i,i}(0) with i = 1,11. It is necessary to get several times (at least 5-7, according to (Bordjand et al., 1998)) the discrete state model according to then find the maximum values and minimum of each parameter. The Gauss probability distribution of each model parameter can be applied as formular following (Kanashenkov & Merkulov, 2004):

\[ p_{i}(0) = \frac{(X_{max} - X_{min})^2}{36} \quad (33) \]

The filter error matrix P(k = 0) whose diagonal elements are not zero:

p_{1,1} = 6.25.10^{-6} (m^2/s^2); \quad p_{2,2} = 1.1.10^{-7} (m^2/s^2); \quad p_{3,3} = 1.128 (m/s)^2; \quad p_{4,4} = 0.528 (m^2/s^2); \quad p_{5,5} = 2.5.10^{-4} (m/s)^2; \quad p_{6,6} = 7.11 (m/s)^2; \quad p_{7,7} = 0.278 (m/s)^2; \quad p_{8,8} = 25 (m/s^2); \quad p_{9,9} = 0.4 (m/s^2); \quad p_{10,10} = 0.46 (m/s^2); \quad p_{11,11} = 0.13 (m/s^2).

To determine the state vector initial values evaluated \( \hat{x}(0) \), that is \( x_i(0), i = 1,11 \), according to (Kanashenkov & Merkulov, 2004) we use the following expression:

\[ x_i(0) = \frac{X_{max} - X_{min}}{2} \quad (34) \]

The initial values of the elements \( x_i(0) \) are not available, they will be very different when changing the initial conditions of filtering phase coordinates in self-guidance.

Thus, the evaluation filter has been synthesized according to the criterion of least squared error in the form of a discrete linear Kalman filter (15) - (20). The dimensions and properties of the matrices as well as their elements have been clearly defined.

3. Simulation and Evaluation of Results

To evaluate the quality of the algorithm for parameters estimating of the separating target as well as of the target angular coordinate system, applied to the optimal Kalman filter algorithm. This article will show simulations of the angular coordinate system of one side maneuvering forms of the target in the group in the horizontal plane. Since the state error itself is a random process, to compare the quality criteria of the algorithms it is necessary to follow the Monte-Carlo test method (Angelova, Semerdjiev, Jilkov, Semerdjiev, & Simulation, 2001).

According to this method, the same target maneuvering trajectory is tested, but each time the gauges (measurement models) get different values due to random effects of measurement noise. The mean squared error of evaluation (MSE) is used to evaluate the quality of the filtering algorithm. The major parameters used for the simulation are as follows:

**Missile interceptor:**
- Initial velocity: 1000 (m/s)
- Horizontal distance: 0 (Km)
- Altitude: 0 (Km)
- The guidance law OPPN (Zarchan, 2011).
- Initial orbital tilt angle: 30°.

**Target:**
- Initial velocity: 350 (m/s)
- Horizontal distance: 25 (Km)
- Altitude: 15 (Km)
- Initial orbital tilt angle: 140°.

The two-target normal acceleration in the group generated from equation (10) is a one-side maneuver with the following control signal. Where u is the control signal or maneuvering command for targets 1 and 2 respectively:

\[ u_1 = \begin{cases} 0 & \text{when } t < 2 [s] \\ 10 [m/s^2] & \text{when } t \geq 14 [s] \end{cases} \quad (35) \]
\[ u_2 = \begin{cases} 0 & \text{when } t < 2 [s] \\ 40 [m/s^2] & \text{when } t \geq 14 [s] \end{cases} \quad (36) \]

3.1. Model simulation results for the separating target.

![Figure 1](Image)

**Figure 1**

Trajectory of the missile intercepts the separating target.
The figure shows that both targets fly at the same distance and at the same speed at the beginning, after a period of 2s, target 2 appears as a separating target. Accurate evaluation of maneuver parameters is of great significance for building an algorithm to detect the direction of maneuver of separate targets later. Based on detecting the direction of maneuver, the interceptor can reselect the target in the group to destroy, ensuring the safety of the combat formation.

3.2. Evaluate the parameters of the 2 targets in the group. Target (1)

Evaluation the angle of line-of-sight Missile – Target

Evaluation angle of line-of-sight Missile – Target after 100 Monte Carlo
Target (2)

**Figure 8**
Evaluation the angle of line-of-sight Missile – Target

**Figure 9**
Evaluation the angle of line-of-sight Missile – Target after 100 Monte Carlo

**Figure 10**
Evaluation the angular velocity of line-of-sight Missile – Target after 100 Monte Carlo

**Figure 11**
Evaluation the normal acceleration target (2)
Comment: Through the survey results and running 100 Monte Carlo. We see that the optimal filtering algorithm can evaluate the parameters for the 2 targets in the group. The result evaluation of the algorithm still ensures the determination of state variables and the evaluation of the target parameters.

4. Conclusion

The article presented the results of synthesizing the separation target model recognition algorithm. Basically, it is a synthetic algorithm that states evaluation and evaluates target parameters using an optimal Kalman filter. Simulation results using MATLAB software show that the synthetic filter can accurately estimate the parameters of separate targets. This is an important basis for building guidance laws for intercepting missiles and detecting the direction of maneuver of separate targets.

Conflicts of Interest

The authors declare that they have no conflicts of interest to this work.

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