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RESEARCH ARTICLE

Synthesis of Suboptimal Guidance Law for Anti-Tank Guided Missile with Terminal Impact Angle Constraint Based on the SDRE Technique

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Abstract: In this paper, the homing-phase guidance law is proposed against a stationary target in the planar engagement scenario. In this problem, two technical criteria that the guidance law must achieve are: first, zero terminal miss distance, and second, satisfying the desired impact angle constraint at the final time. This guidance law is synthesized as a nonlinear optimal control problem with an infinite-time horizon and is solved using the state-dependent Riccati equation (SDRE) method. To obtain a guidance law with a finite-time horizon, a state weighting matrix based on the time-to-go is utilized in the SDRE control scheme. The synthesized guidance law is applied to a new generation anti-tank guided missile class, with specific thrust and drag parameters. Nonlinear simulations are conducted to demonstrate the fundamental properties, applicability, and effectiveness of the proposed guidance law.

Keywords: terminal impact angle, SDRE, anti-tank guided missile (ATGM)

1. Introduction

To increase the damage effectiveness of the warhead, anti-tank guided missiles (ATGMs) should attack the target in a top-down direction into the thin armor area of the tank. This helps enhance the combat effectiveness of the missile without increasing the size or weight of the warhead. To achieve the aforementioned objective, the new generation ATGM systems utilize a guidance law with impact angle constraints, allowing the missile to perform a "pop-up" attack mode to enhance target kill probability. Traditional guidance laws such as proportional navigation (PN) have the advantage of simplicity, requiring minimal information for command acceleration. However, these guidance methods do not satisfy the constraints on final conditions, such as impact angle at the target encounter point.

In particular, the work (Kim & Grider, 1973) is one of the first studies on the synthesis of guidance laws to control impact angle at the final phase of missile's trajectory using linear quadratic regulator optimal control technique.

Several research works (Erer & Merttopçuoglu, 2012; Jeong et al., 2004; Kim et al., 1998; Kim et al., 2013; Lee et al., 2013; Park, 2015; Ratnoo & Ghose, 2008; Ratnoo & Ghose, 2010) have been conducted to improve the PN guidance law to solve the problems of controlling impact angle while satisfying constraints such as zero final miss distance. Although the authors have proposed optimal guidance law to solve the impact angle control problem, these synthesized guidance laws are all based on linear engagement kinematics.

Previous works (Lee et al., 2013; Ryoo et al. 2005) have proposed closed-form optimal guidance laws that achieve specified impact angle as well as zero terminal miss distance. Chi et al. (2021) proposed a practical optimal guidance law that can handle terminal angle and acceleration constraints while providing robustness against uncertainty in autopilot dynamics.

Qin et al. (2022) proposed the guidance laws based on the fast terminal error dynamics method with impact angle/time constraints and field-of-view angle constraint.

Yang et al. (2016) designed a time-varying biased PN guidance law to achieve the impact angle constraints without violating the look angle.

However, in these works, the authors often assume that the missile's velocity is constant. The proposed guidance laws based on that assumption can be effective only if the velocity variation of the missile is relatively small. However, ATGMs usually have a low initial speed, and the speed increases rapidly after launch. On the other hand, ATGMs have a short flight time, which means neglecting velocity variation would result in significant guidance errors.

The state-dependent Riccati equation (SDRE) technique is an effective and flexible algorithm for synthesizing nonlinear feedback controller through the design of state-dependent weighting matrices (Cloutier, 1997). In many researches (Çimen, 2012; Lin & Xin, 2019; Ratnoo & Ghose, 2009; Tyan & Shen, 2010), the authors

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used the SDRE technique to synthesize the guidance laws for the missile; however, they did not emphasize the constraints on impact angle for ATGM. On the other hand, these guidance laws have not been applied to the specific class of ATGM.

In this paper, the authors propose the development of a suboptimal guidance law for nonlinear target and missile kinematics model taking into account impact angle constraint. The guidance law was proposed in this paper to attack a stationary ground target in the 2D plane, with desired terminal angles constraints. The guidance law is built using SDRE technique with the matrices of the state equations depending on the state variables. Then, the guidance law was applied numerical simulation to ATGM with different initial conditions and impact angles.

2. Problem Statement

2.1. Model of missile-target relative motion

Consider the engagement geometry in the vertical plane shown in Figure 1, where *OXY* is the inertial coordinate system associated with the missile launcher. At time *t* during flight, *M (x, y)* is the missile coordinate, V_M is the missile velocity, and θ, λ, σ are flight path angle, line-of-sight (LOS) angle, and lead angle, respectively. *R* is the relative distance between the missile and the target and the component of missile acceleration perpendicular to the missile's velocity vector is denoted as a_M. The coordinate of target is T (x_f, y_f) with the expected impact angle θ_f.

The relative kinematic equations between the missile and the target can be described by

$$\dot{r} = -V_M cos\sigma = -V_M cos(\theta - \lambda) \tag{1}$$

$$\dot{\lambda} = -\frac{V_M \sin\sigma}{r} \tag{2}$$

$$\dot{\theta} = \frac{a_M}{V} \tag{3}$$

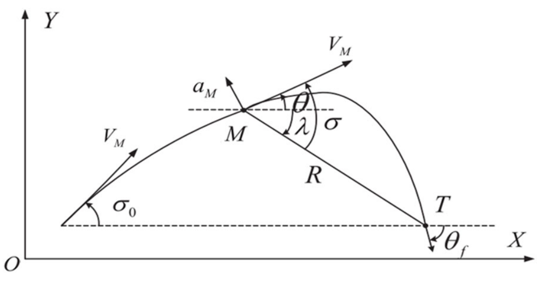
$$\dot{\sigma} = \dot{\theta} - \dot{\lambda} \tag{4}$$

where the dot operator represents the derivative with respect to time.

2.2. Problem formulation for synthesizing guidance law based on the SDRE technique

SDRE is a control algorithm constructed based on the principles of state feedback for nonlinear systems. The controller receives the input signals in the form of the system state and the reference signal

Figure 1
Engagement geometry



and then computes and converts them into control signals for the process.

SDRE is an effective approach for designing control laws for nonlinear systems. The algorithm utilizes the state information of the system and combines it with a reference signal to calculate the control inputs required for the process.

Considering a nonlinear dynamical system, it can be described by the following state equations:

$$\dot{X} = A(X)X + B(X)U \tag{5}$$

where the matrices *A(X)*, *B(X)* are dependent on the state variables. The objective is to find the control signal *u(t)* that control the system from an initial state $x(0) = x_0$ to a desired final state $x(t_f) = 0$ while minimizing a quadratic cost function:

$$J = \frac{1}{2}\int_{t_0}^{\infty} [x^T Qx + Ru^2(t)]dt \tag{6}$$

where $Q \geq 0$ and $R > 0$ are the state weighting matrix and the input weighting matrix, respectively.

The control signal calculated by the SDRE method is the suboptimal solution of the equation of state (5) and the cost function (6).

The objective of the problem is to synthesize a suboptimal guidance law using the SDRE technique applied to the nonlinear engagement geometric model of missile target and can achieve a specified impact angle as well as zero terminal miss distance.

This means that the state variables at the terminal time reach the desired impact angle $\theta(t_f) = \theta_f$ and the terminal miss distance $r(t_f) = 0$. To hit a stationary tank target, the missile's longitudinal axis at the last moment must point toward the target. Therefore, for successful impact angle control, the LOS angle at the terminal instant must satisfy $\lambda_f = \theta_f$.

To simplify the calculation process, the optimal leading rules are usually synthesized with the following assumptions: It is assumed that the information obtained by the missile and the target to provide the design of the guidance law is ideal. Without loss of generality, distance and angular motions in the horizontal plane are assumed to be zero.

3. Synthesis of Suboptimal Guidance Law for Missiles Based on SDRE Technique

The synthesized guidance law must ensure that the state variables of the system tend to zero when the missile approaches the target. On the other hand, the absolute value of the angle created by the missile velocity vector and the LOS is always less than 90^0 ($|\sigma| < \pi/2$) before the time the missile approaches the target, from equation (1) it follows that the variable will decrease with time. In order for optimal control problems to have solution in the form of analytic expressions, it is recommended to use low-order equations of state variables. Therefore, the distance information in this problem is not considered as a state but is used in a state weight matrix based on the variable t_{go}.

On that basis, we choose state variables as follows:

$$X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} \lambda - \theta_f \\ \sigma \end{bmatrix} \tag{7}$$

From equation (4), $\dot{\sigma} = \frac{a_M}{V_M} - \dot{\lambda} = \frac{(a_M - \dot{\lambda} V_M)}{V_M}$

Set new variable:

$$u = a_M - \dot{\lambda} V_M \tag{8}$$

$$\dot{X} = \begin{bmatrix} (\lambda - \theta_f)' \\ \sigma' \end{bmatrix} = \begin{bmatrix} -V_M \frac{\sin\sigma}{r\sigma} \sigma \\ u/V_M \end{bmatrix} \tag{9}$$

Substituting the variables into equation (9), we get

$$\dot{X} = \begin{bmatrix} (\lambda - \lambda_f)' \\ \sigma' \end{bmatrix} = \begin{bmatrix} 0 & -V_M \frac{\sin\sigma}{r\sigma} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \lambda - \lambda_f \\ \sigma \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{V_M} \end{bmatrix} u \tag{10}$$

From equation (10), the state-dependent matrices can be determined:

$$A(x) = \begin{bmatrix} 0 & -V_M \frac{\sin\sigma}{r\sigma} \\ 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ \frac{1}{V_M} \end{bmatrix} \tag{11}$$

Furthermore, the weight matrices Q and R are two design parameters used to create the desired controller. Without loss of generality, Q and R are chosen as follows:

$$Q = \begin{bmatrix} q_1^2 & 0 \\ 0 & q_2^2 \end{bmatrix}, R = 1 \tag{12}$$

Q can be designed as a function of the parameter to incorporate target information into the logic of the guidance law. Before implementing the SDRE algorithm, it is necessary to check whether the system satisfies the necessary and sufficient conditions for solving the problem using the SDRE algorithm, according to the following steps:

i. Matrix $Q \geq 0$ and $R > 0$. From equation (11), these conditions are satisfied.

ii. Function $f(x) = A(x)x \in C^1$. Using the state variable x in (7), and the matrix expression AðxÞ in equation (11), we get:

$$A(x)x = \begin{bmatrix} -V_M \frac{\sin\sigma}{r} \\ 0 \end{bmatrix} \in C^1 \tag{13}$$

iii. The initial condition $f(0) = 0$. Using equation (13):

$$f(0) = \begin{bmatrix} -V_M \frac{\sin 0}{r} \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{14}$$

iv. Matrix $B(x) \neq 0$. From equation (11), we have B which is a matrix of constants that is always non-zero.

v. For all $x \in C^1$, using equations (11) and (7), we have the following matrix pairs: $[B \quad AB] = \begin{bmatrix} 0 & -\frac{\sin\sigma}{r\sigma} \\ 1/V_M & 0 \end{bmatrix}$, will have rank 2. Thus, in this SDC form {A(x), B(x)} are point-wise controllable for all $x \in C^1$.

Therefore, the SDRE algorithm can be used to design the guidance law controlling the target impact angle for ATGM. Then, the matrix *P(x)* ≥ 0 must satisfy the Riccati equation:

$$A^T(x)P(x) + P(x)A(x) - P(x)BR^{-1}B^TP(x) + Q = 0 \tag{15}$$

In which: $P(x) = \begin{bmatrix} p_{11} & p_{12} \\ p_{12} & p_{22} \end{bmatrix}$

Substituting equations (11) and (12) into (15), we get

$$\begin{bmatrix} 0 & 0 \\ -\frac{V_M \sin\sigma}{r\sigma} & 0 \end{bmatrix} \begin{bmatrix} p_{11} & p_{12} \\ p_{12} & p_{22} \end{bmatrix} + \begin{bmatrix} p_{11} & p_{12} \\ p_{12} & p_{22} \end{bmatrix} \begin{bmatrix} 0 & -V_M \frac{\sin\sigma}{r\sigma} \\ 0 & 0 \end{bmatrix}$$

$$- \begin{bmatrix} p_{11} & p_{12} \\ p_{12} & p_{22} \end{bmatrix} \begin{bmatrix} 0 \\ \frac{1}{V_M} \end{bmatrix} \begin{bmatrix} 0 & \frac{1}{V_M} \end{bmatrix} \begin{bmatrix} p_{11} & p_{12} \\ p_{12} & p_{22} \end{bmatrix} + \begin{bmatrix} q_1^2 & 0 \\ 0 & q_2^2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Performing the matrix multiplications of the above equation to get:

$$\begin{cases} q_1^2 - \frac{p_{12}^2}{V_M^2} = 0 \\ -p_{11} V_M \frac{\sin\sigma}{r\sigma} - \frac{p_{12}p_{22}}{V_M^2} = 0 \\ -2p_{12} V_M \frac{\sin\sigma}{r\sigma} - \frac{p_{22}^2}{V_M^2} + q_2^2 = 0 \end{cases} \tag{16}$$

After solving the system of equations (16), we get the solutions of the matrix *P(x)* as follows:

$$p_{11} = \frac{q_1}{V_M \sin\sigma} \sqrt{q_2^2 - 2q_1 \frac{V_M^2 \sin\sigma}{r\sigma}} \tag{17}$$

$$p_{12} = q_1 V_M \tag{18}$$

$$p_{22} = V_M \sqrt{q_2^2 - 2q_1 \frac{V_M^2 \sin\sigma}{r\sigma}} \tag{19}$$

A nonlinear feedback control law received is in the following form:

$$u = -R^{-1}(x)B^T(x)P(x)x \tag{20}$$

By substituting equations (17), (18), and (19) into (20), we obtain the expression of the nonlinear feedback controller as follows:

$$u = -q_1(\lambda - \theta_f) - \sqrt{q_2^2 - 2q_1 \frac{V_M^2 \sin\sigma}{r\sigma}}\sigma \tag{21}$$

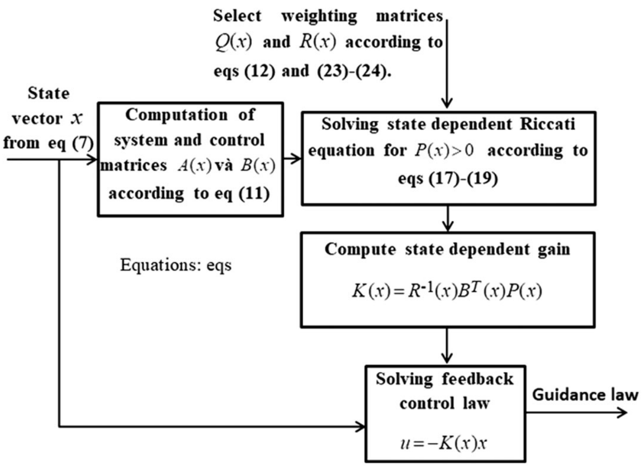
From equations (8) and (21), we obtain the expression for the command acceleration of the missile as:

$$a_M = \dot{\lambda} V_M - q_1(\lambda - \theta_f) - \sqrt{q_2^2 - 2q_1 \frac{V_M^2 \sin\sigma}{r\sigma}}\sigma \tag{22}$$

The method for selecting the weights of the Q matrix follows the principle that the values of state weights should be small when the missile is far away from the target and should increase as the missile approaches the target.

In this problem, the weights of the state variables Q(x) are chosen as a function of the time to go and the relative distance between the missile and the target. As the missile approaches the target, the weights influencing the controlled variable will increase to drive it toward zero (Ratnoo & Ghose, 2009). Since the state variable that needs to be controlled is mainly $x_1 = \lambda - \theta_f$ to ensure

Figure 2
Steps to calculate guidance law by SDRE technique



the desired impact angle, the weights q_1 and q_2 are selected as follows to simplify the expression of the guidance law.

$$q_2 = 0, \tag{23}$$

$$q_1 = -\left(\frac{N}{t_{go}}\right)^2 r; \tag{24}$$

N is a positive coefficient.

Figure 2 illustrates the computational steps involved in the SDRE technique.

By substituting the values from (23), (24) and adding the gravity compensation term into (22), we obtain the expression of the guidance law.

$$a_M = \dot{\lambda} V_M + \frac{N^2}{t_{go}^2} r(\lambda - \theta_f) - \frac{N V_M}{t_{go}} \sqrt{2\frac{\sin\sigma}{\sigma}}\sigma + g\cos\theta \tag{25}$$

To implement this guidance law, the following information need to be provided: missile velocity, LOS range, and angle between the missile and the target, LOS rate, and lead angle σ. Additionally, time to go (t_{go}) needs to be estimated. The guidance law is synthesized assuming a stationary target, so the parameter is estimated using the following formula.

$$t_{go} = \begin{cases} -r/V_c, \text{when}(V_c > V_M/2) \\ 2r/V_M, \text{when}(V_c \le V_M/2) \end{cases} \tag{26}$$

Estimating the parameter t_{go} using both cases helps to avoid errors in situations with large heading errors and closing velocity (V_c). In such cases, closing velocity can be zero or even negative, leading to inaccurate estimation of the parameter t_{go}.

4. Numerical Simulations

For simulation purposes, we utilize a model of the new generation ATGM to verify the effectiveness of the synthesized guidance law. The dynamics of the missile's motion with time-varying velocity are described by the following equation:

$$\begin{aligned} m\dot{V}_M &= T - D - mg\sin\theta \\ V_M\dot{\theta} &= a_M - g\cos\theta \end{aligned} \tag{27}$$

where T and D are the longitudinal thrust and aerodynamic drag force of the missile, respectively. The data on aerodynamic drag and thrust of the ATGM engine used in the simulation are obtained from the referenced paper (Harris & Slegers, 2009). The drag D is determined as follows:

$$D = 0.5\rho S_{ref} C_D V_M^2 \tag{28}$$

The parameters ρ and S_{ref} represent the air density and the aerodynamic cross-sectional area of the missile, respectively. The value of S depends on the size and aerodynamic shape of the missile.

To simplify the simulation process, we assume that the drag coefficient C_D is constant. In this paper, the values of S and the drag coefficient C_D are obtained from the referenced paper (Harris & Slegers, 2009; Abdallah & Ouda, 2018), corresponding to the Javelin ATGM. The guidance gain parameter N in equation (25) is chosen as 2. The above parameters are summarized in Table 1.

For modern ATGMs, a typical example is the Javelin ATGM, which is propelled by two engines: the launch motor and flight motor. The initial thrust is generated by the launch motor to propel the missile out of the launcher to a safe distance. Once the missile has left the launcher, the flight motor is activated and provides thrust for a burn time of 5.2 s to propel the missile to its

Table 1
Guidance and aerodynamic parameters of ATGM

Parameters	Value
N	2
Drag coefficient C_D	0.387
Reference area of the missile: S_{ref}	0.01736 m^2
Gravity acceleration g	9.8 m/s^2
ρ	1.225kg/m3

Table 2
Thrust and mass parameters of ATGM

t (s)	*T (N)*	m (kg)
0	0	11.25
0.3	570	11.16
0.6	650	11.06
1.2	750	10.82
1.8	770	10.58
2.4	650	10.38
4.2	50	10.16
5.2	0	10.15

Table 3
Simulation conditions

Parameters	Value	Unit
Missile initial position (x_0, y_0)	(0, 1)	m
Target position (x_f, y_f)	(1000, 0)	m
Missile initial velocity	13	m/s
Initial launch angle ($\theta_0 = \sigma_0$)	18	deg
Desired impact angles (θ_f)	−90 ~0	deg

Figure 3
(a) Trajectory history; (b) missile lateral acceleration at different distances; (c) missile velocity at different distances; and (d) flight path angle at different distances

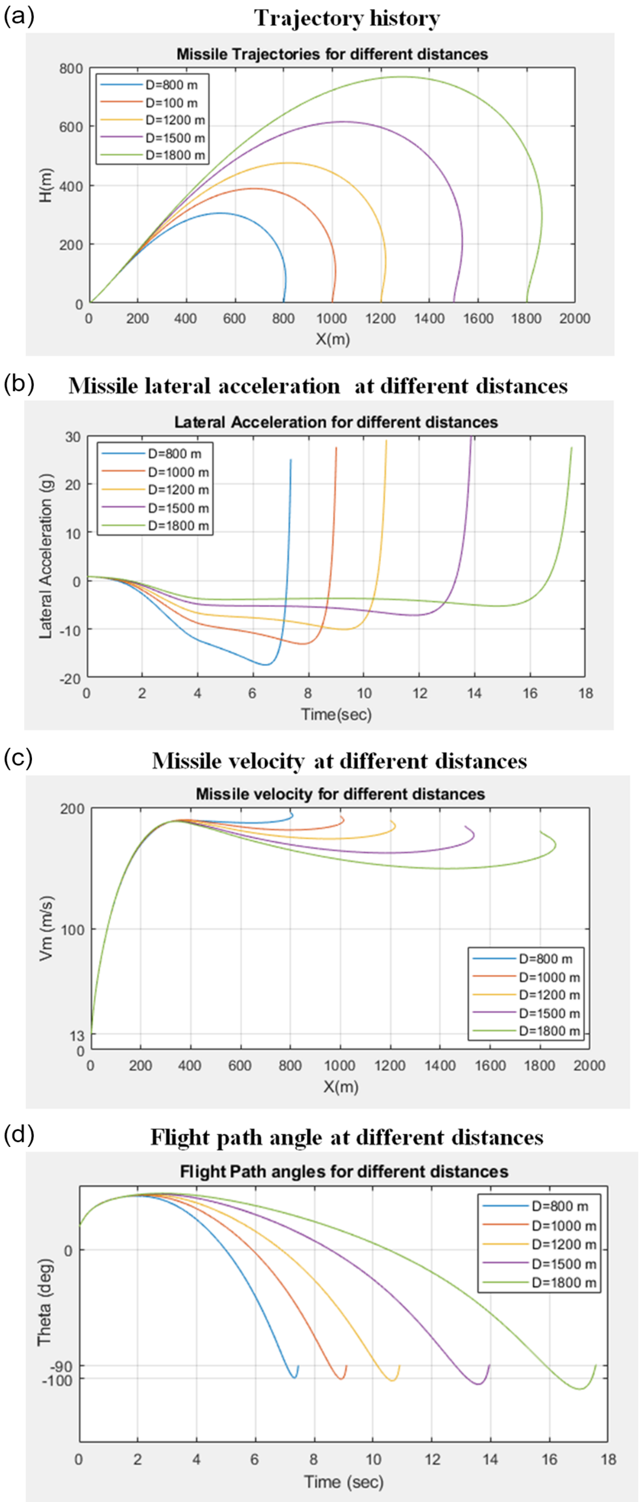
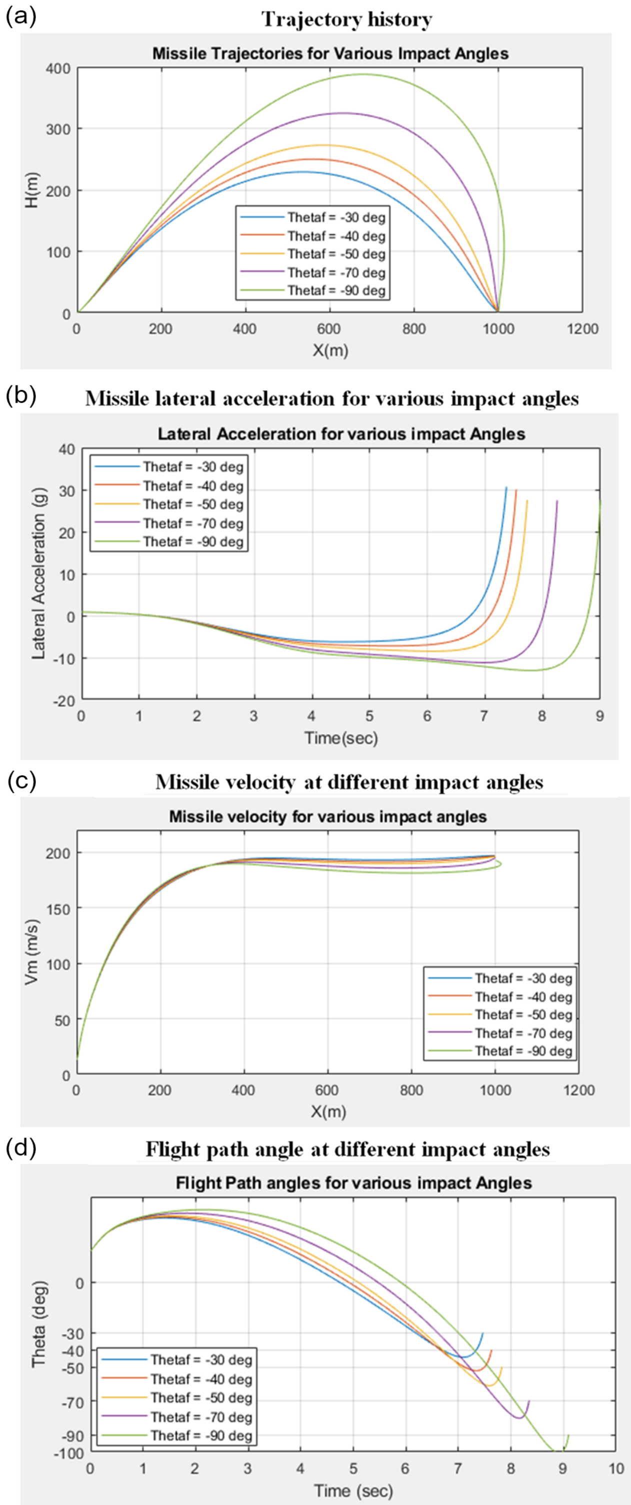


Figure 4
(a) Trajectory history; (b) missile lateral acceleration for various impact angles; (c) missile velocity at different impact angles; and (d) flight path angle at different impact angles



maximum velocity. During the 5.2 s burn time of the flight motor, all values regarding mass and thrust are updated as shown in Table 2 below (Harris & Slegers, 2009).

The missile is launched from a distance of 1 m above the ground at an initial coordinate of $x_0 = 0$ and an initial launch angle $\sigma_0 = 18°$. The initial velocity after leaving the launcher is 13 m/s (Harris & Slegers, 2009). To evaluate the synthesized guidance law's compliance with the specified requirements, we conducted numerical simulations for various combat scenarios and different initial firing conditions. The parameters and conditions for the numerical simulations are presented in Tables 1, 2, and 3.

The simulation process involves solving a system of differential equations comprising equations (1)–(4) and equation (27), with the guidance law expression (25) obtained using the SDRE technique. The parameters of the system of equations are taken from Tables 1 and 2. The initial simulation conditions for the variables and the desired impact angle are obtained from Table 3.

All simulation cases are terminated when the relative distance between the missile and the target becomes less than 0.1 m, and the error of the impact angle to the target from the required value is less than 0.1°.

Scenario 1: Consider the typical situation of attacking a stationary tank target, with a missile launch angle of 18^0 and a desired approach angle to the target of $\theta_f = -90^0$. The target is attacked at different distances ($D = 2000, 1500, 1200, 1000, 800$ m) sequentially. The missile trajectory is depicted in Figure 3(a). The required lateral acceleration is shown in Figure 3(b). The results demonstrate that the guidance law successfully intercepts the target with the desired impact angle. The lateral acceleration tends to approach zero when encountering the target, which is a desirable characteristic. The flight path angle and the missile velocity over time are depicted in Figure 3(c) and (d), respectively. The flight path angle meets the requirement of the desired impact angle at different distances, as shown in Figure 3(d).

Scenario 2: Attacking a stationary target with constraint on the different desired impact angles, namely -30^0, -40^0, -50^0, -70^0, -90^0. The initial launch angle is set to 18^0. Graphs of missile trajectories, required lateral accelerations, missile velocities, and flight path angles for various impact angles are shown in Figure 4(a), (b), (c), and (d), respectively.

As the results are shown in Figure 4(a), (b), (c), and (d), the missile can hit the target accurately with predetermined impact angles. These results show that the synthesized guidance law can achieve the desired impact angles over a wide range of variation.

5. Conclusion

In this study, a guidance law providing the desired impact angle for a stationary tank target was devised using the SDRE technique-based method. The effectiveness of the guidance law is verified on a realistic ATGM model. Simulations are performed for different combat situations and initial firing conditions. The results show that the synthesized guidance law meets the criteria of hitting the target and satisfying the requirements of the desired impact angle. The proposed guidance law is an option for a new generation ATGM.

However, the proposed guidance law only considers engaging fixed target and is limited to the vertical plane. In future studies, the authors will develop the proposed guidance law for the case of engaging moving targets, as well as consider factors such as the seeker's field of view limitation and acceleration controller lag for the practical implementation.

Ethical Statement

This study does not contain any studies with human or animal subjects performed by any of the authors.

Conflicts of Interest

The authors declare that they have no conflicts of interest to this work.

Data Availability Statement

The data that supports the findings of this study are openly available at https://doi.org/10.1016/j.mcm.2009.02.009 and https://doi.org/10.17577/IJERTV7IS090051

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